MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Given $\left(\sqrt{a}x^2+\dfrac{1}{2x^3}\right)^{10}$ if the coefficient of independent term is 105 then a^3 equals to

(1) 2 (2) 8

(3) 9 (4) 6

Answer (2)

Sol. General term = ${}^{10}C_r\left(\sqrt{a}x^2\right)^{10-r}\left(\dfrac{1}{2x^3}\right)^r$

$$={}^{10}C_r\left(\sqrt{a}\right)^{10-r}x^{20-2r}\left(\frac{1}{2}\right)^r x^{-3r}$$

$$={}^{10}C_r\left(\sqrt{a}\right)^{10-r}x^{20-5r}2^{-r}$$

Now, $20 - 5r = 0$

$20 = 5r$

$r = 4$

$$\therefore\ {}^{10}C_4\left(\sqrt{a}\right)^6 2^{-4}=105$$

$$\frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2}\frac{a^3}{16}=105$$

$\Rightarrow$ $a^3 = 8$

Option (2) is correct.

2. Number of 5 letters words made from the word "MATHEMATICS" is equal to

(1) 13540 (2) 13560

(3) 14210 (4) 17310

Answer (2)

Sol. 2M

2A

2T

H, E, I, C, S

Case-I

2 Alike 2 Alike 1 Diff

$${}^3C_2\cdot{}^6C_1\cdot\frac{5!}{2!2!}=18\times 30=540$$

Case-II

2 Alike + 3 Diff

$${}^3C_1\cdot{}^7C_3\times\frac{5!}{2!}=105\times 60=6300$$

Case-III

All different

$${}^8C_5\cdot 5!=6720$$

Total words = 13560

3. If the shortest distance between the lines $\dfrac{x-\lambda}{2}=\dfrac{y-4}{4}=\dfrac{z-7}{8},\ \dfrac{x-2}{1}=\dfrac{y-3}{2}=\dfrac{z-4}{4}$ is $\dfrac{13}{\sqrt{21}}$ then sum of values of λ is

(1) $\dfrac{23}{4}$ (2) $\dfrac{23}{5}$

(3) $\dfrac{21}{4}$ (4) $\dfrac{25}{2}$

Answer (2)

Sol. S.D = $\dfrac{\left|\left(\hat{i}+2\hat{j}+4\hat{k}\right)\times\left((\lambda-2)\hat{i}+\hat{j}+3\hat{k}\right)\right|}{\left|\hat{i}+2\hat{j}+4\hat{k}\right|}=\dfrac{13}{\sqrt{21}}$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 4\\ \lambda-2 & 1 & 3\end{vmatrix}=2\hat{i}+\hat{j}(4\lambda-9)+\hat{k}(1-2\lambda+4)$$

$$=\left|2\hat{i}+\hat{j}(4\lambda-9)+(5-2\lambda)\hat{k}\right|$$

$$\Rightarrow \frac{\sqrt{4+(4\lambda-9)^2+(5-2\lambda)^2}}{\sqrt{21}} = \frac{13}{\sqrt{21}}$$

$\Rightarrow$ $20\lambda^2 + 110 - 92\lambda = 169$

$\Rightarrow$ $20\lambda^2 - 92\lambda - 59 = 0$

4.

2

5   8

11   14   17

Then the sum of elements of 10th row.

(1) 1505   (2) 1438

(3) 1981   (4) 1745

Answer (1)

Sol. The given sequence is

2, (5, 8), (11, 14, 17), (20, 21, 24, 27) …

Before 10th term total number of numbers would be

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 9 × 5 = 45

It is an A.P. with a = 2 and d = 3

T_{46} = 2 + 45 × 3

= 137

Sum of 10th terms = $\frac{10}{2}$ (2 × 137 + (10 – 1)3)

= 1505

5. If $y(x)$ be the solution of differential equation $\sec y \frac{dy}{dx} + 2x \sin y = x^3 \cos y$ and $y(1) = 0$ then $y(\sqrt{3})$ is equal to

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{12}$

(3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

Answer (1)

Sol. $\sec^2 y \frac{dy}{dx} + 2x \tan y = x^3$

$\Rightarrow$ Let $z = \tan y$

$$\frac{dz}{dx} = \sec^2 y \frac{dy}{dx}$$

$$\Rightarrow \frac{dz}{dx} + 2xz = x^3$$

$$\Rightarrow \text{I.F.} = e^{\int 2x\,dx} = e^{x^2}$$

$$\Rightarrow z.\, e^{x^2} = \int e^{x^2}.\, x^3 dx + C$$

$$\Rightarrow \tan y . e^{x^2} = \frac{1}{2}\left(x^2 e^{x^2} - e^{x^2}\right) + C$$

$$\tan(0).\, e' = \frac{1}{2}(1.\, e' - e') + C$$

$\Rightarrow$ $C = 0$

$$\Rightarrow \tan y = \left(\frac{x^2-1}{2}\right)$$

$$f(x) = \tan^{-1}\left(\frac{x^2-1}{2}\right)$$

$$f(\sqrt{3}) = \tan^{-1}\left(\frac{3-1}{2}\right) = \frac{\pi}{4}$$

6. Two points (5, 2) and (2, a). Line passes through these points makes an angle of $\frac{\pi}{4}$ at origin. The product of all values of a is equal to

(1) 8   (2) –4

(3) –2   (4) 1

Answer (2)

Sol. $m_1 = \frac{2}{5}$, $m_2 = \frac{a}{2}$

$$1 = \left[\frac{\frac{2}{5} - \frac{a}{2}}{1 + \frac{2}{5}\cdot\frac{a}{2}}\right] \quad \left(\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|\right)$$

$$\pm 1 = \frac{4-5a}{10+2a}$$

$\oplus$ $10 + 2a = 4 - 5a$

$\Rightarrow 7a = -6$

$\Rightarrow a = \dfrac{-6}{7}$

$\ominus$ $-10 - 2a = 4 - 5a$

$\Rightarrow 3a = 14$

$\Rightarrow a = \dfrac{14}{3}$

$$a_1 \cdot a_2 = -\frac{6}{7} \times \frac{14}{3} = -4$$

7. A circle $x^2 + y^2 = 8$ and a parabola $y^2 = 2x$ are given. Find area bounded by these two curves in first quadrant which lie inside the circle and outside the parabola.

(1) $\pi - \dfrac{2}{3}$

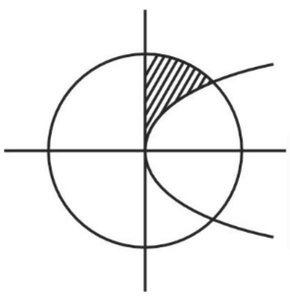
(2) $\pi - \dfrac{1}{3}$

(3) $\pi + \dfrac{1}{3}$

(4) $\pi - \dfrac{4}{3}$

Answer (1)

Sol.



$$\text{Area} = \int_0^2 \left(\sqrt{8 - x^2} - \sqrt{2x}\right) dx$$

$$= \left[\frac{x}{2}\sqrt{8 - x^2} + \frac{8}{2}\sin^{-1}\left(\frac{x}{2\sqrt{2}}\right)\right]_0^2 - \sqrt{2} \times \frac{2}{3}\left[x^{3/2}\right]_0^2$$

$$= 2 + 4\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - \frac{2\sqrt{2}}{3}\left(2\sqrt{2}\right)$$

$$= \left(\pi - \frac{2}{3}\right) \text{ sq. units.}$$

8. If $\alpha \neq a$, $\beta \neq b$, $\gamma \neq c$ and

$$\begin{vmatrix} \alpha & b & c \\ a & \beta & c \\ a & b & \gamma \end{vmatrix} = 0, \text{ then}$$

$$\frac{a}{\alpha - a} + \frac{b}{\beta - b} + \frac{\gamma}{\gamma - c} \text{ is equal to}$$

(1) 3

(2) 2

(3) 0

(4) 1

Answer (3)

Sol. $\begin{vmatrix} \alpha & b & c \\ a & \beta & c \\ a & b & \gamma \end{vmatrix} = 0$

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$$\begin{vmatrix} \alpha - a & b - \beta & 0 \\ 0 & \beta - b & c - \gamma \\ a & b & \gamma \end{vmatrix} = 0$$

Take $(\alpha - a)$, $(\beta - b)$ and $(\gamma - c)$ common from column 1, column 2 and column 3 respectively.

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \dfrac{a}{\alpha - a} & \dfrac{b}{\beta - b} & \dfrac{\gamma}{\gamma - c} \end{vmatrix} = 0$$

$$\Rightarrow \frac{b}{\beta - b} + \frac{\gamma}{\gamma - c} + \frac{a}{\alpha - a} = 0$$

9. Bag X contains five one-rupee coins, four five-rupee coins. Bag Y contains 4 one rupee and 5 five rupees. Bag Z contains 3 one-rupee coin and 6 five-rupee coins. If 1 rupee coin is selected at random, what is the probability it is drawn from bag *Y*?

(1) $\dfrac{1}{3}$

(2) $\dfrac{1}{4}$

(3) $\dfrac{1}{5}$

(4) $\dfrac{1}{6}$

Answer (1)

Sol. By Baye's theorem

Probability (coin drawn from bag Y)

$$=\frac{\frac{1}{3}\cdot\frac{4}{9}}{\frac{1}{3}\cdot\frac{5}{9}+\frac{1}{3}\cdot\frac{4}{9}+\frac{1}{3}\cdot\frac{3}{9}}=\frac{4}{12}=\frac{1}{3}$$

10. If $\dfrac{3\cos 36^\circ - 2\sin 18^\circ}{2\cos 36^\circ - 3\sin 18^\circ}=\dfrac{a\sqrt{5}+b}{c}$, where a and c are co-primes. The value of $a + b + c$ is equal to

(1) 6 (2) 7

(3) 2 (4) 5

Answer (1)

Sol. $\cos 36^\circ = \dfrac{\sqrt{5}+1}{4}$

$\sin 18^\circ = \dfrac{\sqrt{5}-1}{4}$

$\dfrac{3\cos 36^\circ - 2\sin 18^\circ}{2\cos 36^\circ - 3\sin 18^\circ}=\dfrac{\sqrt{5}+3}{2}$

$\therefore$ $b = 3$, $a = 1$, $c = 2$

11. $\lim\limits_{x\to 0^+}\dfrac{e^{\sqrt{\tan x}}-e^{\sqrt{x}}}{\sqrt{\tan x}-\sqrt{x}}$ is equal to

(1) 1 (2) $\dfrac{1}{2}$

(3) 2 (4) $\dfrac{3}{2}$

Answer (1)

Sol. $\lim\limits_{x\to 0^+}\dfrac{e^{\sqrt{x}}\left(e^{\sqrt{\tan x}-\sqrt{x}}-1\right)}{\sqrt{\tan x}-\sqrt{x}}=1$

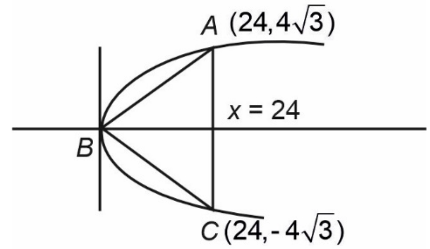
12. A triangle is drawn inside bounded region of $y^2 = 2x$ and $x = 24$. Then maximum area of triangle is

(1) $64\sqrt{3}$ (2) $108\sqrt{3}$

(3) $96\sqrt{3}$ (4) $120\sqrt{3}$

Answer (3)

Sol.



Base is constant i.e., $AC = 8\sqrt{3}$

Area of ΔABC will be maximum when height is maximum

$\therefore$ B is (0, 0)

$\therefore$ $(\text{Area})_{\max} = = \dfrac{1}{2}\times 24\times 8\sqrt{3}=96\sqrt{3}$

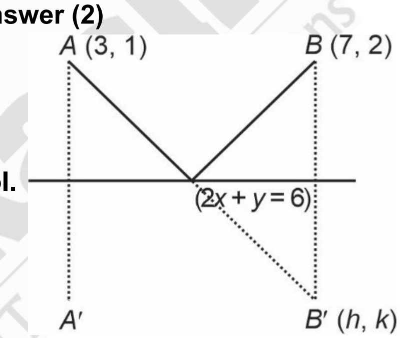
13. A ray of light coming from (3, 1) incident on $2x + y = 6$ and deflected ray passing through (7, 2). If equation of incident ray is $ax + by + 1 = 0$, then $a^2 + b^2 + 3ab =$

(1) $\dfrac{11}{25}$ (2) $-\dfrac{11}{25}$

(3) $-\dfrac{3}{5}$ (4) $\dfrac{3}{5}$

Answer (2)

Sol.



$\dfrac{h-7}{2}=\dfrac{k-2}{1}=-2\dfrac{(14+2-6)}{5}=-4$

$\therefore$ $h = -1$, $k = -2$

$\therefore$ B' (–1, –2)

$\therefore$ $AB' \equiv \dfrac{4}{5}y-\dfrac{3}{5}x+1=0$

$a=\dfrac{4}{5},\ b=\dfrac{-3}{5}$

$a^2+b^2+3ab=\dfrac{16}{25}+\dfrac{9}{25}-\dfrac{36}{25}=\dfrac{-11}{25}$

14.
15.
16.
17.
18.
19.
20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If $|x+1||x+3| - 4|x+2| + 5 = 0$, then sum of squares of solutions is

Answer (16.00)

Sol. Let $t = x + 2$

$\Rightarrow\ |t^2 - 1| - 4|t| + 5 = 0$

(1) $t \in [1, \infty)$

$t^2 - 4t + 4 = 0 \Rightarrow t = 2$

(2) $t \in [0, 1]$

$\Rightarrow\ (t+2)^2 = 10 \Rightarrow t = \sqrt{10} - 2$ or $t = -\sqrt{10} - 2$

No solution

(3) $t \in [-1, 0]$

$\Rightarrow\ (t+2)^2 = 10 \Rightarrow$ Again no solution

(4) $t \in (-\infty, -1)$, $(t+2)^2 = 0$, $t = -2$

$\Rightarrow\ x + 2 = 2 \Rightarrow x = 0$

$x + 2 = -2 \Rightarrow x = -4$

$\Rightarrow$ Sum of squares $= 0^2 + (-4)^2 = 16$

22. For a given GP if sum of $T_2 + T_6 = \frac{70}{3}$ and product $(T_3 \times T_5) = 49$ and common ratio $r > 1$ then find the sum of $(T_4 + T_6 + T_8)$

Answer (91)

Sol. $T_2 + T_6 = \frac{70}{3}$

$ar + ar^5 = \frac{70}{3} \Rightarrow ar(1 + r^4) = \frac{70}{3}$ …(1)

$ar^2 \times ar^4 = 49$

$\Rightarrow\ a^2r^6 = 49$

$\Rightarrow\ (ar^3)^2 = 49$

$\Rightarrow\ ar^3 = \pm 7$ …(2)

Multiply equation (1) by r^2

$ar^3(1 + r^4) = \frac{70}{3} r^2$

$(1 + r^4) = \frac{10}{3} r^2$

$r^4 - \frac{10}{3} r^2 + 1 = 0$

$3r^4 - 10r^2 + 3 = 0$

$3r^4 - 9r^2 - r^2 + 3 = 0$

$3r^2(r^2 - 3) - 1(r^2 - 3) = 0$

$r^2 = 3,\ r^2 = \frac{1}{3}$

Now, $T_4 + T_6 + T_8$

$= ar^3(1 + r^2 + r^4)$

$= 7\,(1 + 3 + 9) = 91$

23. If $\int \frac{dx}{\sqrt[5]{(x-1)^4 (x+3)^6}} = A\left(\frac{ax-1}{bx-3}\right)^B$ then $a + b$ $20AB$ is equal to

Answer (12)

Sol. Let $\left(\frac{x+3}{x-1}\right) = t \Rightarrow x = \left(\frac{3+t}{t-1}\right)$

$\Rightarrow\ dx = \frac{(t-1)(1) - 1(3+t)}{(t-1)^2} = \frac{-4}{(t-1)^2} dt$

$(x-1)^4 (x+3)^6 = \left((x-1)(x+3)\right)^5 \left(\frac{x+3}{x-1}\right)$

$$\Rightarrow\ I = \int \frac{\frac{-4}{(t-1)^2} dt}{t^{\frac{1}{5}} \left(\frac{3+t}{t-1} - 1\right)\left(\frac{3+t}{t-1} + 3\right)}$$

$$I = \int \frac{-4\, dt}{t^{\frac{1}{5}} [t + 3 - (t-1)]\,[3 + t + 3t - 3]}$$

$$I = \int \frac{-4\,dt}{2t^{\frac{1}{5}}(4t)}$$

$$= \frac{(-4)}{8}\int \frac{dt}{t^{\frac{6}{5}}} = \frac{(-4)}{8}\,\frac{\left(\frac{x+3}{x-1}\right)^{\frac{-1}{5}}}{\frac{-1}{5}}$$

$$\Rightarrow \quad I = \frac{5}{2}\left(\frac{x-1}{x+3}\right)^{\frac{1}{5}}$$

Comparing, $A = \frac{5}{2}, B = \frac{1}{5}$, a = 1, b = 1

$\Rightarrow$ $a + b + 20AB = 1+1+20\times\frac{1}{2} = 12$

24. Given $f(x) = \begin{cases} \dfrac{\tan((a+1)x) + \tan x \cdot b}{x} & x < 0 \\ 3 & x = 0 \\ \dfrac{\sqrt{ax+b^2x^2} - \sqrt{ax}}{\sqrt{a}\; bx\sqrt{x}} & x > 0 \end{cases}$

It is continuous function at x = 0, then find $\frac{b}{a}$.

Answer (6)

Sol. $\lim\limits_{x\to 0^-} f(x) = f(0) = \lim\limits_{x\to 0^+} f(x)$

$\Rightarrow$ $a + 1 + b = 3$

$$\lim_{x\to 0^+} \frac{\sqrt{ax+b^2x^2} - \sqrt{ax}}{\sqrt{abx}\sqrt{x}}$$

$$= \lim_{x\to 0^+} \frac{\sqrt{a+b^2x} - \sqrt{a}}{\sqrt{abx}}$$

Rationalising

$$= \lim_{x\to 0} \frac{b^2x}{\sqrt{abx}\left(\sqrt{a+b^2x} + \sqrt{a}\right)}$$

$$3 = \frac{b}{2a}$$

$$\Rightarrow \frac{b}{a} = 6$$

25. If complex number $\frac{1+2i\cos\theta}{1-3i\cos\theta}$ is purely imaginary, then find the number of values of θ in the interval $[-2\pi, 2\pi]$.

Answer (08.00)

Sol. $\because$ $\frac{1+2i\cos\theta}{1-3i\cos\theta} = \frac{(1+2i\cos\theta)(1+3i\cos\theta)}{1+9\cos^2\theta}$ = purely imaginary

$\therefore$ $1 - 6\cos^2\theta = 0$

$\Rightarrow$ $\cos^2\theta = \frac{1}{6}$

$\therefore$ $\cos\theta = \pm\frac{1}{\sqrt{6}}$

$\therefore$ Total 8 solutions

26. If solution of differential equation $xdy - ydx = xy(xdy - ydx)$ and $y(1) = 1$ is $\alpha|y| = |x|e^{(xy-\beta)}$ then $(\alpha + \beta)$ is equal to

Answer (02.00)

Sol. $(xdy - ydx) = xy(xdy + ydx)$

$\Rightarrow$ $\frac{dy}{y} - \frac{dx}{x} = xdy + ydx = d(xy)$

$\Rightarrow$ $\ln|y| - \ln|x| = xy + c$

$\Rightarrow$ $\ln\left(\left|\frac{y}{x}\right|\right) = (xy + c)$ $\quad\Rightarrow$ $\left|\frac{y}{x}\right| = Ae^{xy}$, $A > 0$

$y(1) = 1$

$\Rightarrow$ $1 = Ae^1$ $\quad\Rightarrow$ $A = \frac{1}{e}$

$\Rightarrow$ $\left|\frac{y}{x}\right| = e^{(xy-1)}$

$\Rightarrow$ $|y| = |x|e^{(xy-1)}$

27.

28.

29.

30.

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