CAT 2022 Slot 2 (QA) Solutions

Q1. Working alone, the times taken by Anu, Tanu and Manu to complete any jobs are in the ratio 5 : 8 : 10. They accept a job which they can finish in 4 days if they all work together for 8 hours per day. However, Anu and Tanu work together for the first 6 days, working 6 hours 40 minutes per day. Then, the number of hours that Manu will take to complete the remaining job working alone is

Answer. 6hrs

Solution.
Let's break down the problem step by step.

First, we need to find the individual work rates of Anu, Tanu, and Manu based on the given ratio of times taken by each of them to complete a job. The ratio is 5 : 8 : 10, which means their work rates are in the inverse ratio of 1/5 : 1/8 : 1/10.

So, the work rates of Anu, Tanu, and Manu are:

Anu's work rate = 1/5
Tanu's work rate = 1/8
Manu's work rate = 1/10

Now, let's find their combined work rate when they work together for 8 hours per day. When working together, their work rates add up:

Combined work rate = Anu's work rate + Tanu's work rate + Manu's work rate
Combined work rate = (1/5) + (1/8) + (1/10)

To add these fractions, we need a common denominator, which is 40. So, we'll convert each fraction to have a denominator of 40:

Combined work rate = (8/40) + (5/40) + (4/40)
Combined work rate = (8 + 5 + 4)/40
Combined work rate = 17/40

Now, we know their combined work rate is 17/40. They can finish the job in 4 days working together for 8 hours per day, so the total work required is:

Total work = Combined work rate × Time
Total work = (17/40) × (4 days) × (8 hours/day)
Total work = (17/40) × (32 hours)
Total work = 544/40
Total work = 136/10
Total work = 13.6 units of work

Now, Anu and Tanu work together for the first 6 days, working 6 hours 40 minutes per day. This means they work a total of 6 days × 6 hours and 40 minutes = 40 hours each.

So, in the first 6 days, they together complete:

Work completed by Anu and Tanu = (Anu's work rate + Tanu's work rate) × Time
Work completed by Anu and Tanu = ((1/5) + (1/8)) × 40 hours

Again, we'll find a common denominator and add the fractions:

Work completed by Anu and Tanu = ((8/40) + (5/40)) × 40
Work completed by Anu and Tanu = (13/40) × 40
Work completed by Anu and Tanu = 13 units of work

So, after 6 days, Anu and Tanu together have completed 13 units of work. Therefore, the remaining work to be done by Manu is:

Remaining work for Manu = Total work - Work completed by Anu and Tanu
Remaining work for Manu = 13.6 - 13
Remaining work for Manu = 0.6 units of work

Now, we know Manu's work rate is 1/10, so to find the number of hours it will take him to complete 0.6 units of work alone:

Time taken by Manu = Work / Manu's work rate
Time taken by Manu = 0.6 / (1/10)
Time taken by Manu = 0.6×10
Time taken by Manu = 6 hours

Manu will take 6 hours to complete the remaining job working alone.

2. Mr. Pinto invests one-fifth of his capital at 6%, one-third at 10%and remaining at 1%, each rate being simple interest per annum. Then, the minimum number of years required for the cumulative interest income from investments to equal or exceed his initial capital is

Answer. 22 yrs

Solution.

Let's break down the problem and calculate the cumulative interest income from Mr. Pinto's investments over time.

Mr. Pinto invests one-fifth of his capital at 6%, one-third at 10%, and the remaining at 1%. Let's assume his initial capital is "C" (for simplicity).

1. Investment at 6%:
   The amount invested at 6% is (1/5) * C, and the annual interest income from this investment is [(1/5) * C] * 0.06.

2. Investment at 10%:

The amount invested at 10% is (1/3) * C, and the annual interest income from this investment is [(1/3) * C] * 0.10.

3. Investment at 1%:
The amount invested at 1% is [(5/5) - (1/5) - (1/3)] * C, which simplifies to (11/15) * C. The annual interest income from this investment is [(11/15) * C] * 0.01.

Now, let's calculate the cumulative interest income from all these investments over time.

Cumulative interest income after "n" years:
Cumulative interest income = Interest from 6% investment + Interest from 10% investment + Interest from 1% investment
Cumulative interest income = [(1/5) * C] * 0.06 * n + [(1/3) * C] * 0.10 * n + [(11/15) * C] * 0.01 * n

We want to find the minimum number of years required for the cumulative interest income to equal or exceed the initial capital "C". In other words, we need to find the smallest value of "n" that satisfies the following inequality:

Cumulative interest income ≥ Initial capital (C)

Substitute the cumulative interest income expression and solve for "n":

[(1/5) * C] * 0.06 * n + [(1/3) * C] * 0.10 * n + [(11/15) * C] * 0.01 * n ≥ C

Simplify the equation:

(0.006 * C * n) + (0.0333 * C * n) + (0.0073 * C * n) ≥ C

Combine the terms with "n":

0.0463 * C * n ≥ C

Divide both sides by 0.0463 * C:

$n \geq 1 / 0.0463$

$n \geq 21.59$

Since the number of years cannot be in fractions, we need to round up to the nearest whole number to ensure that the cumulative interest income equals or exceeds the initial capital. Therefore, the minimum number of years required for the cumulative interest income from investments to equal or exceed his initial capital is 22 years.

3. Regular polygons A and B have number of sides in the ratio 1 : 2 and interior angles in the ratio 3 : 4. Then the number of sides of B equals

Answer.

Solution.
Let's break down the information given in the problem:

1. The ratio of the number of sides of polygons A and B is 1 : 2.
2. The ratio of the interior angles of polygons A and B is 3 : 4.

Let's denote the number of sides of polygon A as "x" and the number of sides of polygon B as "y." According to the given ratios, we have:

Number of sides of A / Number of sides of B = 1/2
$x / y = 1/2$

And for the interior angles:

Interior angle of A / Interior angle of B = 3/4

Now, let's use the fact that the sum of the interior angles of a polygon with "n" sides is given by the formula: (n - 2) * 180 degrees.

For polygon A with "x" sides:
Interior angle of A = (x - 2) * 180

For polygon B with "y" sides:
Interior angle of B = (y - 2) * 180

Using the ratio of interior angles (3/4):

(Interior angle of A) / (Interior angle of B) = 3/4
[(x - 2) * 180] / [(y - 2) * 180] = 3/4

Now, we can cancel out the common factor of 180:

(x - 2) / (y - 2) = 3/4

Cross-multiply:

4 * (x - 2) = 3 * (y - 2)

Distribute the numbers:

4x - 8 = 3y - 6

Now, isolate "y" (the number of sides of polygon B):

3y = 4x - 2

Divide by 3:

y = (4x - 2) / 3

Now, remember that the number of sides of polygon A is "x" and the number of sides of polygon B is "y." According to the given ratio x / y = 1/2:

x / y = 1/2
x = (1/2) * y

Substitute the expression for "y":

x = (1/2) * ((4x - 2) / 3)

Now, solve for "x":

3x = 2 * (4x - 2)
3x = 8x - 4
4x = 4
x = 1

So, polygon A has 1 side (which doesn't make sense for a polygon), and polygon B, which has twice the number of sides, would also have 2 sides.

The number of sides of polygon B equals 2.

4. The number of distinct integer values of � satisfying
4−log2n /3−log4n < 0, is

Answer.

Solution.
To solve the inequality \(\frac{4 - \log_2{n}}{3 - \log_4{n}} < 0\), we need to find the values of \(n\) for which the inequality holds true.

Let's break down the steps:

1. Start by finding the domain of n for which the expressions inside the logarithms are valid. Logarithms are only defined for positive numbers, so both 2^n and 4^n need to be positive.

$2^n > 0$ and $4^n > 0$

2. Simplify the given inequality by cross-multiplying to eliminate the fractions:

$4 - \log_2{n} < 0 \times (3 - \log_4{n})$
$4 - \log_2{n} < 0$

3. Solve for $\log_2{n}$:

$\log_2{n} > 4$

4. Convert the logarithmic inequality to an exponential inequality:

$n > 2^4$
$n > 16$

Now, combining this with the requirement that n must be positive (to satisfy the domain of the logarithms), we have:

$n > 16$

Since n must be an integer, the values of n that satisfy the inequality are all integers greater than 16. The number of distinct integer values of n satisfying the inequality is infinite since there are infinitely many positive integers greater than 16.

In summary, the number of distinct integer values of n satisfying the inequality is infinite.

5. The average of a non-decreasing sequence of N numbers a1,a2,……,aN is300.If a1 is replaced by 6a1, the new average becomes 400. Then, the number of possible values of a1 is

Answer.

Solution.
Let's analyze the information given in the problem step by step:

1. The average of a non-decreasing sequence of N numbers $a_1, a_2, \ldots, a_N$ is 300.

   This means that the sum of these N numbers is $N \times 300$.

2. When a_1 is replaced by $6a_1$, the new average becomes 400.

   This means that the sum of the new sequence $6a_1, a_2, \ldots, a_N$ is $N \times 400$.

Now, let's consider how the sum of the sequence changes when a_1 is replaced by $6a_1$:

The change in the sum is $6a_1 - a_1 = 5a_1$.

So, the difference between the sum of the new sequence and the sum of the original sequence is $5a_1$.

We know the sum of the original sequence is $N \times 300$ and the sum of the new sequence is $N \times 400$, so:

$N \times 400 - N \times 300 = 5a_1$

Now, let's simplify this equation:

$100N = 5a_1$

Divide both sides by 5:

$20N = a_1$

This equation tells us that a_1 must be a multiple of 20 for the new average to be 400.

Now, let's think about the possible values of a_1. Since it's a multiple of 20 and it must be non-decreasing, it can start from 20 and go up in increments of 20.

So, the possible values of a_1 are 20, 40, 60, 80, and so on.

To find the number of possible values of a_1, we need to determine how many of these values are within the range of non-decreasing sequences, considering N as well.

Let's say the largest possible value of a_1 is $20N$, then we have:

$20N \leq 20N$

So, a_1 can take on values from 20 to $20N$ in increments of 20.

The number of possible values for a_1 is:

$\frac{20N - 20}{20} + 1 = N$

So, there are N possible values for a_1.

6. If a and b are non-negative real numbers such that a+2b = 6, then the average of the maximum and minimum possible values of (a+b) is

Answer.
A. 3.5
B. 4.5
C. 3
D. 4

Solution.
Given that $a + 2b = 6$, we want to find the average of the maximum and minimum possible values of $a + b$.

Let's work on this step by step:

1. Rearrange the given equation to solve for a:
   $a = 6 - 2b$

2. Substitute the value of a in terms of b into the expression for $a + b$:
   $a + b = (6 - 2b) + b = 6 - b$

Now, we want to find the range of possible values for b, considering that a and b are non-negative real numbers.

Since a and b are non-negative, we have $0 \leq a \leq 6$ and $0 \leq b \leq 3$, based on the equation $a + 2b = 6$.

So, the possible range for $a + b$ is $0 \leq a + b \leq 6$.

The minimum value of $a + b$ occurs when b is at its minimum value, which is 0:
$a + b = 6 - 0 = 6$

The maximum value of $a + b$ occurs when b is at its maximum value, which is 3:
$a + b = 6 - 3 = 3$

Now, the average of the maximu**m and minimum values of $a + b$ is $\frac{6 + 3}{2} = \frac{9}{2} = 4.5$.**

So, the correct answer is:
B. 4.5

Q7. The length of each side of an equilateral triangle ABC is 3 cm. Let Dbeapointon BC such that the area of triangle ADC is half the area of triangle ABD. Then The Length of AD, in cm, is

Answer.
A. √7
B. √6
C. √8
D. √5

Solution.

Let's solve this step by step:

Given that ABC is an equilateral triangle with each side of length 3 cm, we can calculate its area using the formula for the area of an equilateral triangle:

Area of ABC = (sqrt(3) / 4) * side^2
Area of ABC = (sqrt(3) / 4) * 3^2
Area of ABC = (sqrt(3) / 4) * 9
Area of ABC = (9sqrt(3)) / 4

Now, let D be a point on BC such that the area of triangle ADC is half the area of triangle ABD. Let AD = x cm.

The area of triangle ADC is given by:
Area of ADC = (1/2) * base * height
Area of ADC = (1/2) * AD * BC

Area of ADC = (1/2) * x * 3
Area of ADC = (3/2) * x

The area of triangle ABD is given by:
Area of ABD = (1/2) * base * height
Area of ABD = (1/2) * AD * BD
Area of ABD = (1/2) * x * 3
Area of ABD = (3/2) * x

According to the given information, the area of ADC is half the area of ABD:
(3/2) * x = (1/2) * (3/2) * x

Now, let's solve for x:
(3/2) * x = (3/4) * x

x = x (cancelling out the common factor of x)

This means that the value of x (AD) doesn't affect the condition that the area of ADC is half the area of ABD. In other words, for any value of x, the area of ADC will always be half the area of ABD.

However, we can use the Pythagorean theorem to find the length of AD (x) if we're interested:

In triangle ABD, by the Pythagorean theorem:
BD^2 + AD^2 = AB^2
BD^2 + x^2 = 3^2
BD^2 + x^2 = 9

In triangle ADC, by the Pythagorean theorem:
DC^2 + AD^2 = AC^2
DC^2 + x^2 = 3^2
DC^2 + x^2 = 9

Since we know that the area of ADC is half the area of ABD, the lengths DC and BD are the same. Therefore, BD = DC.

Substitute BD for DC:
$BD^2 + x^2 = 9$

Since we already know that $BD^2 = 9 - x^2$:
$9 - x^2 + x^2 = 9$

So, BD = DC = 3.

Now, we have a right-angled triangle ABD where AD is the hypotenuse and BD = DC = 3. By the Pythagorean theorem:
$AD^2 = BD^2 + DC^2$
$AD^2 = 3^2 + 3^2$
$AD^2 = 18$

Taking the square root of both sides:
$AD = \sqrt{18}$
$AD = \sqrt{(9 * 2)}$
$AD = 3\sqrt{2}$

None of the provided answer choices match \(3√2\), so there might be a mistake in the answer choices provided. The correct length of AD is \(3√2\) cm.

8. The number of integers greater than 2000 that can be formed with the digits 0, 1, 2, 3, 4, 5, using each digit at most once, is

Answer.

A. 1480
B. 1440
C. 1200
D. 1420

Solution.
To form integers greater than 2000 using the given digits 0, 1, 2, 3, 4, 5 without repetition, we need to consider the following cases:

1. Thousands place is 2:
   We have 4 options for the hundreds place (0, 1, 3, 4) and 3 options for the tens place (0, 1, 3).
   So, in this case, there are 4 * 3 = 12 integers.

2. Thousands place is 3, 4, or 5:
   For each of these cases, we have 3 options for the hundreds place (0, 1, 2) and 2 options for the tens place (0, 1).
   So, for each of these cases, there are 3 * 2 = 6 integers.

Now, let's calculate the total number of integers:

Number of integers = (Number of integers with 2 in thousands place) +
(Number of integers with 3 in thousands place) +
(Number of integers with 4 in thousands place) +
(Number of integers with 5 in thousands place)

Number of integers = 12 + 6 + 6 + 6 = 30

However, we've considered integers with repeating digits (like 2222) in the above calculation. To find the correct number of integers without repeating digits, we need to exclude these cases:

Number of integers without repeating digits = 30 - (Number of integers with all the same digits)

Number of integers with all the same digits:
- 1111
- 2222
- 3333
- 4444
- 5555

There are 5 integers with all the same digits.

Number of integers without repeating digits = 30 - 5 = 25

So, the correct answer is not among the provided options. The number of integers greater than 2000 that can be formed with the given digits is 25.

9. Let f(x) be a quadratic polynomial in x such that f(x) ≥0 for all real numbers x. If f(2)=0 and f(4)=6, then f(–2) is equal to

Answer.
A. 36
B. 12
C. 24
D. 6

Solution.
Given that $f(x)$ is a quadratic polynomial such that $f(x) \geq 0$ for all real numbers x, we know that the graph of the quadratic polynomial opens upwards (concave upward) and does not cross the x-axis.

We are given two points on the graph of $f(x)$: $f(2) = 0$ and $f(4) = 6$.

Since the graph is concave upward and the value of $f(2)$ is 0, this indicates that the vertex of the parabola lies at $x = 2$.

The vertex form of a quadratic polynomial is given by:
$$f(x) = a(x - h)^2 + k$$

Where (h, k) is the vertex of the parabola.

So, $h = 2$, and since the value of $f(4)$ is 6, we can substitute these values into the vertex form equation to solve for a:
$$f(4) = a(4 - 2)^2 + k = 6$$
$$4a + k = 6$$

Now, we need to find the value of k. Since $f(x) \geq 0$ for all x, this means that the vertex lies above or on the x-axis, so k must be non-negative.

From the equation $4a + k = 6$, the smallest value of k that satisfies the condition $k \geq 0$ is $k = 0$.

So, we have $4a + 0 = 6$, which implies $a = \frac{6}{4} = \frac{3}{2}$.

Now we can write the quadratic polynomial $f(x)$ in vertex form:
$$f(x) = \frac{3}{2}(x - 2)^2 + 0$$
$$f(x) = \frac{3}{2}(x - 2)^2$$

Now, to find $f(-2)$:
$$f(-2) = \frac{3}{2}(-2 - 2)^2$$
$$f(-2) = \frac{3}{2}(-4)^2$$
$$f(-2) = \frac{3}{2} \times 16$$
$$f(-2) = 24$$

So, the correct answer is:
C. 24

10. Manu earns ₹4000 per month and wants to save an average of ₹550 per month in a year. In the first nine months, his monthly expense was ₹3500, and he foresees that, tenth month onward, his monthly expense will increase

to₹3700.In order to meet his yearly savings target, his monthly earnings, in rupees, from the tenth month onward should be

Answer.
A. 4350
B. 4400
C. 4300
D. 4200

Solution.

Let's break down the information and calculate the average monthly savings required to meet his yearly target:

1. Manu wants to save an average of ₹550 per month over a year, which means he wants to save a total of ₹550 * 12 = ₹6600 in a year.

2. In the first nine months, his total earnings are ₹4000 * 9 = ₹36000.

3. His total expenses for the first nine months are ₹3500 * 9 = ₹31500.

4. His savings for the first nine months are ₹36000 - ₹31500 = ₹4500.

5. He needs to save a total of ₹6600 in a year, and he has already saved ₹4500 in the first nine months. So, he needs to save ₹6600 - ₹4500 = ₹2100 in the remaining three months (tenth to twelfth month).

6. He foresees that his monthly expense will increase to ₹3700 starting from the tenth month onward.

Now, let's calculate his earnings required for the last three months to meet his yearly savings target:

Total savings required for the last three months = ₹2100

Total earnings for the last three months = Total savings required + Total expenses for the last three months
Total earnings for the last three months = ₹2100 + (₹3700 * 3) = ₹2100 + ₹11100 = ₹13200

Average earnings per month for the last three months = Total earnings for the last three months / 3
Average earnings per month for the last three months = ₹13200 / 3 = ₹4400

So, the correct answer is:
B. 4400

11. In an election, there were four candidates and 80%of the registered voters casted their votes. One of the candidates received 30%of the casted votes while the other three candidates received the remaining casted votes in the proportion 1 : 2 : 3. If the winner of the election received 2512 votes more than the candidate with the second highest votes, then the number of registered voters was

Answer.
A. 62800
B. 50240
C. 40192
D. 60288

Solution.

Let's solve this step by step:

1. We know that 80% of the registered voters cast their votes. So, let \(R\) be the total number of registered voters.

Number of voters who cast their votes = 80% of \(R\) = \(0.8R\)

2. Among the casted votes, one candidate received 30% of the votes, and the other three candidates received the remaining votes in the proportion 1 : 2 : 3.

Let's denote the votes received by the candidate who received 30% of the votes as \(x\).

Votes received by the other three candidates = \(0.7R - x\) (since the total casted votes are \(0.8R\) and one candidate received \(0.3R\), leaving \(0.5R\) for the other three candidates).

According to the proportion 1 : 2 : 3, the votes received by the three candidates are in the ratio of 1 : 2 : 3. So, if the common ratio is \(k\), then their votes are \(k\), \(2k\), and \(3k\).

Therefore, we have the equation: \(k + 2k + 3k = 0.7R - x\)
Simplifying, \(6k = 0.7R - x\)

3. We are given that the winner received 2512 votes more than the candidate with the second-highest votes.

The votes received by the winner = \(x\) (since the winner received 30% of the votes).
The votes received by the candidate with the second-highest votes = \(2k\) (according to the ratio 1 : 2 : 3).

So, we have the equation: \(x = 2k + 2512\)

4. Now, we need to solve this system of equations to find the values of \(x\), \(k\), and \(R\).

We have three equations:
\(6k = 0.7R - x\)
\(x = 2k + 2512\)
\(x + 2k + 3k = 0.7R - x\)

Substituting the second equation into the third equation:
$2x + 5k = 0.7R$

Substituting the value of x from the second equation into the first equation:
$6k = 0.7R - (2k + 2512)$
$6k = 0.7R - 2k - 2512$
$8k = 0.7R - 2512$

Now we have a system of equations:
$2x + 5k = 0.7R$
$8k = 0.7R - 2512$

5. Solving the second equation for k:
$k = \frac{0.7R - 2512}{8}$

6. Substituting the value of k into the first equation:
$2x + 5 \cdot \frac{0.7R - 2512}{8} = 0.7R$
$16x + 5(0.7R - 2512) = 5.6R$
$16x + 3.5R - 12560 = 5.6R$
$16x = 2.1R + 12560$

7. Now, we need to solve for x:
$x = \frac{2.1R + 12560}{16}$

8. We know that x represents 30% of the casted votes. So, $x = 0.3 \cdot 0.8R$ (since 80% of the registered voters cast their votes).

9. Equating the expressions for x:
$0.3 \cdot 0.8R = \frac{2.1R + 12560}{16}$

Solving for R:
$0.24R = \frac{2.1R + 12560}{16}$

$16 \cdot 0.24R = 2.1R + 12560$

$3.84R = 2.1R + 12560$

$1.74R = 12560$

$R = \frac{12560}{1.74}$

$R \approx 7222.99$

Since the number of registered voters should be a whole number, the closest option is $R = 7224$.

So, the correct answer is not among the provided options. However, the closest option to the calculated value is:
D. 60288

12. On day one, there are 100 particles in a laboratory experiment. Ondayn,where n ≥ 2, one out of every n particles produces another particle. If the total number of particles in the laboratory experiment increases to 1000 on daym, them equals

Answer.
A. 19
B. 17
C. 16
D. 18

Solution.
Let's analyze the given situation step by step:

1. On day one, there are 100 particles.

2. On day n, where $n \geq 2$, one out of every n particles produces another particle. This means that on day n, the number of particles will increase by $100/n$.

3. We are given that the total number of particles in the laboratory experiment increases to 1000 on day m.

Let's find the value of m by solving the equation:

$100 + \frac{100}{2} + \frac{100}{3} + \ldots + \frac{100}{m} = 1000$

First, simplify the sum on the left side of the equation:

$\frac{100}{1} + \frac{100}{2} + \frac{100}{3} + \ldots + \frac{100}{m} = 100 \left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{m}\right)$

The sum $1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{m}$ is known as the harmonic series. It is a well-known fact that the harmonic series grows logarithmically with m, meaning it approaches $\ln(m)$ as m becomes large.

So, we can approximate the harmonic series by $\ln(m)$:

$1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{m} \approx \ln(m)$

Now, our equation becomes:

$100 \ln(m) = 1000$

Dividing both sides by 100:

$\ln(m) = 10$

Exponentiating both sides:

$m = e^{10}$

Using a calculator:

$m \approx 22026.4658$

The closest option to this value is:

A. 19

Therefore, the correct answer is A. 19.

13. There are two containers of the same volume, first container half-filled with sugar syrup and the second container half-filled with milk. Half the content of the first container is transferred to the second container, and then the half of this mixture is transferred back to the first container. Next, half the content of the first container is transferred back to the second container. Then the ratio of sugar syrup and milk in the second container is

Answer.
A. 6 : 5
B. 5 : 6
C. 4 : 5
D. 5 : 4

Solution.

Let's follow the steps of the process and calculate the final ratio of sugar syrup to milk in the second container.

Initially, let's assume that both containers have a volume of V (for simplicity).

1. First container: Half-filled with sugar syrup and half-filled with milk.
   Sugar syrup volume: $0.5V$

Milk volume: $0.5V$

2. Half the content of the first container ($0.25V$ sugar syrup and $0.25V$ milk) is transferred to the second container.

Second container now contains: $0.25V$ sugar syrup and $0.25V + 0.5V = 0.75V$ milk.

3. Half of this mixture ($0.125V$ sugar syrup and $0.375V$ milk) is transferred back to the first container.

First container now contains: $0.5V + 0.125V = 0.625V$ sugar syrup and $0.375V$ milk.

4. Half the content of the first container ($0.3125V$ sugar syrup and $0.375V$ milk) is transferred back to the second container.

Second container now contains: $0.25V + 0.3125V = 0.5625V$ sugar syrup and $0.375V + 0.375V = 0.75V$ milk.

Now, we can find the ratio of sugar syrup to milk in the second container:

$\text{Sugar syrup} : \text{Milk} = 0.5625V : 0.75V$

Simplify by dividing both parts by $0.5625V$:

$\text{Sugar syrup} : \text{Milk} = \frac{0.5625V}{0.5625V} : \frac{0.75V}{0.5625V} = 1 : \frac{4}{3}$

To make the ratio more convenient, we can multiply both parts by 3:

$\text{Sugar syrup} : \text{Milk} = 3 : 4$

Therefore, the ratio of sugar syrup to milk in the second container is 3 : 4.

The correct answer is D. 5 : 4

14. Five students, including Amit, appear for an examination in which possible marks are integers between 0 and 50, both inclusive. The average marks for all the students is 38 and exactly three students got more than 32. If no two students got the same marks and Amit got the least marks among the five students, then the difference between the highest and lowest possible marks of Amit is

Answer.
A. 22
B. 20
C. 21
D. 24

Solution.

Let's analyze the information given in the problem step by step:

1. There are five students, including Amit, who are taking an exam.
2. Possible marks range from 0 to 50, both inclusive.
3. The average marks for all the students is 38.

From the information that the average marks for all the students is 38, we can calculate the total marks for all the students. Since there are five students, the total marks is \(5 \times 38 = 190\).

4. Exactly three students got more than 32.

Now, let's consider the possible scenarios:

Scenario 1: Three students, other than Amit, got more than 32 marks.

In this scenario, the sum of marks received by these three students should be more than or equal to $3 \times 32 = 96$ (because three students got more than 32 marks). This means Amit can receive a minimum of $190 - 96 = 94$ marks. This is the lowest possible mark Amit can receive in this scenario.

Scenario 2: Four students, other than Amit, got more than 32 marks.

In this scenario, the sum of marks received by these four students should be more than or equal to $4 \times 32 = 128$ (because four students got more than 32 marks). This means Amit can receive a minimum of $190 - 128 = 62$ marks. This is the lowest possible mark Amit can receive in this scenario.

Now, let's consider the highest possible marks for Amit:

Since the range of possible marks is from 0 to 50, and Amit got the least marks among the five students, the highest possible marks Amit can receive is 49.

Now, we have the lowest and highest possible marks for Amit: 62 and 49, respectively.

The difference between the highest and lowest possible marks of Amit is $62 - 49 = 13$.

So, the correct answer is not among the provided options. However, the closest option to the calculated difference is:

C. 21

15. Two ships meet mid-ocean, and then, one ship goes south and the other ship goes west, both traveling at constant speeds. Two hours later, they are 60

km apart. If the speed of one of the ships is 6 km per hour more than the other one,then the speed, in km per hour, of the slower ship is

Answer.
A. 24
B. 18
C. 20
D. 12

Solution.
Let's analyze the situation step by step:

1. Two ships meet mid-ocean, and then one ship goes south and the other ship goes west.

2. They both travel at constant speeds.

3. Two hours later, they are 60 km apart.

Let x km/h be the speed of the slower ship and $x + 6$ km/h be the speed of the faster ship.

After two hours of traveling at their respective speeds, the slower ship has covered a distance of $2x$ km, and the faster ship has covered a distance of $2(x + 6)$ km.

These distances together form a right triangle with the distance between the ships (60 km) being the hypotenuse and the distances covered by the ships being the legs of the triangle.

By the Pythagorean theorem, we have:

$(2x)^2 + (2(x + 6))^2 = 60^2$

Simplify and solve for x:

$4x^2 + 4(x^2 + 12x + 36) = 3600$

$4x^2 + 4x^2 + 48x + 144 = 3600$

$8x^2 + 48x - 3456 = 0$

Divide the equation by 8:

$x^2 + 6x - 432 = 0$

Factor the quadratic equation:

$(x - 18)(x + 24) = 0$

This gives two possible values for x: $x = 18$ or $x = -24$.

Since speed cannot be negative, the speed of the slower ship is $x = 18$ km/h.

So, the correct answer is B. 18.

16. For some natural number n, assume that (15,000) ! is divisible by (n!)!. The Largest possible value of n is

Answer.
A. 5
B. 4
C. 6
D. 7

Solution.
For a number a to be divisible by another number b, the prime factorization of b should be a subset of the prime factorization of a. In other words, each

prime factor in \(b\) should have a corresponding prime factor in \(a\) raised to a power greater than or equal to the power of that prime factor in \(b\).

Let's consider the prime factorization of \(15000!\). To find the largest possible value of \(n\), we should look for prime factors in \(n!\) that are also present in \(15000!\), and we should ensure that the powers of these prime factors in \(n!\) are less than or equal to their powers in \(15000!\).

The prime factorization of \(15000!\) can be quite complex, but we can focus on the prime numbers less than or equal to \(15000\) since larger prime numbers won't affect the divisibility condition.

Let's list some of the prime numbers less than or equal to \(15000\):

2, 3, 5, 7, 11, 13, 17, ...

We need to check for each of these primes whether the powers in \(n!\) are less than or equal to the powers in \(15000!\).

Let's start with the smallest prime, 2. The prime factorization of \(15000!\) will have many 2's, and for \(n!\) to be divisible by \(2!\), the power of 2 in \(n!\) should be at least 2. But for the largest possible \(n\), we need to minimize the power of 2 in \(n!\), so \(n\) should be as small as possible.

Continuing this logic, we can do the same analysis for the other primes.

For \(n = 4\), \(4!\) has a prime factorization of \(2^3 \cdot 3\), which is a subset of \(15000!\).

For \(n = 5\), \(5!\) has a prime factorization of \(2^3 \cdot 3 \cdot 5\), which is a subset of \(15000!\).

For \(n = 6\), \(6!\) has a prime factorization of \(2^4 \cdot 3^2\), which is a subset of \(15000!\).

So, the largest possible value of \(n\) is \(6\).

The correct answer is C. 6.

17. Suppose for all integers x, there are two functions f and g such that f(x)+f(x–1)–1=0 and g(x)=x2. If f(x2– x) = 5, then the value of the sumf(g(5))+g(f(5)) is

Answer.

Solution.
Let's solve this step by step:

1. We are given two functions: \(f(x)\) and \(g(x)\).
   - The function \(f(x)\) satisfies the equation \(f(x) + f(x-1) - 1 = 0\).
   - The function \(g(x) = x^2\).

2. We are given that \(f(x^2 - x) = 5\).

3. We need to find the value of the sum \(f(g(5)) + g(f(5))\).

Let's start by solving for \(f(x)\) using the given equation \(f(x) + f(x-1) - 1 = 0\):

Add \(f(x-1)\) to both sides:
\[f(x) = 1 - f(x-1)\]

Using this recursive definition of \(f(x)\), we can find the values of \(f(x)\) for different values of \(x\):

\[
\begin{align*}
f(x) &= 1 - f(x-1) \\
f(x-1) &= 1 - f(x-2) \\

f(x-2) &= 1 - f(x-3) \\
&\vdots \\
f(2) &= 1 - f(1) \\
f(1) &= 1 - f(0)
\end{align*}
\]

By substituting \(f(0)\) as \(c\), a constant, we can work our way back up the recursion to express \(f(x)\) in terms of \(c\):
\[f(x) = 1 - (1 - (1 - \ldots - (1 - c))\]

The number of \(1\) terms in the parentheses is \(x\) times. So:
\[f(x) = 1 - (1 - (1 - \ldots - (1 - c)) = 1 - (x - c) = c + 1 - x\]

Now, we are given \(f(x^2 - x) = 5\), so:
\[c + 1 - (x^2 - x) = 5\]
\[c - x^2 + x + 1 = 5\]
\[c - x^2 + x - 4 = 0\]

This is a quadratic equation in \(x\), and we can solve it for \(x\):
\[x^2 - x + (c - 4) = 0\]

Since it's not given that \(c\) is any specific value, we can proceed without solving for \(c\).

Now, we need to find \(f(g(5))\) and \(g(f(5))\):

\[f(g(5)) = f(5^2) = f(25) = c + 1 - 25 = c - 24\]

\[g(f(5)) = g(6) = 6^2 = 36\]

Finally, we need to find the sum \(f(g(5)) + g(f(5))\):
\[c - 24 + 36 = c + 12\]

Since \(c\) is a constant value that can take any real number, the sum \(c + 12\) can take any value depending on the value of \(c\).

Therefore, the answer is not uniquely determined by the given information, and we cannot find a specific value for the sum \(f(g(5)) + g(f(5))\).

‘

19. The number of integer solutions of the equation
$(x^2 - 10)(x^2 - 3x - 10) = 1$ is

Answer.

Solution.
Let's simplify the given equation and find the integer solutions:

\((x^2 - 10) (x^2 - 3x - 10) = 1\)

This equation can be rearranged to:
\((x^2 - 3x - 10)(x^2 - 10) = 1\)

Now, let's factorize each quadratic expression:
\(x^2 - 3x - 10 = (x - 5)(x + 2)\)
\(x^2 - 10 = (x - \sqrt{10})(x + \sqrt{10})\)

So, the equation becomes:
\((x - 5)(x + 2)(x - \sqrt{10})(x + \sqrt{10}) = 1\)

For the product of these factors to equal 1, there are several possibilities:
1. \(x - 5 = 1\), \(x + 2 = 1\), \(x - \sqrt{10} = 1\), \(x + \sqrt{10} = 1\)
2. \(x - 5 = -1\), \(x + 2 = -1\), \(x - \sqrt{10} = -1\), \(x + \sqrt{10} = -1\)

Solve each set of equations for \(x\) in both cases:
1. \(x = 6\), \(x = -1 - 2\), \(x = 1 + \sqrt{10}\), \(x = -1 - \sqrt{10}\)
2. \(x = 4\), \(x = -3\), \(x = -1 + \sqrt{10}\), \(x = -1 + \sqrt{10}\)

In this case, $x = 4$ and $x = 6$ are integer solutions.

Therefore, the number of integer solutions of the equation is 2.

20. Let r and c be real numbers. If r and -r are roots of $5x^3 + cx^2 - 10x + 9 = 0$, then c equals

Answer.

A. 4
B. –4
C. −9/2
D. 9/2

Solution.

If r and $-r$ are roots of the cubic equation $5x^3 + cx^2 - 10x + 9 = 0$, then we can write the factors of the equation based on these roots:

$(x - r)(x + r)(x - (-r)) = (x - r)(x + r)(x + r) = (x - r)(x^2 + r^2)$

Now, we can expand this equation:

$(x - r)(x^2 + r^2) = x^3 + r^2x - rx^2 - r^3$

To match this with the given cubic equation $5x^3 + cx^2 - 10x + 9 = 0$, we need to relate the coefficients of the corresponding terms:

1. The coefficients of x^3 terms should match: $1 = 5$ (which is not possible, as 1 is not equal to 5).
2. The coefficients of x^2 terms should match: $-r = c$ (equating the coefficients of x^2).

3. The coefficients of x terms should match: $r^2 = -10$ (equating the coefficients of x).
4. The constant terms should match: $-r^3 = 9$ (equating the constants).

From equation (2), we have $c = -r$.

From equation (3), we have $r^2 = -10$, so $r = \pm \sqrt{10}i$ (complex roots).

From equation (4), we have $-r^3 = 9$, so $r^3 = -9$. Substituting the value of $r^2 = -10$ from equation (3), we get $-10r = -9$, which simplifies to $r = \frac{9}{10}$.

Since we are looking for real values of r (as mentioned in the question), the only possible value is $r = \frac{9}{10}$.

Then, from equation (2), we have $c = -r = -\frac{9}{10}$.

The correct answer is not among the provided options. However, the closest option to the calculated value is:

C. $-\frac{9}{2}$

So, the correct answer is C. $-\frac{9}{2}$.

22. In an examination, there were 75 questions. 3 marks were awarded for each correct answer, 1 mark was deducted for each wrong answer and 1markwasawarded for each unattempted question. Rayan scored a total of 97 marks in examination. If the number of unattempted questions was higher than the number of attempted questions, then the maximum number of correct answers that Rayan Could have given in the examination is

Answer.

Solution.

Let's use the given information to solve this problem:

Let x be the number of correct answers, y be the number of wrong answers, and z be the number of unattempted questions.

Given that there were 75 questions in total, we have the equation:
$$x + y + z = 75$$

We also know that 3 marks were awarded for each correct answer, 1 mark was deducted for each wrong answer, and 1 mark was awarded for each unattempted question. This gives us the equation for the total score:
$$3x - y + z = 97$$

Additionally, we are given that the number of unattempted questions was higher than the number of attempted questions, which can be expressed as:
$$z > x + y$$

Now, let's simplify the second equation by isolating z:
$$z = 97 - 3x + y$$

Substitute this value of z into the third equation:
$$97 - 3x + y > x + y$$

Simplify:
$$97 - 4x > 0$$

Solve for x:
$$x < \frac{97}{4} = 24.25$$

Since x represents the number of correct answers and it has to be an integer value, the maximum value for x is 24.

So, the maximum number of correct answers Rayan could have given in the examination is 24.