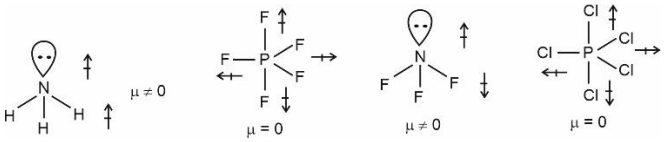
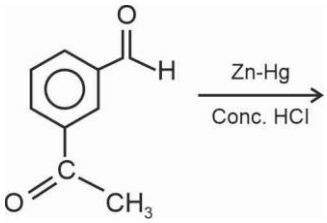
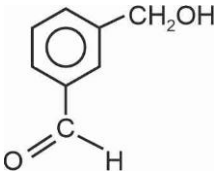
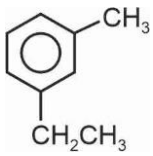
Sol.

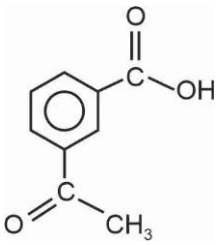


NH_3 has greater dipole moment than NF_3

6. 

(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. This is an example of Clemmensen reduction reaction. In this reaction carbonyl group is reduced to methylene group.

7. Which of the following is the correct order of first ionization enthalpy?

(1) Be < B < O < F < N

(2) B < Be < O < N < F

(3) B < Be < N < F < O

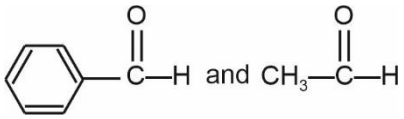
(4) Be < B < N < O < F

Answer (2)

Sol. Be has more value of first ionization enthalpy than B due to fully filled configuration and N has more value of first ionization enthalpy than O due to half filled configuration

The correct order is B < Be < O < N < F

8. Statement-1 : Aldol condensation is caused by acidity of α hydrogen

Statement-2 : Cross aldol is not possible between 

(1) Both statement-1 and statement-2 are correct

(2) Both statement-1 and statement-2 are incorrect

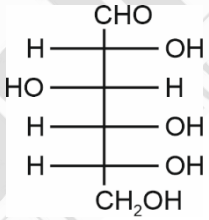
(3) Statement-1 is correct but statement-2 is incorrect

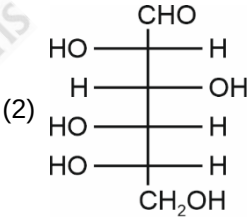
(4) Statement-1 is incorrect but statement-2 is correct

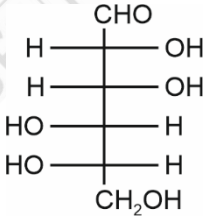
Answer (3)

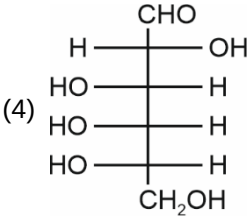
Sol. Aldol reaction is given by those carbonyl compounds which have at least one α hydrogen atom because α-hydrogen of carbonyl compounds is acidic. Benzaldehyde and acetaldehyde will form cross aldol because acetaldehyde has α-hydrogen atom.

9. Select the correct structure of L-glucose.

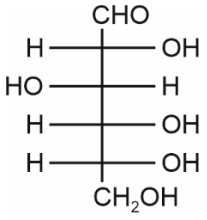
(1) 

(2) 

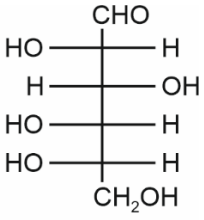
(3) 

(4) 

Answer (2)

Sol. 

D-Glucose



L-Glucose

10. Decreasing order of the field strength of the following ligands will be:

$CO, \overset{\ominus}{C}N, \overset{\ominus}{C}l, H_2O$

(1) $CO > \overset{\ominus}{C}N > H_2O > Cl^-$ (2) $CO > \overset{\ominus}{C}N > Cl^- > H_2O$

(3) $\overset{\ominus}{C}N > CO > H_2O > Cl^-$ (4) $\overset{\ominus}{C}N > CO > Cl^- > H_2O$

Answer (1)

Sol. $CO > \overset{\ominus}{C}N > H_2O > \overset{\ominus}{C}l$

11. Calculate the molarity of NaCl solution, if 5.85 gm of NaCl is dissolved in 500 ml of solution.

(1) 0.1 M (2) 0.2 M

(3) 0.32 M (4) 0.4 M

Answer (2)

Sol. $\text{Molarity} = \frac{\text{Number of moles of solute}}{\text{Volume of solution (in L)}}$

$= \frac{5.85 \times 1000}{58.5 \times 500} = 0.1 \times 2 = 0.2\ \text{M}$

12. Which of the following does not give Lassaigne's test?

(1) Urea (2) Azobenzene

(3) Hydrazine (4) Phenylhydrazine

Answer (3)

Sol. Hydrazine ($NH_2 - NH_2$) does not contain carbon. On fusion with sodium metal, it cannot form NaCN. So hydrazine does not show Lassaigne's test.

13. Among the following, species that have one unpaired $e^{\ominus}$?

(1) $CN^{\ominus}$ (2) O_2^{2-}

(3) O_2^{+} (4) $NO^{\ominus}$

Answer (3)

Sol. Unpaired $e^{\ominus}$

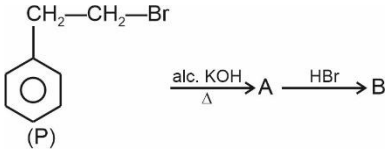
$CN^{\ominus} \rightarrow 14e^{\ominus} \rightarrow$ zero

$O_2^{2-} \rightarrow 18e^{\ominus} \rightarrow$ zero

$O_2^{+} \rightarrow 15e^{\ominus} \rightarrow$ one

$NO^{\ominus} \rightarrow 16e^{\ominus} \rightarrow$ two

14. For a given reaction



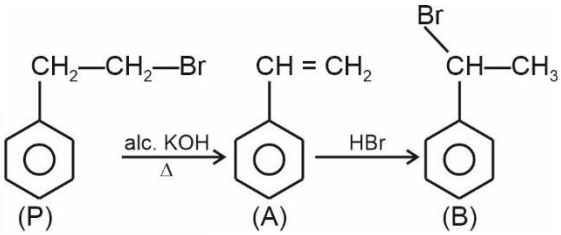
Relation between the molecules P and B are:

(1) Enantiomer (2) Diastereomers

(3) Positional isomers (4) Functional isomers

Answer (3)

Sol. Positional isomers.



15. From the given data, find enthalpy of hydrogenation of ethene in kJ/mol

(a) B.E. of C – C = 350 kJ/mol

(b) B.E. of C = C = 600 kJ/mol

(c) B.E. of H – H = 400 kJ/mol

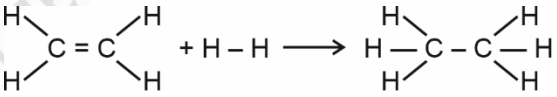
(d) B.E. of C – H = 410 kJ/mol

(1) –170 (2) –580

(3) +170 (4) +580

Answer (1)

Sol.



$\Delta_r H = \Delta H(C = C) + \Delta H(H - H) - \Delta H(C - C) - 2\Delta H(C - H)$

$= 600 + 400 - 350 - 2(410)$

$= -170$ kJ/mol

16. Find out wavelength of a photon having frequency equal to 900 sec^{-1}.

(1) 3.33×10^5 m (2) 3.33×10^5 cm

(3) 3.33×10^7 m (4) 3.33×10^4 m

Answer (1)

Sol. $\nu = \frac{C}{\lambda}$

$\lambda = \frac{C}{\nu}$

$\lambda = \frac{3\times10^{8}\,m\,sec^{-1}}{900\ sec^{-1}}$

$= \frac{3\times10^{8}}{900}$

$= \frac{3\times10^{6}}{9}$

$= \frac{1}{3}\times10^{6}$

$= 0.333 \times 10^{6}$

$= 3.33 \times 10^{5}$ m

17. Why NH_4Cl is added before NH_4OH for the ppt. of Fe^{3+} ions?

(1) To decrease OH^- ion concentration

(2) To increase Cl^- ion concentration

(3) To increase NH_4^+ ion concentration

(4) To decrease H^+ ion concentration

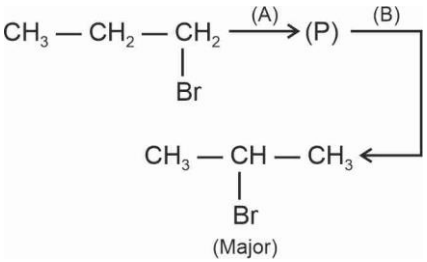
Answer (1)

Sol. $NH_4OH \rightleftharpoons NH_4^+ + OH^-$

$NH_4Cl \rightleftharpoons NH_4^+ + Cl^-$

Solid NH_4Cl is added to NH_4OH solution to decrease the OH^- ion concentration due to common ion effect.

18. Consider the following sequence of reactions and identify the unknown reagents (A) and (B) respectively.



(1) (A) :Dil. aq NaOH at 20°C
(B) :HBr, CH_3COOH

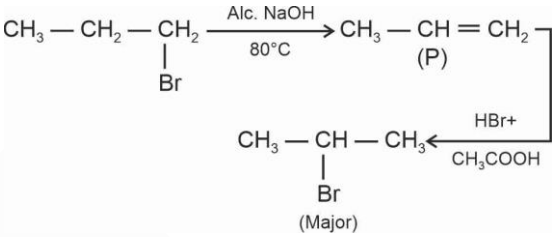
(2) (A) : Dil. aq NaOH at 20°C
(B) : Br_2, $CHCl_3$

(3) (A) : Alc. NaOH at 80°C
(B) : HBr, CH_3COOH

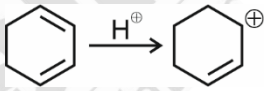
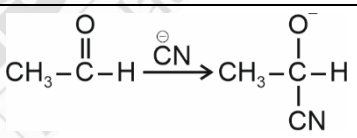
(4) (A) : Alc. NaOH at 80°C
(B) : Br_2, $CHCl_3$

Answer (3)

Sol.



19. Match the following

(i)	Nitrobenzene	(a)	+R
(ii)	Aniline	(b)	–R
(iii)		(c)	+E
(iv)		(d)	–E

(1) (i) → (b), (ii) → (a), (iii) → (c), (iv) → (d)

(2) (i) → (a), (ii) → (b), (iii) → (c), (iv) → (d)

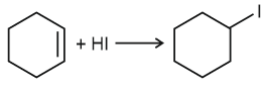
(3) (i) → (c), (ii) → (b), (iii) → (a), (iv) → (d)

(4) (i) → (d), (ii) → (c), (iii) → (a), (iv) → (b)

Answer (1)

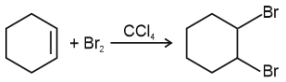
Sol. (i) → (b), (ii) → (a), (iii) → (c), (iv) → (d)

20. Which of the following is not possible major product?

(1) 

(2) $CH_3-(CH_2)_2-NH_2 \xrightarrow[HX]{NaNO_2} CH_3(CH_2)_2-NO_2 + N_2$

(3) $CH_3-CH=CH_2 + HBr \longrightarrow CH_3-\underset{\underset{Br}{|}}{CH}-CH_3$

(4) 

Answer (2)

Sol. $CH_3-CH_2-CH_2-NH_2 \xrightarrow[HX]{NaNO_2} \underset{\underset{(Major)}{}}{\underset{\underset{OH}{|}}{CH_3-CH-CH_3}} + N_2$

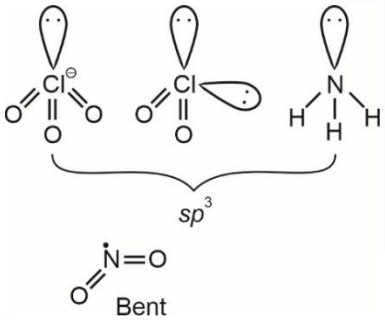
SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. How many of the following compounds are sp^3 hybridised?

$ClO_3^-, ClO_2^-, NH_3, NO_2$

Answer (3)

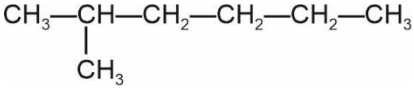
Sol. 

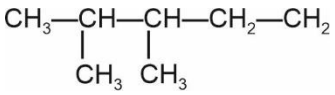
22. Total number of structural isomers possible for a compound with molecular formula C_7H_{16} are:

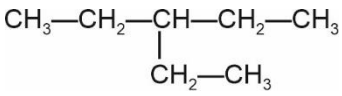
Answer (5)

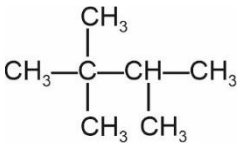
Sol. C_7H_{16} has DoU = 0

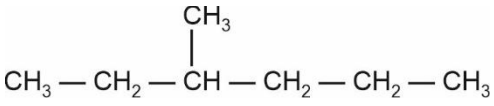
(i) $CH_3-CH_2-CH_2-CH_2-CH_2-CH_2-CH_3$

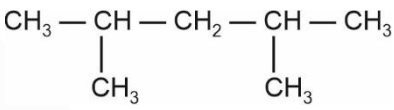
(ii) 

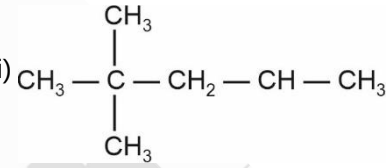
(iii) 

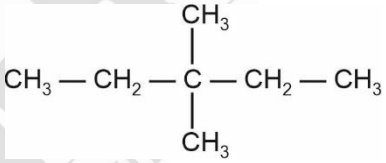
(iv) 

(v) 

(vi) 

(vii) 

(viii) 

(ix) 

23. The de-Broglie wavelength of an electron in 4th orbit of hydrogen atom is _____ πa_0 (a_0 = Bohr radius).

Answer (8)

Sol. $\because \quad \lambda_{de\text{-}Broglie} = \frac{2\pi r}{n} = \frac{2\pi}{n} \times 0.529 \frac{n^2}{z}$ Å

or, $\lambda_{de\text{-}Broglie} = 2\pi \times n \times a_0$ Å

$= 2\pi \times 4 \times a_0$ Å

$= 8\pi a_0$ Å

24. 50 mL of $KMnO_4$ solution is used for titration with 20 mL of 2M oxalic acid solution in Acidic medium. The molarity of $KMnO_4$ solution is $x \times 10^{-2}$ M. The value of x is

Answer (32)

Sol. $\underset{\text{n.f.}=5}{MnO_4^{\ominus}}(aq) + \underset{\text{n.f.}=2}{C_2O_4^{2-}}(aq) \xrightarrow{H^+} Mn^{2+} + CO_2\uparrow$

$5\times M_{KMNO_4}\times 50 = 2\times 20\times 2$

$M_{KMnO_4} = \frac{8}{25} = 32\times 10^{-2}M$

x = 32

25. A solution having non-volatile solute in water shows elevation in boiling point of 2°C. Find out vapour pressure of solution (in mm Hg) (Nearest integer)
Vapour pressure of pure water = 760 mm Hg
K_b of water = 0.52 K.kg mole^{-1}

Answer (711)

Sol. $\Delta T_b = (K_b)(m)$

2 = (0.52) (m)

m = 3.846

$X_{Solute} = \frac{m}{m+55.5} = 0.0648$

$\frac{760-X}{760} = 0.0648$

$\Rightarrow$ $P_{solution}$ = 710.74 mm Hg

$\approx$ 711 mm Hg

26. $MnO_2 + KOH + O_2 \longrightarrow A$
'A' disproportionate into 'B' and 'C'. Find the sum of magnetic moment (spin only) (in B.M.) of B and C (Nearest integer)

Answer (4)

Sol. $2MnO_2 + 4KOH + O_2 \longrightarrow \underset{(A)}{2K_2MnO_4} + 2H_2O$

$\underset{(A)}{3MnO_4^{2-}} + 4H^+ \xrightarrow{\text{Disproportionation}} 2MnO_4^- + MnO_2 + 2H_2O$

B and C are MnO_4^- and MnO_2

Mn in MnO_2 has +4 oxidation state hence it has $(n-1)d^3\,ns^0$ electronic configuration

unpaired e = 3

Mag. moment : 3.87 B.M. by $\sqrt{n(n+2)}$

$KMnO_4/MnO_4^-$ is diamagnetic hence magnetic moment = 0 because it has no unpaired electron.

Hence, sum of mag. moment = 3.87 B.M.

Nearest integer = 4

27. How many of the following coordination compounds have even number of unpaired electrons?
$[V(H_2O)_6]^{2+}$, $[Fe(H_2O)_6]^{2+}$, $[Cu(H_2O)_6]^{2+}$, $[Ni(H_2O)_6]^{2+}$, $[Cr(H_2O)_6]^{2+}$

Answer (3)

Sol. $[V(H_2O)_6]^{2+} \Rightarrow d^2sp^3 \Rightarrow n = 3$

$[Fe(H_2O)_6]^{2+} \Rightarrow sp^3d^2 \Rightarrow n = 4$

$[Cu(H_2O)_6]^{2+} \Rightarrow sp^3d^2 \Rightarrow n = 1$

$[Ni(H_2O)_6]^{2+} \Rightarrow sp^3d^2 \Rightarrow n = 2$

$[Cr(H_2O)_6]^{2+} \Rightarrow sp^3d^2 \Rightarrow n = 4$

28. Consider the following reaction sequence :

$A \underset{k_2}{\overset{k_1}{\rightleftharpoons}} B \xrightarrow{k_3} C$

Overall $k = \frac{k_1k_2}{k_3}$

if $E_{a_1} = 300\ kJ/mole$

$E_{a_2} = 200\ kJ/mole$

Overall, $(E_a)_{eff}$ = 400 kJ/mole

Find out E_{a_3} (in kJ/mole)

Answer (100)

Sol. $(E_a)_{eff} = E_{a_1} + E_{a_2} - E_{a_3}$

400 = 300 + 200 – E_{a_3}

E_{a_3} = 100 kJ/mole

29. x g of ethylamine on reaction with $NaNO_2$ and HCl, produces 2.24 L of N_2(g) at NTP. The value of 2x will be

Answer (9)

Sol. $NaNO_2 + HCl \longrightarrow NaCl + HNO_2$

$C_2H_5NH_2 + HNO_2 \longrightarrow C_2H_5OH + N_2\uparrow + H_2O$

Mole of N_2(g) produced = $\frac{2.24}{22.4}$ = 0.1 mol

So, mole of $C_2H_5NH_2$ used = 0.1 mol

Mass of $C_2H_5NH_2$ = 45 × 0.1 = 4.5 g

So, 2x = 2 × 4.5

= 9

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. If $f(x)=\begin{cases}x-2, & 0<x\le 2\\ -2, & -2\le x\le 0\end{cases}$ and

$h(x) = f(|x|) + |f(x)|$ then

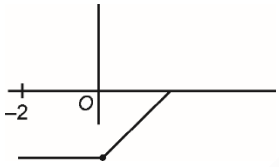
find $\int_0^k h(x)dx$ is equal to ($k > 0$)

(1) 0 (2) $\frac{k}{2}$

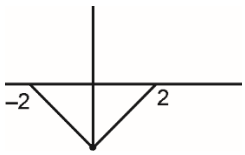
(3) $2k$ (4) k

Answer (1)

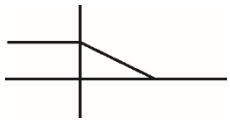
Sol. Graph of $f(x)$



$f(|x|)$



$\Rightarrow f(|x|)=\begin{cases}-2-x, & x<0\\ x-2, & x>0\end{cases}$



$|f(x)|=\begin{cases}2, & x<0\\ 2-x, & x>0\end{cases}$

$\Rightarrow h(x) = f(|x|) + |f(x)| = \begin{cases}-x, & x<0\\ 0, & x>0\end{cases}$

$\Rightarrow \int_0^k h(x)dx=\int_0^k 0dx=0$

2. Let three urn A, B, C : A = 7 red, 5 black

B = 5 red, 7 black

C = 6 red, 6 black

Urn is selected and black ball is taken. Then the probability that the selected urn is A is equal to

(1) $\frac{7}{18}$ (2) $\frac{5}{17}$

(3) $\frac{7}{19}$ (4) $\frac{5}{18}$

Answer (4)

Sol. Urn A has 7 red, 5 black balls

Urn B has 5 red, 7 black balls.

Urn C has 6 red, 6 black balls

If ball drawn is black then probability that it is chosen from urn A.

$$=\frac{\frac{1}{3}\times\frac{5}{12}}{\frac{1}{3}\times\frac{5}{12}+\frac{1}{3}\times\frac{7}{12}+\frac{1}{3}\times\frac{6}{12}}$$

$$=\frac{\frac{5}{36}}{\frac{5}{36}+\frac{7}{36}+\frac{6}{36}}$$

$$=\frac{\frac{5}{36}}{\frac{18}{36}}=\frac{5}{18}$$

3. $\int_{-\pi/2}^{\pi/2}\frac{\sin^2 x}{1+2^x}dx =$

(1) $\left(\frac{\pi}{4}\right)$ (2) $\frac{\pi}{8}$

(3) 4π (4) $\frac{\pi}{2}$

Answer (1)

Sol. $I = \int_0^{\pi/2} \left(\frac{\sin^2 x}{1+2^x} + \frac{\sin^2 (x)}{1+2^{-x}} \right) dx$

$I = \int_0^{\pi/2} \sin^2 x \, dx$

$I = \int_0^{\pi/2} \cos^2 x \, dx$

$2I = \int_0^{\pi/2} 1 dx$

$I = \frac{\pi}{4}$

4. If $f(x) = \frac{2x^2 - 3x + 8}{2x^2 + 3x + 8}$ then sum of maximum and minimum values of $f(x)$ is

(1) $\frac{136}{55}$ (2) $\frac{146}{55}$

(3) $\frac{146}{11}$ (4) $\frac{136}{11}$

Answer (2)

Sol. $y = \frac{2x^2 - 3x + 8}{2x^2 + 3x + 8}$, $2x^2 + 3x + 8 > 0 \; \forall x \in R$

$\Rightarrow x^2(2y - 2) + x(3y + 3) + 8y - 8 = 0$

Since $x \in R$, the equation has real roots

$\Rightarrow$ Discriminant is greater than or equal to 0

$\Rightarrow (3y + 3)^2 - 4(2y - 2)(8y - 8) \geq 0$

$\Rightarrow 9(y + 1)^2 - 64y(y - 1)^2 \geq 0$

$\Rightarrow (3y + 3)^2 - (8y - 8)^2 \geq 0$

$\Rightarrow (11y - 5)(-5y + 11) \geq 0$

$\Rightarrow \left(y - \frac{5}{11} \right) \left(y - \frac{11}{5} \right) \leq 0$

$\Rightarrow y \in \left[\frac{5}{11}, \frac{11}{5} \right]$

$\Rightarrow$ Sum of y_{max} and $y_{min} = \frac{5}{11} + \frac{11}{5}$

$= \frac{121 + 25}{55}$

$= \left(\frac{146}{55} \right)$

5. The coefficient of x^7 in $(1 - x - x^2 + x^3)^6$ equals to

(1) 132 (2) 144

(3) –132 (4) –144

Answer (4)

Sol. Coefficient of x^7 in $(1 - x)^6 (1 - x^2)^6$

${}^6C_1 \, {}^6C_3 - {}^6C_3 {}^6C_2 + {}^6C_5 \, {}^6C_1$

$120 - 15 \times 20 + 6 \times 6$

$120 - 300 + 36$

$= -144$

6. If $(\bar{z})^2 + |z| = 0$ and if α is sum of roots and β is product of non-zero roots, then $4(\alpha^2 + \beta^2)$ is

(1) $\frac{1}{4}$ (2) 1

(3) 4 (4) 2

Answer (3)

Sol. $(\bar{z})^2 + |z| = 0$

Let $z = x + iy$

$\Rightarrow (x - iy)^2 + \sqrt{x^2 + y^2} = 0$

$\Rightarrow (x^2 - y^2) + \sqrt{x^2 + y^2} - 2xyi = 0$

$\Rightarrow x^2 - y^2 + \sqrt{x^2 + y^2} = 0$ and $2xy = 0$

$\Rightarrow x = 0$ and $y \neq 0$

Case I

$\Rightarrow -y^2 + |y| = 0 \Rightarrow |y| = y^2 \Rightarrow y = \pm 1$

Cas II

$x \neq 0$ and $y = 0$

$\Rightarrow x^2 + |x| = 0 \Rightarrow x = 0$ only not possible

$\Rightarrow x = 0, y = 0$ satisfies

$\Rightarrow z = i, -i, 0$ are solution

$\alpha = i - i = 0$

$\beta = (i)(-i) = -1 \Rightarrow 4(\alpha^2 + \beta^2) = 4$

7. If α & β are roots of $ax^2 + bx + c = 0$ then equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}$ is

(1) $cx^2 + bx + a = 0$ (2) $bx^2 + ax + c = 0$

(3) $ax^2 + bx + c = 0$ (4) $cx^2 + ax + b = 0$

Answer (1)

Sol. $ax^2 + bx + c = 0 \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix}$

$\alpha + \beta = \frac{-b}{a}$

$\alpha\beta = \frac{c}{a}$

Now $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = -\frac{b}{c}$

$\frac{1}{\alpha\beta} = \frac{a}{c}$

$x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha\beta} = 0$

$x^2 + \frac{b}{c}x + \frac{a}{c} = 0$

$cx^2 + bx + a = 0$

8. Let $f(x) = \begin{cases} \frac{1-\cos\alpha x}{x^2}; & x < 0 \\ 2 \quad ; & x = 0 \\ \frac{\beta\sqrt{1-\cos x}}{x}; & x > 0 \end{cases}$

is continuous at $x = 0$. Then $\alpha^2 + \beta^2$ equals to

(1) 10 (2) 12

(3) 13 (4) 9

Answer (2)

Sol. Given $f(x)$ is continuous at $x = 0$

$\therefore \lim\limits_{x\to0^-} f(x) = f(0) = \lim\limits_{x\to0^+} f(x)$

When $x < 0$, $x = 0 - h$

$\therefore \lim\limits_{h\to0} \frac{1-\cos(\alpha(0-h))}{(0-h)^2}$

$= \lim\limits_{h\to0} \frac{1-\cos(h\alpha)}{h^2}$

$= \lim\limits_{h\to0} \left(\frac{1-\cos(\alpha h)}{\alpha^2.h^2}\right)\alpha^2$

$= \alpha^2 \lim\limits_{h\to0} \frac{1-\cos(\alpha h)}{(\alpha h)^2}$

$= \frac{\alpha^2}{2}$ ...(1)

When $x > 0$

$x = 0 + h$

$\lim\limits_{h\to0} \frac{\beta\sqrt{1-\cos h}}{h} = \lim\limits_{h\to0} \frac{\beta\sqrt{\frac{1-\cosh}{h^2}.h^2}}{h}$

$= \frac{\beta}{\sqrt{2}}$ ...(2)

as $f(0) = 2$ ...(3)

$\therefore$ From (1), (2) and (3)

$\frac{\alpha^2}{2} = 2, \quad \frac{\beta}{\sqrt{2}} = 2$

$\alpha = 2, \quad \beta = 2\sqrt{2}$

$\alpha^2 + \beta^2 = 4 + 8 = 12$

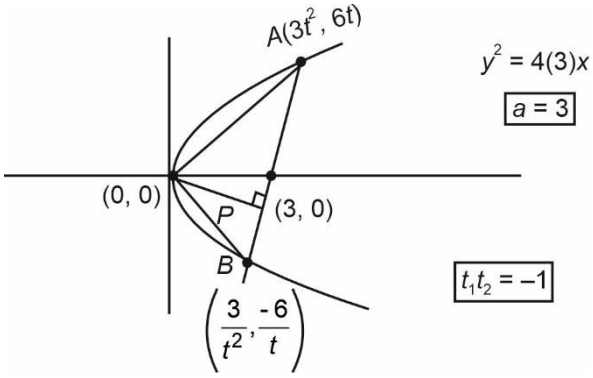
9. If the length of focal chord of $y^2 = 12x$ is 15 and if the distance of the focal chord from origin is P then $10P^2$ is equal to

(1) 36 (2) 25

(3) 72 (4) 144

Answer (3)

Sol.



$\Rightarrow AB = 15$

$$\left(3t^2-\frac{3}{t^2}\right)^2+\left(6t+\frac{6}{t}\right)^2=225$$

$$\Rightarrow 9\left(t^2-\frac{1}{t^2}\right)^2+36\left(t+\frac{1}{t}\right)^2=225$$

$$\Rightarrow 9\left(t+\frac{1}{t}\right)^2\left[\left(t-\frac{1}{t}\right)^2+4\right]=225$$

$$\Rightarrow 9\left(t+\frac{1}{t}\right)^2\left(t+\frac{1}{t}\right)^2=225$$

$$\Rightarrow t+\frac{1}{t}=\left(\frac{225}{9}\right)^{1/4}=(25)^{1/4}=\sqrt{5}$$

Equation of $AB \equiv (y-0)=\frac{2}{\left(t-\frac{1}{t}\right)}(x-3) \Rightarrow \left|t-\frac{1}{t}\right|=1$

$\Rightarrow y = 2x - 6 \Rightarrow y - 2x + 6 = 0$

Distance from origin $\Rightarrow P=\frac{6}{\sqrt{5}} \Rightarrow 10P^2=\frac{10\times 36}{5}$

$= 72$

10. Numbers –3, 4, 7, –6, α, β

Mean = 2, Variance = 23, then

Mean deviation about mean equals to

(1) $\frac{13}{8}$ (2) $\frac{13}{3}$

(3) $\frac{13}{7}$ (4) $\frac{13}{9}$

Answer (2)

Sol. Mean $= \frac{-3+4+7+(-6)+\alpha+\beta}{6}=2$

$= 2 + \alpha + \beta = 2 \times 6$

$\Rightarrow \alpha + \beta = 10$

Variance $= \frac{\Sigma xi^2}{n}-\left(\frac{\bar{x}}{n}\right)^2=23$

$$=\frac{\Sigma xi^2}{n}=23+4$$

$$=\Sigma xi^2=27\times 6$$

$= 9 + 16 + 49 + 36 + \alpha^2 + \beta^2 = 162$

$\Rightarrow \alpha^2 + \beta^2 = 52$

$\Rightarrow$ We get α and β as 4 and 6

So, mean deviation about mean

$$=\frac{|-3-2|+|4-2|+|7-2|+|-6-2|+|4-2|+|6-2|}{6}$$

$$=\frac{5+2+5+8+2+4}{6}$$

$$=\frac{26}{6}=\frac{13}{3}$$

11. If $\frac{dy}{dx}=\frac{2x^2+2x+3}{x^4+2x^3+3x^2+2x+2}$

and $y(-1) = -\frac{\pi}{4}$

then $y(0)$ is

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{4}$

(3) $\frac{\pi}{2}$ (4) $\frac{\pi}{6}$

Answer (2)

Sol. $\int dy=\int\frac{2x^2+2x+3}{x^4+2x^3+3x^2+2x+2}dx$

$$= \int \frac{2x^2 + 2x + 3}{(x^2+1)(x^2+2x+2)}\,dx$$

$$= \int \frac{1}{x^2+2x+2}dx + \int \frac{1}{x^2+1}\,dx$$

$$= \int \frac{1}{1+(x+1)^2}dx + \tan^{-1}x + C$$

$y = \tan^{-1}(x+1) + \tan^{-1}x + C$

$y(-1) = -\frac{\pi}{4}$

$-\frac{\pi}{4} = 0 - \frac{\pi}{4} + C$

$\Rightarrow \quad C = 0$

$\therefore \quad y = \tan^{-1}(x+1) + \tan^{-1}(x)$

Now $y(0) = \tan^{-1}(1) + \tan^{-1}(0) = \frac{\pi}{4}$

12. If $\vec{c}$ is a variable unit vector and $\vec{c}$ makes angle of 45° with $\vec{b}$ and 60° with $\vec{a}$ with $\vec{b} = \hat{i} - \hat{k}$ and $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$ then $|\vec{c} + 2\vec{a} - 3\vec{b}|$ is

(1) 19 (2) 20

(3) $\sqrt{19}$ (4) $\sqrt{20}$

Answer (3)

Sol. $\vec{c}$ is unit vector

$\vec{b} = \hat{i} - \hat{k}$

$\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$

$|\vec{a}| = 3, \ |\vec{b}| = \sqrt{2}, \ |\vec{c}| = 1$

$|\vec{c} + 2\vec{a} - 3\vec{b}|^2 = |\vec{c}|^2 + 4|\vec{a}|^2 + 9|\vec{b}|^2 + 4\vec{a}.\vec{c} - 12\vec{a}.\vec{b} - 6\vec{b}.\vec{c}$

$= 1 + 36 + 18 + 4|\vec{a}||\vec{c}|\cos 60° - 12[3] - 6|\vec{b}||\vec{c}|\cos 45°$

$= 55 + 12 \times \frac{1}{2} - 36 - 6\sqrt{2} \times \frac{1}{\sqrt{2}}$

$= 55 + 6 - 36 - 6$

$= 19$

$|\vec{c} + 2\vec{a} - 3\vec{b}| = \sqrt{19}$

13. If the system of equations

$A + \sqrt{2}\sin x\, B + \sqrt{2}\cos x\, C = 0$

$A + \sin x\, B - \cos x\, C = 0$

$A + \cos x\, B + \sin x\, C = 0$ has non-trivial solution then the value of $x, x \in \left(0, \frac{\pi}{2}\right)$ is

(1) $\frac{5\pi}{12}$ (2) $\frac{\pi}{12}$

(3) $\frac{5\pi}{24}$ (4) $\frac{\pi}{8}$

Answer (3)

Sol. For non-trivial solution

$$\begin{vmatrix} 1 & \sqrt{2}\sin x & \sqrt{2}\cos x \\ 1 & \sin x & -\cos x \\ 1 & \cos x & \sin x \end{vmatrix} \text{ is zero}$$

$\Rightarrow \quad 1 - 1\left(\sqrt{2}\sin^2 x - \sqrt{2}\cos^2 x\right) + 1\left(-2\sqrt{2}\sin x \cos x\right) = 0$

$\Rightarrow \quad 1 + \sqrt{2}(\cos 2x) - \sqrt{2}\sin 2x = 0$

$\Rightarrow \quad \sqrt{2}(\cos 2x - \sin 2x) = -1$

$\Rightarrow \quad \cos\left(2x + \frac{\pi}{4}\right) = \frac{-1}{2}$

$x \in \left(0, \frac{\pi}{2}\right)$

$2x \in (0, \pi)$

$2x + \frac{\pi}{4} \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

$\Rightarrow \quad \cos\left(2x + \frac{\pi}{4}\right) = \frac{-1}{2} \Rightarrow 2x + \frac{\pi}{4} = \frac{2\pi}{3}$

$2x = \frac{2\pi}{3} - \frac{\pi}{4} = \frac{5\pi}{12}$

$\Rightarrow \quad x = \frac{5\pi}{24}$

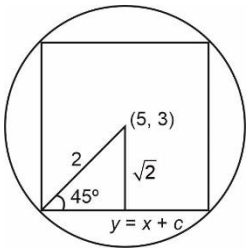
14. A line L_1 having equation $y = x + 3$. A square is inscribed in a circle $x^2 + y^2 - 10x - 6y + 30 = 0$ such that one side of square is parallel to L_1. Find $\sum_{i=1}^{4}\left(x_i^2 + y_i^2\right)$ where (x_i, y_i) $i \in \{1, 2, 3, 4\}$ are the vertices of square.

(1) 152 (2) 162

(3) 172 (4) 182

Answer (1)

Sol.



Distance of (5, 3) to the line $y = x + c$ is $\sqrt{2}$

$$\Rightarrow \frac{|3-5-c|}{\sqrt{2}} = \sqrt{2}$$

$|c + 2| = 2$

$\Rightarrow c = 0$

$c = -4$

So, the lines are $y = x$ and $y = x - 4$

Now, solving these lines with the circle

$y = x$ and $x^2 + y^2 - 10x - 6y + 30 = 0$

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$x = 3, y = 3$

$x = 5, y = 5$

$y = x - 4$ and $x^2 + y^2 - 10x - 6y + 30 = 0$

$2x^2 - 24x + 70 = 0$

$x^2 - 12x + 35 = 0$

$x = 5, y = 1$

$x = 7, y = 3$

$$\sum_{i=1}^{4} x_i^2 + y_i^2 = 9 + 9 + 25 + 25 + 25 + 1 + 49 + 9 = 152$$

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The number of rational numbers in the expansion of $(2^{1/5} + 5^{1/3})^{15}$ is

Answer (02)

Sol. $T_{r+1} = {}^{15}C_r (5^{1/3})^r (2^{1/5})^{15-r}$, $r \in \{0, 1, \ldots.15\}$

$= {}^{15}C_r \, 5^{\left(\frac{r}{3}\right)} \cdot 2^{\left(3-\frac{r}{5}\right)}$, $\quad r \in \{0, 1, \ldots 15\}$

For rational terms,

$\frac{r}{3} \in$ integer and $\frac{r}{5} \in$ integer

$\Rightarrow$ 3 and 5 divides $r \Rightarrow$ 15 divides r

$\Rightarrow r = 0$ and 15

$\Rightarrow$ only 2 rational terms.

22. In ΔABC there are 18 points, on side AB there are P_1, P_2, P_3, P_4, P_5 points, on BC there are P_6, P_7 …P_{11} points and on CA P_{12}…P_{18} points. By joining any three points from P_1, P_2 …P_{18} form a triangle. Then number of triangles possible are

Answer (751)

Sol. Total ways to select three points out of 18 points = ${}^{18}C_3$

Total ways to select 3 points from $P_1 \ldots P_5 = {}^5C_3$

Total ways to select 3 points from $P_6 \ldots P_{11} = {}^6C_3$

Total ways to select 3 points from $P_{12} \ldots P_{18} = {}^7C_3$

Total number of triangles possible

$= {}^{18}C_3 - {}^5C_3 - {}^6C_3 - {}^7C_3$

$= 751$

23. If $\lim\limits_{x\to 1}\dfrac{(5x+1)^{1/3}-(x+5)^{1/3}}{(2x+3)^{1/2}-(x+4)^{1/2}}=\dfrac{m(5)^{1/2}}{n(2n)^{2/3}}$

Then $8m + 12n$ is

Answer (100)

Sol. $\lim\limits_{x\to 1}\dfrac{(5x+1)^{1/3}-(x+5)^{1/3}}{(2x+3)^{1/2}-(x+4)^{1/2}}$

$$\lim_{x\to 1}\frac{\frac{1}{3}(5x+1)^{-2/3}\cdot 5-\frac{1}{3}(x+5)^{-2/3}}{2\times\frac{1}{2}(2x+3)^{-1/2}-\frac{1}{2}(x+4)^{-1/2}}$$

$$=\frac{\frac{1}{3}\times\frac{5}{(6)^{2/3}}-\frac{1}{3}\times\frac{1}{(6)^{2/3}}}{\frac{1}{2}\times\frac{2}{(5)^{1/2}}-\frac{1}{2}\times\frac{1}{(5)^{1/2}}}$$

$$=\frac{\frac{4}{3\times(6)^{2/3}}}{\frac{1}{2.(5)^{1/2}}}=\frac{8(5)^{1/2}}{3(6)^{2/3}}=\frac{m(5)^{1/2}}{n(2n)^{2/3}}$$

$\Rightarrow$ $m = 8, n = 3$

$8m + 12n = 64 + 36 = 100$

24. In a G.P. $T_1 = 2$, $T_2 = P$, $T_3 = Q$, these are also terms of A.P (7^{th}, 8^{th} and 13^{th} term).

If 5^{th} term of G.P = n^{th} term of A.P3. Then n is

Answer (27)

Sol. $T_1 = 2$ $\qquad a = 2$

$T_2 = P$ $\qquad 2r = P \Rightarrow r = \dfrac{P}{2}$

$T_3 = Q$ $\qquad 2r^2 = Q \Rightarrow r^2 = \dfrac{Q}{2}$

$a' + 6d = 2$ $\qquad \ldots(1)$

$a' + 7d = P$ $\qquad \ldots(2)$

$a' + 12d = Q$ $\qquad \ldots(3)$

$d = 2(r - 1)$

$2r(r - 1) = 5d$

$$\frac{5d}{d}=\frac{-2r(r-1)}{2(r-1)}$$

$r = 5 \Rightarrow d = 8$

$a + 48 = 2$

$a = -46$

$2.3^4 = -46 + (n - 1) \times 8$

$\Rightarrow n = 27$

25. Domain of $\sin^{-1}\left(\dfrac{3x-22}{2x-19}\right)+\log_e\left(\dfrac{3x^2-8x+5}{x^2-3x-10}\right)$

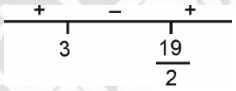
is $(\alpha, \beta]$. Then $3\alpha + 10\beta$ equals to

Answer (97)

Sol. $-1\le\dfrac{3x-22}{2x-19}\le 1$

$$\frac{3x-22-2x+19}{2x-19}\le 0$$

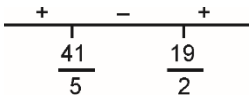
$$\frac{x-3}{2x-19}\le 0$$



$$\left[3,\frac{19}{2}\right)$$

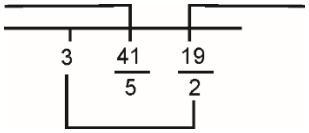
$$\frac{3x-22+2x-19}{2x-19}\ge 0$$

$$\frac{5x-41}{2x-19}\ge 0$$



$$\left(-\infty,\frac{41}{5}\right]\cup\left(\frac{19}{2},\infty\right)$$

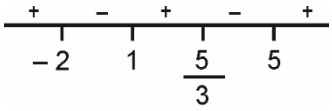
Taking intersection



$\rightarrow \left[3, \frac{41}{5}\right]$

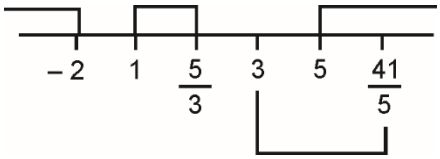
$\frac{3x^2-8x+5}{x^2-3x-10} > 0$

$\frac{(3x-5)(x-1)}{(x-5)(x+2)} > 0$



$(-\infty, -2) \cup \left[1, \frac{5}{3}\right] \cup (5, \infty)$

Taking intersection of individual domains



$\left(5, \frac{41}{5}\right]$

$3\alpha + 10\beta = 3\times5 + 10\times\frac{41}{5}$

= 15 + 82 = 97

26. If $a = \frac{1}{2!} + \frac{{}^2C_2}{3!} + \frac{{}^3C_2}{4!} + \frac{{}^4C_2}{5!} + ...$

$b = 1 + \frac{{}^1C_0 + {}^1C_1}{1!} + \frac{{}^2C_0 + {}^{+2}C_1 + {}^2C_2}{2!} + ...$

Then $\frac{2b}{a^2}$ equals to

Answer (8)

Sol. $a = \frac{1}{2} + \sum_{n=2}^{\infty} \frac{{}^nC_2}{(n+1)!}$

$= \frac{1}{2} + \sum_{n=2}^{\infty} \frac{\frac{n(n+1)}{2}}{(n+1)!}$

$= \frac{1}{2} + \sum_{n=2}^{\infty} \frac{1}{2} \times \frac{1}{(n-1)!}$

$= \frac{1}{2} + \frac{1}{2}(e-1)$

$= \frac{e}{2}$

$b = 1 + \frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + ...$

$b = e^2$

$\frac{2b}{a^2} = \frac{2\times e^2}{\frac{e^2}{4}} = 8$

27. If $A = \begin{bmatrix} 1 & 2 & \alpha \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ and Det(Adj $(A - 2A^T)$ Adj $(2A - A^T)) = 2^8$ then $\det(A)^2$ is

Answer (16.00)

Sol. $|\text{Adj}(A - 2A^T)\ \text{Adj}(2A - A^T)| = 2^8$

$P = A - 2A^T$

$Q = 2A^T - A \Rightarrow Q^T = 2A^T - A = -P$

$|\text{adj}(P)\ \text{adj}\ Q| = 2^8, \Rightarrow |Q^T| = |-P| \Rightarrow |Q| = -|P|$

$|P|^2|Q|^2 = 2^8 \Rightarrow |PQ| = -2^4$

$\Rightarrow |P|(-|P|) = -2^4 \Rightarrow |P| = 4$ and $|Q| = -4$

$|A - 2A^T| = 4$

$A - 2A^T = \begin{bmatrix} 1 & 2 & \alpha \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \\ \alpha & 1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & \alpha \\ -3 & 0 & -1 \\ -2\alpha & -1 & -2 \end{bmatrix}$

$\Rightarrow |A - 2A^T| = 1 + 3\alpha = 4 \Rightarrow \alpha = 1 \Rightarrow |A| = -4 \Rightarrow |A|^2 = 16$

28.

29.

30.

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