PART : MATHEMATICS

1. If the function $f(x)=\dfrac{\sin 3x+\alpha\sin x-\beta\cos 3x}{x^3}$, $x\in R$ is continuous at $x=0$, then $f(0)$ is

(1) –4 (2) –1 (3) 1 (4) 4

Ans. (1)

Sol. $$\frac{3x-\frac{(3x)^3}{3!}+\alpha\left(x-\frac{x^3}{3!}\right)-\beta\left(1-\frac{x^2}{2!}\right)}{x^3}$$

$\beta=0,\quad \alpha=-3$

$$=\lim_{x\to 0}\frac{-3^3\frac{x^3}{6}+3.\frac{x^3}{6}}{x^3}=\frac{3-3^3}{2\times 3}$$

$$=\frac{1-9}{2}=-4$$

2. The value of $\displaystyle\int_{-\pi}^{\pi}\frac{2y(1+\sin y)}{1+\cos^2 y}\,dy$ is

(1) π^2 (2) $-\pi^2$ (3) $2\pi^2$ (4) $3\pi^2$

Ans. (1)

Sol. $$I=\int_{-\pi}^{\pi}\frac{2y}{1+\cos^2 y}dy+\int_{-\pi}^{\pi}\frac{2y\sin y}{1+\cos^2 y}dy$$

$$I=0+\int_{0}^{\pi}\frac{4y\sin y}{1+\cos^2 y}dy \text{ \_\_\_\_\_\_\_\_ (1)}$$

$$I=4\int_{0}^{\pi}\frac{(\pi-y)\sin(\pi-y)}{1+\cos^2(\pi-y)}dy$$

$$I=4\int_{0}^{\pi}\frac{(\pi-y)\sin y}{1+\cos^2 y}dy \text{ \_\_\_\_\_\_\_\_ (2)}$$

From (1) and (2)

$$2I=4\int_{0}^{\pi}\frac{\pi\sin y}{1+\cos^2 y}dy$$

Let $\cos y=t$

$-\sin dy=dt$

$$2I=4\pi\int_{1}^{-1}\frac{-dt}{1+t^2}$$

$$I=4\pi\int_{0}^{1}\frac{dt}{1+t^2}\,dt$$

$$I=4\pi\left(\tan^{-1}t\right)_0^1$$

$$I=4\pi\left(\frac{\pi}{4}-0\right)=\pi^2$$

3. If $\theta \in \left[0, \frac{\pi}{4}\right]$ is a solution of equation $4\cos\theta - 3\sin\theta = 1$, then $\cos\theta$ is equal to

(1) $\frac{4(\sqrt{13}-1)}{25}$ (2) $\frac{2(\sqrt{11}+1)}{5}$ (3) $\frac{4+2\sqrt{102}}{25}$ (4) $\frac{4+6\sqrt{6}}{25}$

Ans. (4)

Sol. $(4\cos\theta-1)^2 = 9\sin^2\theta$

$25\cos^2\theta - 8\cos\theta - 8 = 0 \Rightarrow \cos\theta = \frac{4 \pm 6\sqrt{6}}{25}$

As $\theta \in \left[0, \frac{\pi}{4}\right] \Rightarrow \cos\theta = \frac{4+6\sqrt{6}}{25}$

4. If $y = y(x)$ is a solution of differential equation $\frac{dy}{dx} + 2y = \sin 2x$, $y(0) = \frac{3}{4}$ then $y\left(\frac{\pi}{8}\right)$ is equal to

(1) $e^{\frac{\pi}{4}}$ (2) $e^{-\frac{\pi}{4}}$ (3) $e^{\frac{\pi}{8}}$ (4) $e^{-\frac{\pi}{8}}$

Ans. (2)

Sol. I.F. $= e^{2x}$

G.S. $y.e^{2x} = \int e^{2x}.\sin 2x dx + C$

$y.e^{2x} = \frac{e^{2x}}{(2^2+2^2)}(2.\sin 2x - 2.\cos 2x) + C$

$x = 0,\ y = \frac{3}{4} \Rightarrow \frac{3}{4} = \left(-\frac{1}{4}\right) + C \Rightarrow C = 1$

$y.e^{2x} = \frac{1}{4}e^{2x}(\sin 2x - \cos 2x) + 1$

$x = \frac{\pi}{8} \Rightarrow y = e^{-\frac{\pi}{4}}$

5. Let A and B are two matrices of order 3 and $|A| = 2$, $|B| = 3$ then value of $\left|AA^T \text{adj}(2A)^{-1} \text{adj}(AB)^{-1} \text{adj}(4B)AA^T\right|$ is

(1) 128 (2) 32 (3) 64 (4) 144

Ans. (3)

Sol. $\left|AA^T \text{adj}(2A)^{-1} \text{adj}(AB)^{-1} \text{adj}(4B)AA^T\right|$

$= |A||A^T||(2A)^{-1}|^2|(AB)^{-1}|^2|4B|^2|A||A^T|$

$= |A|^2|A|^2 \frac{1}{2^6} \frac{1}{|A|^2} \frac{1}{|A|^2} \frac{1}{|B|^2} 4^6 |B|^2$

$= 2^6 = 64$

6. The angle between the vectors $\vec{Q}$ and resultant of $\left(2\vec{Q} + 2\vec{P}\right) + \left(2\vec{Q} - 2\vec{P}\right)$ is

(1) 0° (2) $\tan^{-1} 2$ (3) $\tan^{-1} 5$ (4) $\tan^{-1} 1$

Ans. (1)

Sol. one vector is $\vec{Q}$ and other vector is $4\vec{Q}$, angle between them = 0º

7. If $m = \frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + ... + \frac{1}{\sqrt{99}+\sqrt{100}}$

and $n = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{99.100}$

then point p(m, n) satisfy the equation

(1) $11x - 100y = 0$
(2) $11x + 100y = 0$
(3) $11(x-2) + 100(y-3) = 0$
(4) $11(x-2) - 100(y-3) = 0$

Ans. **(1)**

Sol. $m = (\sqrt{2}-1) + (\sqrt{3}-\sqrt{2}) + (\sqrt{4}-\sqrt{3}) + ... + (\sqrt{100}-\sqrt{99})$

$m = 10 - 1 = 9$

$n = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{99.100}$

$= \left(1-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{4}\right) + ... + \left(\frac{1}{99}-\frac{1}{100}\right)$

$= 1 - \frac{1}{100} = \frac{99}{100}$

8. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{36\sin x}{3\cos x + 5\sin x} dx$ is equal to

(1) $\frac{\pi}{7} - \frac{4}{7}\ell n2$ (2) $\frac{3\pi}{17} - \frac{24}{17}\ell n(2)$ (3) $\frac{20\pi}{17} - \frac{24}{17}\ell n4$ (4) $\frac{13\pi}{7} - \frac{4}{7}\ell n(4)$

Ans. **(3)**

Sol. $36\sin x = A(3\cos x + 5\sin x) + B(-3\sin x + 5\cos x)$

$36 = 5A - 3B$

$0 = 3A + 5B$

$\Rightarrow \quad A = 5 \times \frac{8}{17}, \quad B = -3 \times \frac{8}{17}$

$= \int_{-\pi/4}^{\pi/4} \left(5 \times \frac{8}{17} - 3 \times \frac{8}{17} \frac{-3\sin x + 5\cos x}{3\cos x + 5\sin x}\right) dx$

$= \frac{20\pi}{17} - \frac{24}{17}\ell n\left(\left|\frac{3+5}{3-5}\right|\right)$

9. Number of ways of getting sum 16 on throwing a dice four times is

Ans. **(125)**

Sol. $x_1 + x_2 + x_3 + x_4 = 16 \qquad x_i \in \{1, 2, 3, 4, 5, 6\}$

For I = 1, 2, 3, 4

Coefficient of x^{16} in : $(x^1 + x^2 + x^3 + ... + x^6)^4$

: $x^4(x^0 + x^1 + x^2 + x^3 + ...x^5)^4$

Coefficient x^{12} in : $\left(\frac{1-x^6}{1-x}\right)^4$

: $(1-x^6)^4(1-x)^{-4}$

: $(1-{}^4C_1x^6+{}^4C_2x^{12}......)(1-x)^{-4}$

x^{12} : $(1-x)^{-4}-4x^6(1-x)^{-4}+6x^{12}(1-x)^{-4}$

: ${}^{12+4-1}C_{4-1}-4\times{}^{6+4-1}C_{4-1}+6\,{}^{4-1}C_{4-1}$

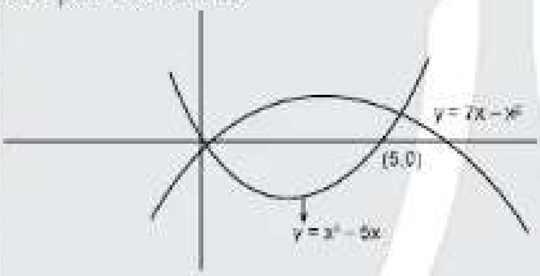
: ${}^{15}C_3-4\,{}^9C_3+6\times{}^3C_3$

: $455-336+6=125$

10. Area bounded by the curves $y=x^2-5x$ and $y=7x-x^2$ is

Ans. **(72)**

Sol. Intersection point of curves:



$x^2-5x=7x-x^2$

$2x^2=12\,x$

$\Rightarrow$ $x=0,6$

Point are (0,0) and (6,6)

Required area = $\int_0^6(12x-2x^2)\,dx$

$= \quad 12\left(\frac{x^2}{2}\right)_0^6-2\left(\frac{x^3}{3}\right)_0^6$

$=6^3-6^3\times2/3=72$

11. The term independent of x in $(1-x+2x^2)\left(3x^2+\frac{1}{x^3}\right)^9$ is $3^7\lambda$ then value of λ is

(1) 24 (2) 28 (3) 56 (4) 9

Ans. **(2)**

Sol. Coefficient x^0 in $(1-x+2x^2)\left(3x^2+\frac{1}{x^3}\right)^9$

$=\left(3x^2+\frac{1}{x^3}\right)^9-x\left(3x^2+\frac{1}{x^3}\right)^9+2x^2\left(3x^2+\frac{1}{x^3}\right)^9$

General term of $\left(3x^2+\frac{1}{x^3}\right)^9$

$={}^9C_r(3x^2)^{9-r}\left(\frac{1}{x^3}\right)^r$

$={}^9C_r\,3^{9-r}\,x^{18-5r}$

for term independent of x ; $18-5r=-2\Rightarrow r=4$

term independent of x is ${}^9C_4\;3^5\times2$

12. If the lines $\frac{x-3}{3}=\frac{2y-1}{4\lambda+1}=\frac{4-z}{1}$ and $\frac{x-3}{3\mu}=\frac{1-2y}{-4}=\frac{z-4}{7}$ are perpendicular then the value of $9\mu + 4\lambda$ is

(1) 4 (2) 5 (3) 3 (4) 6

Ans. (4)

Sol. lines are : $\frac{x-3}{3}=\frac{y-1/2}{2\lambda+1/2}=\frac{z-4}{-1}$

and $\frac{x-3}{3\mu}=\frac{y-1/2}{2}=\frac{z-4}{7}$

$\therefore$ lines are perpendicular

$3\mu(3) + (2\lambda + 1/2)2 + (-1)7 = 0$

$9\mu + 4\lambda + 1 - 7 = 0$

$9\mu + 4\lambda = 6$

13. If $f(x) = x^5 + 2x^3 + 3x + 1$, $g(x)$ is a function such that $f(g(x)) = x$, then the value of $\frac{g(7)}{g'(7)}$ is equal to

Ans. (14.00)

Sol. Put $x = 1 \Rightarrow f(1) = 7 \Rightarrow g(7) = 1$

$g(f(x)) = x \quad \Rightarrow g'(f(x)) = \frac{1}{5x^4+6x^2+3}$

Put $x = 1 \quad \Rightarrow g'(7) = \frac{1}{14}$

$\frac{g(7)}{g'(7)} = 14$

14. Given a circle of radius one such that it touches the line parallel to coordinate axes passing through (3, 2). Find minimum distance of circle from the the point (5, 5)

(1) 2 (2) 4 (3) 3 (4) 5

Ans. (2)

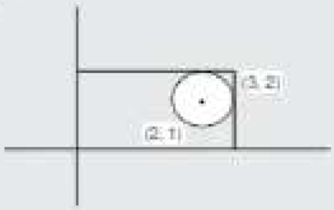
Sol. Centre of circle is (2, 1)

minimum distance of circle

from (5, 5)

$= \sqrt{(5-2)^2+(5-1)^2} - 1$

$= 5 - 1 = 4$



15. The sum of all real roots of the equation $|x|\,|x-2| - |x-1| - 6 = 0$ is

(1) 1 (2) 2 (3) 3 (4) 4

Ans. (4)

Sol. $|x|\,|x-2| - |x-1| = 6$

Case.1 $x \le 0$

$x^2 - 2x + x - 1 = 6$

$x^2 - x - 7 = 0$ sum of roots = 1

Case.2 $0 < x < 1$

$2x - x^2 + x - 1 = 6$

$x^2 - 3x + 7 = 0 \Rightarrow$ roots are imaginary

Case.3 $1 \le x \le 2$

$2x - x^2 - x + 1 - 6 = 0$

$x^2 - x + 5 = 0 \Rightarrow$ roots are imaginary

Case.4 $x > 2$

$x^2 - 2x - x + 1 = 6$

$x^2 - 3x - 5 = 0 \Rightarrow$ sum of roots = 3

sum of all real roots = 1 + 3 = 4

16. If f(t) is a function such that f(1) = 1, $\underset{t\to x}{\text{Lt}} \frac{t^2f(x) - x^2f(t)}{t - x} = 1$, then value of 2f(2) + 3f(3) is equal to

Ans. (24.00)

Sol. $\underset{t\to x}{\text{Lt}} \frac{2tf(x) - x^2f'(t)}{1} = 1$

$2xf(x) - x^2 f'(x) = 1$

Let y = f(x)

$\frac{dy}{dx} - \frac{2}{x}y = \frac{-1}{x^2}$, I.F. $= e^{-2\ell nx} = \frac{1}{x^2}$

$y.\frac{1}{x^2} = \frac{x^{-3}}{3} + C$ x = 1, y = 1

$C = \frac{2}{3}$

$f(x) = \frac{1}{3x} + \frac{2x^2}{3}$

$f(2) = \frac{1}{6} + \frac{8}{3}$ $2f(2) = \frac{1}{3} + \frac{16}{3} = \frac{17}{3}$

$3f(3) = \frac{1}{3} + 2(9) = \frac{1 + 54}{3} = \frac{55}{3}$

$2f(2) + 3f(3) = \frac{72}{3} = 24$

17. Let $F(x) = \sin x - \frac{2}{\pi}(x^2 + x) + 3x$, $x \in \left(0, \frac{\pi}{2}\right)$

Statement-1 : f(x) is an increasing function

Statement-2 : f'(x) is an increasing function

(1) Both statements are correct

(2) Both statements are incorrect

(3) Statement 1 is correct and statement 2 is incorrect

(4) Statement 1 is incorrect and statement 2 is correct

Ans. (3)

Sol. $F(x) = \sin x - \frac{2}{\pi}(x^2 + x) + 3x$

$F'(x) = \cos x - \frac{2}{\pi}(2x + 1) + 3$

$F'(x) = \cos x - \frac{4\pi}{\pi} - \frac{2}{\pi} + 3$

$\Rightarrow F'(x) > 0 \ \forall \ x \in (0, \pi/2) \therefore \frac{4x}{\pi} \in (0, 2)$

$\Rightarrow$ F(x) is an increasing function

Now $F''(x) = -\sin x - \frac{2}{\pi} < 0, \quad \forall x \in (0, \pi/2)$

$\Rightarrow F'(x)$ is decreasing function

$\Rightarrow$ statement 1 is correct but statement 2 is incorrect

18. If the system of equations $11x + y + \lambda z = -5$, $2x + 3y + 5z = 3$, $8x - 19y - 39z = \mu$, has infinite number of solutions then the value of $(\lambda - 1)\,\mu$ is

(1) 141 (2) 93 (3) 39 (4) 131

Ans. (2)

Sol. Multiply by 2 $\quad 22x + 2y + 2\lambda z = -10$

Multiply by 7 $\quad 14x + 21y + 35z = 21$

subtract $\quad 8x - 19y + (2\lambda - 35)z = -31$

It is identical to other equation as far as coefficient of x, y are

$\Rightarrow \quad 2\lambda - 35 = -39, \quad -31 = \mu$

$\lambda = -2, \mu = -31$

$(\lambda - 1)(\mu) = 3 \times 31 = 93$

19. If A(1, –1, 2), B(5, 7, –6), C(3, 4, –10), D(–1, –4, 2) then area of quadrilateral ABCD is

Ans. (54)

Sol. $\overline{AC} = 2\hat{i} + 5\hat{j} - 12\hat{k}$, $\overline{BD} = -6\hat{i} - 11\hat{j} + 8\hat{k}$,

$$\overline{AC} \times \overline{BD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 5 & -12 \\ -6 & -11 & 8 \end{vmatrix} = -92\hat{i} + 56\hat{j} + 8\hat{k}$$

$$= 4\left(-23\hat{i} + 14\hat{j} + 2\hat{k}\right)$$

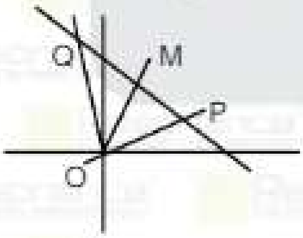
$$\text{Area} = \frac{1}{2}\left|\overline{AC} \times \overline{BD}\right| = 2\sqrt{23^2 + 14^2 + 4} = 54$$

20. Let O be origin. If OP, OQ are two perpendicular lines forming an isosceles triangle with line PQ = $3x + 4y - 12 = 0$ then $OP^2 + OQ^2 + PQ^2$ is

(1) $\frac{1152}{25}$ (2) $\frac{576}{125}$ (3) $\frac{729}{25}$ (4) $\frac{1024}{125}$

Ans. (1)

Sol.



$OM = \frac{12}{5}$, $\quad MP = MQ = \frac{12}{5}$

$PQ = \frac{24}{5} \quad \Rightarrow OP = \frac{24}{5\sqrt{2}} = OQ$

$OP^2 + OQ^2 + PQ^2 = \frac{1152}{25}$