1. The number of ways in which the word PANDEMIC can be arranged such that the vowels appear together is

(A) $6 \times (3!)(5!)$ (B) $5 \times (3!)(5!)$
(C) $4 \times (3!)(5!)$ (D) $1 \times (3!)(5!)$

2. Consider the functions $f(x) = x^2 - x - 1$ and $g(x) = x + 1$, both defined for all real values of x. Let $\alpha_1 > 0$ be the positive real root and $\alpha_2 < 0$ be the negative real root of the equation $f(x) = 0$. Let $\beta_1 > 0$ be the positive real root and $\beta_2 < 0$ be the negative real root of the equation $f(g(x)) = 0$. After identifying the exact values of $\alpha_1, \alpha_2, \beta_1$ and β_2, identify which one of the following four statements is *incorrect*.

(A) $\alpha_1 - \beta_1 = \alpha_2 - \beta_2 = 1$
(B) $\alpha_1 + \beta_2 = \alpha_2 + \beta_1 = 0$
(C) $\alpha_1 + \beta_1 = -(\alpha_2 + \beta_2) = \sqrt{5}$
(D) $\alpha_1 + \alpha_2 = -(\beta_1 + \beta_2) = -1$

3. Let the function $f(x) = 1 - \sqrt{1 - x^2}$ be defined only over all x belonging in $[0, 1]$. Then $f(1 - f(x))$ equals

(A) x (B) $1 - x$
(C) x^2 (D) $1 - x^2$

4. Suppose $f(x)$ is increasing, concave and twice differentiable and $g(x)$ is decreasing, convex and twice differentiable. Then the function $G(x) = g(f(x))$ is

(A) increasing and convex
(B) decreasing and convex
(C) increasing and concave
(D) decreasing and concave

5. Let A and B be two non-singular matrices of the same order and let C be a matrix such that $C = BAB^{-1}$. Then for any scalar λ, the value of $\det(C + \lambda I)$ (where I is the identity matrix) is

(A) $\det A$ (B) $\det B$
(C) $\det(A + \lambda I)$ (D) $\det(B + \lambda I)$

6. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a twice differentiable function satisfying $f''(x) > 0$ for all $x \in \mathbb{R}$. Furthermore, assume that $f(1) = 1$ and $f(2) = 2$. Then,

(A) $0 < f'(2) < 1$ (B) $f'(2) > 1$
(C) $f'(2) = 1$ (D) $f'(2) = 0$

7. The value of $\lim_{x \to e} \frac{\log_e x - 1}{x - e}$ is

(A) 0 (B) e
(C) $\frac{1}{e}$ (D) None of these

8. Let $f : [0, \infty) \to \mathbb{R}$ be a function such that $f(0) = 0$ and $f''(x) > 0$ for all $x > 0$. Then the function $g : (0, \infty) \to \mathbb{R}$, defined by $g(x) = \frac{f(x)}{x}$, is

(A) increasing in $(0, \infty)$
(B) decreasing in $(0, \infty)$
(C) increasing in $(0, 1]$ and decreasing in $(1, \infty)$
(D) decreasing in $(0, 1]$ and increasing in $(1, \infty)$

9. Let $f : A \to B$ be a function where $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 2\}$. How many onto functions can one generate?

(A) $5^2 - 1$ (B) $5^2 - 2$
(C) $2^5 - 1$ (D) $2^5 - 2$

10. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \dfrac{x}{1+x^2}$ for all $x \in \mathbb{R}$. Then,

(A) $-1 \leq f(x) \leq 1$ (B) $-1 \leq f(x) \leq 1/2$
(C) $-1/2 \leq f(x) \leq 1$ (D) $-1/2 \leq f(x) \leq 1/2$

11. If a 3×3 matrix A has rank 3 and a 3×4 matrix B has rank 3, then the rank of AB is

(A) 3 (B) 4 (C) 6 (D) 7

12. Let $A = \begin{pmatrix} 2 & 0 & 3 & 1 & -1 \\ 2 & 3 & 1 & 0 & -1 \\ 3 & 1 & 2 & 0 & -1 \\ 1 & 2 & 3 & -1 & 0 \\ 2 & 1 & -1 & 0 & 3 \end{pmatrix}$. Which one is an eigenvalue of A?

(A) 2 (B) 1 (C) 3 (D) 5

13. Let A be a 5×5 non-null singular matrix. Then which of the following statement is true?

(A) $A\mathbf{x} = 0$ has only a trivial solution
(B) $A\mathbf{x} = 0$ has 5 solutions
(C) $A\mathbf{x} = 0$ has no solution
(D) $A\mathbf{x} = 0$ has infinitely many solutions

14. A family has two children. What is the probability that both are boys given that at least one is a boy?

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

15. Consider two boxes, one containing one black ball and one white ball, the other containing two white balls and one black ball. A box is selected at random, and a ball is selected at random from the selected box. What is the probability that the ball is black?

(A) $\frac{5}{12}$ (B) $\frac{2}{5}$ (C) $\frac{1}{6}$ (D) $\frac{5}{11}$

16. The function $f : \mathbb{R} \to \mathbb{R}$, defined by $f(x) = (x^2 + 1)^{2022}$, is

(A) one-one but not onto

(B) onto but not one-one

(C) both one-one and onto

(D) neither one-one nor onto

17. Consider an economy with two goods X and Y. Let the utility function be given by $u(x, y) = A\sqrt{xy}$ where $A > 0$, $x \geq 0$ is the amount of good X consumed and $y \geq 0$ is the amount of good Y consumed. Suppose that the budget constraint is given by $P_X x + P_Y y \leq M$ where $M > 0$ is the money income of the consumer and P_X and P_Y are the prices of the goods X and Y, respectively. Let $P_X = P_Y > 1$ and let (x^*, y^*) be the equilibrium quantities of this consumer who maximizes utility subject to the budget constraint. Then,

(A) it must always be that $x^* > y^*$

(B) it must always be that $x^* = y^*$

(C) it must always be that $x^* < y^*$

(D) it must always be that $x^* + y^* = M$

18. Consider the utility function $u(x_1, x_2) = 3x_1 + 2x_2$ of a consumer defined for all $x_1 \geq 0$ and $x_2 \geq 0$. Let the price

of good 1 be $p_1 > 0$ and that of good 2 be $p_2 > 0$. Let $M > 0$ be the money income of the consumer. Consider the optimization problem $\max_{x_1 \geq 0, x_2 \geq 0} 3x_1 + 2x_2$ subject to $2x_1 + 3x_2 \leq M$. The associated Lagrangian function for this maximization problem is $L(x_1, x_2; \lambda) = 3x_1 + 2x_2 + \lambda[M - 2x_1 - 3x_2]$; where λ denotes the non-negative Lagrangian multiplier. Then the equilibrium solution $(x_1^*, x_2^*, \lambda^*)$ to this Lagrangian function maximization problem is

(A) $(x_1^* = \frac{M}{2}, x_2^* = 0, \lambda^* = \frac{2}{3})$
(B) $(x_1^* = \frac{M}{2}, x_2^* = 0, \lambda^* = \frac{3}{2})$
(C) $(x_1^* = 0, x_2^* = \frac{M}{3}, \lambda^* = \frac{2}{3})$
(D) $(x_1^* = 0, x_2^* = \frac{M}{3}, \lambda^* = \frac{3}{2})$

19. Consider a two good economy where the two goods are X and Y and consider two consumers A and B. In a month when the price of good X was Rs. 2 and that of good Y was Rs. 3, consumer A consumed 3 units of good X and 8 units of good Y and consumer B consumed 6 units of both goods. In the next month, when the price of good X was Rs. 3 and that of good Y was Rs. 2, consumer A consumed 8 units of good X and 3 units of good Y and consumer B consumed 4 units of good X and 9 units of good Y. Given this information which one of the following statements is correct?

(A) Both consumers satisfy the weak axiom of revealed preference
(B) Neither consumer satisfies the weak axiom of revealed preference
(C) Consumer A satisfies the weak axiom of revealed preference but not consumer B
(D) Consumer B satisfies the weak axiom of revealed preference but not consumer A

20. Let the production function be $Y(L, K) = min\{2L, K\}$, where L and K are the amounts of labor and capital, respectively. Consider the cost function $C(L, K) = wL + rK$, where $w > 0$ denotes the price of labor and $r > 0$ denotes the price of capital. Suppose that (L^*, K^*) is the combination of labor and capital at which cost is minimized subject to the constraint $Y(L, K) \geq \bar{Y}$. Then,

(A) $L^* = \bar{Y}$ and $K^* = \bar{Y}/2$

(B) $L^* = \bar{Y}$ and $K^* = \bar{Y}$

(C) $L^* = \bar{Y}/2$ and $K^* = \bar{Y}$

(D) None of the other options is correct

21. Suppose that there are two firms 1 and 2 that produce the same good. Let the inverted demand function be $P(q_1, q_2) = 1 - q_1 - q_2$, where firm 1 produces $q_1 \geq 0$ and firm 2 produces $q_2 \geq 0$. Suppose that the cost function of firm $i \in \{1, 2\}$ is given by $c_i(q_i) = \kappa_i q_i$, where $\kappa_i \in \left(0, \frac{1}{2}\right)$. Note that there is no fixed cost for either firm. Then, the Cournot equilibrium profit of firm 2 is

(A) $\frac{(1-\kappa_1+\kappa_2)^2}{9}$ (B) $\frac{(1-\kappa_2+\kappa_1)^2}{9}$

(C) $\frac{(1-2\kappa_1+\kappa_2)^2}{9}$ (D) $\frac{(1-2\kappa_2+\kappa_1)^2}{9}$

22. A non-transitive preference relation can be represented by a utility function

(A) Always

(B) Only if preferences are complete

(C) Only if preferences are complete and convex

(D) Never

23. Which of the following statements is correct in a two-good world?

(A) Diminishing marginal utility of both goods is sufficient for diminishing marginal rate of substitution

(B) Diminishing marginal utility of both goods is necessary for diminishing marginal rate of substitution

(C) Diminishing marginal utility of at least one good is necessary for diminishing marginal rate of substitution

(D) Diminishing marginal utility of at least one good is neither necessary nor sufficient for diminishing marginal rate of substitution

24. Rahul consumes two goods, X and Y, in amounts x and y, respectively. Rahul's utility function is $U(x, y) = \min\{x, y\}$. Rahul makes Rs 200; the price of X and price of Y are both Rs 2. Rahul's boss is thinking of sending him to another town where the price of X is Rs 2 and the price of Y is Rs 3. The boss offers no raise in pay. Rahul, who understands compensating and equivalent variations perfectly, complains bitterly. He says that although he doesn't mind moving for its own sake and the town is just as pleasant as the old, having to move is as bad as a cut in pay of Rs A in his current location. He also says that he would not mind moving if, when he moved, he got a raise of Rs B. What are A and B equal to?

(A) $A = 30$, $B = 70$ (B) $A = 40$, $B = 50$

(C) $A = 50$, $B = 75$ (D) $A = 60$, $B = 60$

25. Let $U(x, y) = -[(10-x)^2 + (10-y)^2]$ be a utility function of some consumer. All prices are equal to 1, and income is 40. Then the optimal values of x and y will be

(A) 10, 10 (B) 0, 0
(C) 5, 5 (D) None of these

26. Consider a production function be $F(K, L) = \min\{\frac{K}{a}, \frac{L}{b}\}$; $a, b > 0$ and $a \neq b$. For any given $K = \overline{K} > 0$, the marginal productivity of labor is

(A) 0
(B) $\frac{1}{a}$ if $L < (\frac{a}{b})\overline{K}$ and 0 otherwise
(C) $\frac{1}{b}$ if $L < (\frac{b}{a})\overline{K}$ and 0 if $L > (\frac{b}{a})\overline{K}$
(D) None of the above

27. Let $e_i(p_0)$ be the price elasticity of demand for a good X of consumer i $(i = 2, \cdots, N)$ at price p_0, given its demand function. Consumers do not consume identical amounts of X at p_0. Then the price elasticity of demand at price p_0 for the aggregate demand function for X is

(A) $\sum_i (e_i(p_0))^2$ (B) $\sum_i e_i(p_0)$
(C) $\frac{\sum_i e_i(p_0)}{N}$ (D) None of these

28. There are m identical competitive firms in an industry. Every firm has the (total) cost function $C(q) = q^2 + 1$, where q is the level of its output, $q \geq 0$. Industry demand for the product is given by $D(P) = a - bP$, where P is price, and $a, b > 0$. Then the short-run equilibrium output of each firm is

(A) 0 (B) $\frac{a}{m+2b}$
(C) $\frac{a}{\frac{m}{2}+b}$ (D) $\frac{a}{m+\frac{b}{2}}$

29. Suppose the (total) cost function for a monopolist is $C = 3q^2 + 800$, where q is its output. The inverse market demand function is $p = 280 - 4q$. What is the price elasticity of demand at the profit maximizing price?

(A) -4.5 (B) -3.5 (C) -2.5 (D) -1.5

30. Consider the Solow growth model with constant average saving propensity s, rate of depreciation δ, and labor supply growth rate n. There is no technological progress. Then, at steady state, the capital-output ratio is

(A) $\frac{s}{n+\delta}$ (B) $\frac{n}{\delta+n}$
(C) $\frac{\delta}{s+n}$ (D) $\frac{1}{s+n+\delta}$