CAT 2021 QA Solution Slot 2

Q. 1)
For all possible integers n satisfying $2.25 \leq 2 + 2^{n+2} \leq 202$, then the number of integer values of $3+3^{n+1}$ is:

Solution.
The given inequality is:
$$2.25 \leq 2^{n+2} \leq 202$$

Let's break down this inequality into two parts.

From $2.25 \leq 2^{n+2}$:
This translates to $9/4 \leq 2^{n+2}$
Dividing both sides by 4, we get: $9/16 \leq 2^n$
Now, $2^4 = 16$. So, the inequality becomes:
$$9/2^4 \leq 2^n$$
From the above inequality, the smallest integer n for which 2^n exceeds $9/16$ is n = 0 (since $2^0 = 1$)

From $2^{n+2} \leq 202$:
This translates to $2^{n+2} \leq 2^8$ because $2^8 = 256$ which is the next power of 2 after 202.
From this, $n+2 \leq 8$, so $n \leq 6$.

Combining the two results, the possible integer values of n range from 0 to 6.

Now, for the number of integer values of 3^{n+1}:
Since n ranges from 0 to 6, $n+1$ ranges from 1 to 7. Therefore, the integer values of 3^{n+1} for n in this range are: $3^1, 3^2, 3^3, ... , 3^7$.

So, there are 7 integer values of 3^{n+1}

Q. 2)
Three positive integers x, y and z are in arithmetic progression. If y-x > 2 and xyz = 5(x+ y+ z), then z-x equals
[1] 8
[2] 12
[3] 14
[4] 10

Solution.
Given:
$$|x^2 - 4x - 13| = r$$

We know that $x^2 - 4x - 13$ is a quadratic, and the graph of a quadratic is a parabola. The absolute value function essentially takes any negative portion of the function and reflects it above the x-axis.

The equation $|x^2 - 4x - 13| = r$ will have three real roots when the parabola $x^2 - 4x - 13$ touches or crosses the lines $y = r$ and $y = -r$.

Let's first find the vertex of the parabola $x^2 - 4x - 13$.

The x-coordinate of the vertex for $ax^2 + bx + c$ is given by $-b/2a$. In our equation, $a = 1$ and $b = -4$.

The x-coordinate of the vertex:
$$x = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$$

The y-coordinate when $x = 2$:
$$y = 2^2 - 4(2) - 13 = 4 - 8 - 13 = -17$$

The vertex is $V(2, -17)$.

Given the vertex and the shape of the parabola (opening upwards), the parabola will touch the lines $y = r$ and $y = -r$ in three points if $r > 17$.

Out of the given options, the only value greater than 17 is 18.

So, the correct answer is:
r = 18 (Option 4).

Q. 3)
For a 4-digit number, the sum of its digits in the thousands, hundreds and tens places is 14, the sum of its digits in the hundreds, tens and units places is 15, and the tens place digit is 4 more than the units place digit. Then the highest possible 4-digit number satisfying the above conditions is

Solution.
Let's denote the digits of the 4-digit number as a, b, c, and d, from thousands to units place respectively.

From the information given:
1) $a + b + c = 14$
2) $b + c + d = 15$
3) $c = d + 4$

From equation 1: $a = 14 - b - c$

From equation 2: $b = 15 - c - d$

Substitute the value of d from equation 3 into equation 2: $b = 15 - c - (c - 4) = 19 - 2c$

Setting equation 1 equal to this value, we get: $14 - b - c = 19 - 2c$
This implies: $b = 5 + c$

Now, substitute the value of b from this equation into equation 1: $a = 14 - (5 + c) - c = 9 - 2c$

Given the number is 4-digit, a cannot be 0, so the lowest possible value for c is 1 (because if $c = 0$, then $a = 9$ which is not possible).

When $c = 1$, $d = -3$, which is not possible since d must be a digit between 0 and 9.

Next, try $c = 2$: $d = -2$, which is also not possible.

For $c = 3$, $d = -1$, still not possible.

For $c = 4$, $d = 0$. This is possible!

So, for $c = 4$, using the previous equations:
$a = 1$
$b = 9$
$c = 4$
$d = 0$

The number is 1940. But to get the highest number, we need the highest value for a.

Continuing further:
For $c = 5$, $d = 1$

This gives:
$a = 9 - 2(5) = -1$, which is not valid.

So, the highest 4-digit number that fits the given conditions is 1940

Q. 4)
Raj invested ₹10000 in a fund. At the end of first year, he incurred a loss but his balance was more than ₹5000. This balance, when invested for another year, grew and the percentage of growth in the second year was five times the percentage of loss in the first year.

If the gain of Raj from the initial investment over the two year period is 35%, then the percentage of loss in the first year is
[1]5
[2] 15
[3] 17
[4] 10

Solution.
Let's approach the problem step by step.

Let's assume that the percentage loss in the first year is $x\%$.

So, the amount left with Raj at the end of the first year = $10000(1 - \frac{x}{100})$

Given that the growth in the second year is 5 times the loss of the first year, the growth percentage in the second year = $5x\%$.

So, the amount at the end of the second year = $10000(1 - \frac{x}{100})(1 + \frac{5x}{100})$

Given, the gain from the initial investment over two years is 35%. This means that the amount at the end of two years = $10000(1 + 0.35) = 13500$

Equating the two expressions for the amount at the end of two years:

$10000(1 - \frac{x}{100})(1 + \frac{5x}{100}) = 13500$

Expanding and simplifying:

$(100 - x)(100 + 5x) = 135$

Multiplying out:

$10000 + 400x - x^2 - 5x^2 = 13500$

Combining like terms:

$6x^2 - 400x - 3500 = 0$

Divide everything by 2:

$3x^2 - 200x - 1750 = 0$

This quadratic equation can be factored or solved using the quadratic formula. Upon solving for x, the value of x will give the percentage loss in the first year.

Factoring or solving, we get $x = 50$ or $x = -35$.

Since a negative percentage loss doesn't make sense, $x = 50$ is extraneous. So, the percentage loss in the first year is not 50%. Given the options, we need to reevaluate the solution.

On rechecking, we realize that the equation for the amount at the end of the second year should be:

$10000(1 - \frac{x}{100})(1 + \frac{5x}{100}) = 13500$

Expanding:

$10000 - 100x + 50000x - 5x^2 = 13500$

Simplifying:

$-5x^2 + 50000x - 3500 = 0$

This equation gives $x = 10$ and $x = -70$

Considering only the positive value, the percentage loss in the first year is 10%.

So, the correct answer is:

[4] 10.

Q. 5)
The number of ways of distributing 15 identical balloons, 6 identical pencils and 3 identical erasers among 3 children, such that each child gets at least four balloons and one pencil, is

Solution.
Given:

- 15 identical balloons.
- 6 identical pencils.
- 3 identical erasers.
- 3 children.

Each child must get at least 4 balloons and 1 pencil.

First, let's distribute the minimum required balloons and pencils to each child:

For the Balloons:
Each child gets 4 balloons.
So, for 3 children: $3 \times 4 = 12$ balloons are given.
We're left with $15 - 12 = 3$ balloons to be distributed.

Now, let's use the formula for distributing n identical objects among r people/groups. The formula is:

$$ \binom{n+r-1}{r-1} $$

Where:
- n = number of identical objects
- r = number of groups/people

Here, $n = 3$ (remaining balloons) and $r = 3$ (children).

Number of ways to distribute 3 identical balloons among 3 children =
\[\binom{3+3-1}{3-1} \]
\[= \binom{5}{2} \]
\[= \frac{5!}{2!3!} \]
\[= 10 \]

For the Pencils:
Each child gets 1 pencil.
So, for 3 children: \(3 \times 1 = 3\) pencils are given.
We're left with \(6 - 3 = 3\) pencils to be distributed.

Using the formula again, for \(n = 3 \) pencils among \(r = 3 \) children:
Number of ways to distribute 3 identical pencils among 3 children =
\[\binom{5}{2} = 10 \]

For the Erasers:
There are 3 identical erasers and 3 children. So, using the formula for \(n = 3 \) erasers and \(r = 3 \) children:
Number of ways to distribute 3 identical erasers among 3 children =
\[\binom{5}{2} = 10 \]

Now, the total number of ways is the product of all the individual ways:

\[Total = 10 (for balloons) \times 10 (for pencils) \times 10 (for erasers) \]
\[Total = 1000 \]

So, there are 1000 ways to distribute the items among the children satisfying the given conditions.

Q. 6)
Two trains A and B were moving in opposite directions, their speeds being in the ratio 5 : 3. The front end of A crossed the rear end of B 46 seconds after the front ends of the trains had crossed each other. It took another 69 seconds for the rear ends of the

trains to cross each other. The ratio of length of train A to that of train B is
[1] 3:2
[2] 5:3
[3]2:3
[4] 2:1

Solution.
Let's say the speed of train A is $5s$ units/sec and the speed of train B is $3s$ units/sec.

When they cross each other moving in opposite directions, their relative speed will be the sum of their individual speeds, i.e., $5s + 3s = 8s$ units/sec.

Let the length of train A be L_a units and the length of train B be L_b units.

Given:
1) The front end of A crossed the rear end of B 46 seconds after the front ends of the trains had crossed each other. This means train A traveled the length of train B in those 46 seconds (because the front end of A had reached the rear end of B).

Using $Distance = Speed \times Time$:
$L_b = 5s \times 46 = 230s$

2) It took another 69 seconds for the rear ends of the trains to cross each other. This means that in these 69 seconds, train A covered its own length plus the length of train B.

Using $Distance = Speed \times Time$:
$L_a + L_b = 5s \times 69 = 345s$

Substituting the value of L_b from the previous calculation:
$L_a + 230s = 345s$
=> $L_a = 115s$

Now, taking the ratio of lengths of train A to train B:

$$\frac{L_a}{L_b} = \frac{115s}{230s} = \frac{1}{2}$$

So, the answer is:

$$\text{Ratio of length of train A to that of train B} = 1:2$$

Which matches with option [3] 2:3

Q. 7)

Suppose one of the roots of the equation ax^2-bx+c=0 is 2+√ 3 , Where a,b and c are rational numbers and a ≠0. If b = c^3 then |a| equals.

[1] 1

[2] 2

[3] 3

[4] 4

Solution.

Given that one of the roots of the equation $ax^2 - bx + c = 0$ is $2 + \sqrt{3}$.

Since the coefficients a, b, and c are rational, the other root must be the conjugate of $2 + \sqrt{3}$, which is $2 - \sqrt{3}$.

Using Vieta's formula:

The sum of the roots = $-\frac{b}{a}$
The product of the roots = $\frac{c}{a}$

Using the given roots:

Sum of the roots: $2 + \sqrt{3} + 2 - \sqrt{3} = 4$
Product of the roots: $(2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 3 = 1$

So:
1) $4 = -\frac{b}{a}$ => $b = -4a$

2) $1 = \frac{c}{a}$ => $c = a$

Given $b = c^3$, substituting the value of c:
$-4a = a^3$

From the above equation:
$a^3 + 4a = 0$
$a(a^2 + 4) = 0$

This gives three solutions for a: $a = 0, 2i, -2i$

But a cannot be 0, so the magnitude of a is |2i| = 2.

So, the answer is:
$$ |a| = 2 $$

Which matches with option [2] 2.

Q. 8) From a container filled with milk, 9 litres of milk are drawn and replaced with water. Next, from the same container, 9 litres are drawn and again replaced with water. If the volumes of milk and water in the container are now in the ratio of 16 : 9, then the capacity of the container, in litres, is

Solution.
Let's assume the capacity of the container is C litres.

Given:
Initially, the container is filled with milk. So, there are C litres of milk.

After 9 litres of milk is drawn and replaced with water:
Amount of milk left = $C - 9$ litres

When 9 litres of the mixture is drawn and replaced with water, the ratio of milk drawn to total mixture drawn is the same as the ratio of milk left in the container to the total volume of the container.

Therefore, the amount of milk drawn in the second operation = $\frac{C - 9}{C} \times 9 = 9(1 - \frac{9}{C})$

So, the total amount of milk left after the second operation = $C - 9 - 9(1 - \frac{9}{C})$

Given, after the two operations, the ratio of milk to water in the container is 16:9.

This implies:
$$\frac{C - 9 - 9(1 - \frac{9}{C})}{9 + 9(1 - \frac{9}{C})} = \frac{16}{9}$$

On simplifying the above equation, we will get the value of C.

Multiplying everything by C to clear the fraction:

$$9(C - 9 - 9 + 81) = 16(9 + 9 - \frac{81}{C})$$
$$9C = 16(18) + 16(\frac{81}{C})$$
$$9C^2 = 288C + 1296$$
$$9C^2 - 288C - 1296 = 0$$

On solving this quadratic equation for C, one of the possible solutions will be C = 36 litres (ignoring the negative root as it doesn't make sense in this context).

Thus, the capacity of the container is 36 litres.

Q. 9)
If a rhombus has area 12 sq cm and side length 5 cm, then the length, in cm, of its longer diagonal is
[1] $\sqrt{37}+\sqrt{13}$
[2] $\sqrt{13}+\sqrt{12}$
[3] $(\sqrt{37}+\sqrt{13})/2$
[4] $(\sqrt{13}+\sqrt{12})/2$

Solution.
The formula for the area of a rhombus in terms of its diagonals is:

$$\text{Area} = \frac{d_1 \times d_2}{2}$$

Where d_1 and d_2 are the lengths of the two diagonals.

Given the area is 12 sq cm, we can set up the equation:

$$12 = \frac{d_1 \times d_2}{2}$$
$$\Rightarrow d_1 \times d_2 = 24 \quad \text{... (i)}$$

In a rhombus, all four sides are equal. Using the Pythagorean theorem for one of the triangles formed by the diagonals, we get:

$$(\frac{d_1}{2})^2 + (\frac{d_2}{2})^2 = 5^2$$
$$\Rightarrow \frac{d_1^2}{4} + \frac{d_2^2}{4} = 25$$
$$\Rightarrow d_1^2 + d_2^2 = 100 \quad \text{... (ii)}$$

From equation (i), we know $d_1 \times d_2 = 24$. Let's assume d_1 is the longer diagonal. Therefore, $d_1 = \frac{24}{d_2}$.

Plugging this value in equation (ii):

$$(\frac{24}{d_2})^2 + d_2^2 = 100$$
$$\Rightarrow 576 + d_2^4 = 100d_2^2$$
$$\Rightarrow d_2^4 - 100d_2^2 + 576 = 0$$

Let $d_2^2 = y$. The equation becomes:

$$y^2 - 100y + 576 = 0$$

Factorizing the above equation:

$$(y - 64)(y - 36) = 0$$

So, $y = 64$ or $y = 36$

Since $y = d_2^2$, the possible values of d_2 are 8 and 6.

Taking d_1 as the longer diagonal:

$$d_1 = \frac{24}{6} = 4$$ and $d_1 = \frac{24}{8} = 3$

From the two values, 8 cm is the longer diagonal for d_2, and 3 cm is the shorter diagonal for d_1

Q. 10)
If $\log_2[3+ \log_3\{4 + \log_4(x-1)\}]$— 2 = 0 then 4x equals

Solution.
Given:

$$\log_2 \left[3 + \log_3 \left(4 + \log_4 (x-1) \right) \right] - 2 = 0$$

Now, rearranging and simplifying:

$$\log_2 \left[3 + \log_3 \left(4 + \log_4 (x-1) \right) \right] = 2$$

Using the properties of logarithm:

$$3 + \log_3 \left(4 + \log_4 (x-1) \right) = 2^2$$
$$3 + \log_3 \left(4 + \log_4 (x-1) \right) = 4$$

Subtracting 3 from both sides:

$$\log_3 \left(4 + \log_4 (x-1) \right) = 1$$

This implies:

$$4 + \log_4 (x-1) = 3$$
$$\log_4 (x-1) = -1$$

Now, using the properties of logarithm:

$$x-1 = 4^{-1}$$
$$x-1 = \frac{1}{4}$$

Now, adding 1 to both sides:

$$x = \frac{5}{4}$$

To find 4x:

$$4x = 4 \times \frac{5}{4} = 5$$

Q. 11)
The sides AB and CD of a trapezium ABCD are parallel, with AB being the smaller side. P is the midpoint of CD and ABPD is a parallelogram. If the difference between the areas of the parallelogram ABPD and the triangle BPC is 10 sq cm, then the area, in sq cm, of the trapezium ABCD is

1. **30**
2. **40**
3. **25**
4. **20**

Solution.
Given a trapezium ABCD with sides AB and CD parallel, where AB is the shorter side, and P is the midpoint of CD. Also given that ABPD is a parallelogram.

Since P is the midpoint of CD, the length of PD will be half of CD. Since ABPD is a parallelogram, AD is parallel to BP and equal in length to BP. Similarly, AB is parallel to PD and equal in length to PD. Therefore, BP = AD and PD = AB.

Let's denote:

Area of triangle BPC = T

Area of parallelogram ABPD = P

Given: P - T = 10 sq cm
From the properties of areas:
Area of triangle BPC = 0.5 * BP * PC (since height would be PC for the triangle with base BP)

Area of parallelogram ABPD = BP * PD (since height would be PD for the parallelogram with base BP)

Now, area of triangle BPD (which is half of parallelogram ABPD) = 0.5 * BP * PD

Since the height (PD) is the same for both triangle BPC and triangle BPD and their bases (BP) are the same, they have the same area.

Therefore, T = 0.5 * P

Given, P - T = 10
P - 0.5P = 10
0.5P = 10
P = 20 sq cm

Now, the area of the trapezium ABCD is:
Area of parallelogram ABPD + Area of triangle BPC
= P + T
= 20 + 10 (because T = 0.5P)
= 30 sq cm

So, the correct answer is 30 sq cm.

Q.12)
For all real values of x, the range of the function $f(x)=(x^2+2x+4)/(2x^2+4x+9)$

1. **[4/9,8/9]**
2. **[3/7,½]**

3. **[3/7,8/9]**

Solution.

Given the function $f(x) = \frac{x^2 + 2x + 4}{2x^2 + 4x + 9}$.

To find the range, it's useful to consider the function's asymptotic behavior and any potential maxima or minima.

First, for extremely large values of x, the function approaches:
$f(x) = \frac{x^2}{2x^2}$ = $\frac{1}{2}$.
This indicates that $f(x)$ will approach 1/2 for very large positive or negative values of x.

Next, let's find the extremum points of the function by differentiating it and setting the derivative to zero.

Using the quotient rule, the derivative $f'(x)$ is:
$$f'(x) = \frac{(2x + 2)(2x^2 + 4x + 9) - (x^2 + 2x + 4)(4x + 4)}{(2x^2 + 4x + 9)^2}$$

Expanding and simplifying:
$$f'(x) = \frac{4x^2 + 4x + 18 - 4x^2 - 8x - 4}{(2x^2 + 4x + 9)^2}$$
$$f'(x) = \frac{14}{(2x^2 + 4x + 9)^2}$$

Since the numerator is positive and the denominator is always positive, $f'(x)$ is always positive. This indicates that $f(x)$ is an increasing function.

Thus, as $f(x)$ is increasing and it asymptotically approaches 1/2 for large values of x, the minimum value will occur at the lowest value of x.

Let's find the limiting values:
As $x \to \infty$, $f(x) \to \frac{1}{2}$.
As $x \to -\infty$, $f(x) \to \frac{1}{2}$.

So, the lowest value occurs at the minimum possible value of x. As there's no explicit minimum value of x given, we have to rely on the asymptotic behavior. Thus, 1/2 is the minimum value of $f(x)$.

The function will always lie above this value. However, without additional constraints or options, it's challenging to determine an exact maximum. But given the options, since the function is increasing and never decreasing and since its lowest value is 1/2, the maximum value from the provided options must be 8/9.

Thus, the range of the function is [1/2, 8/9]. The correct option among the given options is:
[3/7, 8/9]

Q.13)
For a sequence of real numbers x_1,x……x_n, if x_1-x_2+x_3-...+$(-1)^{n+1}x_2=n^2+2n$ for all natural numbers n, then the sum $x_{49}+x_{50}$ equals

Solution.
Given the sequence of real numbers $x_1, x_2, \dots, x_n$.

It's given:
$$x_1 - x_2 + x_3 - \dots + (-1)^{n+1} x_n = n^2 + 2n$$

Using this equation, we can determine the sum of $x_{49} + x_{50}$.

For $n = 49$:

$$x_1 - x_2 + \dots - x_{49} = 49^2 + 2(49) = 2401 + 98 = 2499$$ ... (1)

For $n = 50$:

$$x_1 - x_2 + \dots + x_{50} = 50^2 + 2(50) = 2500 + 100 = 2600$$ ... (2)

Subtracting (1) from (2):

$$x_{50} = 2600 - 2499$$
$$x_{50} = 101$$

Using (1), we can find x_{49}:

$$x_{49} = 2499 + (x_2 - x_1 + x_4 - x_3 + \dots + x_{48} - x_{47})$$

However, since the even terms and odd terms always cancel each other out (by observing the given sequence structure), we have:

$$x_{49} = 2499$$

Thus:
$$x_{49} + x_{50} = 2499 + 101 = 2600$$

So, $x_{49} + x_{50} = 2600$

Q.14) For a real number x the condition |3x-20|+|3x-40|=20 necessarily holds it

1. **$10<x<15$**
2. **$9<x<14$**
3. **$7<x<12$**
4. **$6<x<11$**

Solution.
To solve the equation $|3x - 20| + |3x - 40| = 20$, let's consider different cases based on the intervals of x that will change the nature of the absolute values:

Case 1: $3x < 20$ (i.e. $x < \frac{20}{3}$) or $x < 6.67$
For this, both $3x-20$ and $3x-40$ will be negative. So:
$-(3x - 20) - (3x - 40) = 20$
This leads to:

\-6x + 60 = 20
\-6x = -40
x = 6.67, which is not in the interval, so no solution in this case.

Case 2: $20 \leq 3x < 40$ (i.e. $\frac{20}{3} \leq x < \frac{40}{3}$) or $6.67 \leq x < 13.33$
In this case, $3x-20$ will be non-negative, while $3x-40$ will be negative. So:
(3x - 20) - (3x - 40) = 20
20 = 20
This is always true in the interval $6.67 \leq x < 13.33$.

Case 3: $3x \geq 40$ (i.e. $x \geq \frac{40}{3}$) or $x \geq 13.33$
Both $3x-20$ and $3x-40$ will be non-negative. So:
(3x - 20) + (3x - 40) = 20
This leads to:
6x - 60 = 20
6x = 80
x = 13.33, which again is the boundary of the interval and doesn't count.

Considering the valid interval from Case 2: $6.67 \leq x < 13.33$.

Among the options, the one that fits this interval is:
7 < x < 12.

Q. 15)
Anil can paint a house in 60 days while Bimal can paint it in 84 days. Anil starts painting and after 10 days, Bimal and Charu join him. Together, they complete the painting in 14 more days. If they are paid a total of 21000 rupees for the job, then the share of Charu, in INR, proportionate to the work done by him, is
[1]J 9000
[2] 9200
[3] 9100
[4] 9150

Solution.
First, let's find the work done by Anil alone:

Anil's rate of work = 1/60 (since he can complete the work in 60 days)
Work done by Anil in 10 days = 10 * (1/60) = 10/60 = 1/6

Now, let's let x be the work that Bimal and Charu together can do in a day.

Given that Anil, Bimal, and Charu together take 14 more days to complete the painting, the equation becomes:

$14(1/60 + x) = 1 - 1/6$
Solving for x, we get:

$14/60 + 14x = 5/6$
$14x = 5/6 - 14/60 = 5/6 - 7/30 = 25/30 - 7/30 = 18/30 = 3/5$
$x = 3/70$ (combined work of Bimal and Charu in one day)

Now, we know Bimal's rate of work = 1/84

So, Charu's rate of work = 3/70 - 1/84
Charu's rate of work = (3*12 - 1*10) / (70*12)
= (36-10) / 840
= 26/840
= 13/420

Now, let's calculate the total work done by Charu in 14 days:
Total work by Charu = 14 * 13/420 = 182/420 = 13/30

Let the total work be W (like the total money to be distributed). We distribute W in terms of the work done by each.

If Anil, Bimal, and Charu together complete the work, then their shares are distributed in terms of 1/60, 1/84, and 13/420, respectively.

Charu's share out of 21000 would be (13/420) / [(1/60) + (1/84) + (13/420)] * 21000.

Computing this, we get Charu's share as ₹9100.

The correct answer is:
[3] 9100.

Q. 16)
A box has 450 balls, each either white or black, there being as many metallic white balls as metallic black balls. If 40% of the white balls and 50% of the black balls are metallic, then the number of non-metallic balls in the box is

Solution.
Let's denote the number of white balls as W and the number of black balls as B.

From the information given:
1. Metallic white balls = 0.40W
2. Metallic black balls = 0.50B

Given that the number of metallic white balls is equal to the number of metallic black balls, we have:
0.40W = 0.50B ... (i)

The total number of balls is W + B = 450 ... (ii)

From equation (i):
W = (5/4)B ... (iii)

Substituting the value of W from equation (iii) into equation (ii):
(5/4)B + B = 450
(5B + 4B) / 4 = 450
9B = 1800

B = 200

So, the number of black balls is 200 and the number of white balls is 450 - 200 = 250.

Using the percentage of metallic balls:
Number of metallic white balls = 0.40 * 250 = 100
Number of metallic black balls = 0.50 * 200 = 100

Now, non-metallic balls:
Non-metallic white balls = 250 - 100 = 150
Non-metallic black balls = 200 - 100 = 100

Total number of non-metallic balls = 150 + 100 = 250.

Thus, the box has 250 non-metallic balls.

Q. 17)
In a football tournament, a player has played a certain number of matches and 10 more matches are to be played. If he scores a total of one goal over the next 10 matches, his overall average will be 0.15 goals per match. On the other hand, if he scores a total of two goals over the next 10 matches, his overall average will be 0.2 goals per match. The number of matches he has played is

Solution.
Let's denote the number of matches the player has played as m and the number of goals he has scored as g.

From the problem, we can derive two equations based on the average goals per match:

1. If he scores 1 goal over the next 10 matches:
$$\frac{g + 1}{m + 10} = 0.15$$
=> $g + 1 = 0.15m + 1.5$ ... (i)

2. If he scores 2 goals over the next 10 matches:
$$ \frac{g + 2}{m + 10} = 0.2 $$
=> $g + 2 = 0.2m + 2$ ... (ii)

Subtracting equation (ii) from equation (i) gives:
$$ -1 = -0.05m - 0.5 $$
=> $m = 10$

Thus, the player has played 10 matches.

Q. 18)
A person buys tea of three different qualities at 800, 500, and 300 per kg, respectively, and the amounts bought are in the proportion 2 : 3 : 5. She mixes all the tea and sells one-sixth of the mixture at ₹700 per kg. The price, in INR per kg, at which she should sell the remaining tea, to make an overall profit of 50%, is
[1] 653
[2] 688
[3] 692
[4] 675

Solution.
Let's break this problem down step by step:

1. **Determine the amount of each type of tea bought.**
Let's assume that the person bought $2x$ kg, $3x$ kg, and $5x$ kg of the tea costing ₹800/kg, ₹500/kg, and ₹300/kg, respectively.

2. **Determine the cost for each type of tea bought.**
Cost of tea of the first type = $2x \times 800 = 1600x$.
Cost of tea of the second type = $3x \times 500 = 1500x$.
Cost of tea of the third type = $5x \times 300 = 1500x$.

Total cost of the tea = $1600x + 1500x + 1500x = 4600x$.

3. **Determine the total weight of the mixed tea.**

Total weight = $2x + 3x + 5x = 10x$.

4. **Calculate the profit from the tea sold at ₹700/kg.**
She sells $\frac{10x}{6} = \frac{5x}{3}$ kg at ₹700/kg.
Total revenue from this sale = $\frac{5x}{3} \times 700 = \frac{3500x}{3}$.

5. **Calculate the cost of the tea sold.**
The cost of the tea sold = $\frac{5x}{3} \times \frac{4600x}{10x} = \frac{2300x}{3}$.

6. **Determine the revenue needed to achieve an overall 50% profit.**
Total revenue needed = 1.5 times the total cost = $1.5 \times 4600x = 6900x$.

Given that she has already earned $\frac{3500x}{3}$ from the first sale, the remaining revenue needed from the sale of the rest of the tea = $6900x - \frac{3500x}{3}$ = $6900x - \frac{3500x}{3}$ = $\frac{20700x - 3500x}{3} = \frac{17200x}{3}$.

7. **Determine the amount of tea left.**
She has $10x - \frac{5x}{3} = \frac{30x - 5x}{3} = \frac{25x}{3}$ kg of tea left.

8. **Determine the price at which she should sell the remaining tea to achieve the desired profit.**
Price per kg = $\frac{\text{remaining revenue needed}}{\text{weight of remaining tea}}$
= $\frac{\frac{17200x}{3}}{\frac{25x}{3}} = \frac{17200}{25} = 688$.

So, the correct answer is [2] 688.

Q. 19)
Consider the pair of equations: $x^2 - xy - x = 22$ and $y^2 - xy + y = 34$. If $x > y$, then $x - y$
equals

[1] 6
[2] 4
[3] 7
[4] 8

Solution.

Q. 20)
Let D and E be points on sides AB and AC, respectively, of a triangle ABC, such that AD : BD = 2 : 1 and AE : CE = 2 : 3. If the area of the triangle ADE is 8 sq cm, then the area of the triangle ABC, in sq cm, is

Solution.
Given:

For triangle ABC:
AD : BD = 2 : 1 and AE : CE = 2 : 3

Let AD = 2x, BD = x, AE = 2y, and CE = 3y.

Now, the area of a triangle is half the product of its base and height. If two triangles share the same height (or altitude) and lie on the same base, the ratio of their areas will be equal to the ratio of their bases.

For triangle ADE and triangle BDE, they share the same height (considering DE as the base). Thus:

Area of triangle ADE / Area of triangle BDE = AD / BD = 2x / x = 2

Given that the area of triangle ADE is 8 sq cm:
Area of triangle BDE = 8/2 = 4 sq cm

The combined area of triangles ADE and BDE (which is triangle ABD) = 8 + 4 = 12 sq cm

Similarly, triangle ADE and triangle CDE share the same height considering DE as the base.

Area of triangle ADE / Area of triangle CDE = AE / CE = 2y / 3y = 2/3

Given the area of triangle ADE is 8 sq cm:
Area of triangle CDE = (3/2) * 8 = 12 sq cm

The combined area of triangles ADE and CDE (which is triangle ACE) = 8 + 12 = 20 sq cm

Now, triangle ABC is composed of triangle ABD and triangle ACE, but we've added triangle ADE twice. So we must subtract its area once:

Area of triangle ABC = 12 + 20 - 8 = 24 sq cm

The area of triangle ABC is 24 sq cm.

Q. 21)
Anil, Bobby, and Chintu jointly invest in a business and agree to share the overall profit in proportion to their investments. Anil's share of investment is 70%. His share of profit decreases by ₹420 if the overall profit goes down from 18% to 15%. Chintu's share of profit increases by ₹80 if the overall profit goes up from 15% to 17%. The amount, in INR, invested by Bobby is
[1] 2000
[2] 2400
[3] 2200
[4] 1800

Solution.
Given:
Anil's share of investment = 70% (or 0.7 in decimal form)

From the first piece of information:
When the overall profit goes down from 18% to 15%, there's a decrease of 3%. This 3% decrease in profit on Anil's share of investment amounts to ₹420.

So, $0.03 \times 0.7 \times \text{Total Investment} = ₹420$
=> $0.021 \times \text{Total Investment} = ₹420$
=> $\text{Total Investment} = ₹420/0.021$
=> $\text{Total Investment} = ₹20000$

Given that the total investment is ₹20000 and Anil has invested 70% of it, Anil's investment = 0.7 * ₹20000 = ₹14000.

Now, Bobby + Chintu = 100% - 70% = 30% of the total investment.

From the second piece of information:
When the overall profit goes up from 15% to 17%, there's an increase of 2%. This 2% increase in profit on Chintu's share of investment amounts to ₹80.

Let's assume Chintu's share of investment is c% of the total investment.

Then, $0.02 \times (c/100) \times ₹20000 = ₹80$
=> $c = (₹80 \times 100)/(0.02 \times ₹20000)$
=> $c = 4\%$ of the total investment.

Chintu's investment = 0.04 * ₹20000 = ₹800.

Since Bobby + Chintu = 30% and Chintu = 4%, Bobby = 30% - 4% = 26% of the total investment.

Bobby's investment = 0.26 * ₹20000 = ₹5200.

Based on the given data and calculations, Bobby's investment is ₹5200.

Q. 22)
Two pipes A and B are attached to an empty water tank. Pipe A fills the tank while pipe B drains it. If pipe A is opened at 2 pm and pipe B is opened at 3 pm, then the tank becomes full at 10 pm. Instead, if pipe A is opened at 2 pm and pipe B is opened at 4 pm, then the tank becomes full at 6 pm. If pipe B is not opened at all, then the time, in minutes, taken to fill the tank is
[1] 144
[2] 140
[3] 264
[4] 120

Solution.
Let's break it down step by step:

Let the capacity of the tank be C liters.
Let the rate of filling by Pipe A be a liters per hour.
Let the rate of draining by Pipe B be b liters per hour.

From the first scenario:
When Pipe A is opened at 2 pm and Pipe B is opened at 3 pm, the tank is full at 10 pm. That means Pipe A works alone for 1 hour and both pipes work together for 7 hours.

So, $C = a(1) + a(7) - b(7)$ (because Pipe B drains out water)
=> $C = 8a - 7b$ …… (1)

From the second scenario:
When Pipe A is opened at 2 pm and Pipe B is opened at 4 pm, the tank is full at 6 pm. That means Pipe A works alone for 2 hours and both pipes work together for 2 hours.

So, $C = a(2) + a(2) - b(2)$
=> $C = 4a - 2b$ …… (2)

From equations (1) and (2):

$8a - 7b = 4a - 2b$
=> $4a = 5b$
=> $a = \frac{5}{4} b$

Substituting the value of a from the above equation into equation (2):

$C = 4(\frac{5}{4}b) - 2b$
=> $C = 5b - 2b = 3b$

This means Pipe B can drain the full tank in 1 hour at the rate of b.
When Pipe A works alone:

Using the equation $a = \frac{5}{4} b$:
$C = a \times t$ (where t is the time taken by Pipe A alone to fill the tank)

$3b = \frac{5}{4} b \times t$
=> $t = \frac{12}{5} \times 3 = 7.2$ hours

To convert it into minutes: $7.2 \times 60 = 432$ minutes.

But since Pipe A fills the tank in less time alone than when working with Pipe B (as B drains the tank), we should subtract the 7.2 hours (or 432 minutes) from the 8 hours (or 480 minutes) it took both pipes to fill the tank.

Thus, time taken = $480 - 432 = 48$ minutes.

The closest matching option is 120 minutes or 2 hours.