MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

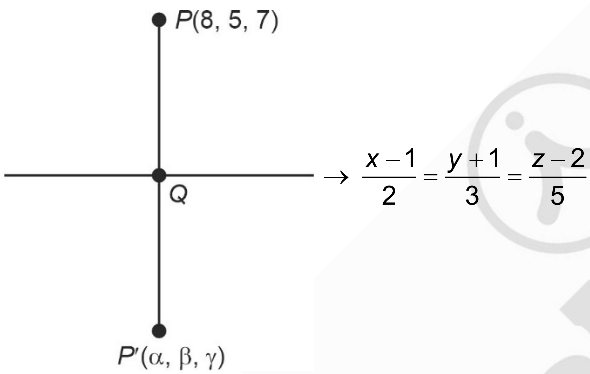
1. Let image of point (8, 5, 7) with respect to line $\frac{x-1}{2}=\frac{y+1}{2}=\frac{z-2}{5}$ is (α, β, γ). Then $\alpha + \beta + \gamma$ is equal to

(1) 10 (2) 12

(3) 9 (4) 14

Answer (4)

Sol. Given point (8, 5, 7)



Let Q be general point.

$(x, y, z) = (2\lambda + 1, 3\lambda - 1, 5\lambda + 2)$

$\therefore$ Now D.R. of $P.Q$

$PQ \Rightarrow (2\lambda + 1 - 8, 3\lambda - 1 - 5, 5\lambda + 2 - 7)$

$= (2\lambda - 7, 3\lambda - 6, 5\lambda - 5)$ ...(1)

$\therefore$ D.R. of line <2, 3, 5> ....(2)

From (1) and (2)

$2(2\lambda - 7) + 3(3\lambda - 6) + 5(5\lambda - 5) = 0$

$4\lambda - 14 + 9\lambda - 18 + 25\lambda - 25 = 0$

$38\lambda - 57 = 0$

$\lambda = \frac{57}{38} \Rightarrow \boxed{\lambda = \frac{3}{2}}$

$Q \equiv \left(2\left(\frac{3}{2}\right)+1,\ 3\left(\frac{3}{2}\right)-1,\ 5\left(\frac{3}{2}\right)+2\right)$

$Q \equiv \left(4,\ \frac{7}{2},\ \frac{19}{2}\right)$

$\therefore \frac{8+\alpha}{2} = 4, \frac{5+\beta}{2} = \frac{7}{2}, \frac{7+\gamma}{2} = \frac{19}{2}$

$\Rightarrow \alpha = 0, \beta = 2, \gamma = 12$

$\therefore (\alpha, \beta, \gamma) \equiv (0, 2, 12)$

$\therefore \alpha + \beta + \gamma = 0 + 2 + 12 = 14$

2. The 50[th] word in the dictionary using the letters B, B, H, J, O is

(1) OBBJH (2) OBBHJ

(3) JHBBO (4) BBHOJ

Answer (1)

Sol. Number of words staring with ‘B’ = 4!

= 24

Number of words staring with ‘H’ = $\frac{4!}{2!}$

= 12

Number of words staring with ‘J’ = 12

49[th] word = OBBHJ

50[th] word = OBBJH

3. $\left(\frac{3^{\frac{1}{5}}}{x}+\frac{2x}{5^{\frac{1}{3}}}\right)^{12}$. Find which term is constant.

(1) 4[th] (2) 5[th]

(3) 6[th] (4) 7[th]

Answer (4)

Sol. $\left(\frac{3^{\frac{1}{5}}}{x}+\frac{2x}{5^{\frac{1}{3}}}\right)^{12}$

$$T_{r+1}={}^nC_r\left(\frac{3^{\frac{1}{5}}}{x}\right)^{n-r}\left(\frac{2x}{5^{\frac{1}{3}}}\right)^{r}$$

$$\left(3^{\frac{1}{5}}\right)^{n-r}x^{r-n}\frac{2^r.x^r}{5^{\frac{r}{3}}}$$

For constant term

$r - n + r = 0$

$\Rightarrow\quad 2r - n = 0$

We have $n = 12$

$\Rightarrow\quad 2r - 12 = 0$

$r = 6$

So 7th term is constant.

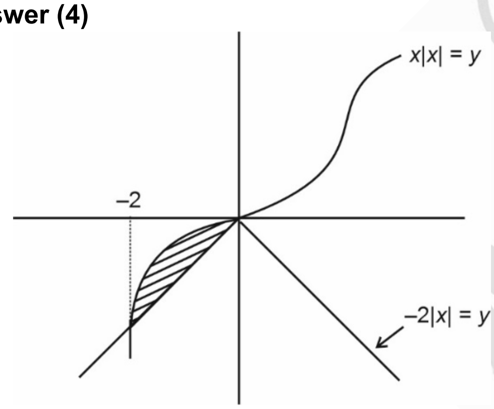
4. Area bounded by $y = -2|x|$ and $y = x|x|$ is

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$

(3) $\frac{1}{2}$ (4) $\frac{4}{3}$

Answer (4)

Sol.



$$\text{Area}=\left|\int_{-2}^{0}(-x^2-(2x))dx\right|$$

$$=\left|\left[\frac{-x^3}{3}-x^2\right]_{-2}^{0}\right|$$

$$=\left|\frac{8}{3}-4\right|=\frac{4}{3}\text{ sq. unit}$$

5. $A=\begin{bmatrix}\alpha & \alpha & \alpha\\ \beta & \alpha & -\beta\\ -\alpha & \alpha & \alpha\end{bmatrix}$

B is formed by co-factor of A matrix, then find out determinant of AB.

(1) $4\alpha^3(2\alpha+\beta)^5$

(2) $12\alpha^4(\alpha+\beta)^2$

(3) $8\alpha^6(\alpha+\beta)^3$

(4) $18\alpha^8(\alpha+\beta)^3$

Answer (3)

Sol. $A=\begin{bmatrix}\alpha & \alpha & \alpha\\ \beta & \alpha & -\beta\\ -\alpha & \alpha & \alpha\end{bmatrix}$

$$|A|=\begin{bmatrix}2\alpha & 0 & 0\\ \beta & \alpha & -\beta\\ -\alpha & \alpha & \alpha\end{bmatrix}$$

$= 2\alpha\,(\alpha^2 + \alpha\beta)$

$= 2\alpha^2\,(\alpha + \beta)$

Now, $\beta = (\text{adj}A)^T$

Determinant of $A{\cdot}B = |A{\cdot}B|$

$= |A{\cdot}(\text{adj}A)^T|$

$= |A|{\cdot}|A|^2$

$= |A|^3$

$|A|^3 = 8\alpha^6(\alpha + \beta)^3$

6. Consider a equation $P(x)=ax^2+bx+c=0$. If $a, b, c\in A$, were $A=\{1, 2, 3, 4, 5, 6\}$. Then the probability that $P(x)$ has real and distinct roots?

(1) $\frac{1}{4}$ (2) $\frac{1}{16}$

(3) $\frac{25}{108}$ (4) $\frac{19}{108}$

Answer (4)

Sol. $b^2 - 4ac > 0$

$\Rightarrow\ b < 2$ not possible

$\Rightarrow\ b = 3\ \Rightarrow ac < \frac{9}{4}$

$(a, c) \in \{(1, 1), (1, 2), (2, 1)\} \Rightarrow 3$ cases

$\Rightarrow\ b = 4\ \Rightarrow ac < 4\ \Rightarrow ac = \{1,\ 2, 3\}$

$(a, c) \in \{(1, 1), (1, 2), (2, 1), (3, 1), (1, 3)\} = 5$ ways

$\Rightarrow b = 5 \Rightarrow ac < \frac{25}{4} \Rightarrow ac = \{1, 2, 3, 4, 5, 6\}$

$(a, c) \in \{(1, 1), (1, 2), (2, 1), (3, 1), (1, 3), (2, 2),$
$(4, 1), (1, 4), (3, 2), (2, 3), (5, 1), (1, 5),$
$(1, 6), (6, 1)\}\ \Rightarrow 14$ ways

$\Rightarrow b = 6 \Rightarrow ac < 9 \Rightarrow ac \in \{1, 2, 3, 4, 5, 6, 7, 8\}$

$(a, c) \in \{(1, 1), (1, 2), (2, 1), (3, 1), (1, 3), (2, 2),$
$(4, 1), (1, 4), (3, 2), (2, 3), (5, 1), (1, 5),$
$(1, 6), (6, 1), (2, 4), (4, 2)\}\ \Rightarrow 16$ ways

$\Rightarrow 3 + 5 + 14 + 16 = 38$ cases

$\Rightarrow$ Probability = $\frac{38}{6^3} = \frac{19}{108}$

7. If $f : R \to R$ and $g : R \to R$ defined such that

$f(x) = |x| - 1$

$$g(x) = \begin{cases} e^x & ;\ x > 0 \\ x - 1 & ;\ x \le 0 \end{cases}$$

Then,

(1) Both f and g is one-one

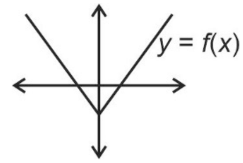
(2) f is one-one and g is many one

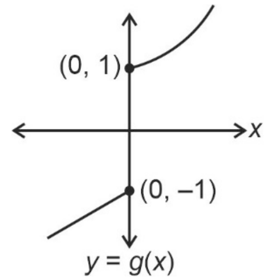
(3) f is many one and g is one-one

(4) f and g both are many one

Answer (3)

Sol.





By horizontal line test $f(x)$ is many one and $g(x)$ is one-one.

Option (3) is correct.

8. A line L is perpendicular to $y = 2x + 10$ such that it touches the parabola $y^2 = 4(x - g)$. Then the distance between point of contact and origin is equal to

(1) $\sqrt{165}$ (2) $\sqrt{175}$

(3) $\sqrt{185}$ (4) $\sqrt{190}$

Answer (3)

Sol. L: $2y + x = c$

$y^2 = 4(x - 9)$

Now

$$\left(\frac{c - x}{2}\right)^2 = 4(x - 9)$$

$x^2 - 2(c + 8)x + c^2 + 144 = 0$

$D = 0$

$\Rightarrow\ c = 5$

$\therefore\ L: 2y + x = 5$

Parabola and L meets at (13, –4)

Now, distance = $\sqrt{185}$

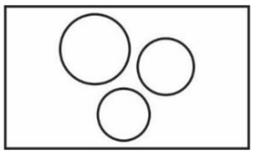
9. If $S = \{2, 4, 8, 16, \ldots, 512\}$. If S is broken in 3 equal subsets A, B and C such that $A \cap B = B \cap C = C \cap A = \phi$ and $A \cup B \cup C = S$ then maximum number of ways to break is

(1) ${}^{9}C_3$ (2) $\dfrac{9!}{(3!)3}$

(3) $\dfrac{9!}{(3!)4}$ (4) $\dfrac{9!}{(3!)^2}$

Answer (2)

Sol. $S = \{2^1, 2^2, 2^3, \ldots, 2^9\}$



$A \cap B = B \cap C = A \cap C = \phi$

and $= A \cup B \cup C = S$

$\Rightarrow$ A, B, C are disjoint mutually exhaustive and exclusive

$$\Rightarrow {}^{9}C_3 \cdot {}^{6}C_3 \cdot {}^{3}C_3 = \frac{9!}{6!3!} \times \frac{6!}{3!3!} \times (1)$$

$$= \frac{9!}{3!3!3!} = 1680$$

10. If $y = \dfrac{2\cos 2\theta + \cos\theta}{\cos 3\theta + \cos^2\theta + \cos\theta}$

Then value of $y'' + y' + y$ is

(1) $\sec\theta(1 - \tan^3\theta)$

(2) $\tan\theta(\sec^3\theta + 2\tan^2\theta)$

(3) $\sec\theta(2\sec^2\theta + \tan\theta)$

(4) $\cot\theta(\sec^3\theta + 2\tan\theta)$

Answer (3)

Sol. $y = \dfrac{2\cos 2\theta + \cos\theta}{\cos 3\theta + \cos^2\theta + \cos\theta}$

$$y = \frac{2\cos 2\theta + \cos\theta}{2\cos 2\theta \cdot \cos\theta + \cos^2\theta}$$

$$y = \frac{2\cos 2\theta + \cos\theta}{\cos\theta(2\cos 2\theta + \cos\theta)}$$

$$y = \frac{1}{\cos\theta}$$

$y = \sec\theta$

$y' = \sec\theta \tan\theta$

$y'' = \sec^3\theta + \tan\theta \cdot (\sec\theta \tan\theta)$

$= \sec^3\theta + \sec \tan^2\theta$

$y'' + y' + y = \sec^3\theta + \sec\theta\tan^2\theta + \sec\theta \tan\theta + \sec\theta$

$= \sec\theta\,(\sec^2\theta + 1) + \sec\theta \tan\theta(\tan\theta + 1)$

$= \sec\theta\,(\sec^2\theta + 1 + \tan^2\theta + \tan\theta)$

$= \sec\theta\,(2\sec^2\theta + \tan\theta)$

11. If $2x^2 - x + 2 = 0$ and one root is a then $\displaystyle\lim_{x \to \frac{1}{a}} \frac{16(1 - \cos(2x^2 - x + 2))}{(ax - 1)^2}$ equals

(1) $\dfrac{32(1-a^2)^2}{a^4}$ (2) $\dfrac{8(1-a^2)^2}{a^3}$

(3) $\dfrac{16(1-a^2)^2}{a^4}$ (4) $\dfrac{20(1-a^2)^2}{a^3}$

Answer (1)

Sol. $2x^2 - x + 2 = 0$ ⟨ roots a and $\dfrac{1}{a}$

$$\lim_{x \to \frac{1}{a}} \frac{16[1 - \cos(2x^2 - x + 2)]}{a^2\left(x - \frac{1}{a}\right)^2}$$

$$= \lim_{x \to \frac{1}{a}} \frac{16\left[1 - \cos\left[2(x-a)\left(x - \frac{1}{a}\right)\right]\right]}{a^2 4\left(x - \frac{1}{a}\right)^2 (x-a)^2}(x-a)^2 . 4$$

$$= \frac{32}{a^2}\left(\frac{1}{a} - a\right)^2$$

$$= \frac{32(1-a^2)^2}{a^4}$$

12. If $\frac{dy}{dx}+\frac{y\cdot 2x}{(1+x^2)^2}=xe^{\frac{1}{1+x^2}}$ and $y(0) = 0$. Given $f(x)=y(x)e^{\frac{-1}{1+x^2}}$, then the area bounded between these two curves equals to _____

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$

(3) $\frac{7}{6}$ (4) 2

Answer (1)

Sol. $\frac{dy}{dx}+\frac{y\cdot 2x}{(1+x^2)^2}=x\cdot e^{\frac{1}{1+x^2}}$

I.F. = $e^{\int\frac{2x}{1+x^2}dx}$

Put $x^2 = t$

$2xdx = dt$

$= e^{\int\frac{dt}{(1+t)^2}}$

$= e^{-\frac{1}{1+t}}$

$= e^{-\frac{1}{1+x^2}}$

$y\cdot e^{-\frac{1}{1+x^2}}=\int x\cdot e^{-\frac{1}{1+x^2}}e^{\frac{1}{1+x^2}}dx$

$y\cdot e^{-\frac{1}{1+x^2}}=\int x\cdot dx$

$y\cdot e^{-\frac{1}{1+x^2}}=\frac{x^2}{2}+c$

$y=\frac{x^2}{2}e^{\frac{1}{1+x^2}}+c\cdot e^{\frac{1}{1+x^2}}$

$y(0) = 0$

$c = 0$

$\therefore\ f(x)=\left(e^{\frac{1}{1+x^2}}\cdot\frac{x^2}{2}\right)e^{-\frac{1}{1+x^2}}$

$f(x)=\frac{x^2}{2}$ ...(1)

Given $y = x$ ...(2)

$\therefore$ Area bounded between (1) and (2)

$\therefore\ A=\int_0^2\left(x-\frac{x^2}{2}\right)dx$

$=\left(\frac{x^2}{2}-\frac{x^3}{6}\right)_0^2$

$=\left(\frac{4}{2}-\frac{8}{6}\right)$

$=2-\frac{4}{3}$

$\Rightarrow\ \frac{2}{3}$

Option (1) is correct.

13. Find the differential equation of circle whose centre lies on $y = x$ and passes through (0, 1).

(1) $-x^2+y^2-2xy+2x-1+\frac{dy}{dx}(x^2+y^2-2+2y)=0$

(2) $-x^2-y^2-2xy+2x-1+\frac{dy}{dx}(x^2+y^2)=0$

(3) $-x^2-y^2-2xy+2x-1+\frac{dy}{dx}(x^2-y^2)=0$

(4) $x^2+y^2-2xy+2x-1+\frac{dy}{dx}(x^2+y^2-2)=0$

Answer (1)

Sol. The centre lies on $y = x$

$\therefore$ Centre of circle is of form (a, a)

$\therefore$ It passes through (0, 1)

$\therefore$ The equation of circle will be

$(x-a)^2+(y-a)^2=a^2+(a-1)^2$

$\Rightarrow\ x^2+y^2-2ax-2ay=-2a+1$ ...(1)

Differentiating w.r.t. x & eliminating 'a',

$$a=\frac{x+y\frac{dy}{dx}}{1+\frac{dy}{dx}}$$

Putting value of 'a' in equation (1), we get

$$-x^2+y^2-2xy+2x-1+\frac{dy}{dx}(x^2+y^2-2+2y)=0$$

14. $\beta(m,n)=\int_0^1 x^m\left(1-x^m\right)^{n-1}dx$

$$a\times\beta(-b,c)=\int_0^1\left(1-x^{10}\right)^{20}dx$$

Then $(a+b+c)$ is equal to

(1) 210

(2) 230

(3) 250

(4) 270

Answer (1)

Sol. $I=\int_0^1\left(1-x^{10}\right)^{20}dx$

Applying integration by parts

$$I=\left[x\left(1-x^{10}\right)^{20}\right]_0^1+200\int_0^1 x^{10}\left(1-x^{10}\right)^{19}dx$$

$$I=200\int_0^1 x^{10}\left(1-x^{10}\right)^{19}dx=a\times\beta(-b,c)$$

$\Rightarrow$ $a = 200$

$b = -10$

$c = 20$

$(200 - 10 + 20) = 210$

15. If $|\vec{a}|=2$, $|\vec{b}|=3$ and $\vec{a}=\vec{b}\times\vec{c}$ then minimum value of $|\vec{c}-\vec{a}|^2$ is

(1) 13

(2) 5

(3) $\frac{40}{9}$

(4) $\frac{20}{9}$

Answer (3)

Sol. $|\vec{a}|=2$, $|\vec{b}|=3$

Also, $\vec{a}=\vec{b}\times\vec{c}$

$\Rightarrow$ $\vec{a}\cdot\vec{b}=0$ and $\vec{a}\cdot\vec{c}=0$

$$|\vec{a}-\vec{c}|^2=|\vec{a}|^2+|\vec{c}|^2-2\vec{a}\cdot\vec{c}$$

$$=4+|\vec{c}|^2$$

$$|\vec{a}|=|\vec{b}\times\vec{c}|=|\vec{b}|\sin\theta|\vec{c}|$$

$$\Rightarrow\ (\sin\theta)|\vec{c}|=\frac{2}{3}$$

$$\Rightarrow\ \sin^2\theta=\frac{4}{9|\vec{c}|^2}$$

$$\Rightarrow\ |\vec{c}|^2=\frac{4}{9\sin^2\theta}$$

$$|\vec{a}-\vec{c}|^2=4+\frac{4}{9\sin^2\theta}$$

For $|\vec{a}-\vec{c}|^2$ to be minimum

$\Rightarrow$ $\sin\theta = 1$

$$\Rightarrow\ 4+\frac{4}{9}=\left(\frac{40}{9}\right)$$

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let $4^{1+x}+4^{1-x},\ \frac{K}{2},\ 16^x+16^{-x}$ are in AP then least value of K is

Answer (10)

Sol. $4^{1+x}+4^{1-x},\ \frac{K}{2},\ 16^x+16^{-x}$

$$2\times\frac{K}{2}=4^{1+x}+4^{1-x}+16^x+16^{-x}$$

$$K=\underbrace{4.4^x+\frac{4}{4^x}}_{\geq 8}+\underbrace{4^{2x}+4^{-2x}}_{\geq 2}$$

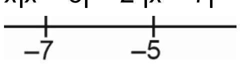
$\Rightarrow\ K\geq 10 \Rightarrow K = 10$

22. The number of real solution

$x|x + 5| + 2\,|x + 7| – 2 = 0$ is

Answer (03.00)

Sol. $x|x + 5| + 2\,|x + 7| – 2 = 0$



(i) $d\ x\geq -5 \Rightarrow x(x+5)+2(x+7)-2=0$

$x^2+7x+12=0 \Rightarrow x=-3,-4$

(ii) $x\in(-7,-5)$

$x(-x-5)+2(x+7)-2=0$

$-x^2-3x+12=0$

$\Rightarrow x^2+3x-12=0$

$\Rightarrow x=\frac{-3-\sqrt{57}}{2}$ satisfy

(iii) $x\leq -7$

$\Rightarrow x(-x-5)+2(-x-7)-2=0$

$-x^2-7x-16=0 \Rightarrow x^2+7x+16=0$

No solution

23. If $f(t)=\int_0^{\pi}\frac{2x}{1-\cos^2 t\sin^2 x}dx$, then the value of $\int_0^{\pi}\frac{\pi^2}{f(t)}dt$ is equal to

Answer (2)

Sol. $f(t)=\int_0^{\pi}\frac{2x}{1-\cos^2 t\sin^2 x}dx$

$$f(t)=2\int_0^{\pi}\frac{(\pi-x)}{1-\cos^2 t\sin^2 x}dx$$

$$2f(t)=2\int_0^{\pi}\frac{\pi}{1-\cos^2 t\sin^2 x}dx$$

$$f(t)=\pi\int_0^{\pi}\frac{\sec^2 x}{\sec^2 x-\cos^2 t\tan^2 x}dx$$

$\tan x = k$

$\sec^2 x\ dx = dk$

$$f(t)=\pi\int\frac{dk}{1+\sin^2 t\,k^2}$$

$$f(t)=\pi\times\frac{1}{\sin t}\left[\tan^{-1}(\sin t\times\tan x)\right]_0^{\pi/2}+\left[\tan^{-1}(\sin t\tan x)\right]_{\frac{\pi}{2}}^{\pi}$$

$$=\frac{\pi}{\sin t}(\pi)=\frac{\pi^2}{\sin t}$$

$$\Rightarrow \int_0^{\pi}\frac{\pi^2}{\frac{\pi^2}{\sin t}}dt=\int_0^{\pi}\sin t\ dt=2$$

24.

25.

26.

27.

28.

29.

30.

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