MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. For $8^{2x} – 168^{x} + 48 = 0$, the sum of values of x is equal to

(1) $1+\log_6 8$ (2) $1+\log_8 6$

(3) $\log_8 6$ (4) 16

Answer (2)

Sol. Let $8^x = t$

$t^2 - 16t + 48 = 0$

$\Rightarrow (t - 12)(t - 4) = 0$

$t = 12$ or $t = 4$

$\Rightarrow 8^x = 12, 4$

$\Rightarrow x_1 = \log_8 12$ and $x_2 = \log_8 4$

$x_1 + x_2 = \log_8 12 + \log_8 4$

$= \log_8 48$

$= 1 + \log_8 6$

2. If $I(x) = \int \frac{6dx}{\sin^2 x(1+\cot x)^2}$ and $I(0) = 3$ then $I\left(\frac{\pi}{12}\right)$ is equal to

(1) $\frac{21-9\sqrt{3}}{3-\sqrt{3}}$ (2) $\frac{21+9\sqrt{3}}{3-\sqrt{3}}$

(3) $\frac{21}{3-\sqrt{3}}$ (4) $\frac{3+\sqrt{3}}{3-\sqrt{3}}$

Answer (1)

Sol. $I(x) = \int \frac{6dx}{\sin^2 x(1+\cot x)^2}$

$$I(x) = \int \frac{6dx}{(\sin x + \cos x)^2}$$

$$I(x) = \int \frac{6dx}{\sin^2 x + \cos^2 x + 2\sin x \cos x}$$

$$I(x) = \int \frac{6\sec^2 x dx}{\tan^2 x + 1 + 2\tan x}$$

$$I(x) = \int \frac{6\sec^2 x\, dx}{(1+\tan x)^2}, \quad t = \tan x$$

$\Rightarrow dt = \sec^2 x dx$

$$= \int \frac{6dt}{(1+t)^2} = \frac{-6}{(1+t)} + c$$

$$\Rightarrow I(x) = \frac{-6}{1+\tan x} + c$$

$$I(0) = \frac{-6}{1+0} + c = 3 \Rightarrow c = 9$$

$$I(x) = 9 - \frac{6}{1+\tan x} = 9 - \frac{6}{1+\tan\left(\frac{\pi}{12}\right)}$$

$$I\left(\frac{\pi}{12}\right) = 9 - \frac{6}{1+(2-\sqrt{3})} = \frac{9(3-\sqrt{3})-6}{(3-\sqrt{3})}$$

$$= \left(\frac{21-9\sqrt{3}}{3-\sqrt{3}}\right)$$

3. Let $f(x) = \cos x - x + 1$, $x \in [0, \pi]$, then

(1) $f(x)$ is increasing in $(0, \pi)$

(2) $f(x)$ is decreasing in $(0, \pi)$

(3) $f(x)$ is increasing in $(0, \pi/2)$ and decreasing in $(\pi/2, \pi)$

(4) $f(x)$ is decreasing in $(0, \pi/2)$ and increasing in $(\pi/2, \pi)$

Answer (2)

Sol. $f'(x) = -\sin x - 1 < 0 \;\forall\; x \in (0, \pi)$

$\therefore f(x)$ is decreasing $\forall\; x \in (0, \pi)$

4. If $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$, if sum of diagonal elements of A^{13} is 3^n then n is equal to

(1) 5 (2) 7

(3) 9 (4) 13

Answer (2)

Sol. Trace = Sum of eigen values

tr (A^{13}) = Sum of eigen values

$$\Rightarrow \begin{bmatrix} 2-\lambda & -1 \\ 1 & 1-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)(1-\lambda) + 1 = 0 \Rightarrow \lambda^2 - 3\lambda + 3 = 0 \begin{matrix} \nearrow \lambda_1 \\ \searrow \lambda_2 \end{matrix}$$

$\Rightarrow \lambda_1 + \lambda_2 = 3,$ $\lambda_1\lambda_2 = 3$

To get $(A^{13})'$ trace $\Rightarrow \lambda_1^{13} + \lambda_2^{13}$

$$\lambda_1^2 + \lambda_2^2 = (\lambda_1 + \lambda_2)^2 - 2\lambda_1\lambda_2$$

$$= 9 - 6 = 3$$

$$\lambda_1^3 + \lambda_2^3 = (\lambda_1 + \lambda_2)\left(\lambda_1^2 + \lambda_2^2 - \lambda_1\lambda_2\right) = (3)(3-3) = 0$$

$$\lambda_1^3 = -\lambda_2^3 \Rightarrow \lambda_1^3\lambda_2^3 = 27 \Rightarrow -\lambda_1^6 = 27$$

$$\Rightarrow \lambda_1^6 = \lambda_2^6 \Rightarrow \lambda_1^{12} = \lambda_2^{12} \Rightarrow \lambda_1^{12} = 27^2$$

$$\lambda_1\lambda_1^{12} + \lambda_2\lambda_2^{12} = \lambda_1^{12}(\lambda_1 + \lambda_2)$$

$$= (27)^2 \cdot 3 = 3^6 \cdot 3^1 = 3^7$$

5. 3 blue balls and 4 yellow balls are in a box. 3 balls are drawn on random. Let variance and mean be x and y respectively then value of $3x + 4y$ is

(1) 5.21 (2) 8.39

(3) 7.34 (4) 6.54

Answer (3)

Sol. Let z denote the number of blue balls in sample of 3 balls drawn from a box containing 3 blue balls and 4 yellow balls.

So z can be 0, 1, 2, 3

$P(z = 0) = P$(no blue ball)

$$= \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} = \frac{64}{343}$$

$$P(z=1) = 3\left(\frac{4}{7} \times \frac{4}{7} \times \frac{3}{7}\right) = \frac{144}{343}$$

$$P(z=2) = 3\left(\frac{4}{7} \times \frac{3}{7} \times \frac{3}{7}\right) = \frac{108}{343}$$

$$P(z=3) = \frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} = \frac{27}{243}$$

z	0	1	2	3
$P(z)$	$\frac{64}{343}$	$\frac{144}{343}$	$\frac{108}{343}$	$\frac{27}{343}$

Mean = $\Sigma z \cdot P(z) = 0 \cdot \frac{64}{343} + \frac{144}{343} + 2 \cdot \frac{108}{343} + 3 \cdot \frac{27}{343}$

$$y = \frac{441}{343}$$

Variance = $\sigma_x^2 = \sum x^2 \cdot [P(x)] - (\text{mean})^2$

$$x = \left[\frac{144}{343} + 4 \cdot \frac{108}{343} + 9 \cdot \frac{27}{343}\right] - \left(\frac{441}{343}\right)^2$$

$$3x + 4y = 3\left[\frac{819}{343} - \left(\frac{441}{343}\right)^2\right] + 4 \cdot \frac{441}{343}$$

$$= \frac{2457}{343} - 4.96 + 5.14$$

= 7.16 + 0.18

= 7.34

6. If $C_1 = (x-\alpha)^2 + (y-\beta)^2 = r_1^2$,

$C_2 : (x-6)^2 + \left(y - \frac{15}{2}\right)^2 = r_2^2$ touches each other at (6, 6). If line joining centres of C_1 and C_2 is divided by (6, 6) in 2 : 1 internally, then $(\alpha + \beta) + 4\left(r_1^2 + r_2^2\right)$ is equal to

(1) 54

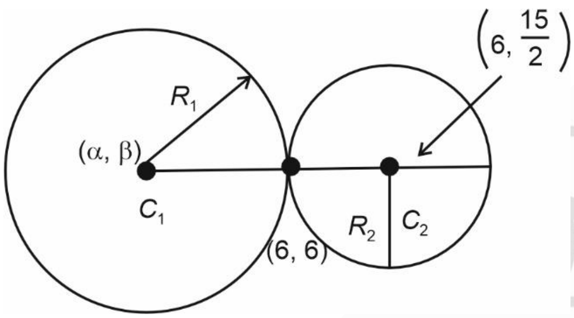
(2) 36

(3) 18

(4) 27

Answer (1)

Sol.



$$\frac{\alpha + 2.6}{3} = 6, \quad \frac{\beta + \frac{2.15}{2}}{3} = 6$$

$\Rightarrow$ $\alpha = 6, \beta = 3$

$\Rightarrow$ Also, $\frac{R_1}{R_2} = \frac{2}{1}$

$$R_2 = \sqrt{(6-6)^2 + \left(6 - \frac{15}{2}\right)^2} = \frac{3}{2}$$

$\Rightarrow R_1 = 2R_2 = 3$

$\Rightarrow \alpha + \beta + 4\left(r_1^2 + r_2^2\right) = 6 + 3 + 4\left(3^2 + \frac{9}{4}\right)$

= 54

7. $R = (a, b) : a + 5b = 42$ and $a, b \in IN$ has m elements and $\sum_{n=1}^{m}(1 + i^{n!}) = x + iy$. (Where $i = \sqrt{-1}$) find $x + y + m$

(1) 20 (2) 12

(3) 8 (4) 13

Answer (1)

Sol. $R = (a, b) : a + 5b = 42$

Then R = {(2, 8), (7, 7), (12, 6), (17, 5), (22, 4), (27, 3), (32, 2), (37, 1)}

$m = 8$

and $\sum_{n=1}^{m}(1 + i^{n!}) = x + iy$

$\therefore \sum_{n=1}^{8}(1 + i^{n!}) = 8 + (i + i^2 + i^6 + 1 + 1 + 1 + 1 + 1)$

$= 11 + i$

$\therefore$ $x = 11, y = 1$

$\therefore$ $x + y + m = 20$

8. If $y = \int \frac{e^{\tan x}}{\cos^2 x(1 + e^{2\tan x})}dx$ and $y(0) = 6$, then $y\left(\frac{\pi}{4}\right)$ is equal to

(1) $\tan^{-1}(e) - \frac{\pi}{4}$ (2) $\tan^{-1}(e) + 6 - \frac{\pi}{4}$

(3) $\tan^{-1}(e) - 6 + \frac{\pi}{4}$ (4) $\tan^{-1}\left(\frac{1}{e}\right) + \frac{\pi}{4} - 6$

Answer (2)

Sol. Put $e^{\tan x} = t$

$\Rightarrow e^{\tan x}.\sec^2 x dx = dt$

$y = \int \frac{dt}{1 + t^2} \Rightarrow \tan^{-1}(e^{\tan x}) + c = y$

$\therefore y = \tan^{-1}(e^{\tan x}) + c$

$y(0) = 6$

$$6 = \tan^{-1}(1) + c \quad \Rightarrow c = 6 - \frac{\pi}{4}$$

$$\therefore \; y = \tan^{-1}\left(e^{\tan x}\right) + 6 - \frac{\pi}{4}$$

$$y\left(\frac{\pi}{4}\right) = \tan^{-1}(e) + 6 - \frac{\pi}{4}$$

9. The area bounded by $y = \min\{\sin x, \cos x\}$ and x-axis in $-\pi \le x \le \pi$ interval is equal to (in sq. units)

(1) 4 (2) 8

(3) $2-\sqrt{2}$ (4) $4-2\sqrt{2}$

Answer (1)

Sol. $f(x) = \min\{\sin x, \cos x\}$

$$\Rightarrow f(x) = \begin{cases} \cos x, & x \in \left(-\pi, -\frac{3\pi}{4}\right) \\ \sin x, & x \in \left(-\frac{3\pi}{4}, \frac{\pi}{4}\right) \\ \cos x, & x \in \left(\frac{\pi}{4}, \pi\right) \end{cases}$$

Area bounded by $f(x)$ and x-axis

$$= \int_{-\pi}^{\pi} |f(x)| dx = \int_{-\pi}^{-\frac{3\pi}{4}} |\cos x|\, dx + \int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} |\sin x\, dx| + \int_{\frac{\pi}{4}}^{\pi} |\cos x)\, dx$$

$$= \frac{1}{\sqrt{2}} + 2 + 2 - \frac{1}{\sqrt{2}} = 4 \text{ sq. unit}$$

10. Let $f(\theta) = \dfrac{\sin^4\theta + 3\cos^2\theta}{\sin^4\theta + \cos^2\theta}$, then range of $f(\theta) \in [a, b]$. The sum of infinite G.P., where first term is 64 and common ratio is $\dfrac{a}{b}$ is equal to

(1) 32 (2) 64

(3) 96 (4) 108

Answer (3)

Sol. $$f(\theta) = 1 + \frac{2\cos^2\theta}{\sin^4\theta + \cos^2\theta}$$

$$= 1 + \frac{2\cos^2\theta}{\left(1-\cos^2\theta\right)^2 + \cos^2\theta}$$

$$= 1 + \frac{2\cos^2\theta}{1+\cos^4\theta - 2\cos^2\theta + \cos^2\theta}$$

$$= 1 + \frac{2\cos^2\theta}{1+\cos^4\theta - \cos^2\theta}$$

$$= 1 + \frac{2}{\cos^2\theta + \frac{1}{\cos^2\theta} - 1}$$

$$f(\theta)_{\max} = 1 + \frac{2}{2-1} = 3$$

$f(\theta)_{\min} = 1 + 0 = 1$

$\therefore \; a = 1, b = 3$

$$\therefore \; r = \frac{1}{3}, a = 64$$

$$\therefore \; S_\infty = \frac{64}{1-\frac{1}{3}} = \frac{64\times 3}{2} = 96$$

11.

1
2 3
4 5 6
7 8 9 10

If this pattern continue then which row number, the number 5437 lies

(1) 103 (2) 104

(3) 102 (4) 105

Answer (2)

Sol. Number of term :

{1}, {2}, {3} ….

To find term number 5437 will lie

Let r^{th} term have it.

$$1+2+...+r=\frac{r(r+1)}{2}$$

$$\Rightarrow \frac{(r-1)r}{2} \le 5437 \le \frac{r(r+1)}{2}$$

$\Rightarrow$ $r(r + 1) \ge 2.5437$

If $r = 104$, $104 \times 105 > 2.5437$

12. If $A=\begin{bmatrix} 2 & a & 1 \\ 1 & 3 & 1 \\ 0 & 5 & b \end{bmatrix}$ and $A^3 = 4A^2 - A - 21I$ then $(2a + 3b)$ is equal to

(1) 33 (2) 23

(3) 13 (4) 7

Answer (3)

Sol. $A=\begin{bmatrix} 2 & a & 1 \\ 1 & 3 & 1 \\ 0 & 5 & b \end{bmatrix}$

A satisfy characteristic equation

$$\Rightarrow \begin{vmatrix} 2-\lambda & a & 1 \\ 1 & 3-\lambda & 1 \\ 0 & 5 & b-\lambda \end{vmatrix}=0$$

$$(2-\lambda)\left[\lambda^2-(b+3)\lambda+3b-5\right)-(a-3+\lambda)=0$$

$$-\lambda^3+(b+3+2)\lambda^2+\lambda(-2b-6-3b+4)$$

$$+ 6b - 10 - a + 3 = 0$$

$$\lambda^3-(b+5)\lambda^2+\lambda(5b+2)+(a+7-6b)=0$$

$$A^3-(b+5)A^2+\lambda(5b+2)+a+7-6b=0$$

$\Rightarrow$ $b + 5 = 4$

$5b + 2 = 1$ $\Rightarrow$ $b = -1$

$a + 7 - 6b = 21$ $\Rightarrow a = 8$

$2a + 3b = 16 - 3 = 13$

13. If sum of two positive numbers is 24 then the probability of product of numbers is not less than $\frac{3}{4}$ times the maximum possible product of a and b then probability of such event is $\frac{m}{n}$ (m, n are O prime) then $n - m$ is

(1) 1 (2) 3

(3) 5 (4) 7

Answer (1)

Sol. Take two numbers as a and b

$a + b = 24$

$a = 24 - b$

Now product of these numbers.

$f(b) = b \times (24 - b)$

$= 24b - b^2$

$f'(b) = 24 - 2b$

$\Rightarrow$ $f'(b) = 0$

$\Rightarrow$ $b = 12$

$\Rightarrow$ $f''(12) = -2 < 0$

So, at $b = 12$ product is maximum

$\Rightarrow$ $a = b = 12$

Maximum possible product = 144

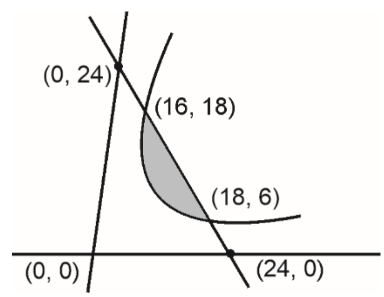
Now $ab \ge \frac{3}{4}\cdot 144 = 36.3 = 108$

So, probability

$=\frac{12\sqrt{2}}{24\sqrt{2}}$

$=\frac{1}{2}=\frac{m}{n}$

$\Rightarrow n - m = 1$



14. Let $\sin\pi = \frac{-3}{5}$, $\pi < x < \frac{3\pi}{2}$, then 80($\tan^2 x$ – $\cos x$) is equal to

(1) 109 (2) 108

(3) 9 (4) 8

Answer (1)

Sol. $\tan x = \frac{3}{4}$, $\cos x = \frac{-4}{5}$

$$\therefore\ 80\left(\frac{9}{16}+\frac{4}{5}\right) = 80\left(\frac{45+64}{80}\right)$$

= 109

15. If |z + 2| = 1, Im $\left(\frac{z+1}{z+2}\right) = \frac{1}{5}$ then, $Re(z) < -2$ is equal to

(1) $\frac{24}{25}$ (2) $\frac{2}{5}$

(3) $\frac{12}{5}$ (4) $\frac{3}{5}$

Answer (1)

Sol. $z = x + iy$, $x, y \in \mathbb{R}$

$$\text{Im}\left[\frac{(x+1)+iy}{(x+2)+iy}\right] = \frac{1}{5}$$

$$\Rightarrow \text{Im}\left(\frac{((x+2)-iy)((x+1)+iy)}{(x+2)^2+y^2}\right) = \frac{1}{5}$$

$$= \frac{(x+2)y - y(x+1)}{(x+2)^2+y^2} = \frac{1}{5}$$

$5y = (x + 2)^2 + y^2$

$\Rightarrow (x + 2)^2 + y^2 - 5y = 0$

$(x + 2)^2 + y^2 - 1 = 0$ $\Rightarrow y = \frac{1}{5} = 0.2$

$$\Rightarrow (x+2)^2 = \frac{24}{25}$$

16. The value of

$$\lim_{x\to 0} \frac{1-\sqrt{\cos x}\cdot\sqrt[2]{\cos 2x}\cdot\sqrt[3]{\cos 3x}+\ldots\sqrt[n]{\cos nx}}{x^2} \text{ is}$$

equal to

(1) $\frac{n^2+n+1}{4}$ (2) $\frac{n^2-n-1}{4}$

(3) $\frac{n^2+n-1}{4}$ (4) $\frac{n^2-n+1}{4}$

Answer (3)

Sol. The given form is $\left(\frac{0}{0}\right)$ form. Applying L' Hospital

$$L = \frac{\frac{1}{2}\sqrt{\cos x}(-\sin x)\left(\sqrt[2]{\cos 2x}\cdot\sqrt[3]{\cos 3x}\ldots\right)}{2x}$$

$$\frac{-\frac{1}{2}\sqrt{\cos 2x}(-\sin 2x)\times 2\left(\sqrt{\cos x}\cdot\sqrt[3]{\cos 3x}\ldots\right)}{2x}$$

$$-\frac{1}{3}(\cos 3x)^{2/3}(-\sin 3x)\times 3\left(\sqrt{\cos x}\ldots\right)-\ldots$$

$$\ldots-\frac{1}{n}(-\sin nx)(n)(\cos nx)^{1/n-1}$$

$$= \frac{1}{2}\left(\frac{1}{2}+2+3+\ldots n\right) = \frac{1}{2}\left[\frac{(n)(n+1)}{2}-\frac{1}{2}\right] = \frac{1}{4}\left(n^2+n-1\right)$$

17. If $I(n) = \int_0^1 (1-x^k)^n dx, k \in N$, and $147I(20)$ = $148I(21)$, then k is equal to

(1) 17

(2) 21

(3) 7

(4) 15

Answer (3)

Sol. $I(n) = \int_0^1 (1-x^k)^n dx$

$$I(21)=\int_0^1 (1-x^k)^{21}dx$$

$$=\int_0^1 (1-x^k)(1-x^k)^{20}dx=\int_0^1 (1-x^k)^{20}dx$$

$$-\int_0^1 x^k(1-x^k)^{20}dx$$

$$I(21)=I(20)-\int_0^1 x^k\cdot(1-x^k)^{20}$$

$$=I(20)-\int_0^1 x\cdot \underbrace{x^{k-1}(1-x^k)^{20}}dx$$

$$I(21)=I(20)-\left[\frac{(1-x^k)^{21}}{-21k}x\Big|_0^1-\int_0^1\frac{(1-x^k)^{21}}{-21k}dx\right]$$

$$I(21)=I(20)+\frac{1}{21k}I(21)$$

$\Rightarrow$ $I(21)(21k+1)=21kI(20)$

$\Rightarrow$ $21k=147 \Rightarrow k=7$

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Find the number of 3 digit numbers which are not divisible by 3 and made using the digits {2, 3, 5, 7, 4} and repetition is not allowed.

Answer (36.00)

Sol. Possible triplets for which number is divisible by 3

(2, 3, 7), (2, 3, 4), (3, 5, 7), (3, 5, 4)

$\therefore$ Number of required numbers = ${}^5C_3\cdot 3!-4\times 3!$

= 3! × (6)

= 6 × 6 = 36

22. Let $f(x)=(2x-3)^{2/3}(x+2)$. The number of critical points of $f(x)$ is equal to

Answer (2.00)

Sol. $f'(x)=(2x-3)^{2/3}+\dfrac{(x+2)}{(2x-3)^{1/3}}\times\dfrac{2}{3}\times 2$

$$=(2x-3)^{2/3}+\frac{4}{3}\frac{(x+2)}{(2x-3)^{1/3}}$$

$$=\frac{(2x-3)+\frac{4}{3}(x+2)}{(2x-3)^{1/3}}$$

$$\Rightarrow \frac{6x-9+4x+8}{3(2x-3)^{1/3}}$$

$$=\frac{10x-1}{(3)(2x-3)^{1/3}}$$

$\therefore$ $f'(x)=0$ at $x=\dfrac{1}{10}$

$f(x)$ is nondifferentiable at $x=\dfrac{3}{2}$

$\therefore$ 2 critical points are there.

23.

24.

25.

26.

27.

28.

29.

30.

❑ ❑ ❑