MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. If $\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x} = \frac{1}{12}\tan^{-1}(3\tan x) + c$

then the maximum value of $a\sin x + b\cos x$ is

(1) $\sqrt{10}$ (2) $\sqrt{20}$

(3) $2\sqrt{10}$ (4) $2\sqrt{5}$

Answer (3)

Sol. $I = \int \frac{\sec^2 x dx}{b^2 + a^2 \tan^2 x}$

$\tan x = t \Rightarrow \sec^2 x dx = dt$

$I = \int \frac{dt}{b^2 + a^2 t^2} = \frac{1}{ba}\tan^{-1}\left(\frac{at}{b}\right)$

$\Rightarrow I = \frac{1}{ab}\tan^{-1}\left(\frac{a}{b}\tan x\right) + c$

$\Rightarrow ab = 12 \Rightarrow a^2 = 36$

$\frac{a}{b} = 3 \Rightarrow b^2 = 4$

$\Rightarrow$ Maximum value of $a\sin x + b\cos x$ is

$\sqrt{a^2 + b^2} = \sqrt{40} = 2\sqrt{10}$

2. Range of $\frac{1}{7 - \sin 5x}$ equals to

(1) $\left[\frac{1}{7}, \frac{1}{5}\right]$ (2) $\left[\frac{1}{7}, \frac{1}{6}\right]$

(3) $\left[\frac{1}{8}, \frac{1}{5}\right]$ (4) $\left[\frac{1}{8}, \frac{1}{6}\right]$

Answer (4)

Sol. We know that,

$-1 \le \sin 5x \le 1$

$-1 \le -\sin 5x \le 1$

$6 \le 7 - \sin 5x \le 8$

$\frac{1}{8} \le \frac{1}{7 - \sin 5x} \le \frac{1}{6}$

$\therefore$ Range is $\left[\frac{1}{8}, \frac{1}{6}\right]$

3. There are letters to be delivered to 5 different location, then find the probability that letter is delivered to exactly 2 correct address assuming each letter is delivered to unique address.

(1) $\frac{1}{6}$ (2) $\frac{1}{5}$

(3) $\frac{1}{12}$ (4) $\frac{1}{4}$

Answer (3)

Sol. $\Rightarrow$ Select any two correct address

$\Rightarrow \left({}^5C_2\right)$

Remaining 3 have to be dearranged

$\Rightarrow 3!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right)$

$= 6\left(1 - 1 + \frac{1}{2} - \frac{1}{6}\right) = 3 - 1 = 2$ ways

$\Rightarrow {}^5C_2 \cdot 2 = 10$

Probability $= \frac{10}{5!} = \frac{1}{12}$

4. The 315th word in dictionary arranged in order for the word 'NAGPUR' is

(1) NRAGPU (2) NRPGUA

(3) NPRGUA (4) NRAPGU

Answer (4)

Sol. Letters N,A,G,P,U,R.

Total words start with letter A

5! = 120

Words start with G

5! = 120

Words with N at first place and A at 2nd place

4! = 24

Words with N at first and G at 2nd place

4! = 24

Words with N at first and P at 2nd place

4! = 24

So total words 120 + 120 + (24)3

= 312

313th word = NRAGPU

314th word = NRAGUP

315th word = NRAPGU

So, 315th word = NRAPGU

5. Let A = [1, 2, 3, 4, 5], m be the number of relation such as $4x \le 5y$ XRY and n be the minimum number of elements to be added from $A \times A$ to make symmetric relation. Then the value of $n + m$.

(1) 26 (2) 25

(3) 24 (4) 23

Answer (2)

Sol. A = [1, 2, 3, 4, 5]

XRY when $4x \le 5y$

So R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 2), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 4), (4, 5), (5, 4), (5, 5)}

m = 16

As (1, 2) $\in R$ then (2, 1) is to be added

(1, 3) $\in R$ So (3, 1) will be added

(1, 4) $\in R$ So (4, 1) $\in R$

(1, 5) $\in R$ So (5, 1) $\in R$

(2, 3) $\in R$ So (3, 2) $\in R$

(2, 4) $\in R$ So (4, 2) $\in R$

(2, 5) $\in R$ So (5, 2) $\in R$

(3, 4) $\in R$ So (4, 3) $\in R$

(3, 5) $\in R$ So (5, 3) $\in R$

to make R symmetric

So n = 9

$m + n$ = 25

6. If the area bounded by the region (x, y) such that $\left\{(x, y)\,\middle|\,\frac{a}{x^2} < y < \frac{1}{x}\right.$ such that $1 < x < 2$, $0 < a < 1$} is $\left(\ln 2 - \frac{2}{7}\right)$ sq. units then $(7a - 3)$ is equal to

(1) 0 (2) 1

(3) 2 (4) 4

Answer (2)

Sol. $\Rightarrow \int_1^2 \left(\frac{1}{x} - \frac{a}{x^2}\right) dx = \left(\ln|x| + \frac{a}{x}\right)_1^2$

$$\left(\ln 2 + \frac{a}{2}\right) - (\ln 1 + a) = \ln 2 - \frac{a}{2}$$

$$= \ln 2 - \frac{2}{7} = \ln 2 - \frac{a}{2}$$

$\Rightarrow a = \frac{4}{7}$

$\Rightarrow 7a - 3 = 1$

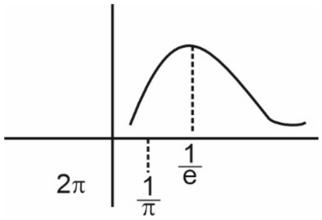
7. If the function $f(x) = \left(\frac{1}{x}\right)^{2x}$ $x > 0$, attains the maximum value of $x = \frac{1}{e}$, then

(1) $e^{\pi} < \pi^{e}$ (2) $e^{2\pi} < (2\pi)^{e}$

(3) $(2e)^{\pi} > (\pi)^{2e}$ (4) $e^{\pi} > \pi^{e}$

Answer (4)

Sol. $f\left(\frac{1}{\pi}\right) < f\left(\frac{1}{e}\right)$



$(\pi)^{\frac{2}{\pi}} < e^{\frac{2}{e}} \Rightarrow \pi^{2e} < e^{2\pi}$

$e^{\pi} > \pi^{e}$

8. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \left(\left(\vec{a} \times \left(\hat{i} + \hat{j}\right) \times \hat{i}\right) \times \hat{i}\right)$ then the square of projection of $\vec{a}$ on $\vec{b}$ is

(1) $\frac{1}{3}$ (2) 2

(3) $\frac{1}{2}$ (4) $\frac{2}{5}$

Answer (4)

Sol. $\vec{a} \times \left(\hat{i} + \hat{j}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$

$= -\hat{i} + \hat{j} + 3\hat{k}$

$\left(\left(\vec{a} \times \left(\hat{i} + \hat{j}\right)\right) \times \hat{i}\right) = -\hat{k} + 3\hat{j}$

$\left(\left(\left(\vec{a} \times \left(\hat{i} + \hat{j}\right)\right) \times \hat{i}\right) \times \hat{i}\right) = -\hat{j} - 3\hat{k}\left(\vec{b}\right)$

$\therefore$ Projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a} \cdot \vec{b}}{\left|\vec{b}\right|}$

$= \frac{-2}{\sqrt{10}}$

Square of projection = $\frac{4}{10} = \frac{2}{5}$

9. $\lim\limits_{n \to \infty} \frac{\sum(n^4 - 2n^3 + n^2)}{\sum((3n)^4 + n^3 - n^2)}$ is equal to

(1) $\frac{1}{81}$ (2) $\frac{1}{72}$

(3) $\frac{1}{57}$ (4) $\frac{1}{93}$

Answer (1)

Sol. $\lim\limits_{n \to \infty} \frac{\sum(n^4 - 2n^3 + n^2)}{\sum((3n)^4 + n^3 - n^2)} = \lim\limits_{n \to \infty} \frac{\sum n^4}{\sum (3n)^4}$

(As Σn^2 will dominate and has highest powers of n)

$= \lim\limits_{n \to \infty} \frac{1}{34} \frac{\sum n^4}{\sum n^4}$

$= \frac{1}{81}$

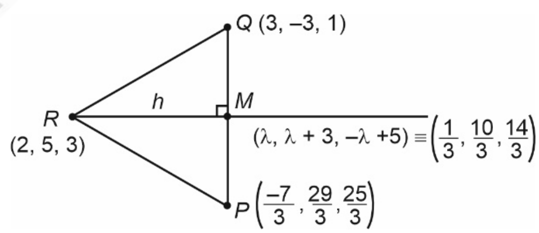
10. If (α, β, γ) is the mirror image of Q (3, –3, 1) in the line $\frac{x-0}{1} = \frac{y-3}{1} = \frac{z-5}{-1}$ and R (2, 5, 3). If the area of ΔPQR is λ, then $\frac{\lambda^2}{546}$ equals to

(1) $\frac{125}{81}$ (2) $\frac{25}{81}$

(3) $\frac{1}{81}$ (4) $\frac{5}{81}$

Answer (1)

Sol.



$(\lambda - 3) + 1(\lambda + 6) - 1(-\lambda + 4) = 0$

$3\lambda - 1 = 0 \Rightarrow \lambda = \frac{1}{3}$

Area of $\Delta PQR = 2\times\frac{1}{2}\ (QM\cdot MR)$

$$(QM)(MR)=\left(\sqrt{\left(3-\frac{1}{3}\right)^2+\left(-3\frac{-10}{3}\right)^2+\left(1-\frac{14}{3}\right)^2}\right)$$

$$\left(\sqrt{\left(2-\frac{1}{3}\right)^2+\left(5-\frac{10}{3}\right)^2+\left(3-\frac{14}{3}\right)^2}\right)$$

$$(QM)(MR)=\frac{1}{9}\left(\sqrt{64+19^2+11^2}\right)\left(\sqrt{25+25+25}\right)$$

$$=\frac{\sqrt{546\times125}}{9}$$

$$=\frac{5}{9}\sqrt{546\times5}$$

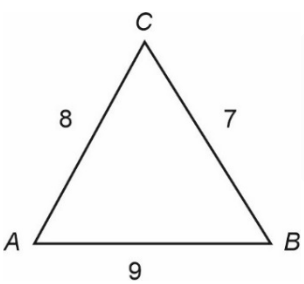
11. Sides of a triangle are $AB = 9$, $BC = 7$, $AC = 8$. Then $\cos 3C$ equals to

(1) $\frac{-262}{343}$ (2) $\frac{181}{247}$

(3) $\frac{81}{93}$ (4) $\frac{-283}{285}$

Answer (1)

Sol.



$$\cos C=\frac{8^2+7^2-9^2}{2\times8\times7}=\frac{32}{2\times8\times7}=\frac{2}{7}$$

$\cos 3C = 4\cos^3 C - 3\cos C$

$$=4\times\frac{8}{343}-\frac{6}{7}=\frac{32-6\times49}{343}$$

$$=\frac{-262}{343}$$

12. The locus of P such that the ratio of distance P from $A(3, 1)$ and $B(1, 2)$ is 5 : 4 is

(1) $81x^2 - 92x + 81y^2 - 180y = 35$

(2) $81x^2 + 92x + 81y^2 - 19y = 35$

(3) $81x^2 - 48x + 81y^2 + 20y = 35$

(4) $81x^2 - 90x + 81y^2 - 180y = 35$

Answer (4)

Sol. Take point $P(x, y)$

5 P 4
(3, 1) (1, 2)

$$x=\frac{5+12}{9},\ y=\frac{10+4}{9}$$

$$P=\left(\frac{17}{9},\frac{14}{9}\right)\ \text{(internally)}$$

for externally division.

$$x=-\frac{7}{9},\ y=\frac{6}{9}$$

$$P'=\left(\frac{-7}{9},\ \frac{6}{9}\right)$$

Locus of P is the circle whose diameter is PP'

$$\left(x-\frac{-17}{9}\right)\left(x+\frac{7}{9}\right)+\left(y-\frac{14}{9}\right)\left(y-\frac{6}{9}\right)=0$$

$(9x - 17)(9x + 7) + (9y - 14)(9y - 6) = 0$

So $81x^2 - 90x + 81y^2 - 180y = 35$

13. If $\left|\frac{z_1-2z_2}{\frac{1}{2}-\bar{z}_1z_2}\right|=2$ then

(1) z_1 lie on circle with radius 1 and z_2 lie on circle with radius 2

(2) z_1 lie on circle with radius 1 and z_2 lie on circle with radius 1

(3) z_1 lie on circle with radius $\frac{1}{2}$ and z_2 lie on circle with radius 1

(4) z_1 lie on circle with radius 1 and z_2 lie on circle with radius $\frac{1}{2}$

Answer (4)

Sol. $|z_1 - 2z_2| = |1 - 2\bar{z}_1 z_2|$

$\Rightarrow \ (z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2) = (1 - 2\bar{z}_1 z_2)(1 - 2z_1\bar{z}_2)$

$\Rightarrow \ |z_1|^2 + 4|z_2|^2 - 2z_1\bar{z}_2 - 2\bar{z}_1 z_2$

$= 1 - 2z_1\bar{z}_2 - 2\bar{z}_1 z_2 + 4|z_1|^2|z_2|^2$

$\Rightarrow \ |z_1|^2 + 4|z_2|^2 - 4|z_1|^2|z_2|^2 - 1 = 0$

$\left(|z_1|^2 - 1\right)\left(4|z_2|^2 - 1\right) = 0$

$\Rightarrow \ |z_1| = 1$ and $|z_2| = \frac{1}{2}$

14. If the orthocentre of triangle formed by (8, 3), (5, 1) and (*h*, *k*) is (6, 1), then (*h*, *k*) lie on

(1) $x^2 + y^2 = 64$

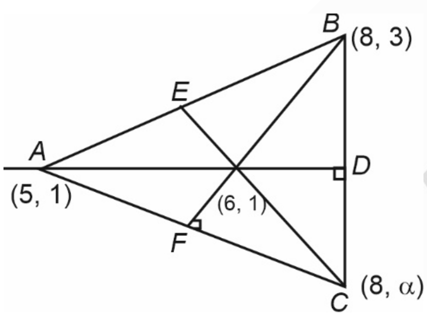
(2) $x^2 + y^2 = 68$

(3) $x^2 + y^2 = 65$

(4) $x^2 + y^2 = 71$

Answer (2)

Sol.

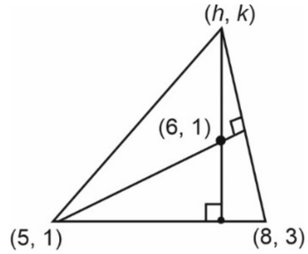


Slope of BF = 1

$\Rightarrow$ Slope of $AC \equiv \left(\frac{\alpha - 1}{8 - 5}\right) = -1$

$\Rightarrow \alpha - 1 = -3$

$\Rightarrow \alpha = -2$



(*h*, *k*) lie on $(y - 1) = \frac{-3}{2}(x - 6)$

$2y - 2 + 3x - 18 = 0$

$2y + 3x = 20$ ...(1)

(*h*, *k*) lies on circumcircle eg. of circumcircle is $x^2 + y^2 = 68$

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If α, β are the roots of the equation $x^2 - \sqrt{2}x - 8 = 0$ and $A_n = \alpha^n + \beta^n$, $n \in N$, then the value of $\frac{A_{10} - \sqrt{2}A_9}{2A_8}$

Answer (4)

Sol. $x^2 - \sqrt{2}x - 8 = 0$

$A_{10} - \sqrt{2} \cdot A_9 - 8A_8 = 0$

$\Rightarrow \ \frac{A_{10} - \sqrt{2} \cdot A_9}{A_8} = 8$

$\Rightarrow \ \frac{A_{10} - \sqrt{2} \cdot A_9}{2 \cdot A_8} = 4$

22. If ${}^{n+1}C_{r+1} : {}^{n}C_{r} : {}^{n-1}C_{r-1} = 55 : 35 : 21$

The value of $n + r$ is

Answer (16)

Sol. $\frac{n+1}{r+1} \times {}^{n}C_r : {}^{n}C_r : \frac{r}{n}\, {}^{n}C_r = 55 : 35 : 21$

$\Rightarrow \frac{n+1}{r+1} = \frac{55}{35}$ and $\frac{n}{r} = \frac{35}{21}$

$\Rightarrow \frac{n+1}{r+1} = \frac{11}{7}$ and $\frac{n}{r} = \frac{5}{3}$

$\Rightarrow$ $7n + 7 = 11r + 11$

$7n - 11r = 4$ … (1)

$3n - 5r = 0$ … (2)

Solving (1) and (2)

$r = 6$ and $n = 7$

$\Rightarrow$ $n + r = 10 + 6 = 16$

23. If the order of matrix A is 3 and $|A| = 3$ then the value of det($adj(-4adj\,(-3adj\,(2A^{-1})))$ is $2^m.\,3^n$. The value of $m + 2n$ =

Answer (44)

Sol. $|adj(-4adj(-3adj(2A^{-1})))|$

$= |-4adj(-3adj(2A^{-1}))|^2$

$= 4^6\,|-3adj(2A^{-1})|^4$

$= 4^6.3^{12}\,|adj(2A^{-1})|^4$

$= 4^6.3^{12}\,|2A^{-1}|^8$

$= 4^6.3^{12}.2^{24}\,|A^{-1}|^8$

$= 4^6.3^{12}.2^{24}.\,\frac{1}{|A|^8} = 3^{12}.\frac{2^{36}}{3^8}$

$= 3^4.\,2^{36}$

$m = 36$ $n = 4 \Rightarrow m + 2n = 36 + 8 = 44$

24. If $\int_0^3 \left([x^2] + \left[\frac{x^2}{2} \right] \right) dx$

$= a + b\sqrt{2} + c\sqrt{6} - \sqrt{3} - \sqrt{5} - \sqrt{7}$ (a, b, $c \in I$) then ($a + b + c$) equals

Answer (23.00)

Sol. $\int_0^3 \left([x^2] + \left[\frac{x^2}{2} \right] \right) dx = \int_0^1 0dx + \int_1^{\sqrt{2}} 1dx + \int_{\sqrt{2}}^{\sqrt{3}} 3dx +$

$\int_{\sqrt{3}}^{2} 4dx + \int_{2}^{\sqrt{5}} 6dx + \int_{\sqrt{5}}^{\sqrt{6}} 7dx + \int_{\sqrt{6}}^{\sqrt{7}} 9dx + \int_{\sqrt{7}}^{\sqrt{8}} 10dx + \int_{\sqrt{8}}^{3} 12dx$

$= 31 - 6\sqrt{2} - \sqrt{3} - \sqrt{5} - \sqrt{7} - 2\sqrt{6}$

$\Rightarrow$ $a = 31$, $b = -6$, $c = -2$

$\Rightarrow$ $a + b + c = 23$

25.

26.

27.

28.

29.

30.

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