

# MATHEMATICS

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

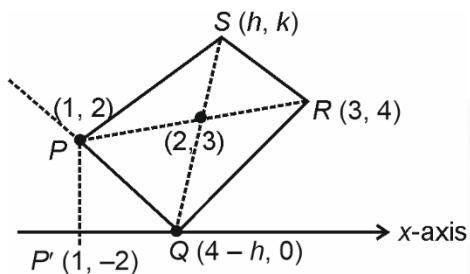
**Choose the correct answer :**

1. A ray of light passing through (1, 2) after reflecting on x-axis at point Q passes through R(3, 4). If S(h, k) is such that PQRS is a parallelogram, then find the value of  $hk^2$ .
 

(1) 90	(2) 84
(3) 96	(4) 108

**Answer (2)**

**Sol.**



$$\therefore k = 6 \text{ (using diagram)}$$

P lies on RQ

$$\frac{4}{h-1} = \frac{6}{2}$$

$$\Rightarrow 4 = 3h - 3$$

$$\Rightarrow 3h = 7$$

$$\Rightarrow h = \frac{7}{3}$$

$$hk^2 = \frac{7}{3} \times 36 = 84$$

2. Tetrahedral dice having outcomes (1, 2, 3, 4) has 3 outcomes  $a, b, c$  (which are visible). Probability that  $ax^2 + bx + c = 0$  has real roots is  $\frac{m}{n}$ . ( $m, n$  are coprime). Then  $m + n = ?$ 

(1) 4	(2) 5
(3) 6	(4) 7

**Answer (2)**

**Sol.**  $a, b, c \in \{1, 2, 3, 4\}$

And  $a, b, c$  are distinct

For real roots of  $ax^2 + bx + c = 0$

$$b^2 \geq 4ac$$

So, we get  $(a, b, c)$  as (1, 3, 2),

(2, 3, 1), (1, 4, 2), (2, 4, 1), (1, 4, 3) and (3, 4, 1)

So, total 6 values of  $(a, b, c)$  are possible for required condition.

$$\text{So, required probability} = \frac{6}{4!} = \frac{6}{24}$$

$$= \frac{1}{4} = \frac{m}{n}$$

$$m + n = 5$$

3. A circle passes through (0, 0) and (1, 0) and touches the circle  $x^2 + y^2 = 9$ . Then the locus of the centre of the circle is
 

(1) Circle
(2) Parabola
(3) Hyperbola
(4) Straight line

**Answer (1)**

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**Sol.** Circle will touch internally.

$$C_1 C_2 = |r_1 - r_2|$$

$$\sqrt{h^2 + k^2} = 3 - \sqrt{h^2 + k^2}$$

$$\Rightarrow 2\sqrt{h^2 + k^2} = 3$$

$$\Rightarrow x^2 + y^2 = \frac{9}{4}$$

4.  $\vec{A}, \vec{B}$  and  $\vec{C}$  are given as

$$\vec{A} = \alpha \hat{i} + 4 \hat{j} + 5 \hat{k}$$

$$\vec{B} = 2 \hat{i} + 5 \hat{j} + 6 \hat{k}$$

$$\vec{C} = \vec{A} + \vec{B}$$

$$|\vec{C}| = |\vec{A} - \vec{B}|$$

The value of  $\alpha$  and  $|\vec{C}|^2$  is equal to

$$(1) 25, 731$$

$$(2) -25, 669$$

$$(3) -25, 731$$

$$(4) 25, 669$$

**Answer (3)**

$$\text{Sol. } |\vec{C}| = |\vec{A} + \vec{B}|$$

$$|\vec{C}| = |\vec{A} - \vec{B}|$$

$$\Rightarrow \vec{A} \cdot \vec{B} = 0$$

$$\Rightarrow 2\alpha + 20 + 30 = 0$$

$$\Rightarrow \alpha = -25$$

$$|\vec{C}|^2 = |\vec{A}|^2 + |\vec{B}|^2 + 2\vec{A} \cdot \vec{B}$$

$$= \alpha^2 + 16 + 25 + 4 + 25 + 36$$

$$= 731$$

5. If set  $A = \{Z : |Z - 1| \leq 1\}$  and set  $B = \{Z : |Z - 5| \leq |Z - 1|\}$ , If  $Z = a + ib$ , (where  $a, b \in I$ ) then sum of modulus squares of  $A \cap B$  is

$$(1) 0$$

$$(2) 2$$

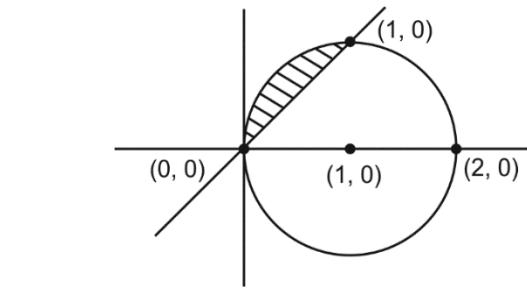
$$(3) 4$$

$$(4) 5$$

**Answer (2)**

$$\text{Sol. } |Z - 1| \leq 1$$

$$|Z - 5| \leq |Z - 1|$$



$$\Rightarrow Z = a + ib, a, b \in I$$

$$\Rightarrow (a, b) = \{(0, 0), (1, 1)\}$$

$$\Rightarrow |Z| = \sqrt{a^2 + b^2} \Rightarrow \sqrt{a^2 + b^2} \leftarrow \{0, \sqrt{2}\}$$

$$\text{Sum of squares of modulus} = 0^2 + (\sqrt{2})^2 = 2$$

$$6. \text{ If } \frac{1}{(1+d)(1+2d)} + \frac{1}{(1+2d)(1+3d)} + \dots + \frac{1}{(1+9d)(1+10d)} = 1.$$

The value of  $50d$  is ( $d > 0$ )

$$(1) 50 \quad (2) 60$$

$$(3) 25 \quad (4) 30$$

**Answer (3)**

$$\text{Sol. } \frac{1}{(1+d)(1+2d)} + \frac{1}{(1+2d)(1+3d)} + \frac{1}{(1+3d)(1+4d)} + \dots + \frac{1}{(1+9d)(1+10d)} = 1.$$

$$\frac{1}{d} \left[ \left( \frac{1}{1+d} - \frac{1}{1+2d} \right) + \left( \frac{1}{1+2d} - \frac{1}{1+3d} \right) + \dots + \left( \frac{1}{1+9d} - \frac{1}{1+10d} \right) \right] = 1$$

$$\frac{1}{d} \left[ \frac{1}{1+d} - \frac{1}{1+10d} \right] = 1$$

$$\frac{9d}{d(1+d)(1+10d)} = 1 \Rightarrow 9 = (10d+1)(d+1)$$

$$\therefore 10d^2 + 11d - 8 = 0$$

$$\Rightarrow d = \frac{1}{2}$$

$$\therefore 50d = \frac{1}{2}$$

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Chirag Falor  
4 Year Classroom  
**1** AIR  
JEE (Adv.)  
2020



Tanishka Kabra  
4 Year Classroom  
**1** AIR  
INDIA  
AIR 16 CRL  
JEE (Adv.)  
2022

7. If  $f(x) = \begin{cases} \left(\frac{8}{7}\right)^{\tan 8x} & , \quad x \in \left(0, \frac{\pi}{2}\right) \\ a - 8 & , \quad x = \frac{\pi}{2} \\ (1 + |\cot x|)^{\frac{b|\tan x|}{a}} & , \quad x \in \left(\frac{\pi}{2}, \pi\right) \end{cases}$

If the function  $f(x)$  is continuous at  $x = \frac{\pi}{2}$  then  $a^2 + b^2$  is equal to

- (1) 97                                      (2) 85  
 (3) 81                                      (4) 100

**Answer (4)**

**Sol.**  $\lim_{x \rightarrow \frac{\pi}{2}^-} f(x) = f\left(\frac{\pi}{2}\right) = \lim_{x \rightarrow \frac{\pi}{2}^+} f(x)$  for continuity at  $x = \frac{\pi}{2}$

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{2}^-} \left(\frac{8}{7}\right)^{\left(\frac{\tan 8x}{\tan 7x}\right)} \quad \text{Let } x = \frac{\pi}{2} - h$$

$$\Rightarrow \lim_{h \rightarrow 0} \left(\frac{8}{7}\right)^{\frac{\tan(4\pi - 8h)}{\tan(3\pi + \frac{\pi}{2}h)}} = \lim_{h \rightarrow 0} \left(\frac{8}{7}\right)^{\frac{\tan(-8h)}{\cot(h)}} = \left(\frac{8}{7}\right)^0 = 1$$

$$\Rightarrow a - 8 = 1 \Rightarrow a = 9$$

$$\lim_{x \rightarrow \frac{\pi}{2}^+} (1 + |\cot x|)^{\frac{b|\tan x|}{a}}, \quad x = \frac{\pi}{2} + h$$

$$\lim_{h \rightarrow 0} (1 - \tan h)^{-\frac{b \cot h}{9}} = \lim_{h \rightarrow 0} (1 - \tan h)^{-\frac{b \cot h}{9}}$$

$$= \lim_{h \rightarrow 0} (1 - \tan h)^{\left(\frac{-1}{\tan h}\right) \cdot (-\tan h) \cdot \left(\frac{-b \cot h}{9}\right)}$$

$$= e^{\frac{b}{9}} = 1 \quad \Rightarrow b = 0$$

$$\Rightarrow a^2 + b^2 = 81 + 0 = 81$$

8. If  $\cos \theta \cos(60 - \theta) \cos(60 + \theta) \leq \frac{1}{8}$ . Find the sum of values of  $\theta$  for which  $\cos 3\theta$  is maximum

- (1)  $6\pi$                                       (2)  $4\pi$   
 (3)  $3\pi$                                         (4)  $7\pi$

**Answer (1)**

**Sol.**  $\cos \theta \cos(60 - \theta) \cos(60 + \theta) \leq \frac{1}{8}$

$$\Rightarrow \frac{1}{4} \cos 3\theta \leq \frac{1}{8}$$

$$\cos 3\theta \leq \frac{1}{2}$$

$\therefore \cos 3\theta \leq \text{maximum}$

$$\Rightarrow \cos 3\theta = \frac{1}{2}, \quad 3\theta = 2n\pi \pm \frac{\pi}{3}, \quad n \in \mathbb{I}$$

$$\therefore \theta = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}$$

$$= \frac{54\pi}{9} = 6\pi$$

9. If  $(x^2 + y^2)dy = 5xy dx$ . Find the general solution of DE.

(1)  $\frac{5}{8} \ln \left| \frac{y-2x}{y+2x} \right| = \frac{1}{4} \ln \left| \frac{y}{x} \right| - \ln |x| + C$

(2)  $\frac{5}{8} \ln \left| \frac{y+2x}{y-2x} \right| = \frac{1}{4} \ln \left| \frac{y}{x} \right| - 2 \ln |x| + C$

(3)  $\frac{5}{8} \ln \left| \frac{y-2x}{y+2x} \right| = -\frac{1}{4} \ln \left| \frac{y}{x} \right| + \ln |x^2| + C$

(4)  $\frac{5}{4} \ln \left| \frac{y-2x}{y+2x} \right| = \frac{1}{4} \ln \left| \frac{y}{x} \right| + 2 \ln |x| + C$

**Answer (1)**

**Sol.**  $\frac{dy}{dx} = \frac{5xy}{x^2 + y^2}$

$$\text{let } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$v + x \frac{dv}{dx} = \frac{5x \cdot vx}{x^2 + v^2 x^2} = \frac{5v}{1+v^2}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{5v}{1+v^2} - v \Rightarrow \frac{4v-v^3}{1+v^2} = \frac{v(4-v^2)}{1+v^2}$$

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$$\int \frac{1+v^2}{v(2-v)(2+v)} dv = \int \frac{dv}{x} \Rightarrow \int \frac{1+v^2}{v(v-1)(v+2)} = -\int \frac{dx}{x}$$

$$\text{Let } \frac{1+v^2}{v(v-2)(v+2)} = \frac{A}{v} + \frac{B}{v-2} + \frac{C}{v+2}$$

$$B = \frac{5}{8}, \quad A = \frac{1}{-4}, \quad C = \frac{5}{8}$$

$$\therefore \int \frac{1+v^2}{v(2-v)(2+v)} dv = -\int \frac{dx}{x}$$

$$= \int \left( -\frac{1}{4v} + \frac{5}{8(v-2)} + \frac{5}{8(v+2)} \right) dv = -\int \frac{dx}{x}$$

$$= -\frac{1}{4} \ln|v| + \frac{5}{8} \ln \left| \frac{(v-2)}{(v+2)} \right| = -\ln|x| + C$$

$$\Rightarrow -\frac{1}{4} \ln \left| \frac{y}{x} \right| + \frac{5}{8} \ln \left| \frac{y-2x}{y+2x} \right| = -\ln|x| + C$$

10. Find the maximum area (in sq. units) of a line passing through (4, 3)

(1) 30

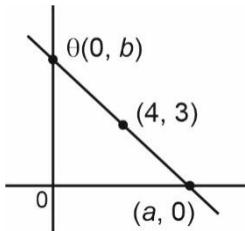
(2) 31

(3) 24

(4) 32

**Answer (3)**

**Sol.**



$$\frac{x}{a} + \frac{y}{b} = 1$$

As line passes through (4, 3)

$$\frac{4}{a} + \frac{3}{b} = 1$$

$$4b + 3a = ab$$

$$3a - ab = -4b$$

$$a = \frac{-4b}{3-b} = \frac{4b}{b-3}$$

$$\text{Now area of triangle} = \left| \frac{1}{2} \times a \times b \right|$$

$$= \left| \frac{1}{2} \times \frac{4b}{b-3} \times b \right|$$

$$\Rightarrow f(b) = \frac{2b^2}{b-3}$$

$$\Rightarrow f'(b) = \frac{2b(b-6)}{(b-3)^2} = 0$$

[At  $b = 6$  we get maximum area]

$$\text{i.e. } \frac{1}{2} \times \frac{4 \times 6}{3} \times 6 = 24 \text{ sq. units}$$

$$11. \int \frac{2+\tan x}{3+\tan x} dx = \frac{1}{2} \lambda x - \frac{1}{10} \log |a \sin x + b \cos x|.$$

Then the value of  $\lambda + \frac{a}{b}$  is equal to

$$(1) \frac{13}{5} \quad (2) \frac{15}{26}$$

$$(3) \frac{26}{15} \quad (4) \frac{5}{13}$$

**Answer (3)**

$$\text{Sol. } \int \frac{2+\tan x}{3+\tan x} dx$$

$$= \int \left( 1 - \frac{1}{3+\tan x} \right) dx$$

$$= x - \int \frac{\cos x}{3\cos x + \sin x} dx$$

$$\text{Consider } I' = \int \frac{\cos x}{3\cos x + \sin x} dx$$

$$\cos x = A(3\cos x + \sin x) + B(-3\sin x + \cos x)$$

$$A - 3B = 0$$

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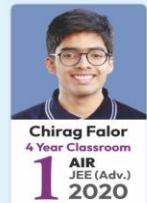
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$$3A + B = 1$$

$$\Rightarrow B = \frac{1}{10}, A = \frac{3}{10}$$

$$I' = \frac{3}{10}x + \frac{1}{10}\log|3\cos x + \sin x|$$

$$I = \frac{7x}{10} - \frac{1}{10}\log|3\cos x + \sin x|$$

$$= \frac{1}{2}\left(\frac{7x}{5}\right) - \frac{1}{10}\log|3\cos x + \sin x|$$

$$a = 1, b = 3, \lambda = \frac{7}{5}$$

$$\therefore \lambda + \frac{a}{b}$$

$$= \frac{7}{5} + \frac{1}{3} = \frac{26}{15}$$

12. If  $y^2 = 4x$  and  $x^2 + y^2 = 5$ , then the area of smaller part of the circle cut by parabola is

$$(1) \frac{2}{3} + \frac{5\pi}{2} + \sin^{-1} \frac{1}{\sqrt{5}}$$

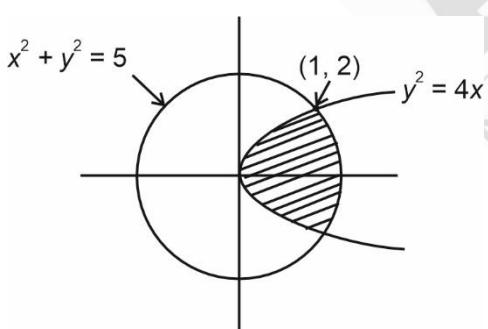
$$(2) \frac{2}{3} + \frac{5\pi}{2} - 5 \sin^{-1} \frac{1}{\sqrt{5}}$$

$$(3) \frac{2}{3} + \frac{5\pi}{4} + \sin^{-1} \frac{1}{\sqrt{5}}$$

$$(4) \frac{2}{3} + \frac{5\pi}{4} - 5 \sin^{-1} \frac{1}{\sqrt{5}}$$

**Answer (2)**

**Sol.**



$$\text{Area} = 2 \left[ \int_0^1 2\sqrt{x} dx + \int_1^{\sqrt{5}} \sqrt{5-x^2} dx \right]$$

$$= 2 \left[ \left( \frac{4}{3}x^{\frac{3}{2}} \right)_0^1 + \left( \frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right) \right)_1^{\sqrt{5}} \right]$$

$$= 2 \left[ \frac{4}{3} + \frac{5\pi}{4} - 1 - \frac{5}{2}\sin^{-1}\left(\frac{1}{\sqrt{5}}\right) \right]$$

$$= \left( \frac{2}{3} + \frac{5\pi}{2} - 5\sin^{-1} \frac{1}{\sqrt{5}} \right) \text{sq. unit}$$

13. If  $\frac{ydy}{dx} + 3 = \frac{2dy}{dx}$  is a parabola passing through  $(1, 0)$ . Then, the vertices of the parabola satisfy the equation

$$(1) 3x + 2y = 6$$

$$(2) 3x + 2y = -6$$

$$(3) 3x + 2y = 9$$

$$(4) 3x + 2y = -9$$

**Answer (3)**

**Sol.**  $ydy + 3dx = 2dy$

$$\Rightarrow \frac{y^2}{2} + 3x = 2y + c \Big|_{(1, 0)}$$

$$\Rightarrow c = 3$$

$$\therefore (y-2)^2 = -6\left(x - \frac{5}{3}\right)$$

$$\therefore \text{Vertex} = \left(\frac{5}{3}, 2\right)$$

14. If  $\alpha, \beta$  are the roots of the equation  $x^2 - 2\sqrt{2}x + 1 = 0$  then equation whose roots are  $\alpha^4 + \beta^4$  and  $\frac{\alpha^6 + \beta^6}{6}$  is

$$(1) x^2 - 66x + 1110 = 0$$

$$(2) x^2 - 33x + 1122 = 0$$

$$(3) x^2 - 34x + 1122 = 0$$

$$(4) x^2 - 67x + 1122 = 0$$

**Answer (4)**

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**Sol.**  $x^2 - 2\sqrt{2}x + 1 = 0$

$$\alpha \quad \beta$$

$$\alpha^4 + \beta^4, \frac{\alpha^6 + \beta^6}{6}$$

$$\Rightarrow \alpha + \beta = 2\sqrt{2}, \alpha\beta = 1$$

$$\Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = 8 \Rightarrow \alpha^2 + \beta^2 = 6$$

$$\Rightarrow \alpha^4 + \beta^4 + 2\alpha^2\beta^2 = 36 \Rightarrow \alpha^4 + \beta^4 = 34$$

$$\Rightarrow \frac{\alpha^6 + \beta^6}{6} = \frac{(\alpha^2 + \beta^2)}{6}(\alpha^4 + \beta^4 - \alpha^2\beta^2)$$

$$= \binom{6}{6} [34 - 1] = 33$$

$\Rightarrow$  roots of the equation are 33, 44

$\Rightarrow$  equation is

$$x^2 - (33 + 34)x + 33 \cdot 34 = 0$$

$$\Rightarrow x^2 - 67x + 1122 = 0$$

15. If  $f(x) = x^2 - 8$ ,  $g(x) = \frac{x}{x-9}$  and  $a = f(g(10))$  and  $b = g(f(3))$  and  $e$  and  $l$  be eccentricity and length of

latus rectum of conic  $\frac{x^2}{|a|} + \frac{y^2}{|b|} = 1$ , then  $(92l + 46e^2)$  is

- (1) 48                          (2) 46  
 (3) 45                          (4) 92

### Answer (2)

**Sol.**  $f(x) = x^2 - 8$

$$g(x) = \frac{x}{x-9}$$

$$\Rightarrow f(g(10)) = f(10) = a = 92$$

$$g(f(3)) = g(1) = b = -\frac{1}{8}$$

$\Rightarrow$  conic :

$$\frac{x^2}{92} + \frac{y^2}{1/8} = 1$$

$$l(L \cdot R) = \frac{2(1/8)}{\sqrt{92}} = \frac{1}{4\sqrt{92}} \Rightarrow l^2 = \frac{1}{16(92)}$$

$$e^2 = 1 - \frac{1/8}{92} = \frac{92 \times 8 - 1}{92 \times 8}$$

$$\Rightarrow 92l^2 + 46e^2 = \frac{1}{16} + \frac{735}{16}$$

$$= \frac{736}{16} = 46$$

16.

17.

18.

19.

20.

### SECTION - B

**Numerical Value Type Questions:** This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The remainder when  $(428)^{2024}$  is divided by 21 is

### Answer (1)

**Sol.**  $(428)^{2024} \equiv 8^{2024} \pmod{21}$

$$(8^2) \equiv 1 \pmod{21}$$

$$(8)^{2024} \equiv 1 \pmod{21}$$

$\therefore$  remainder = 1.

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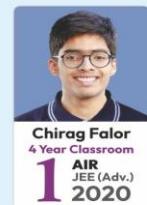


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22. If  $A$  is  $3 \times 3$  matrix,

$$\det(3\text{adj}(2\text{adj}A)) = 2^{-13} \cdot 3^{-10} \text{ and}$$

$$\det'(3\text{adj}(2A)) = 2^m \cdot 3^n, \text{ then } 2m + 2n \text{ is equal to}$$

**Answer (14.00)**

$$\text{Sol. } P \Rightarrow |3\text{adj}(2A)| = 3^3 \cdot |\text{adj}2A| = 3^3 \cdot |2A|^2 = 3^3 \cdot 2^6 |A|^2$$

$$\begin{aligned} |3\text{adj}(2\text{adj}(A))| &= 3^3 \cdot |\text{adj}(2\text{adj}A)| = 3^3 \cdot |(2\text{adj}A)|^2 \\ &= 3^3 \cdot (2^3)^2 |\text{adj}A|^2 \\ &= 3^3 \cdot 2^6 (|A|^2) = 3^3 \cdot 2^6 |A|^4 \\ &= 2^{-13} \cdot 3^{-10} \end{aligned}$$

$$\Rightarrow |A|^4 = 3^{-13} \cdot 2^{-19}$$

$$\Rightarrow |A|^2 = 3^{-6.5} \cdot 2^{-9.5}$$

$$\Rightarrow P = 3^3 \cdot 2^6 \cdot 3^{-6.5} \cdot 2^{-9.5} = 3^{-3.5} \cdot 2^{-3.5} = 2^m \cdot 3^n$$

$$\Rightarrow m = 3.5, n = 3.5$$

$$\Rightarrow 2m + 2n = 14$$

23. If  $f(x) = 3ax^3 + bx^2 + cx + 41$  and  $f(1) = 41$ ,

$$f'(1) = 2 \text{ and } f''(1) = 4 \text{ then } (a^2 + b^2 + c^2) \text{ is}$$

**Answer (08.00)**

$$\text{Sol. } f(1) = 3a + b + c + 41 = 41 \Rightarrow 3a + b + c = 0$$

$$f'(x) = 9ax^2 + 2bx + c \Rightarrow f'(1) = 9a + 2b + c = 2$$

$$f''(x) = 18ax + 2b \Rightarrow f''(1) = 18a + 2b = 4$$

$$\Rightarrow (4 - 9a) + c = 2$$

$$3a + c + (2 - 9a) = 0$$

$$\Rightarrow c - 6a + 2 = 0 \Rightarrow a = 0$$

$$c - 9a + 2 = 0 \quad c = -2$$

$$b = 2$$

$$\Rightarrow a^2 + b^2 + c^2 = 0 + 4 + 4 = 8$$

24. If domain of  $f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$  is  $R - (\alpha, \beta]$  then  $12\alpha\beta$  is equal to

**Answer (32.00)**

$$\text{Sol. } f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$$

$$\Rightarrow 2x+3 \neq 0 \text{ and } -1 \leq \frac{x-1}{2x+3} \leq 1$$

$$\Rightarrow \frac{x-1-2x-3}{2x+3} \leq 0 \Rightarrow \frac{x+4}{2x+3} \geq 0$$

$$\Rightarrow x \in (-\infty, -4] \cup \left(\frac{-3}{2}, \infty\right) \quad \begin{array}{c|c|c|c} & + & - & + \\ \hline -4 & & & \\ \hline -\frac{3}{2} & & & \end{array}$$

$$\text{Also, } 0 \leq \frac{x-1+2x+3}{2x+3} \Rightarrow \frac{3x+2}{2x+3} \geq 0$$

$$\begin{array}{c|c|c} & + & - & + \\ \hline -\frac{3}{2} & & & \\ \hline -\frac{2}{3} & & & \end{array}$$

$$\Rightarrow x \in \left(-\infty, \frac{-3}{2}\right) \cup \left[\frac{-2}{3}, \infty\right)$$

$$\Rightarrow \text{domain} \Rightarrow x \in (-\infty, -4] \cup \left(\frac{-2}{3}, \infty\right)$$

$$\Rightarrow R - \left(-4, \frac{-2}{3}\right]$$

$$\Rightarrow \alpha = -4, \quad \beta = \frac{-2}{3}$$

$$\Rightarrow 12 \times (-4) \left(\frac{-2}{3}\right) = 32$$

## Aakashians Conquer JEE (Main) 2024

SESSION-1



**\*\*143**  
100 PERCENTILERS  
(PHY. OR CHEM. OR MATHS)

**\*\*936** 99+ PERCENTILERS  
**\*\*4155** 95+ PERCENTILERS  
& Counting  
\*\*(Includes Students from Classroom, Distance & Digital Courses)

\*As per student response sheet and NTA answer key.

### Our Stars



25.  $A = \{2, 4, 6, 8\}$

$B = \{3, 7, 6, 9\}$

$R : A \times B \rightarrow A \times B$  such that

$$(a_1, b_1) R(a_2, b_2) \Rightarrow a_1 + a_2 = b_1 + b_2$$

$$(a_1, b_1) \in A \text{ and } (a_2, b_2) \in B.$$

Then the number of elements in the relation is

**Answer (9)**

**Sol.**  $A = \{2, 4, 6, 8\}$

$B = \{3, 7, 6, 9\}$

$$(a_1, b_1) R(a_2, b_2) \text{ when } a_1 + a_2 = b_1 + b_2$$

$$(2, 6) R(7, 3)$$

$$(2, 4) R(9, 7)$$

$$(4, 8) R(7, 3)$$

$$(4, 6) R(9, 7)$$

$$(6, 2) R(3, 7)$$

$$(6, 4) R(7, 9)$$

$$(6, 8) R(9, 7)$$

$$(8, 4) R(3, 7)$$

$$(8, 6) R(7, 9)$$

So total 9 elements are in the relation.

26. If  $f(m + n) = f(m) + f(n)$ ,  $f(1) = 1$

then  $\sum_{k=1}^{2022} f(\lambda + k) \leq (2022)^2$ , then  $\lambda_{\max} \in I$  is

**Answer (1010)**

**Sol.**  $f(m + n) = f(m) + f(n)$

$$\Rightarrow f(x) = kx$$

$$\therefore f(1) = 1 \Rightarrow k = 1$$

$$\therefore f(x) = x$$

Now,  $\sum_{k=1}^{2022} f(\lambda + k) \leq (2022)^2$

$$\Rightarrow \sum_{k=1}^{2022} (\lambda + k) \leq (2022)^2$$

$$= \underbrace{\lambda + \lambda + \dots + \lambda}_{2022} + (1 + 2 + 3 + \dots + 2022) \leq (2022)^2$$

$$2022\lambda + \frac{2022 \times 2023}{2} \leq (2022)^2$$

$$\lambda \leq \left( 2022 - \frac{2023}{2} \right)$$

$$\Rightarrow \lambda \leq \frac{2021}{2}$$

$$\lambda \leq 1010.5$$

$$\therefore \lambda_{\max} = 1010$$

27.

28.

29.

30.



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TWO YEAR CLASSROOM PROGRAM

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**\*\*143**  
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\*As per student response sheet and NTA answer key.

### Our Stars



**Chirag Falor**  
4 Year Classroom  
**1** AIR JEE (Adv.) 2020



**Tanishka Kabra**  
4 Year Classroom  
**1** AIR-16 CRL INDIA RANK JEE (Adv.) 2022