WBJEE - 2023

Answer Keys by

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MATHEMATICS

Q.No.	❍	✚	✿	☐
01	B	D	D	C
02	A	B	C	A
03	B	A	A	A
04	C	B	C	B
05	B	B	C	A
06	C	D	C	B
07	D	C	A	B
08	D	D	*	D
09	A	B	C	A
10	B	D	B	A
11	B	A	A	D
12	C	*	D	B
13	D	A	D	D
14	C	B	D	B
15	C	B	D	B
16	A	B	A	B
17	A	C	B	C
18	*	D	A	D
19	C	C	A	*
20	D	D	A	D
21	C	B	C	A
22	B	A	A	B
23	A	B	B	A
24	D	D	B	C
25	D	C	D	B
26	D	A	A	B
27	B	C	B	D
28	A	C	D	D
29	A	C	B	C
30	A	A	A	B
31	C	*	B	B
32	A	A	B	A
33	B	D	D	C
34	B	C	C	D
35	A	B	D	C
36	A	D	B	A
37	B	D	D	C
38	D	D	A	*
39	B	A	*	C
40	A	B	A	A
41	D	A	B	B
42	B	A	B	A
43	D	A	B	D
44	B	C	C	C
45	D	A	D	D
46	B	B	C	D
47	C	B	D	D
48	A	D	B	A
49	*	A	A	A
50	D	B	B	B
51	D	A	C	A
52	C	C	A	B
53	D	B	C	A
54	A	D	C	B
55	C	D	B	A
56	C	C	C	C
57	C	C	B	C
58	C	A	A	D
59	B	C	A	D
60	A	C	A	A
61	A	B	C	C
62	B	C	B	C
63	C	B	D	C
64	B	A	D	C
65	A	A	C	B
66	B	B,D	A,C	A,C
67	C	A,C	C,D	*
68	B,C	B,C	*	A,C
69	C,D	B	A,C	A,C
70	A,C	C	A,C	B,D
71	A,C	A,C	B,D	C
72	A,C	C,D	A,C	B,C
73	*	*	B,C	B
74	A,C	A,C	B	C,D
75	B,D	A,C	C	A,C

* No option is correct or ambiguity in the options.

Aakash + BYJU'S

Code - ❍

ANSWERS & HINTS
for
WBJEE - 2023
SUB : MATHEMATICS

CATEGORY - 1 (Q:1 to Q50)

(Carry 1 mark each. Only one option is correct. Negative marks: – ¼)

1. $\lim_{x\to\infty}\left\{x-\sqrt[n]{(x-a_1)(x-a_2)...(x-a_n)}\right\}$ where $a_1, a_2, ..., a_n$ are positive rational numbers. The limit

(A) does not exist (B) is $\frac{a_1+a_2+...a_n}{n}$ (C) is $\sqrt[n]{a_1a_2...a_n}$ (D) is $\frac{n}{a_1+a_2+...+a_n}$

Ans : (B)

Hint : $\underset{x\to\infty}{\text{lt}}\left\{x-\sqrt[x]{(x-a_1)(x-a_2)...(x-a_n)}\right\}$

$$\underset{x\to\infty}{\text{lt}}\left\{x-x^n\sqrt{\left(1-\frac{a_1}{x}\right)\left(1-\frac{a_2}{x}\right)...\left(1-\frac{a_n}{x}\right)}\right\}$$

$$\Rightarrow \underset{x\to\infty}{\text{lt}}\ x\left\{1-\left(1-\frac{a_1}{x}\right)^{1/n}\left(1-\frac{a_2}{x}\right)^{1/n}...\left(1-\frac{a_n}{x}\right)^{1/n}\right\}$$

$$\Rightarrow \underset{x\to\infty}{\text{lt}}\ x\left[1-\left(1-\frac{a_1}{nx}\right)\left(1-\frac{a_2}{nx}\right)...\left(1-\frac{a_n}{xn}\right)\right]$$

$$\Rightarrow \underset{x\to\infty}{\text{lt}}\ \cancel{x}\left[\frac{a_1+a_2+...+a_n}{n\cancel{x}}\right]=\frac{a_1+a_2+...+a_n}{n}$$

2. Suppose $f:\mathbb{R}\to\mathbb{R}$ be given by $f(x)=\begin{cases}1, & \text{if } x=1\\ e^{(x^{10}-1)}+(x-1)^2\sin\frac{1}{x-1}, & \text{if } x\neq 1\end{cases}$ then

(A) $f'(1)$ does not exist

(B) $f'(1)$ exists and is zero

(C) $f'(1)$ exist and is 9

(D) $f'(1)$ exists and is 10

Ans : (A)

Hint : $f : R \to R\ f(x) = \begin{cases} 1 & \text{if } x = 1 \\ e^{(x^{10}-1)} + (x-1)^2 . \sin\left(\frac{1}{x-1}\right) & \text{if } x \neq 1 \end{cases}$

$f(1^-) = 1 + 0 = 1$

$f(1^+) = 1 + 0 = 1$

$f(1) = 1$

$\therefore$ f(x) is continuous at x = 1

$$f'(x) = \left\{ e^{(x^{10}-1)}.(10x^5) + 2(x-1).\sin\left(\frac{1}{x-1}\right) - (x-1)^2 .\cos\left(\frac{1}{x-1}\right).\left(\frac{1}{x-1}\right)^2 \right\}$$

which does not exist.

3. Let $f : [1, 3] \to \mathbb{R}$ be continuous and be derivable in (1, 3) and $f'(x) = [f(x)]^2 + 4 \forall x \in (1, 3)$. Then

(A) f(3) – f(1) = 5 holds (B) f(3) – f(1) = 5 does not hold

(C) f(3) – f(1) = 3 holds (D) f(3) – f(1) = 4 holds

Ans : (B)

Hint : $f'(c) = (f(c))^2 + 4 = \frac{f(3) - f(1)}{2}$

$\Rightarrow f(3) - f(1) = 2(f(c))^2 + 8$

4. f(x) is a differentiable function and given f′(2) = 6 and f′(1) = 4, then $L = \lim_{h \to 0} \frac{f(2 + 2h + h^2) - f(2)}{f(1 + h - h^2) - f(1)}$

(A) does not exist (B) equal to –3 (C) equal to 3 (D) equal to 3/2

Ans : (C)

Hint : $\underset{h \to 0}{\text{lt}} \frac{f(2 + 2h + h^2) - f(2)}{f(1 + h - h^2) - f(1)}$ $[\div (\text{form})]$

by L. Hospital rule.

$$\underset{h \to 0}{\text{lt}} \frac{f'(2 + 2h + h^2)(2h + 2)}{f'(1 + h - h^2)(1 - 2h)} = \frac{f'(2)}{f'(1)} \times \left(\frac{2}{1}\right)$$

$$= \frac{6}{4} \times 2 = 3$$

5. Let $\cos^{-1}\left(\frac{y}{b}\right) = \log_e\left(\frac{x}{n}\right)^n$, then $Ay_2 + By_1 + Cy = 0$ is possible for

(A) $A = 2, B = x^2, C = n$

(B) $A = x^2, B = x, C = n^2$

(C) $A = x, B = 2x, C = 3n + 1$

(D) $A = x^2, B = 3x, C = 2n$

Ans : (B)

Hint : $\frac{-1}{\sqrt{1-\frac{y^2}{b^2}}} \times \frac{1}{b} y_1 = \left(\frac{n}{n \times \frac{x}{n}}\right)$

$\Rightarrow \frac{-1 \ y_1}{\sqrt{b^2 - y^2}} = \frac{n}{x}$

$\Rightarrow y_1 x + n\sqrt{b^2 - y^2} = 0$

$\Rightarrow y_1 + xy_2 + \frac{n}{\cancel{2}\sqrt{b^2 - y^2}}.\left(-\cancel{2}y\right)y_1 = 0$

$\Rightarrow y_1 + xy_2 + \frac{n^2}{x} y = 0$

$\Rightarrow y_1 x + x^2 y_2 + n^2 y = 0$

$\Rightarrow x^2 y_2 + xy_1 + n^2 y = 0$

$A = x^2, B = x, C = n^2$

6. If $I = \int \frac{x^2 dx}{(x \sin x + \cos x)^2} = f(x) + \tan x + c$, then $f(x)$ is

(A) $\frac{\sin x}{x \sin x + \cos x}$

(B) $\frac{1}{(x \sin x + \cos x)^2}$

(C) $\frac{-x}{\cos x(x \sin x + \cos x)}$

(D) $\frac{1}{\sin x(x \cos x + \sin x)}$

Ans : (C)

Hint : $I = \int \frac{x^2}{(x \sin x + \cos x)^2} dx = \int \frac{x}{(x \sin x + \cos x)^2} \times \frac{x}{\cos x} dx$

$I = -\frac{1}{(x \sin x + \cos x)} . \frac{x}{\cos x} + \int \frac{1}{(x \sin x + \cos x)} . \frac{(\cos x + x \sin x)}{\cos^2 x} dx$

$$I = -\frac{1}{(x\sin x + \cos x)}.\frac{x}{\cos x} + \int \sec^2 x dx$$

$$I = -\frac{x}{(x\sin x + \cos x)\cos x} + \tan x + C$$

$$\therefore f(x) = \frac{-x}{\cos x(xisnx + C)}$$

7. If $\int \frac{dx}{(x+1)(x-2)(x-3)} = \frac{1}{k}\log_e\left\{\frac{|x-3|^3|x+1|}{(x-2)^4}\right\} + c$, then the value of k is

(A) 4 (B) 6 (C) 8 (D) 12

Ans : (D)

Hint : $\int \frac{dx}{(x+1)(x-2)(x-3)} = \int\left(\frac{A}{x+1} + \frac{B}{(x-2)} + \frac{C}{x-3}\right)dx$

$$\frac{1}{\cancel{(x+1)}(x-2)(x-3)} = \frac{A(x-2)(x-3) + B(x+1)(x-3) + C(x+1)(x-2)}{\cancel{(x+1)}(x-2)(x-3)}$$

$\Rightarrow 1 = A(x-2)(x-3) + B(x+1)(x-3) + C(x+1)(x-2)$

for x = – 1

$1 = A(-3)(-4) \Rightarrow A = \frac{1}{12}$

for x = 2

$1 = B(3)(-1) \Rightarrow B = -\frac{1}{3}$

for x = 3

$1 = C(4)(1) \Rightarrow C = \frac{1}{4}$

$$\therefore I = \frac{1}{12}\int \frac{1}{x+1}dx - \frac{1}{3}\int \frac{1}{x-2}dx + \frac{1}{4}\int \frac{1}{x-3}dx$$

$$= \frac{1}{12}\ln|x+1| - \frac{1}{3}\ln|x-2| + \frac{1}{4}\ln|x-3| + C$$

$$= \frac{1}{12}\left[\ln|x+1| - \ln|x-2|^4 + \ln|x-3|^3\right] + C$$

$$= \frac{1}{12}\ln\frac{(|x+1|)(|x-3|)^3}{(x-2)^4} - C$$

$\therefore k = 12$

8. The expression $\dfrac{\int_0^n [x]dx}{\int_0^n \{x\}dx}$, where [x] and {x} are respectively integral and fractional part of x and $n \in \mathbb{N}$, is equal to

(A) $\dfrac{1}{n-1}$ (B) $\dfrac{1}{n}$ (C) n (D) n – 1

Ans : (D)

Hint : $\dfrac{\int_0^n [x]dx}{\int_0^n \{x\}dx} = \dfrac{I_1}{I_2}$

$$I_1 = \int_0^1 [x]dx + \int_1^2 [x]dx + \ldots + \int_{n-1}^n [x]dx$$

$$= 0 + 1 + \ldots.. + (n-1) = \frac{n(n-1)}{2}$$

$$I_2 = \int_0^n \{x\}dx = n\int_0^1 \{x\}dx = n\left[\frac{x^2}{2}\right]_0^1 = \frac{n}{2}$$

$$\therefore \frac{I_1}{I_2} = \frac{n(n-1)}{n} = n-1$$

9. The value of $\int_0^{1/2} \dfrac{dx}{\sqrt{1-x^{2n}}}$ is $(n \in \mathbb{N})$

(A) less than or equal to $\dfrac{\pi}{6}$

(B) greater than or equal to 1

(C) less than $\dfrac{1}{2}$

(D) greater than $\dfrac{\pi}{6}$

Ans : (A)

Hint : For 0 to $\dfrac{1}{2}$

$x^2 \geq x^{2n}$

$\Rightarrow -x^2 \leq -x^{2n}$

$\Rightarrow 1-x^2 \leq 1-x^{2n}$

$\Rightarrow \frac{1}{\sqrt{1-x^2}} \geq \frac{1}{\sqrt{1-x^{2n}}}$

$\Rightarrow \int_0^{1/2} \frac{1}{\sqrt{1-x^2}} dx \geq \int_0^{1/2} \frac{1}{\sqrt{1-x^{2n}}} dx$

$\Rightarrow \left[\sin^{-1} x\right]_0^{1/2} \geq I$

$\Rightarrow \boxed{\frac{\pi}{6} \geq I}$

10. If $I_n = \int_0^{\frac{\pi}{2}} \cos^n x \cos nx dx$, then $I_1, I_2, I_3 ...$ are in

(A) A.P. (B) G.P. (C) H.P (D) no such relation

Ans : (B)

Hint : $I_n = \int_0^{\pi/2} \cos^n x . \cos nx \, dx$

$I_1 = \int_0^{\pi/2} \cos x . \cos x dx = \int_0^{\pi/2} \left(\frac{1+\cos 2x}{2}\right) dx = \frac{\pi}{4}$

$I_2 = \int_0^{\pi/2} \cos^2 x . \cos 2x \, dx = \int_0^{\pi/2} -\sin^2 x . \cos 2x \, dx$

$2I_2 = \int_0^{\pi/2} \cos^2 2x = \int_0^{\pi/2} \frac{1+\cos 4x}{2} dx = \frac{\pi}{4}$

$\therefore I_2 = \frac{\pi}{8}$

$I_3 = \int_0^{\pi/2} \cos^3 x . \cos 3x \, dx = \int_0^{\pi/2} \left(\frac{3\cos x + \cos 3x}{4}\right) \cos 3x \, dx$

$$=\frac{3}{4}\int_0^{\pi/2}\cos x.\cos 3x\,dx+\frac{1}{4}\int_0^{\pi/2}\cos^2 3x\,dx$$

$$=\frac{3}{8}\int_0^{\pi/2}(\cos 2x+\cos 4x)dx+\frac{1}{4}\int_0^{\pi/2}\frac{1+\cos 6x}{2}dx$$

$$=\frac{\pi}{16}$$

$\therefore I_1, I_2, I_3$ are in G.P.

11. If $y=\frac{x}{\log_e|cx|}$ is the solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}+\phi\left(\frac{x}{y}\right)$, then $\phi\left(\frac{x}{y}\right)$ is given by

(A) $\frac{y^2}{x^2}$ (B) $-\frac{y^2}{x^2}$ (C) $\frac{x^2}{y^2}$ (D) $-\frac{x^2}{y^2}$

Ans : (B)

Hint : $y=\frac{x}{\ell n|cx|}$

$$\frac{dy}{dx}=\frac{\ell n|cx|-1}{(\ell n|cx|)^2}=\frac{1}{\ell n|cx|}-\frac{1}{(\ell n|cx|)^2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{y}{x}-\left(\frac{y}{x}\right)^2 \quad \therefore \phi\left(\frac{x}{y}\right)=-\frac{y^2}{x^2}$$

12. The function $y=e^{kx}$ satisfies $\left(\frac{d^2y}{dx^2}+\frac{dy}{dx}\right)\left(\frac{dy}{dx}-y\right)=y\frac{dy}{dx}$. It is valid for

(A) exactly one value of k
(B) two distinct values of k
(C) three distinct values of k
(D) infinitely many values of k

Ans : (C)

Hint : $y = e^{kx}$

$$\Rightarrow \frac{dy}{dx}=ke^{kx}=ky \ \Rightarrow \frac{d^2y}{dx^2}=k^2e^{kx}=k^2y$$

Now LHS

$$=\left(\frac{d^2y}{dx^2}+\frac{dy}{dx}\right)\left(\frac{dy}{dx}-y\right)=(k^2y+ky)(ky-y)=k(k+1)(k-1)y^2=k(k^2-1)y^2$$

Now RHS $y\frac{dy}{dx}=ky^2$, A/q $k(k^2-1)y^2=ky^2 \Rightarrow k\left[k^2-2\right]=0 \Rightarrow k=0, k=\pm\sqrt{2}$

13. Given $\frac{d^2y}{dx^2}+\cot x\frac{dy}{dx}+4y\operatorname{cosec}^2x=0$. Changing the independent variable x to z by the substitution z = log tan $\frac{x}{2}$, the equation is changed to

(A) $\frac{d^2y}{dz^2}+\frac{3}{y}=0$ (B) $2\frac{d^2y}{dz^2}+e^y=0$ (C) $\frac{d^2y}{dz^2}-4y=0$ (D) $\frac{d^2y}{dz^2}+4y=0$

Ans : (D)

Hint : $\frac{dz}{dx}=\frac{\frac{1}{2}\sec^2(x/2)}{\tan(x/2)}=\frac{1}{\sin x}=\operatorname{cosec}x$

$\frac{dy}{dx}=\frac{dy}{dz}.\frac{dz}{dx}=\operatorname{cosec}x\frac{dy}{dz}$, $\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dz}\left(\frac{dy}{dx}\right)\left(\frac{dz}{dx}\right)$

$=\frac{d}{dz}\left(\operatorname{cosec}.\frac{dy}{dz}\right)\operatorname{cosec}x$

$=\left(\operatorname{cosec}x\frac{d^2y}{dx^2}-\operatorname{cosec}x.\cot x\frac{dx}{dz}\frac{dy}{dz}\right)\operatorname{cosec}x=\operatorname{cosec}^2x\frac{d^2y}{dz^2}-\operatorname{cosec}x.\cot x\frac{dy}{dz}$

$\therefore\frac{d^2y}{dx^2}+\cot x\frac{dy}{dx}+4y\operatorname{cosec}^2x=0\Rightarrow\operatorname{cosec}^2x\frac{d^2y}{dx^2}-\operatorname{cosec}x.\cot x\frac{dy}{dz}+\cot x.\operatorname{cosec}x.\frac{dy}{dz}+4\operatorname{cosec}^2x=0$

$\Rightarrow\frac{d^2y}{dz^2}+4y=0$

14. Let $f(x)=\begin{cases}x+1, & -1\le x\le 0\\ -x, & 0\le x\le 1\end{cases}$

(A) f(x) is discontinuous in [–1, 1] and so has no maximum value or minimum value in [–1, 1]

(B) f(x) is continuous in [–1, 1] and so has maximum value and minimum value

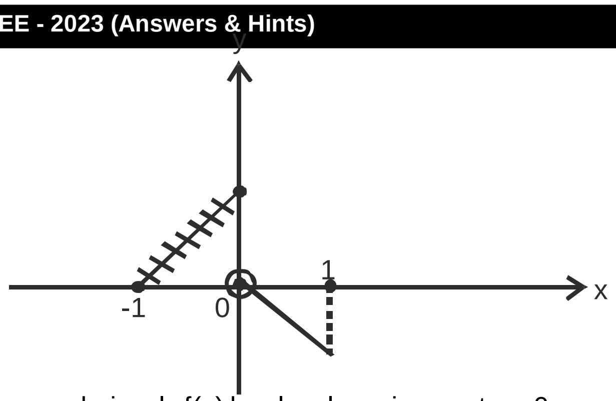
(C) f(x) is discontinuous in [–1, 1] but still has the maximum and minimum value

(D) f(x) is bounded in [–1, 1] and does not attain maximum or minimum value

Ans : (C)

Hint : $f(x)=\begin{cases}x+1 & -1\le x\le 0\\ -x & 0<x\le 1\end{cases}$

$\left.\begin{matrix}f(-1^-)=0\\ f(-1)=0\\ f(-1^+)=0\end{matrix}\right]$ continuous at x = – 1, $\left.\begin{matrix}f(0)=1\\ f(0^-)=1\\ f(0^+)=0\end{matrix}\right]$ discontinuous at x = 0



obviously f(x) has local maximum at x = 0

15. A missile is fired from the ground level rises x meters vertically upwards in t sec, where $x = 100t - \frac{25}{2}t^2$. The maximum height reached is

(A) 100 m (B) 300 m (C) 200 m (D) 125 m

Ans : (C)

Hint : $x = 100t - \frac{25}{2}t^2$

$\frac{dx}{dt} = 100 - 25t$ for maximum and minimum $\frac{dx}{dt} = 0 \Rightarrow t = 4\,\text{sec}$

$\frac{d^2x}{dt^2} = -25 < 0$

$\therefore$ at x = 4 it will have maixmum height

$x_{max} = 100 \times 4 - \frac{25}{2} \times 16 = 400 - 200 = 200\text{m}$

16. If a hyperbola passes through the point $P\left(\sqrt{2}, \sqrt{3}\right)$ and foci at (±2, 0) then the tangent to this hyperbola at P is

(A) $y = x\sqrt{6} - \sqrt{3}$ (B) $y = x\sqrt{3} - \sqrt{6}$ (C) $y = x\sqrt{6} + \sqrt{3}$ (D) $y = x\sqrt{3} + \sqrt{6}$

Ans : (A)

Hint : ae = 2, $a^2 + b^2 = 4$

$\Rightarrow \frac{2}{a^2} - \frac{3}{b^2} = 1 \Rightarrow b^2 = 3$; $a^2 = 1$

$\frac{x^2}{1} - \frac{y^2}{3} = 1$. Tangent at $P\left(\sqrt{2}, \sqrt{3}\right)$ is

$\boxed{\sqrt{6}x - y = \sqrt{3}}$

17. A, B are fixed points with coordinates (0, a) and (0, b) (a > 0, b > 0). P is variable point (x, 0) referred to rectangular axis. If the angle $\angle APB$ is maximum, then

(A) $x^2 = ab$ (B) $x^2 = a + b$ (C) $x = \frac{1}{ab}$ (D) $x = \frac{a+b}{2}$

Ans : (A)

Hint : $\angle APB = \theta = \cos^{1}\left(\frac{x^2 + a^2 + x^2 + b^2 - (a-b)^2}{2\sqrt{x^2 + a^2}\sqrt{x^2 + b^2}}\right)$

For Max ; $\frac{d\theta}{dx} = 0$, $\boxed{x^2 = ab}$

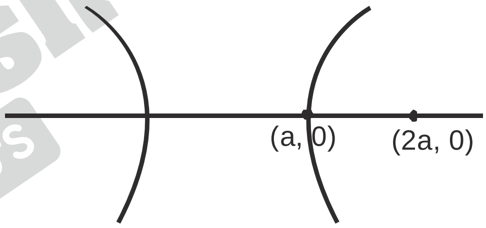
18. The average length of all vertical chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,\ a \le x \le 2a,$ is

(A) $b\left\{2\sqrt{3} + \ell n(2+\sqrt{3})\right\}$ (B) $b\left\{3\sqrt{2} + \ell n(3+\sqrt{2})\right\}$ (C) $a\left\{2\sqrt{5} - \ell n(2+\sqrt{5})\right\}$ (D) $a\left\{5\sqrt{2} + \ell n(5+\sqrt{2})\right\}$

Ans : (**)

Hint : $A_L = \frac{2\int_a^{2a} y\,dx}{\int_a^{2a} dx}$

$= \frac{2\int_a^{2a} \frac{b}{a}\sqrt{x^2 - a^2}\,dx}{\int_a^{2a} dx} = b\left[2\sqrt{3} - \ell n\left(2+\sqrt{3}\right)\right]$



19. The value of 'a' for which the scalar triple product formed by the vectors

$\vec{a} = \hat{i} + a\hat{j} + \hat{k},\ \vec{\beta} = \hat{j} + a\hat{k}$ and $\vec{\gamma} = a\hat{i} + \hat{k}$ is maximum, is

(A) 3 (B) –3 (C) $-\frac{1}{\sqrt{3}}$ (D) $\frac{1}{\sqrt{3}}$

Ans : (C)

Hint : $\Delta = 1 + a^3 - a$ For max ; $\frac{d\Delta}{da} = 0$

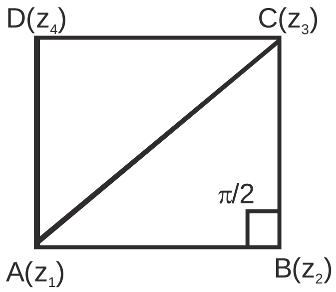
$\frac{d\Delta}{da} = 3a^2 - 1$ $a = \pm\frac{1}{\sqrt{3}}$

$\frac{d^2\Delta}{da^2} = 6a < 0$ for $a = -\frac{1}{\sqrt{3}}$

20. If the vertices of a square are z_1, z_2, z_3 and z_4 taken in the anti-clockwise order, then z_3 =

(A) $-iz_1 - (1 + i)z_2$ (B) $z_1 - (1 + i)z_2$ (C) $z_1 + (1 + i)z_2$ (D) $-iz_1 + (1 + i)z_2$

Ans : (D)



Hint : In ΔABC $\frac{z_1 - z_2}{z_3 - z_2} = \left|\frac{z_1 - z_2}{z_3 - z_2}\right| i^{\pi/2} = \left|\frac{z_1 - z_2}{z_3 - z_2}\right| . i = \frac{AB}{BC} . i = i$

$\Rightarrow z_1 - z_2 = i(z_3 - z_2) \Rightarrow -iz_1 + iz_2 = z_3 - z_2 \Rightarrow z_3 = -iz_1 + (i+1)z_2 = -iz_1 + (1+i)z_2$

21. If the n terms $a_1, a_2, \ldots\ldots\ldots, a_n$ are in A.P. with increment r, then the difference between the mean of their squares & the square of their mean is

(A) $\frac{r^2\left\{(n-1)^2 - 1\right\}}{12}$ (B) $\frac{r^2}{12}$ (C) $\frac{r^2\left(n^2-1\right)}{12}$ (D) $\frac{n^2-1}{12}$

Ans : (C)

Hint : $\frac{a_1^2 + a_2^2 + \ldots + a_n^2}{n} - \left(\frac{a_1 + a_2 + \ldots + a_n}{n}\right)^2$

$$= \frac{a_1^2 + (a_1 + r)^2 + \ldots + \{a_1 + (n-1)r\}^2}{n} - \left(\frac{na_1 + r \cdot \frac{n(n-1)}{2}}{n}\right)^2$$

$$= \frac{na_1^2 + r^2 \cdot \frac{(n-1) \cdot n(2n-1)}{6} + a_1 r(n-1) \cdot n}{n} - a_1^2 - a_1 r(n-1) - \frac{r^2(n-1)^2}{4}$$

$$= \cancel{a_1^2} + r^2 \frac{(n-1)(2n-1)}{6} + \cancel{a_1 r(n-1)} - \cancel{a_1^2} - \cancel{a_1 r(n-1)} - \frac{r^2(n-1)^2}{4}$$

$$= \frac{r^2(n-1)}{2}\left(\frac{2n-1}{3} - \frac{n-1}{2}\right)$$

$$= \frac{r^2(n-1)}{2} \cdot \left(\frac{4n-2-3n+3}{6}\right)$$

$$= \frac{r^2(n-1)(n+1)}{12}$$

$$= \frac{r^2\left(n^2-1\right)}{12}$$

22. If 1, $\log_9(3^{1-x}+2)$, $\log_3(4.3^x-1)$ are in A.P, then x equals

(A) $\log_3 4$ (B) $1-\log_3 4$ (C) $1-\log_4 3$ (D) $\log_4 3$

Ans : (B)

Hint : $2\log_9\left(3^{1-x}+2\right)=\log_3\left(4.3^x-1\right)+1$

$\Rightarrow \log_3\left(3^{1-x}+2\right)=\log_3 3\left(4.3^x-1\right)$

$\Rightarrow 3^{1-x}+2=3\left(4.3^x-1\right)$

$\Rightarrow \dfrac{3}{3^x}+2=4.3^{x+1}-3$

$\Rightarrow \dfrac{3}{3^x}+2=12.3^x-3$

$\Rightarrow 12\cdot\left(3^x\right)^2-5\left(3^x\right)-3=0$

$\Rightarrow \left(4\left(3^x\right)-3\right)\left(3\left(3^x\right)+1\right)=0$

$\because 3^x>0 \qquad \therefore 4\left(3^x\right)-3=0$

$\therefore 3\left(3^x\right)+1\neq 0 \quad \Rightarrow 3^x=\dfrac{3}{4}$

$\Rightarrow x=\log_3\dfrac{3}{4}=\log_3 3-\log_3 4$

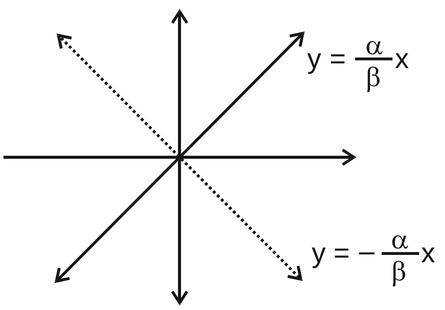
$\Rightarrow x=1-\log_3 4$

23. Reflection of the line $\bar{a}z+a\bar{z}=0$ in the real axis is given by

(A) $az+\overline{az}=0$ (B) $\bar{a}z-a\bar{z}=0$ (C) $az-\overline{az}=0$ (D) $\dfrac{a}{z}+\dfrac{\bar{a}}{\bar{z}}=0$

Ans : (A)

Hint :



Let, $a=\alpha+i\beta$ and $z=x+iy$

$\therefore \bar{a}z+a\bar{z}=0$

$\Rightarrow (\alpha-i\beta)(x+iy)+(\alpha+i\beta)(x-iy)=0$

$\Rightarrow 2(\alpha x+\beta y)=0 \Rightarrow \alpha x+\beta y=0$ which passes through origin $\therefore$ slope $=-\dfrac{\alpha}{\beta}$

$\therefore$ Reflected line's slope $= \dfrac{\alpha}{\beta}$

$\therefore$ Reflected line's equation : $\alpha x - \beta y = 0$

$$\Rightarrow \left(\frac{a+\bar{a}}{2}\right)x - \left(\frac{a-\bar{a}}{2i}\right)y = 0$$

$$\Rightarrow \left(\frac{a+\bar{a}}{2}\right)\left(\frac{z+\bar{z}}{2}\right) - \left(\frac{a-\bar{a}}{2i}\right)\left(\frac{z-\bar{z}}{2i}\right) = 0$$

$$\Rightarrow az + \bar{a}\bar{z} = 0$$

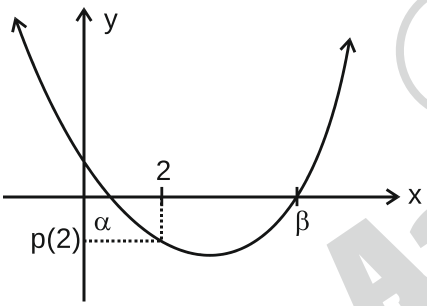
$$\Rightarrow az + \overline{az} = 0$$

24. If one root of $x^2 + px - q^2 = 0$, p and q are real, be less than 2 and other be greater than 2, then

(A) $4 + 2p + q^2 > 0$ (B) $4 + 2p + q^2 < 0$ (C) $4 + 2p - q^2 > 0$ (D) $4 + 2p - q^2 < 0$

Ans : (D)

Hint :



$\alpha < 2$ and $\beta > 2$

Let, $p(x) = x^2 + px - q^2$

$\therefore p(2) < 0$

$\Rightarrow 2^2 + 2p - q^2 < 0$

$\Rightarrow 4 + 2p - q^2 < 0$

25. The number of ways in which the letters of the word 'VERTICAL' can be arranged without changing the order of the vowels is

(A) $6! \times 3!$ (B) $\dfrac{8!}{3}$ (C) $6! \times 3$ (D) $\dfrac{8!}{3!}$

Ans : (D)

Hint : Vowels 'EIA' to be kept in order, so out of 8 places 3 places can be chosen in ${}^8C_3$ ways. Remaining 5 letters can be arranged in 5! ways.

$\therefore$ Total number of ways $= {}^8C_3 \times 5! = \dfrac{8!}{3!}$

26. n objects are distributed at random among n persons. The number of ways in which this can be done so that at least one of them will not get any object is

(A) $n! - n$ (B) $n^n - n$ (C) $n^n - n^2$ (D) $n^n - n!$

Ans : (D)

Hint : Total number of ways of distributing n objects randomly among n persons = n^n

Number of ways in which each person gets exactly one object $= {}^{n}P_{n} = n!$

$\therefore$ Required number of ways $= n^n - n!$

27. Let $P(n) = 3^{2n+1} + 2^{n+2}$ where $n \in \mathbb{N}$. Then

(A) P(n) is not divisible by any prime integer.

(B) there exists prime integer which divides P(n).

(C) P(n) is divisible by 5 for all $n \in \mathbb{N}$.

(D) P(n) is divisible by 3 for all $n \in \mathbb{N}$.

Ans : (B)

Hint : $P(1) = 3^3 + 2^3$ which is divisible by (3+2) = 5 which is prime

$\therefore$ There exists prime integer which divides P(n)

28. Let A be a set containing n elements. A subset P of A is chosen, and the set A is reconstructed by replacing the elements of P. A subset Q of A is chosen again. The number of ways of choosing P and Q such the Q contains just one element more than P is

(A) ${}^{2n}C_{n-1}$ (B) ${}^{2n}C_{n}$ (C) ${}^{2n}C_{n+2}$ (D) 2^{2n+1}

Ans : (A)

Hint : Required number of ways

$= {}^{n}C_{0} \cdot {}^{n}C_{1} + {}^{n}C_{1} \cdot {}^{n}C_{2} + \ldots + {}^{n}C_{n-1} \cdot {}^{n}C_{n}$

$= {}^{2n}C_{n-1}$

29. Let A and B are orthogonal matrices and det A + det B = 0. Then

(A) A + B is singular

(B) A + B is non-singular

(C) A + B is orthogonal

(D) A + B is skew symmetric

Ans : (A)

Hint : $AA^T = I$ and $BB^T = I$

$\Rightarrow \det(AA^T) = 1$ and $\det(BB^T) = 1$

$\det(A) = -\det(B)$ (Given) $\Rightarrow (\det(A))^2 = 1$ $\Rightarrow (\det(B))^2 = 1$

$\therefore \det(A + B) = \det(A(B^T + A^T)B)$

$= -\det(B^T + A^T)$ $\left[\because (\det(A))(\det(B)) = -1\right]$ as $\det(A) = -\det(B)$

$= -\det(B + A)^T$

$= -\det(A + B)^T$

$= -\det(A + B)$

$\Rightarrow 2\det(A + B) = 0$

$\Rightarrow \det(A + B) = 0$

$\therefore (A + B)$ is singular

30. Let $A = \begin{pmatrix} 2 & 0 & 3 \\ 4 & 7 & 11 \\ 5 & 4 & 8 \end{pmatrix}$. Then

(A) det A is divisible by 11

(B) det A is not divisible by 11

(C) det A = 0

(D) A is orthogonal matrix

Ans : (A)

Hint : $\det(A) = 2(56 - 44) + 0 + 3(16 - 35)$

$= 24 - 57$

$= -33$ which is divisible by 11

31. If the matrix M_r is given by $M_r = \begin{pmatrix} r & r-1 \\ r-1 & r \end{pmatrix}$ for r = 1, 2, 3, ... then

det (M_1) + det (M_2) + ... + det (M_{2008}) =

(A) 2007 (B) 2008 (C) $(2008)^2$ (D) $(2007)^2$

Ans : (C)

Hint : det $(M_r) = r^2 - (r-1)^2 = r^2 - (r^2 - 2r + 1) = 2r - 1$

$$\therefore \sum_{r=1}^{2008} \det(M_r) = \sum_{r=1}^{2008} (2r-1) = 1 + 3 + 5 + \cdots + 4015$$

$$= (2008)^2$$

32. Let α, β be the roots of the equation $ax^2 + bx + c = 0$, a, b, c real and $s_n = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+s_1 & 1+s_2 \\ 1+s_1 & 1+s_2 & 1+s_3 \\ 1+s_2 & 1+s_3 & 1+s_4 \end{vmatrix} = k\frac{(a+b+c)^2}{a^4}$$ then k =

(A) $b^2 - 4ac$ (B) $b^2 + 4ac$ (C) $b^2 + 2ac$ (D) $4ac - b^2$

Ans : (A)

Hint : $$\begin{vmatrix} 3 & 1+s_1 & 1+s_2 \\ 1+s_1 & 1+s_2 & 1+s_3 \\ 1+s_2 & 1+s_3 & 1+s_4 \end{vmatrix}$$

$$= \begin{vmatrix} 1+1+1 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}$$

$= \{(1-\alpha)(\alpha-\beta)(1-\beta)\}^2$

$= (1 - (\alpha+\beta) + \alpha\beta)^2 (\alpha-\beta)^2$

$$= \left(1 + \frac{b}{a} + \frac{c}{a}\right)^2 \left(\frac{b^2}{a^2} - \frac{4c}{a}\right)$$

$$= \frac{(a+b+c)^2}{a^2} \times \frac{b^2 - 4ac}{a^2} = (b^2 - 4ac)\frac{(a+b+c)^2}{a^4}$$

$\therefore k = b^2 - 4ac$

33. Let A, B, C are subsets of set X. Then consider the validity of the following set theoretic statement :

(A) $A \cup (B \setminus C) = (A \cup B) \setminus (A \cup C)$ (B) $(A \setminus B) \setminus C = A \setminus (B \cup C)$

(C) $(A \cup B) \setminus A = A \setminus B$ (D) $A \setminus C = B \setminus C$

Ans : (B)

Hint : $(A \setminus B) \setminus C = (A \cap B') \cap C'$

$= A \cap (B' \cap C')$

$= A \cap (B \cup C)'$

$= A \setminus (B \cup C)$

34. Let X be a nonvoid set. If ρ_1 and ρ_2 be the transitive relations on X, then

(A) $\rho_1 \circ \rho_2$ is transitive relation

(B) $\rho_1 \circ \rho_2$ is not transitive relation

(C) $\rho_1 \circ \rho_2$ is equivalence relation

(D) $\rho_1 \circ \rho_2$ is not any relation on X

Ans : (B)

Hint : Fact

35. Let A and B are two independent events. The probability that both A and B happen is $\frac{1}{12}$ and probability that neither A nor B happen is $\frac{1}{2}$. Then

(A) $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}$

(B) $P(A) = \frac{1}{2}, P(B) = \frac{1}{6}$

(C) $P(A) = \frac{1}{6}, P(B) = \frac{1}{2}$

(D) $P(A) = \frac{2}{3}, P(B) = \frac{1}{8}$

Ans : (A)

Hint : $P(A \cap B) = \frac{1}{12}$ $\quad \Rightarrow P(A)P(B) = \frac{1}{12}$ -----------(1)

$P(A' \cap B') = \frac{1}{2}$ $\quad \Rightarrow P(A \cup B) = 1 - \frac{1}{2} = \frac{1}{2}$

$$\Rightarrow P(A) + P(B) - P(A)P(B) = \frac{1}{2}$$

$$\Rightarrow P(A) + P(B) = \frac{1}{2} + \frac{1}{12} = \frac{7}{12}$$

From (1), $\quad P(A)\left(\frac{7}{12} - P(A)\right) = \frac{1}{12}$

$\Rightarrow P(A)^2 - \frac{7}{12}P(A) + \frac{1}{12} = 0$

$\Rightarrow 12P(A)^2 - 7P(A) + 1 = 0$

$\Rightarrow 12P(A)^2 - 4P(A) - 3P(A) + 1 = 0$

$\Rightarrow (3P(A) - 1) - (4P(A) - 1) = 0$

$\therefore P(A) = \frac{1}{3}$ or $P(A) = \frac{1}{4}$

$\therefore \quad P(B) = \frac{1}{12} \times 3 \quad \Rightarrow P(B) = \frac{1}{12} \times 4$

$= \frac{1}{4} \qquad = \frac{1}{3}$

$\therefore P(A) = \frac{1}{3}, \; P(B) = \frac{1}{4}$

36. Let S be the sample space of the random experiment of throwing simultaneously two unbiased dice and E_k = {a, b) ∈ S : ab = k}. If $p_k = P(E_k)$, then the correct among the following is :

(A) $p_1 < p_{10} < p_4$ (B) $p_2 < p_8 < p_{14}$ (C) $p_4 < p_8 < p_{17}$ (D) $p_2 < p_{16} < p_5$

Ans : (A)

Hint : $E_1 = \{(1, 1)\}$, $E_4 = \{(2, 2), (1, 4), (4, 1)\}$ $E_{10} = \{(2, 5), (5, 2)\}$

$p_1 = \frac{1}{36}$, $p_4 = \frac{3}{36} = \frac{1}{12}$, $p_{10} = \frac{2}{36} = \frac{1}{18}$

37. If $\frac{1}{6}\sin\theta, \cos\theta, \tan\theta$ are in G.P., then the solution set of θ is

(A) $2n\pi \pm \frac{\pi}{6}$ (B) $2n\pi \pm \frac{\pi}{3}$ (C) $n\pi + (-1)^n \frac{\pi}{3}$ (D) $n\pi + \frac{\pi}{3}$

Ans : (B)

Hint : $6\cos^3\theta + \cos^2\theta - 1 = 0$

$\Rightarrow (2\cos\theta - 1)(3\cos^2\theta + 2\cos\theta + 1) = 0$

$\Rightarrow \cos\theta = \frac{1}{2}$ $(3\cos^2\theta + 2\cos\theta + 1 \neq 0)$

38. The equation $r^2\cos^2\left(\theta - \frac{\pi}{3}\right) = 2$ represents

(A) a parabola (B) a hyperbola (C) a circle (D) a pair of straight lines

Ans : (D)

Hint : $r^2\left(\frac{\cos\theta}{2} + \frac{\sqrt{3}}{2}\sin\theta\right)^2 = 2$

$\Rightarrow \left(r\cos\theta + \sqrt{3}\,r\sin\theta\right)^2 = 8$

$\Rightarrow \left(x + y\sqrt{3}\right)^2 = 8$

$\Rightarrow \left(x + y\sqrt{3} - 2\sqrt{2}\right)\left(x + y\sqrt{3} + 2\sqrt{2}\right) = 0$

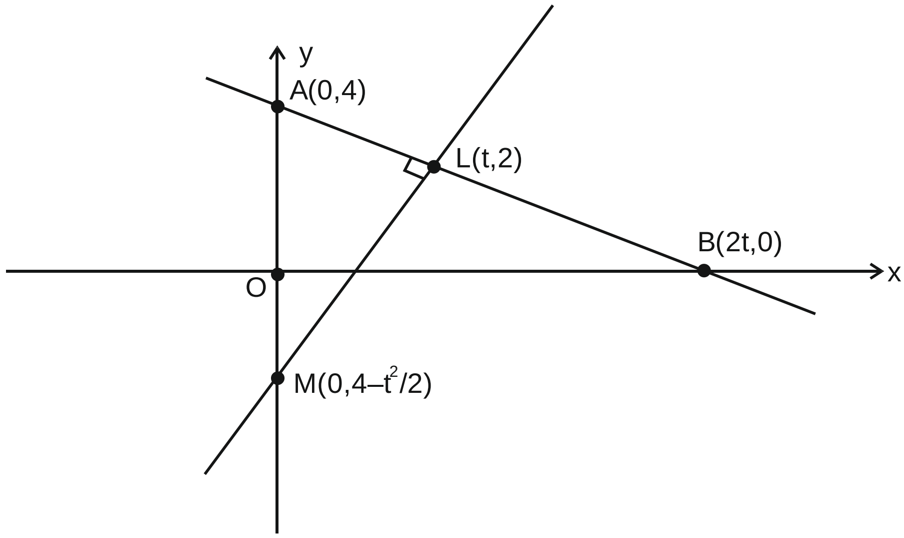
$\Rightarrow$ a pair of straight lines

39. Let A be the point (0, 4) in the xy-plane and let B be the point (2t, 0). Let L be the midpoint of AB and let the perpendicular bisector of AB meet the y-axis M. Let N be the midpoint of LM. Then locus of N is

(A) a circle (B) a parabola (C) a straight line (D) a hyperbola

Ans : (B)

Hint :



Equation of LM $\equiv$

$y - 2 = \frac{t}{2}(x - t)$

$\Rightarrow M\left(0, \frac{4 - t^2}{2}\right)$

Midpoint of LM : N(h, k)

$\Rightarrow$ $2h = t,\ 2k = 4 - \frac{t^2}{2}$

$\Rightarrow$ $x^2 = 2 - y$

40. If $4a^2 + 9b^2 - c^2 + 12ab = 0$, then the family of straight lines $ax + by + c = 0$ is concurrent at

(A) (2, 3) or (–2, –3) (B) (–2, 3) or (2, 3) (C) (3, 2) or (–3, 2) (D) (–3, 2) or (2, 3)

Ans : (A)

Hint : $\because$ $2a + 3b - c = 0$ or $2a + 3b + c = 0$

$\Rightarrow c = \pm(2a + 3b)$

$\because$ $ax + by + c = 0$

$\Rightarrow$ $ax + by \pm (2a + 3b) = 0$

$\Rightarrow$ $a(x \pm 2) + b(y \pm 3) = 0$ (family of lines)

$\Rightarrow$ (–2, –3) or (2, 3)

41. The straight lines $x + 2y - 9 = 0$, $3x + 5y - 5 = 0$ and $ax + by - 1 = 0$ are concurrent if the straight line $35x - 22y + 1 = 0$ passes through the point

(A) (–a, –b) (B) (a, –b) (C) (–a, b) (D) (a, b)

Ans : (D)

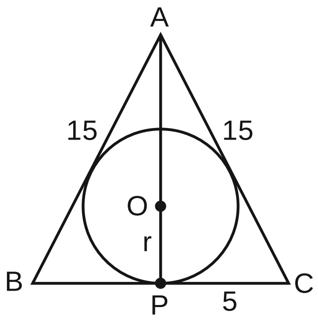
Hint : $\begin{vmatrix} a & b & -1 \\ 1 & 2 & -9 \\ 3 & 5 & -5 \end{vmatrix} = 0 \Rightarrow 35a - 22b + 1 = 0$

42. ABC is an isosceles triangle with an inscribed circle with centre O. Let P be the midpoint of BC. If AB = AC = 15 and BC = 10, then OP equals

(A) $\frac{\sqrt{5}}{\sqrt{2}}$ unit (B) $\frac{5}{\sqrt{2}}$ unit (C) $2\sqrt{5}$ unit (D) $5\sqrt{2}$ unit

Ans : (B)

Hint :



$AP = \sqrt{15^2 - 5^2} = 10\sqrt{2}$

$\Delta = \frac{1}{2} \cdot BC \cdot AP = 50\sqrt{2}$

$s = \frac{15+15+10}{2} = 20$

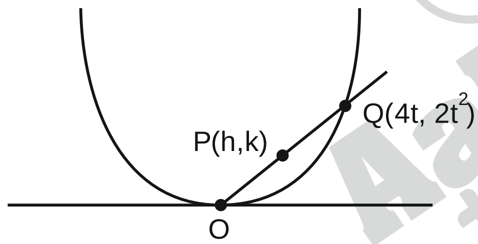
$\because\ r = \frac{\Delta}{s} = \frac{50\sqrt{2}}{20} = \frac{5}{\sqrt{2}}$

43. Let O be the vertex, Q be any point on the parabola $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is

(A) $x^2 = y$ (B) $y^2 = x$ (C) $y^2 = 2x$ (D) $x^2 = 2y$

Ans : (D)

Hint :



OP : PQ = 1 : 3

$\Rightarrow h = t,\ k = \frac{t^2}{2}$

$\Rightarrow k = \frac{h^2}{2}$

44. The tangent at point (a cos θ, b sin θ), $0 < \theta < \frac{\pi}{2}$, to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the x-axis at T and y-axis at T_1.

Then the value of $\min\limits_{0<\theta<\frac{\pi}{2}} (OT)(OT_1)$ is

(A) ab (B) 2ab (C) 0 (D) 1

Ans : (B)

Hint : Tangent : x(b cosθ) + y(a sinθ) = ab

$\therefore OT \cdot OT_1 = \frac{ab}{\cos\theta \sin\theta} = \frac{2ab}{\sin 2\theta}$

45. Let A(2 sec θ, 3 tan θ) and B(2 sec φ, 3 tan φ) where $\theta + \phi = \frac{\pi}{2}$ be two points on the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$. If (α, β) is the point of intersection of normals to the hyperbola at A and B, then β is equal to

(A) $\frac{12}{3}$ (B) $\frac{13}{3}$ (C) $-\frac{12}{3}$ (D) $-\frac{13}{3}$

Ans : (D)

Hint : E_{qn} of Normal at A

$\frac{4x}{2\sec\theta} + \frac{9y}{3\tan\theta} = 13$

at B

$$\frac{4x}{2\operatorname{cosec}\theta}+\frac{9y}{3\cot\theta}=13$$

$\Rightarrow (2\cos\theta)x + (3\cot\theta)y = 13$ ———— (1)

& $(2\sin\theta)x + (3\tan\theta)y = 13$ ———— (2)

$\Rightarrow y = -\frac{13}{3}$

46. If the lines joining the focii of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where a > b, and an extremity of its minor axis is inclined at an angle 60°, then the eccentricity of the ellipse is

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{7}}{3}$ (D) $\frac{1}{\sqrt{3}}$

Ans : (B)

Hint : $\frac{b}{ae}$ = tan60° $\Rightarrow \frac{b}{a} = e\sqrt{3}$

$\because e^2 = 1-\frac{b^2}{a^2} = 1-3e^2$

$\Rightarrow e = +\frac{1}{2} \quad \left(e \neq -\frac{1}{2}\right)$

47. If the distance between the plane ax – 2y + z = k and the plane containing the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is $\sqrt{6}$, then $|k|$ is

(A) 36 (B) 12 (C) 6 (D) $2\sqrt{3}$

Ans : (C)

Hint : E_{qn} of plane

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0$$

$\Rightarrow x - 2y + z = 0$ ———— (1)

& given $\alpha x - 2y + z = k$ ———— (2)

(1) // (2) $\Rightarrow \alpha = 1, k \neq 0$

$\therefore \frac{|k|}{\sqrt{6}} = \sqrt{6} \Rightarrow |k| = 6$

48. The angle between a normal to the plane 2x – y + 2z – 1 = 0 and the X-axis is

(A) $\cos^{-1}\frac{2}{3}$ (B) $\cos^{-1}\frac{1}{5}$ (C) $\cos^{-1}\frac{3}{4}$ (D) $\cos^{-1}\frac{1}{3}$

Ans : (A)

Hint : Normal : 2, –1, 2

X-axis : 1, 0, 0

$\therefore\ \theta = \cos^{-1}\dfrac{2}{3}$

49. Let $f(x) = [x^2] \sin \pi x$, $x > 0$. Then

(A) f is discontinuous everywhere.

(B) f is continuous everywhere.

(C) f is continuous at only those points which are perfect squares.

(D) f is continuous at only those points which are not perfect squares.

Ans : (Ambiguity in the options)

Hint : $[x^2] \sin \pi x$

$\downarrow$

discontinuous

at all points where x^2 is integer

If x^2 is integer and x is also integer then f(x) will be continuous, but if x^2 is integer and x is not integer then f(x) will be discontinuous.

50. If $y = \log^n x$, where $\log^n$ means $\log_e \log_e \log_e$ ... (repeated n times), then

$x \log x \log^2 x \log^3 x \ldots\ldots \log^{n-1} x \log^n x \dfrac{dy}{dx}$ is equal to

(A) log x (B) x (C) 1 (D) $\log^n x$

Ans : (D)

Hint : $\because \dfrac{dy}{dx} = \dfrac{1}{x\log^{n-1}x \quad \log^{n-2} x \ldots. \quad \log x}$

CATEGORY - 2 (Q.51 to 65)

(Carry 2 marks each. Only one option is correct. Negative marks : –½)

51. $\int_0^{2\pi} \theta \sin^6\theta \cos\theta\, d\theta$ is equal to

(A) $\dfrac{\pi}{16}$ (B) $\dfrac{3\pi}{16}$ (C) $\dfrac{16\pi}{3}$ (D) 0

Ans : (D)

Hint : Let I = $\int_0^{2\pi} \theta \sin^6\theta \cos\theta\, d\theta$ ....(I)

applying King's Property,

$\int_0^{2\pi} (2\pi - \theta)\sin^6\theta.\cos\theta\, d\theta$ ....(II)

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adding (I) and (II), $2I = 2\pi\int_0^{2\pi} \sin^6\theta \cos\theta\, d\theta$

$$\Rightarrow I = \pi\int_0^{2\pi} \sin^6\theta\cos\theta \quad .....(III)$$

Using Queen's Property, $\int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx$, when f(2a–x) = f(x)

applying Queen's Property on (III), $I = 2\pi\int_0^{\pi}\sin^6\theta\cos\theta\,d\theta$

again, using Queen's Property, $\int_0^{2a} f(x)\,dx = 0$, when, f(2a – x) = –f(x)

$\boxed{I = 0}$

52. If x = sin θ and y = sin kθ, then $(1–x^2)y_2 – xy_1 – \alpha y = 0$, for α =

(A) k (B) –k (C) $–k^2$ (D) k^2

Ans : (C)

Hint : $x = \sin\theta$

$y = \sin k\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{k\cos k\theta}{\cos\theta} = y_1 \quad ....(I)$$

$$y_2 = \frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\times\frac{d\theta}{dx} = \left[\frac{-k^2\sin(k\theta).\cos\theta + k\cos(k\theta)\sin\theta}{\cos^2\theta}\right]\times\frac{1}{\cos\theta}$$

$$y_2 = \frac{k\cos(k\theta)\sin\theta - k^2\sin(k\theta)\cos\theta}{\cos^3\theta} \quad ......(II)$$

also, $1 - x^2 = 1 - \sin^2\theta = \cos^2\theta$ ....(III)

So, putting the values of $1–x^2$, y_1 and y_2 from (I), (II), (III) we get

$(1–x^2)y_2 – xy_1 = \alpha y$

$$\Rightarrow (\cos^2\theta)\left[\frac{k\cos(k\theta)\sin\theta - k^2\sin(k\theta).\cos\theta}{\cos^3\theta}\right] - \frac{k\sin\theta\cos(k\theta)}{\cos\theta} = \alpha(\sin k\theta)$$

$$\Rightarrow \frac{k\cos(k\theta)\sin\theta - k^2\sin(k\theta)\cos\theta - k\sin\theta\cos(k\theta)}{\cos\theta} = \alpha\sin(k\theta)$$

$$\Rightarrow -k^2\sin(k\theta) = \alpha\sin(k\theta)$$

So, $\boxed{\alpha = -k^2}$

53. In the interval $(-2\pi, 0)$, the function $f(x) = \sin\left(\frac{1}{x^3}\right)$

(A) never changes sign

(B) changes sign only once

(C) changes sign more than once but finitely many times

(D) changes sign infinitely many times

Ans : (D)

Hint : $x \in (-2\pi, 0)$, $f(x) = \sin\left(\frac{1}{x^3}\right)$

$\because -2\pi < x < 0$

$\Rightarrow -8\pi^3 < x^3 < 0$

$\Rightarrow -\infty < \frac{1}{x^3} < -\frac{1}{8\pi^3}$

Hence, $\sin\left(\frac{1}{x^3}\right)$ will take all values from –1 to 1, and will change its sign infinitely many times since it is a periodic function

54. The average ordinate of y = sin x over [0, π] is

(A) $\frac{2}{\pi}$ (B) $\frac{3}{\pi}$ (C) $\frac{4}{\pi}$ (D) π

Ans : (A)

Hint : As sin x is a continuous function in $[0, \pi]$

Average ordinate will be $= \frac{1}{\pi - 0}\int_0^{\pi} \sin x\, dx = \frac{[-\cos x]_0^{\pi}}{\pi} = \frac{2}{\pi}$

55. The portion of the tangent to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, a > 0 at any point of it, intercepted between the axes

(A) varies as abscissa (B) varies as ordinate

(C) is constant (D) varies as the product of abscissa and ordinate

Ans : (C)

Hint : for the given curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, a > 0

Parametric Coordinates are $x = a\cos^3\theta$

$y = a\sin^3\theta$

$\frac{dy}{dx}$ =slope of tangent at any point (x, y) $= \frac{dy/d\theta}{dx/d\theta}$

$= \frac{3a\sin^2\theta.\cos\theta}{3a\cos^2\theta(-\sin\theta)} = -\tan\theta$

Equation of tangent at $\left(a\cos^3\theta, a\sin^3\theta\right)$

$y - a\sin^3\theta = -\tan\theta\left(x - a\cos^3\theta\right)$

its x-intercept = a cos θ

its y-intercept = a sin θ

So, the tangent cuts the axes at A(a cos θ, 0) and B(0, a sin θ) respectively.

$AB = \sqrt{(a\cos\theta)^2 + (a\sin\theta)^2} = a$, which is constant

56. If the volume of the parallelopiped with $\vec{a}\times\vec{b}$, $\vec{b}\times\vec{c}$ and $\vec{c}\times\vec{a}$ as coterminous edges is 9 cu. units., then the volume of the parallelopiped with $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{b}\times\vec{c}\right)$, $\left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right)$ and $\left(\vec{c}\times\vec{a}\right)\times\left(\vec{a}\times\vec{b}\right)$ as coterminous edges is

(A) 9 cu. units (B) 729 cu. units (C) 81 cu. units (D) 243 cu. units

Ans : (C)

Hint : Volume of Parallelopiped whose coterminous edges are $\vec{a}$, $\vec{b}$ and $\vec{c} = \left[\vec{a}\vec{b}\vec{c}\right]$

So, Given that $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] = 9 = [\vec{a}\ \ \vec{b}\ \ \vec{c}]$ (1)

$\therefore \left[\vec{a}\vec{b}\vec{c}\right] = 3$ ...... (II)

Now volume of parallelopiped whose coterminous edges are

$\left(\vec{a}\times\vec{b}\right)\times\left(\vec{b}\times\vec{c}\right)$, $\left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right)$, $\left(\vec{c}\times\vec{a}\right)\times\left(\vec{a}\times\vec{b}\right)$

$\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{b}\times\vec{c}\right)\ \ \left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right)\ \ \left(\vec{c}\times\vec{a}\right)\times\left(\vec{a}\times\vec{b}\right)\right]$

$\left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right]^2$

$=(9)^2$ (from (1))

= 81

57. Given $f(x) = e^{\sin x} + e^{\cos x}$. The global maximum value of f(x)

(A) does not exist

(B) exists at a point in $\left(0, \frac{\pi}{2}\right)$ and its value is $2e^{\frac{1}{\sqrt{2}}}$

(C) exists at infinitely many points

(D) exists at x = 0 only

Ans : (C)

Hint : $f(x) = e^{\sin x} + e^{\cos x}$

$f'(x)=0$

$\Rightarrow e^{\sin x}.\cos x + e^{\cos x}(-\sin x) = 0$

$\Rightarrow e^{\sin x - \cos x} = \tan x$

$\boxed{x = \pi/4}$ or in general $\boxed{x = n\pi + \frac{\pi}{4}}$

Now, $f''(x) < 0$ at $x = \pi/4$ or $x = n\pi + \pi/4$

So, Global maximum exists at infinitely many points

58. Consider a quadratic equation $ax^2 + 2bx + c = 0$ where a, b, c are positive real numbers. If the equation has no real root, then which of the following is true?

(A) a, b, c cannot be in A.P. or H.P. but can be in G.P.

(B) a, b, c cannot be in G.P. or H.P. but can be in A.P.

(C) a, b, c cannot be in A.P. or G.P. but can be in H.P.

(D) a, b, c cannot be in A.P., G.P. or H.P.

Ans : (C)

Hint : For no real root

$(2b)^2 - 4ac < 0$

$b^2 < ac$

$b < \sqrt{ac} \because a, b, c > 0$

If in AP, then a + c = 2b

i.e., x = –1 a solution

$\therefore$ a, b, c not in AP

$\because b^2 < ac$

$\therefore$ a, b, c not in GP

For HP $\quad b = \frac{2ac}{a+c} = \frac{2\sqrt{ac}}{\sqrt{\frac{a}{c}} + \sqrt{\frac{c}{a}}}$

$\sqrt{\frac{a}{c}} + \sqrt{\frac{c}{a}} \geq 2$

$\therefore \frac{2\sqrt{ac}}{\sqrt{\frac{a}{c}} + \sqrt{\frac{c}{a}}} \leq 2$ $\therefore$ a, b, c in HP is possible

59. Let $a_1, a_2, a_3, ..., a_n$ be positive real numbers. Then the minimum value of $\frac{a_1}{a_2} + \frac{a_2}{a_3} + ... + \frac{a_n}{a_1}$ is

(A) 1 (B) n (C) ${}^nC_2$ (D) 2

Ans : (B)

Hint : $\frac{\frac{a_1}{a_2} + \frac{a_2}{a_3} + ... + \frac{a_n}{a_1}}{n} \geq \sqrt[n]{\frac{a_1}{a_2} \times \frac{a_2}{a_3} \times ..... \frac{a_n}{a_1}}$

AM≥GM (property)

$\therefore$ Minimum value = n

It is possible when $a_1 = a_2 = a_3 = ... = a_n$

60. Let $A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ and $P = \begin{pmatrix} 0 & 1 & 0 \\ x & 0 & 0 \\ 0 & 0 & y \end{pmatrix}$ be an orthogonal matrix such that $B = PAP^{-1}$ holds.

Then

(A) x = 1 = y (B) x = 1, y = 0 (C) x = 0, y = 1 (D) x = –1, y = 0

Ans : (A)

Hint : $B = PAP^{-1}$

$P^{-1}BP = P^{-1}PAP^{-1}P$

$$\begin{bmatrix} 0 & 0 & xy \\ x & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore x = 1 = y$

61. Let ρ be a relation defined on set of natural numbers $\mathbb{N}$, as $\rho = \{(x,y) \in \mathbb{N} \times \mathbb{N} : 2x+y = 41\}$. Then domain A and range B are

(A) $A \subset \{x \in \mathbb{N} : 1 \le x \le 20\}$ and $B \subset \{y \in \mathbb{N} : 1 \le y \le 39\}$

(B) $A = \{x \in \mathbb{N} : 1 \le x \le 15\}$ and $B = \{y \in \mathbb{N} : 2 \le y \le 30\}$

(C) $A \equiv \mathbb{N}$, $B \equiv \mathbb{Q}$

(D) $A \equiv \mathbb{Q}$, $B \equiv \mathbb{Q}$

Ans : (A)

Hint : 2x+y = 41

y = 41 – 2x

$\therefore$ x = {1,2,3,4, ... 20}

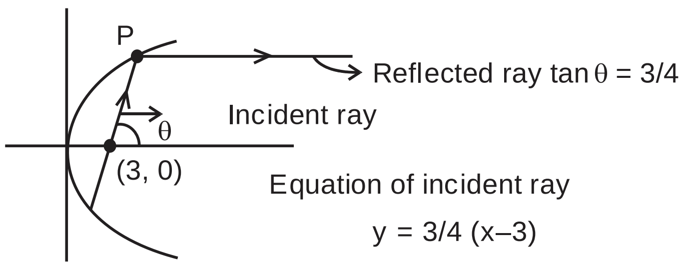
y = {1,3,5,..... 39}

62. From the focus of the parabola $y^2=12x$, a ray of light is directed in a direction making an angle $\tan^{-1}\frac{3}{4}$ with x-axis. Then the equation of the line along which the reflected ray leaves the parabola is

(A) y = 2 (B) y = 18 (C) y = 9 (D) y = 36

Ans : (B)

Hint : Reflected Ray passes parallel to the Axis of the parabola



For point P

$\frac{y^2}{12}=\frac{4y}{3}+3 \quad \therefore y = 18$

63. The locus of points (x, y) in the plane satisfying $\sin^2 x+\sin^2 y =1$ consists of

(A) a circle centered at origin

(B) infinitely many circles that are all centered at the origin

(C) infinitely many lines with slope ± 1

(D) finitely many lines with slope ± 1

Ans : (C)

Hint : $\sin^2 y = \cos^2 x$

$\sin y = \pm \cos x$

If $\sin y = \cos x = \sin\left(\frac{\pi}{2}-x\right)$

$\Rightarrow y = n\pi + (-1)^n\left(\frac{\pi}{2}-x\right)$

If $\sin y = -\cos x = \sin\left(x-\frac{\pi}{2}\right)$

$\Rightarrow y = n\pi + (-1)^n\left(x-\frac{\pi}{2}\right)$

64. The value of $\lim_{n\to\infty}\left[\left(\frac{1}{2.3}+\frac{1}{2^2.3}\right)+\left(\frac{1}{2^2.3^2}+\frac{1}{2^3.3^2}\right)+...+\left(\frac{1}{2^n.3^n}+\frac{1}{2^{n+1}.3^n}\right)\right]$ is

(A) $\frac{3}{8}$ (B) $\frac{3}{10}$ (C) $\frac{3}{14}$ (D) $\frac{3}{16}$

Ans : (B)

Hint : $\lim_{n\to\infty}\sum_{x=1}^{n}\left(\frac{1}{2^x\times 3^x}+\frac{1}{2^{x+1}\times 3^x}\right)$

$= \lim_{n\to\infty}\sum_{x=1}^{n}\frac{1}{6^x}\left(1+\frac{1}{2}\right)$

$= \frac{3}{2}\times\underbrace{\lim_{n\to\infty}\left(\frac{1}{6}+\frac{1}{6^2}+......\frac{1}{6^n}\right)}_{\text{infinite GP}} = \frac{3}{2}\times\frac{1/6}{1-1/6}=\frac{3}{10}$

65. The family of curves $y = e^{a\sin x}$, where 'a' is arbitrary constant, is represented by the differential equation

(A) $y\log y = \tan x\,\frac{dy}{dx}$ (B) $y\log x = \cot x\,\frac{dy}{dx}$

(C) $\log y = \tan x\,\frac{dy}{dx}$ (D) $\log y = \cot x\,\frac{dy}{dx}$

Ans : (A)

Hint : $y = e^{a\sin x}$

$\Rightarrow \log y = a \sin x$

$$\Rightarrow \frac{d}{dx}\left(\frac{\log y}{\sin x}\right) = \frac{d}{dx}(a) \quad \Rightarrow \frac{\sin x \left({dy}/{dx}\right)}{y} - \log y \times \cos x = 0$$

$$\Rightarrow y \log y = \tan x \times \frac{dy}{dx}$$

CATEGORY - 3 (Q66 to Q75)

(Carry 2 marks each. One or more options are correct. No negative marks)

66. Let f be a non-negative function defined on $\left[0, \frac{\pi}{2}\right]$. If $\int_0^x (f'(t) - \sin 2t)\,dt = \int_x^0 f(t)\tan t\,dt$, $f(0) = 1$, then $\int_0^{\frac{\pi}{2}} f(x)\,dx$ is

(A) 3 (B) $3 - \frac{\pi}{2}$ (C) $3 + \frac{\pi}{2}$ (D) $\frac{\pi}{2}$

Ans : (B)

Hint : $f'(x) - \sin 2x = -f(x)\tan x$

$\Rightarrow f'(x) + \tan x\, f(x) = \sin 2x$

$$\Rightarrow \frac{df(x)}{dx} + \tan x . f(x) = \sin 2x$$

If $= e^{\int \tan x\, dx} = e^{\ln|\sec x|} = |\sec x| = \sec x$ As, $x \in \left[0, \frac{\pi}{2}\right]$

Solution is,

$f(x) \times \sec x = \int \sin 2x \sec x\, dx = \int 2 \sin x \cos x \sec x\, dx = 2\int \sin x\, dx$

$f(x) \sec x = -2\cos x + C$

Put $x = 0$; $f(0) = 1$

$f(0) \sec 0 = -2 \cos 0 + c$

$\Rightarrow 1 = -2 + C \quad \therefore C = 3$

$\therefore$ Required solution is;

$f(x) \sec x = -2 \cos x + 3$

$f(x) = -2\cos^2 x + 3 \cos x$

$$= -\cancel{2}\left(\frac{1 + \cos 2x}{\cancel{2}}\right) + 3\cos x = -1 - \cos 2x + 3\cos x$$

$f(x) = -1 - \cos 2x + 3\cos x$

Now $\int_0^{\frac{\pi}{2}} f(x)\,dx$

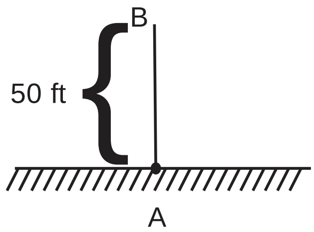
$$\int_0^{\frac{\pi}{2}} (-1 - \cos 2x + 3\cos x)\,dx = \left[-x - \frac{\sin 2x}{2} + 3\sin x\right]_0^{\frac{\pi}{2}} = \left(-\frac{\pi}{2} + 3\right) - 0 = 3 - \frac{\pi}{2}$$

67. A balloon starting from rest is ascending from ground with uniform acceleration of 4 ft/sec^2. At the end of 5 sec, a stone is dropped from it. If T be the time to reach the stone to the ground and H be the height of the balloon when the stone reaches the ground, then

(A) T = 6 sec (B) H = 112.5 ft (C) T = 5/2 sec (D) 225 ft

Ans : (C)

Hint :



Let after 5 sec ballon is at B

Given : a = 4ft/sec^2 u(Initial velocity) = 0

t = 5 sec

Distance covered by ballon in 5 sec;

$$S = ut + \frac{1}{2}at^2 = \frac{1}{2}\times 4\times 5^2 = 50 \text{ ft}$$

Let Speed of balloon at B be V

V = u + at = 0 + 4×5 = 20 ft/sec

So, speed of ball when it's dropped at point B is v_1 = 20ft/sec.

$$S = v_1 t + \frac{1}{2}a_1 t^2$$

$$\Rightarrow -50 = 20t + \frac{1}{2}(-32)t^2 \Rightarrow t = \frac{5}{2} \text{ sec}$$

68. If $f(x) = 3\sqrt[3]{x^2} - x^2$, then

(A) f has no extrema (B) f is maximum at two points x = 1 and x = –1

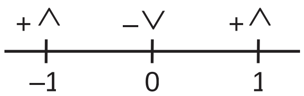
(C) f is minimum at x = 0 (D) f has maximum at x = 1 only

Ans : (B, C)

Hint : $f(x) = 3(x)^{\frac{2}{3}} - x^2$

Differentiate both sides wrt. x

$$= f'(x) = 3.\frac{2}{3}x^{-1/3} - 2x = 2x^{-1/3} - 2x = 2\left(x^{-1/3} - x\right) = 2\left(\frac{1-x^{4/3}}{x^{1/3}}\right)$$



∴ f has maximum at two points x = –1 x = 1

f has minimum at x = 0

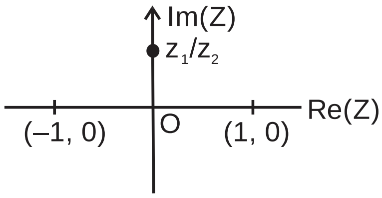
69. If z_1 and z_2 are two complex numbers satisfying the equation $\left|\frac{z_1+z_2}{z_1-z_2}\right|=1$, then $\frac{z_1}{z_2}$ may be

(A) real positive (B) real negative (C) zero (D) purely imaginary

Ans : (C, D)

Hint : $\left|\frac{z_1+z_2}{z_1-z_2}\right|=1$

$|z_1/z_2 + 1| = |z_1/z_2 - 1|$



Distance of $\frac{z_1}{z_2}$ from –1 and 1 are equal

So, locus of $\frac{z_1}{z_2}$ is perpendicular bisector of line joining (–1,0) & (1, 0)

$\therefore \frac{z_1}{z_2} \Rightarrow$ purely imaginary or 0

70. A letter lock consists of three rings with 15 different letters. If N denotes the number of ways in which it is possible to make unsuccessful attempts to open the lock, then

(A) 482 divides N (B) N is the product of two distinct prime numbers.
(C) N is the product of three distinct prime numbers. (D) 16 divides N

Ans : (A, C)

Hint : Number of unsuccessful attempts = $15^3 - 1 = 3374 = 2\times1687 = 2\times7\times241$

Divisible by 482

N is the product of three distinct prime number

71. If R and R^1 are equivalence relations on a set A, then so are the relation

(A) R^{-1} (B) $R \cup R^1$ (C) $R \cap R^1$ (D) All of these

Ans : (A, C)

Hint : If R & R^1 are equivalence relation

$R^{-1} \Rightarrow$ Equivalence

$R \cap R^1 \Rightarrow$ Equivalence

$R \cup R^1$ may or may not equivalence

72. Let f be a strictly decreasing function defined on $\mathbb{R}$ such that f(x) > 0, $\forall x \in \mathbb{R}$. Let $\frac{x^2}{f(a^2+5a+3)} + \frac{y^2}{f(a+15)} = 1$ be an ellipse with major axis along the y-axis. The value of 'a' can lie in the interval (s)

(A) $(-\infty, -6)$ (B) $(-6, 2)$ (C) $(2, \infty)$ (D) $(-\infty, \infty)$

Ans : (A, C)

Hint : f $\Rightarrow$ strictly decreasing function $\forall x \in R$

$f(x) > 0 \Rightarrow f'(x) < 0$

$$\frac{x^2}{f(a^2+5a+3)} + \frac{y^2}{f(a+15)} = 1$$

As major axis is y-axis

$f(a+15) > f(a^2+5a+3)$

$\Rightarrow a+15 < a^2 + 5a + 3$ ($\because$ f is decreasing)

$\Rightarrow a^2+4a-12>0$

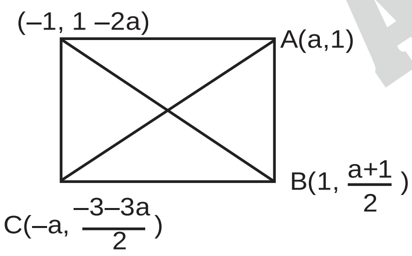
$\Rightarrow a < -6$ or $a > 2$

73. A rectangle ABCD has its side parallel to the line y=2x and vertices A,B,D are on lines y=1, x=1 and x=–1 respectively. The coordinate of C can be

(A) (3, 8) (B) (–3, 8) (C) (–3, –1) (D) (3,–1)

Ans : (No option(s) matched)

Hint : Case I



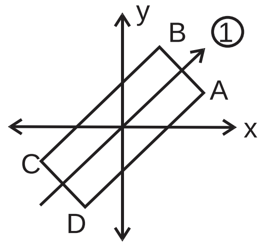
line y = 2x ---- (1)

AD parallel to y = 2x

$\Rightarrow M_{AD} = 2$ & $M_{AB} = -\frac{1}{2}$

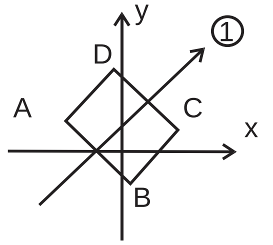
for a = 3

A(3,1), B(1,2), C(–3,–6), D(–1, –7)



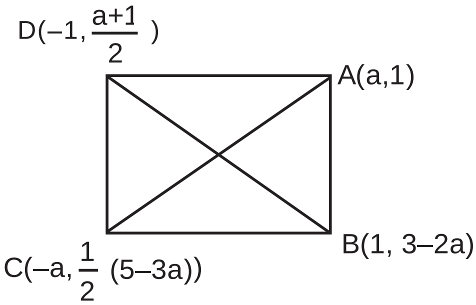
for a = –3

C(3, 3), A(–3,1), B(1, –1), D(–1, 5)

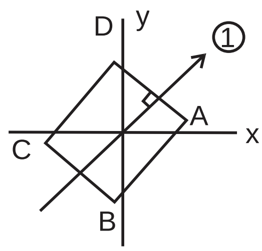


Case II AD perpendicular to line (1)

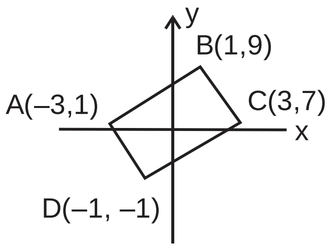
$\Rightarrow M_{AD} = \frac{1}{2}$, $M_{AB} = 2$ $\Rightarrow b = 3 - 2a$ $\Rightarrow d = \frac{a+1}{2}$



for a = 3 , C(–3, –2)



for a = –3, C(3, 7)



* In both cases if abscissa of C : 3 or –3

then co-ordinates of C can be (3,3) or (3,7) or (–3,–6) or (–3, –2)

74. Let $f(x) = x^m$, m being a non-negative integer. The value of m so that the equality $f'(a+b) = f'(a) + f'(b)$ is valid for all a, b > 0 is

(A) 0 (B) 1 (C) 2 (D) 3

Ans : (A, C)

Hint : Let $f(x) = x^m$, $m \geq 0$, $m \in I$

$f'(a+b) = f'(a) + f'(b)$, a,b > 0

$\Rightarrow m(a+b)^{m-1} = ma^{m-1} + m.b^{m-1}$

$(a+b)^{m-1} = a^{m-1} + b^{m-1}$ $f'(x) = mn^{m-1}$

for m = 0 $\Rightarrow 0 = 0$

$(a+b)^{-1} = a^{-1} + b^{-1}$

Not satisfied

for m = 1

$(a+b)^0 = a^{1-1} + b^{1-1}$

1 = 2

Not satisfied

for m = 2

$(a+b)^{2-1} = a^{2-1} + b^{2-1}$

a+b = a+b (satisfied)

for m = 3

$(a+b)^{3-1} = a^{3-1} + b^{3-1}$

Not satisfied

75. Which of the following statements are true ?

(A) If f(x) be continuous and periodic with periodicity T, then I= $\int_{a}^{a+T} f(x)dx$ depend on 'a'.

(B) If f(x) be continuous and periodic with periodicity T, then I= $\int_{a}^{a+T} f(x)dx$ does not depend on 'a'.

(C) Let f(x) = $\begin{cases} 1, \text{ if } x \text{ is rational} \\ 0, \text{ if } x \text{ is irrational} \end{cases}$, then f is periodic of the periodicity T only if T is rational

(D) f defined in (C) is periodic for all T

Ans : (B, D)

Hint : (A) $I = \int_{a}^{a+T} f(x)dx = \int_{0}^{T} f(x)dx$

Independent of a

(B) $I = \int_{a}^{a+T} f(x)dx = \int_{0}^{T} f(x)dx$ does not depend on 'a'

(C) $f(x) = \begin{cases} 1, & x \in Q \\ 0, & x \notin Q \end{cases}$ is periodic for all T with undefined fundamental period.

(D) f defined on (C) is periodic for all T with undefined fundamental period.

❑ ❑ ❑

WBJEE - 2023

Answer Keys by

Aakash Institute, Kolkata Centre

PHYSICS & CHEMISTRY

Q.No.	□	✚	✿	❍
01	B	B	B	B
02	A	*	A	D
03	A	C	D	B
04	B	B	B	** A & C
05	C	A	B	C
06	A	B	D	A
07	C	B	D	A
08	*	D	D	B
09	B	A	A	C
10	A	** A & C	A	D
11	C	C	C	B
12	B	B	D	B
13	D	A	A	A
14	B	B	B	D
15	B	C	A	D
16	C	D	B	D
17	A	B	C	A
18	** A & C	D	C	C
19	A	A	A	B
20	B	D	B	A
21	B	C	*	C
22	B	D	A	A
23	C	A	C	B
24	D	A	B	A
25	D	B	D	C
26	D	B	B	*
27	A	C	B	B
28	A	A	C	B
29	C	C	A	A
30	D	A	** A & C	C
31	C	C	A	A
32	C	A	A	A
33	A	A	C	A
34	A	A	C	C
35	A	C	A	C
36	B,C,D	A,D	A,B,C,D	A.B
37	A,D	A,B	A,B,C	A,B,C,D
38	A,B	A,B,C,D	B,C,D	A,B,C
39	A,B,C,D	A,B,C	A,D	B,C,D
40	A,B,C	B,C,D	A,B	A,D
41	C	D	D	A
42	A	C	A	D
43	D	A	B	C
44	A	B	B	B
45	D	B	D	D
46	B	C	A	A
47	D	D	B	B
48	C	B	C	B
49	A	C	C	D
50	D	B	C	A
51	B	B	A	C
52	C	B	D	C
53	C	C	D	D
54	B	C	A	C
55	C	A	A	A
56	B	D	B	A
57	B	B	D	D
58	B	B	C	C
59	D	D	D	A
60	C	A	B	B
61	A	A	C	D
62	A	B	B	C
63	B	D	B	D
64	B	C	B	B
65	D	C	C	C
66	D	A	B	B
67	A	D	C	B
68	B	C	D	B
69	C	D	C	C
70	C	A	A	B
71	C	B	A	B
72	B	B	A	A
73	B	A	C	A
74	A	A	B	C
75	A	C	B	B
76	A,C,D	A	A,D	B,C
77	A	B,C	A,D	A,D
78	B,C	A,D	A,C,D	A,D
79	A,D	A,D	A	A,C,D
80	A,D	A,C,D	B,C	A

* No option is correct

** Both option is correct

Code - ☐

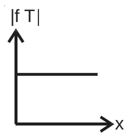
ANSWERS & HINTS
for
WBJEE - 2023
SUB : PHYSICS & CHEMISTRY

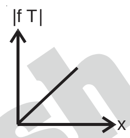
PHYSICS

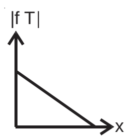
CATEGORY - 1 (Q1 to Q30)

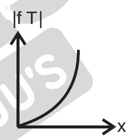
(Carry 1 mark each. Only one option is correct. Negative mark : – ¼)

1. In a simple harmonic motion, let f be the acceleration and T be the time period. If x denotes the displacement, then |fT| vs. x graph will look like,

(A) 

(B) 

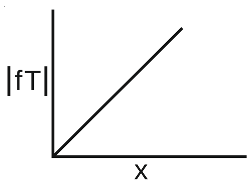
(C) 

(D) 

Ans : (B)

Hint : $F.T = \omega^2 x \cdot \frac{2\pi}{\omega} = 2\pi\omega x$

$\because$ F.T = $2\pi\omega$ x



2. The displacement of a plane progressive wave in a medium, travelling towards positive x-axis with velocity 4m/s at t = 0 is given by y = 3 sin $2\pi\left(-\frac{x}{3}\right)$. Then the expression for the displacement at a later time t = 4 sec will be

(A) $y = 3\sin 2\pi\left(-\frac{x-16}{3}\right)$

(B) $y = 3\sin 2\pi\left(\frac{-x-16}{3}\right)$

(C) $y = 3\sin 2\pi\left(\frac{-x-1}{3}\right)$

(D) $y = 3\sin 2\pi\left(\frac{-x-1}{3}\right)$

Ans : (A)

Hint : Let y = 3sin[ωt – k x]

$$\frac{\text{at } t = 0}{y = 3\sin(-kx)}$$

$$k = \frac{2\pi}{3}$$

$$v = \frac{\omega}{k}$$

$$4 = \frac{\omega}{2\pi/3} \quad \Rightarrow \quad \omega = \frac{8\pi}{3}$$

$$\therefore \quad y = 3\sin\left[\frac{8\pi}{3}t - \frac{2\pi}{3}x\right]$$

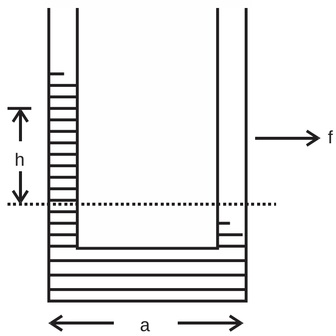
$\therefore$ t = 4

$$y = 3\sin\left[\frac{8\pi}{3}\times 4 - \frac{2\pi}{3}x\right]$$

$$= 3\sin 2\pi\left[\frac{-x+16}{3}\right]$$

$$y = 3\sin\left[2\pi\left(-\frac{x-16}{3}\right)\right]$$

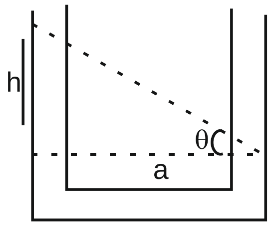
3. A shown in the figure, a liquid is at same levels in two arms of a U-tube of uniform cross-section when at rest. If the U-tube moves with an acceleration 'f' towards right, the difference between liwuid height between two arms of the U-tube will be, (acceleration due to gravity = g)



(A) $\frac{f}{g}a$ (B) $\frac{g}{f}a$ (C) a (D) 0

Ans : (A)

Hint :



$$\tan\theta = \frac{\text{acceleration}}{g}$$

$$\frac{h}{a} = \frac{\text{acceleration}}{g}$$

$$\frac{h}{a} = \frac{f}{g}$$

$$h = \frac{fa}{g}$$

4. Six molecules of an ideal gas have velocities 1, 3, 5, 5, 6 and 5 m/s respectively. At any given temperature, if $\overline{V}$ and V_{rms} represent average and rms speed of the molecules, then

(A) $\overline{V} = 5$ m/s (B) $V_{rms} > \overline{V}$

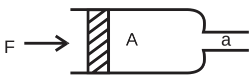
(C) $V_{rms}^2 < \overline{V}^2$ (D) $V_{rms} = \overline{V}$

Ans : (B)

Hint : $\overline{V} = \frac{1+3+5+5+6+5}{6} = \frac{25}{6} = 4.16$

$$\overline{V}_{rms} = \sqrt{\frac{1+9+25\times3+36}{6}} = \sqrt{\frac{121}{6}} = \frac{11}{\sqrt{6}} = 4.48$$

$$V_{rms} > \overline{V}$$

5. 

As shown in the figure, a pump is designed as horizontal cylinder with a piston having area A and an outlet orifice having an area 'a'. The piston moves with a constant velocity under the action of force F. If the density of the liquid is ρ, then the speed of the liquid emerging from the orifice is, (assume A >> a)

(A) $\sqrt{\frac{F}{\rho A}}$ (B) $\frac{a}{A}\sqrt{\frac{F}{\rho A}}$ (C) $\sqrt{\frac{2F}{\rho A}}$ (D) $\frac{A}{a}\sqrt{\frac{2F}{\rho A}}$

Ans : (C)

Hint : by principle of continuity

AV = av

by Bernoulis principle

$$P+\frac{1}{2}\rho V^2 = P_0+\frac{1}{2}\rho v^2$$

$$\left[\frac{F}{A}+P_0\right]+\frac{1}{2}\rho V^2 = P_0+\frac{1}{2}\rho v^2$$

$$\frac{F}{A}+\frac{1}{2}\rho\left[\frac{av}{A}\right]^2 = \frac{1}{2}\rho v^2$$

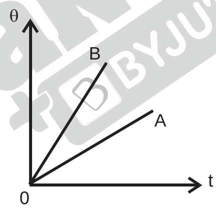
$$\frac{F}{A}=\frac{1}{2}\rho v^2\left[1-\frac{a^2}{A^2}\right]$$

$$v=\sqrt{\frac{2F}{\rho A\left[1-\frac{a^2}{A^2}\right]}}$$

$a^2 << A^2$

$$v=\sqrt{\frac{2F}{\rho A}}$$

6. Two substance A and B of same mass are heated at constant rate. The variation of temperature θ of the substance with time t is shown in the figure. Choose the correct staement



(A) Specific heat of A is greater than that of B

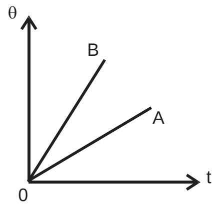
(B) Specific heat of B is greater than that of A

(C) Both have same specific heat

(D) None of the above is true

Ans : (A)

Hint :



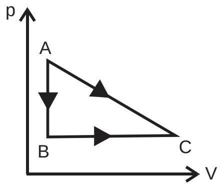
$\Delta H = mC\Delta\theta$

$$\frac{dH}{dt}=mC\frac{d\theta}{dt} \qquad \frac{dH}{dt}=\text{a constant}$$

$$\therefore \frac{d\theta}{dt}\propto\frac{1}{C}$$

i.e. slope $\propto \frac{1}{C}$ $\qquad \therefore C_B < C_A$

7. A given quantity of gas is taken from A to C in two ways; a) directly from A $\rightarrow$ C along a straight line and b) in two steps, from A $\rightarrow$ B and then from B $\rightarrow$ C. Work done and heat absorbed along the direct path A $\rightarrow$ C is 200 J and 280J respectively



(A) 80J (B) 0 (C) 160J (D) 120J

Ans : (C)

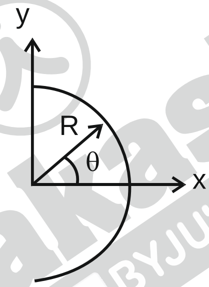
Hint : $\Delta W = 200J$ $\Delta Q = 280$ J for path AC

$\therefore$ $\Delta U = \Delta Q - \Delta W = 280 - 200 = 80J$

(ΔU) is same for both paths.

$\therefore$ $\Delta Q = \Delta W + \Delta U = 80 + 80 = 160J$

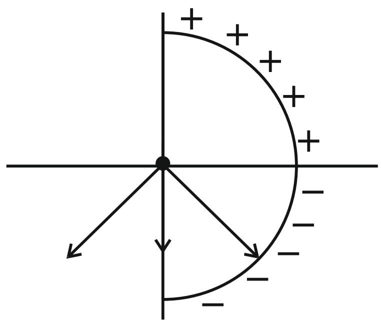
8. A thin glass rod is bent in a semicircle of radius R. A change is non-uniformly distributed along the rod with a linear charge density $\lambda = \lambda_0$ sin(λ_0 is a positive constant). The electric field at the centre P of the semicircle is,



(A) $-\dfrac{\lambda_0}{8\pi\varepsilon_0 R}\hat{j}$ (B) $\dfrac{\lambda_0}{8\pi\varepsilon_0 R}\hat{j}$ (C) $\dfrac{\lambda_0}{8\pi\varepsilon_0 R}\hat{i}$ (D) $-\dfrac{\lambda_0}{8\pi\varepsilon_0 R}\hat{i}$

Ans : (None)

Hint :



$$\lambda = \lambda_0 \sin\theta$$

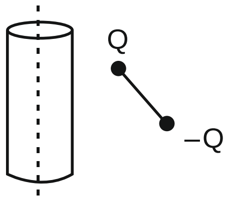
$$dE_y = \frac{k\lambda}{R}\sin\theta d\theta$$

$$= \frac{k\lambda_0}{R}\int_{-\pi/2}^{+\pi/2}\sin^2\theta\, d\theta = \frac{k\lambda_0}{2R}\int_{-\pi/2}^{+\pi/2}(1-\cos 2\theta)d\theta$$

$$\therefore \vec{E} = -\frac{\lambda_0}{8\varepsilon_0 R}\hat{j}$$

None of the options are matching.

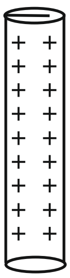
9. Consider a positively charged infinite cylinder with uniform volume charge density r > 0. An electric dipole consisting of +Q and – Q charges attached to opposite ends of a massless rod is oriented as shown in the figure. At the instant as shown in the figure, the dipole will experience,

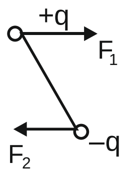


(A) a force to the left and no torque
(B) a force to the right and a clockwise torque
(C) a force to the right and a counter clockwise torque
(D) non froce but only a clockwise torque

Ans : (B)

Hint :



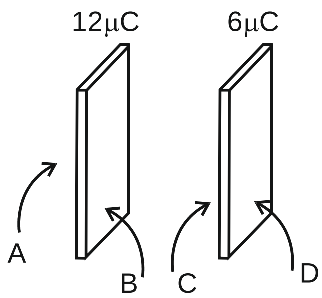


$E \propto \frac{1}{r}$

$F_1 > F_2$

∴ F_{net} towards right τ_{net} clockwise.

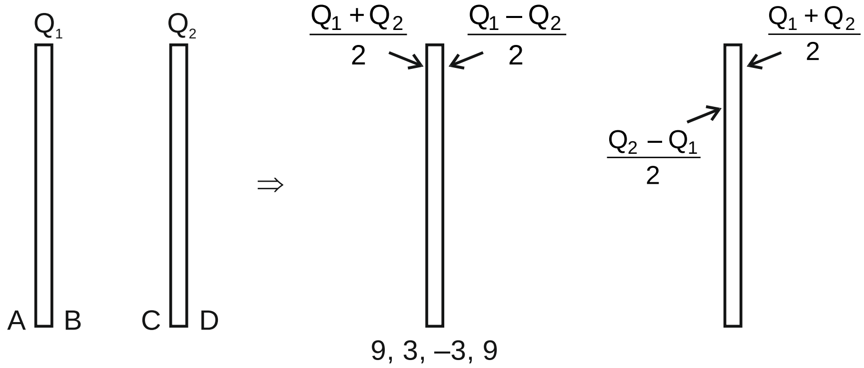
10. 12 mC and 6 mC charges are given to the two conducting plates having same cross-sectional area and placed face to face close to each other as shown in the figure. The resulting charge distribution in mC on surface A, B, C and D are respectively,



(A) 9, 3, –3, 9
(B) 3, 9, –9, 3
(C) 6, 6, – 6, 12
(D) 6, 6, 3, 3

Ans : (A)

Hint :

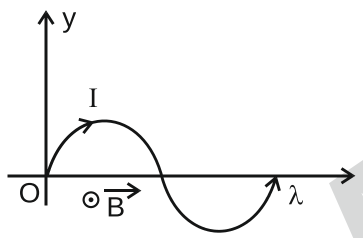


11. A wire carrying a steady current I is kept in the x-y plane along the curve $y = A\sin\left(\frac{2\pi}{\lambda}x\right)$. A magnetic field B exists in the z-direction. The magnitude of the magnetic force in the portion of the wire between x = 0 and x = λ is

(A) 0 (B) $2I\lambda B$ (C) $I\lambda B$ (D) $I\lambda B/2$

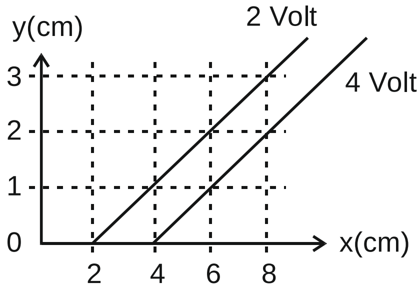
Ans : (C)

Hint :



Magnetic force = $I\lambda B$

12. The figure represents two equipotential lines in x-y plane for an electric field. The x-component E_x of the electric field in space between these equipotential lines is,



(A) 100 V/m (B) –100 V/m (C) 200 V/m (D) –200 V/m

Ans : (B)

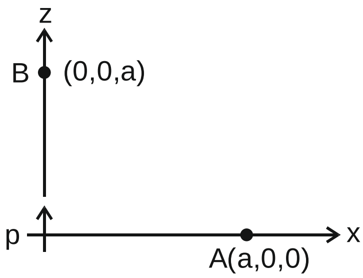
Hint : $E_x = -\frac{dv}{dx} = -\left(\frac{4-2}{2}\right)$ = –1 V/cm = –100 V/m

13. An electric dipole of dipole moment $\vec{p}$ is placed at the origin of the co-ordinate system along the z-axis. The amount of work required to move a charge 'q' from the point (a, 0, 0) to the point (0, 0, a) is,

(A) $\frac{pq}{4\pi\varepsilon_0 a}$ (B) 0 (C) $\frac{-pq}{4\pi\varepsilon_0 a^2}$ (D) $\frac{pq}{4\pi\varepsilon_0 a^2}$

Ans : (D)

Hint :



$W = q(V_B - V_A)$

$= q\left(\frac{p}{4\pi\in_0}\cdot\frac{\cos 0}{a^2} - \frac{p\cos 90^\circ}{4\pi\in_0 a^2}\right)$

$= \frac{pq}{4\pi\varepsilon_0 a^2}$

14. The electric field of a plane electromagnetic wave of wave number k and angular frequency ω is given by $\vec{E} = E_0(\hat{i} + \hat{j})\sin(kz - \omega t)$. Which of the following gives the direction of the associated magnetic field $\vec{B}$?

(A) $\hat{k}$ (B) $-\hat{i} + \hat{j}$ (C) $-\hat{i} - \hat{j}$ (D) $\hat{i} - \hat{k}$

Ans : (B)

Hint : $\vec{E}\cdot\vec{B} = 0$ and in the plane of XY.

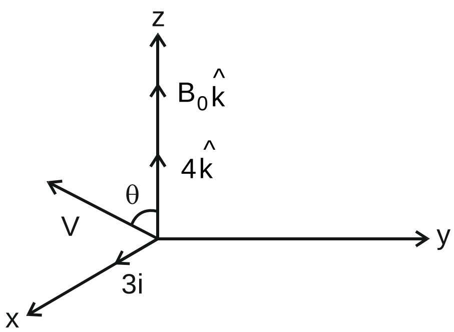
15. A charged particle in a uniform magnetic field $\vec{B} = B_0\hat{k}$ starts moving from the origin with velocity $\nu = 3\hat{i} + 4\hat{k}$ m/s. The trajectory of the particle and the time t at which it reaches 2 m above x-y plane are,

(A) Circular path, $\frac{1}{2}$ sec. (B) Helical path, $\frac{1}{2}$ sec.

(C) Circular path, $\frac{2}{3}$ sec. (D) Helical path, $\frac{2}{3}$ sec.

Ans : (B)

Hint :

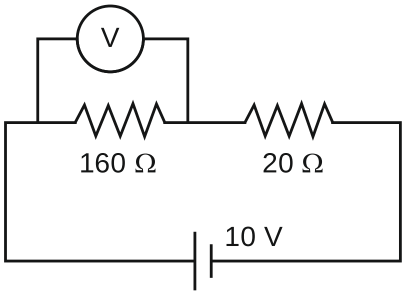


Velocity along z-direction will be const.

$t = \frac{s}{V_z} = \frac{2}{4} = \frac{1}{2}$sec

Path will be helical.

16. In an experiment on a circuit as shown in the figure, the voltmeter shows 8V reading. The resistance of the voltmeter is,



(A) 20 Ω (B) 320 Ω (C) 160 Ω (D) 1.44 kΩ

Ans : (C)

Hint : Voltage across 20Ω = 2V

Main current = $\frac{2}{20}$ = 0.1A

Current through 160Ω = $\frac{8}{160} = \frac{1}{20}$ = 0.05A

Also, 8V = 0.05 × R

R = $\frac{8}{0.05}$ = 160Ω

17. An interference pattern is obtained with two coherent sources of intensity ratio n : 1. The ratio $\frac{I_{Max} - I_{Min}}{I_{Max} + I_{Min}}$ will be maximum if

(A) n = 1 (B) n = 2 (C) n = 3 (D) n = 4

Ans : (A)

Hint : $\frac{I_1}{I_2} = n$

$I_1 = nI_2$

$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = \left(\sqrt{nI_2} + \sqrt{I_2}\right)^2 = \left(\sqrt{n} + 1\right)^2 \left(\sqrt{I_2}\right)^2$

$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = \left(\sqrt{nI_2} - \sqrt{I_2}\right)^2 = \left(\sqrt{n} - 1\right)^2 \left(\sqrt{I_2}\right)^2$

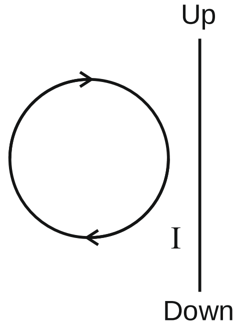
$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{\left(\sqrt{n}+1\right)^2 (I_2) - \left(\sqrt{n}-1\right)^2 I_2}{\left(\sqrt{n}+1\right)^2 I_2 + \left(\sqrt{n}-1\right)^2 I_2} = \frac{\left(\sqrt{n}+1\right)^2 - \left(\sqrt{n}-1\right)^2}{\left(\sqrt{n}+1\right)^2 + \left(\sqrt{n}-1\right)^2}$$

$$= \frac{\left(n+1+2\sqrt{n}\right) - \left(n+1-2\sqrt{n}\right)}{n+1+2\sqrt{n}+n+1-2\sqrt{n}} = \frac{4\sqrt{n}}{2(n+1)} = \frac{2\sqrt{n}}{n+1}$$

∴ decreases with increasing n.

∴ It will be maximum if n = 1

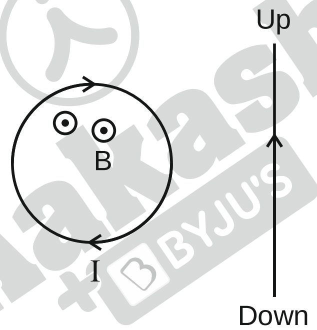
18. A circular coil is placed near a current carrying conductor, both lying on the plane of the paper. The current is flowing through the conductor in such a way that the induced current in the loop is clockwise as shown in the figure. The current in the wire is,



(A) time dependent and downward.
(B) steady and upward.
(C) time dependent and upward.
(D) An alternating current.

Ans : (A, C)

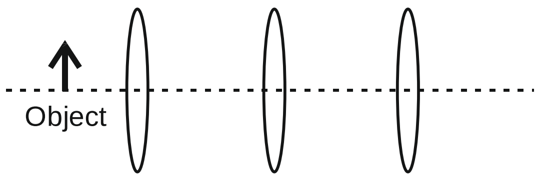
Hint : If current is increasing in upward direction, so magnetic field is increasing in out of plane in order to oppose it induced current will be in clockwise direction



Similarly if current is decreasing in downward direction, so magnetic field is decreasing into the plane in order to support it induced current will be in clockwise direction.

So both option A and C can be correct Bonus.

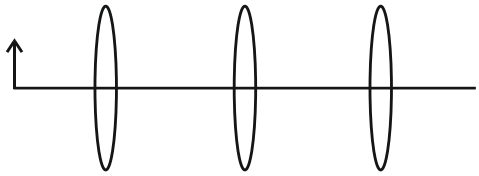
19. Three identical convex lenses each of focal length f are placed in a straight line separated by a distance f from each other. An object is located at f/2 in front of the leftmost lens. Then,



(A) Final image will be at f/2 behind the rightmost lens and its magnification will be –1.
(B) Final image will be at f/2 behind the rightmost lens and its magnification will be +1.
(C) Final image will be at f behind the rightmost lens and its magnification will be –1.
(D) Final image will be at f behind the rightmost lens and its magnification will be +1.

Ans : (A)

Hint :



For first lens

$u = -\frac{f}{2}$

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{1}{v} - \frac{1}{\frac{-f}{2}} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{2}{f} \Rightarrow \frac{1}{v} = \frac{-1}{f}$

$v = -f$

$m_1 = \frac{v}{u} = \frac{-f}{\frac{-f}{2}} = 2$

For second lens

$u = -(f + f) = -2f$

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{-2f} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{2f} \Rightarrow \frac{1}{v} = \frac{1}{2f}$

$v = 2f$

$m_1 = \frac{v}{u} = \frac{2f}{-2f} = -1$

For third lens

$u = f$

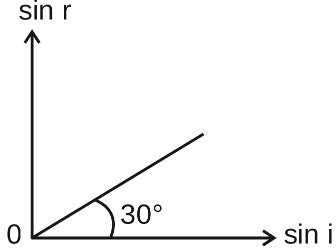
$f = f$

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{2}{f} \Rightarrow v = \frac{f}{2}$

$m_3 = \frac{v}{u} = \frac{\frac{f}{2}}{f} = \frac{1}{2}$

Total magnification = $m_1 m_2 m_3 = 2 \times (-1) \times \frac{1}{2} = -1$

20. A ray of monochromatic light is incident on the plane surface of separation between two media X and Y with angle of incidence 'i' in medium X and angle of refraction 'r' in medium Y. The given graph shows the relation between sin i and sin r. If V_X and V_Y are the velocities of the ray in media X and Y respectively, then which of the following is true?



(A) $V_X = \frac{1}{\sqrt{3}} V_Y$

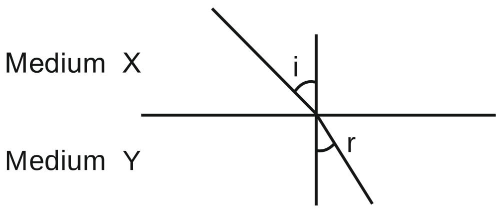
(B) $V_X = \sqrt{3}\, V_Y$

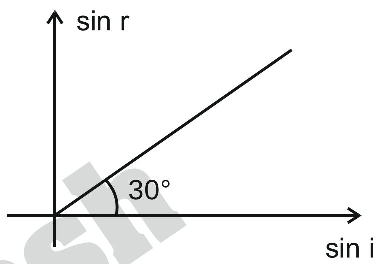
(C) Total internal reflection can happen when the light is incident in medium X.

(D) $\nu_X = \sqrt{3}\, \nu_Y$, where ν_X and ν_Y are frequencies of the light in medium X and Y respectively.

Ans : (B)

Hint :





y = mx

$\mu_1 \sin i = \mu_2 \sin r$

$\frac{C}{V_X} \sin i = \frac{C}{V_Y} \sin r$

$\frac{\sin i}{\sin r} = \frac{V_X}{V_Y} = \sqrt{3}$

$V_Y = \frac{V_X}{\sqrt{3}}$

$V_X = \sqrt{3}\ V_Y$

sin r = (tan 30°) sin i

$\frac{\sin i}{\sin r} = \frac{1}{\tan 30°} = \frac{\sqrt{3}}{1}$

21. If the potential energy of a hydrogen atom in the first excited state is assumed to be zero, then the total energy of n = ∞ state is,

(A) 3.4 eV (B) 6.8 eV (C) 0 (D) ∞

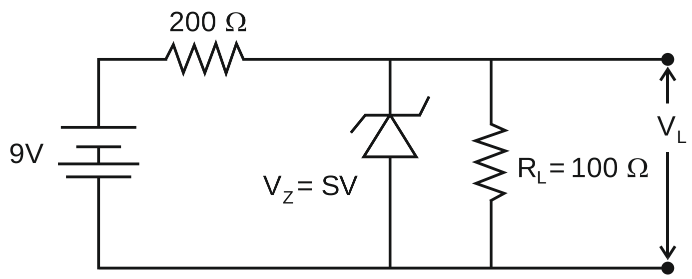
Ans : (B)

Hint : Potential energy in 1st emited stage (U_2) = –6.8eV. When U_2 is assumed to be zero then potential energy in (n = ∞)

will be 6.8 eV

so, total energy for (n = ∞) = 6.8eV

22.

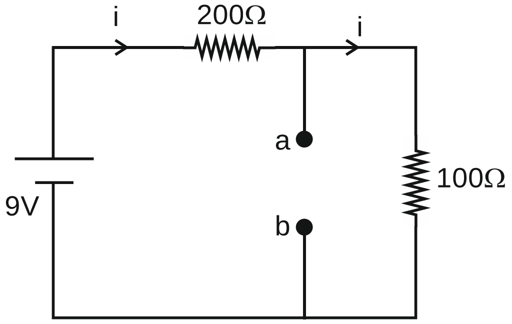


In the given circuit, find the voltage drop V_L in the load resistance R_L.

(A) 5 V (B) 3 V (C) 9V (D) 6V

Ans : (B)

Hint :



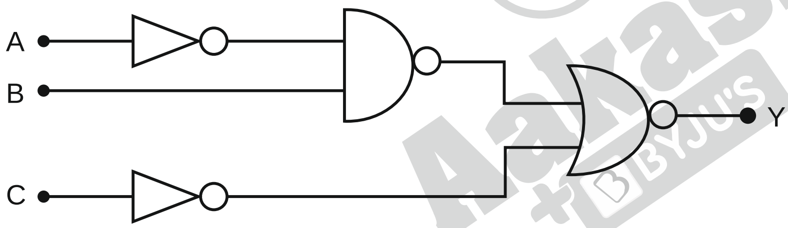
$i = \frac{9}{300} A$

So, $V_a - V_a = \frac{9}{300} \times 100 = 3V$

So, diode is not activated,

So, voltage across load is 3V.

23.

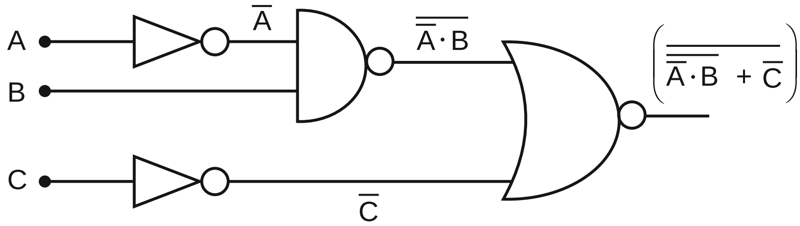


Consider the logic circuit with inputs A, B, C and output Y. How many combinations of A, B and C gives the output Y = 0 ?

(A) 8 (B) 5 (C) 7 (D) 1

Ans : (C)

Hint :



$\overline{\left(A+\overline{B}\right)+\overline{C}} = \overline{A+\overline{B}}\cdot C = \left(\overline{A}\cdot B\cdot C\right)$

$\therefore$ y = 1

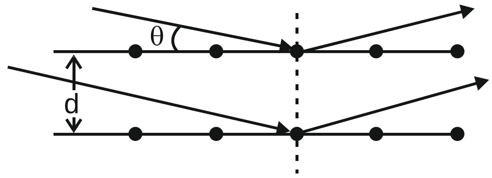
if A = 0, B = 1, C = 1

For rest of all cases Y = O

Total cases = 8

$\therefore$ Ans (8 – 1) = 7

24. X-rays of wavelength λ gets reflected from parallel planes of atoms in a crystal with spacing d between two planes as shown in the figure. If the two reflected beams interfere constructively, then the condition for maxima will be, (n is the order of interference fringe)



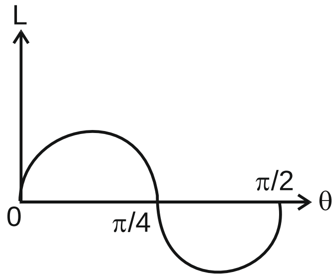
(A) $d \tan\theta = n\lambda$ (B) $d \sin\theta = n\lambda$ (C) $2d \cos\theta = n\lambda$ (D) $2d \sin\theta = n\lambda$

Ans : (D)

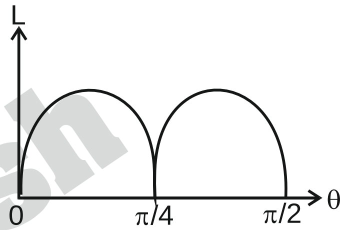
Hint : $2d \sin\theta = n\lambda$

25. A particle of mass m is projected at a velocity u, making an angle θ with the horizontal (x – axis). If the angle of projection θ is varied keeping all other parameters same, then magnitude of angular momentum (L) at its maximum height about the point of projection varies with θ as,

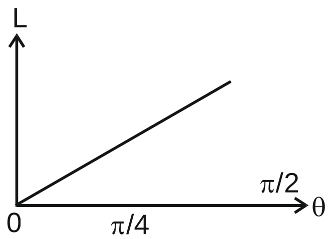
(A)



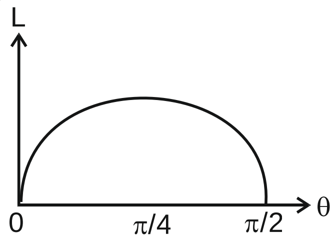
(B)



(C)

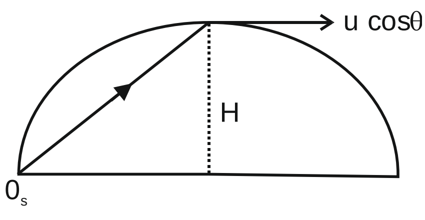


(D)



Ans : (D)

Hint :



$L_0 = mu\cos\theta\, H$

$$= mu\cos\theta \frac{u^2 \sin^2\theta}{2g}$$

$$= \frac{mu^3 \sin^2\theta \cos\theta}{2g}$$

L is zero for $\theta = 0^\circ$ and $\theta = \frac{\pi}{2}$

26. A body of mass 2 kg moves in a horizontal circular path of radius 5 m. At an instant, its speed is $2\sqrt{5}$ m/s and is increasing at the rate of 3 m/s^2. The magnitude of force acting on the body at the instant is,

(A) 6 N (B) 8 N (C) 14 N (D) 10 N

Ans : (D)

Hint : F = ma

$$= m\sqrt{a_c^2 + a_T^2}$$

$$= m\sqrt{\left(\frac{20}{5}\right)^2 + 9}$$

$$= 2\sqrt{16+9} = 2\times 5 = 10\text{N}$$

27. In an experiment, the length of an object is measured to be 6.50 cm. This measured value can be written as 0.0650 m. The number of significant figures on 0.0650 m is

(A) 3 (B) 4 (C) 2 (D) 5

Ans : (A)

Hint :

28. A mouse of mass m jumps on the outside edge of a rotating ceiling fan of moment of inertia I and radius R. The fractional loss of angular velocity of the fan as a result is

(A) $\frac{mR^2}{I+mR^2}$ (B) $\frac{I}{I+mR^2}$ (C) $\frac{I-mR^2}{I}$ (D) $\frac{I-mR^2}{I+mR^2}$

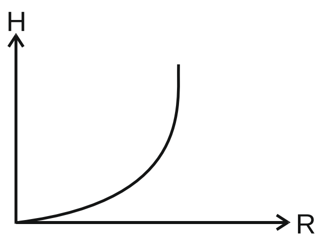
Ans : (A)

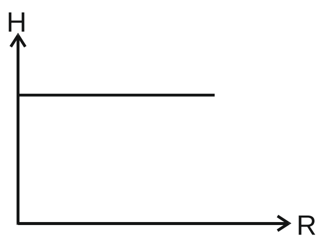
Hint : $I\omega_0 = \left(I + mR^2\right)\omega$ $\omega_0 \rightarrow$ Initial angular velocity

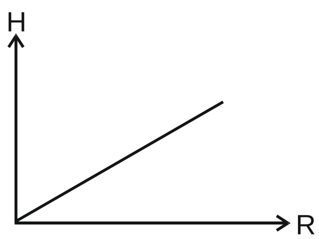
$$\omega = \frac{I\omega_0}{I+mR^2}$$ $\omega \rightarrow$ Final angular velocity

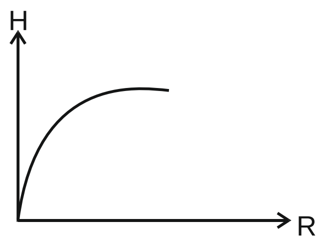
So, $\frac{\omega_0 - \omega}{\omega_0} = I - \frac{I}{I+mR^2} = \frac{mR^2}{I+mR^2}$

29. Acceleration due to gravity at a height H from the surface of a planet is the same as that at a depth of H below of surface. If R be the radius of the planet, then H vs. R graph for different planets will be

(A)


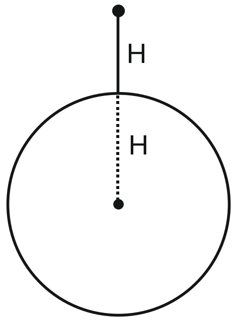
(B)


(C)


(D)


Ans : (C)

Hint :



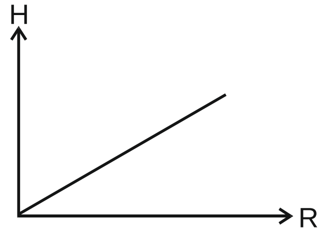
$$\frac{GM}{(R+H)^2} = \frac{GM(R-H)}{R^3}$$

$$(R+H)^2(R-H) = R^3$$

$$\cancel{R^3} - RH^2 + HR^2 - H^3 = \cancel{R^3}$$

$$H^2 - R^2 + RH = 0$$

$$H = \frac{(\sqrt{5}-1)}{2}R$$



30. A uniform rope of length 4 m and mass 0.4 kg is held on a frictionless table in such a way that 0.6 m of the rope is hanging over the edge. The work done to pull the hanging part of the rope on the to the table is, (Assume g = 10 m/s^2)

(A) 0.36 J (B) 0.24 J (C) 0.12 J (D) 0.18 J

Ans : (D)

Hint : $W = \frac{mgL}{2} = \frac{0.4}{4} \times 0.6 \times 10 \times \frac{0.6}{2}$

$= 0.1 \times 0.6 \times 10 \times 0.3$

$= 0.18\,J$

Category 2 (Q. 31 to 35)

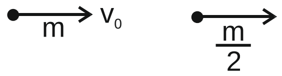
(Carry 2 marks each. Only one option is correct. Negative marks – ½)

31. There are n elastic balls placed on a smooth horizontal plane. The masses of the balls are $m, \frac{m}{2}, \frac{m}{2^2}, ... \frac{m}{2^{n-1}}$ respectively. If the first ball hits the second ball with velocity v_0, then the velocity of the n^{th} ball will be,

(A) $\frac{4}{3}v_0$ (B) $\left(\frac{4}{3}\right)^n v_0$ (C) $\left(\frac{4}{3}\right)^{n-1} v_0$ (D) v_0

Ans : (C)

Hint : 1st Collision



$$V_1 = \frac{2 \times m}{\frac{m}{2} + m} V_0 = \frac{4}{3} V_0$$

2nd Collision

$$V_2 = \frac{2 \times \frac{m}{2}}{\frac{m}{2} + \frac{m}{2^2}} \times V_0 = \left(\frac{4}{3}\right)^2 V_0$$

3rd Collision

$$\left(\frac{4}{3}\right)^3 V_0$$

. . . . .(n – 1) collision, $\therefore V_{n-1} = \left(\frac{4}{3}\right)^{n-1} V_0$

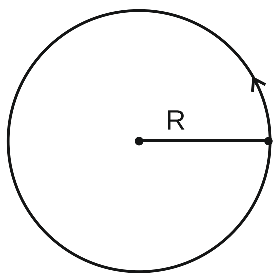
32. An earth's satellite near the surface of the earth takes about 90 min per revolution. A satellite orbiting the moon also takes about 90 min per revolution. Then which of the following is true?

(A) $\rho_m < \rho_e$ (B) $\rho_m > \rho_e$

(C) $\rho_m = \rho_e$ (D) No conclusion can be made about the densities

Ans : (C)

Hint :



$$m\left(E_g\right) = m\omega^2 R$$

$$m \cdot 4\pi \frac{G\rho R}{3} = m\omega^2 R$$

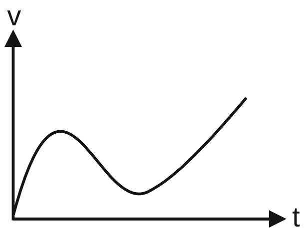
$$\Rightarrow \omega^2 \propto \rho$$

$$\Rightarrow T \propto \frac{1}{\sqrt{\rho}}$$

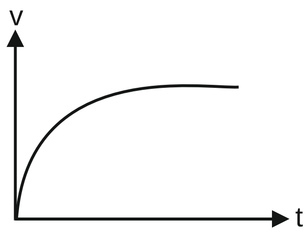
If T are equal, so will be ρ

33. A bar magnet falls from rest under gravity through the centre of a horizontal ring of conducting wire as shown in figure. Which of the following graph best represents the speed (v) vs. time (t) graph of the bar magnet?

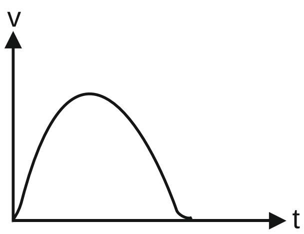
(A)



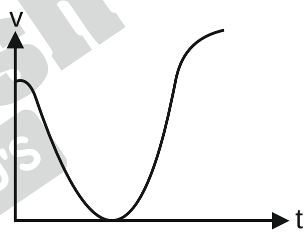
(B)



(C)



(D)



Ans : (A)

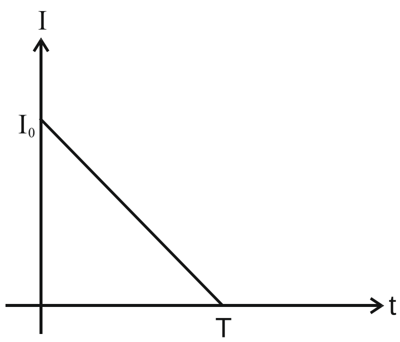
Hint :

34. An amount of charge Q passes through a coil of resistance R. If the current in the coil decreases to zero at a uniform rate during time T, then the amount of heat generated in the coil will be,

(A) $\frac{4Q^2R}{3T}$ (B) $\frac{2Q^2R}{3T}$ (C) $\frac{Q^2R}{4R}$ (D) Q^2RT

Ans : (A)

Hint :



Given, $\frac{1}{2}I_0T = Q \Rightarrow I_0 = \frac{2Q}{T}$

Equation of $I(t) \Rightarrow \frac{I}{I_0} + \frac{t}{T} = 1$

$$I = I_0\left(1 - \frac{t}{T}\right) = \frac{2Q}{T}\left(1 - \frac{t}{T}\right)$$

$$\text{Heat} = \int_0^T I^2 R\,df$$

$$= R\int_0^T \frac{4Q^2}{T^2}\left(1 - \frac{t}{T}\right)^2$$

$$= \frac{4Q^2R}{T^2}\left[\int_0^T dt + \frac{1}{T^2}\int_0^T t^2 dt - \frac{2}{T}\int_0^T t\,dt\right] = \frac{4Q^2R}{T^2}\left[\cancel{T} + \frac{T}{3} - \cancel{T}\right] = \frac{4Q^2R}{3T}$$

35. A modified gravitational potential is given by $V = -\frac{GM}{r} + \frac{A}{r^2}$. If the constant A is expressed in terms of gravitational constant (G), mass (M) and velocity of light (c), then from dimensional analysis, A is,

(A) $\frac{G^2M^2}{c^2}$ (B) $\frac{GM}{c^2}$ (C) $\frac{1}{c^2}$ (D) Dimensionless

Ans : (A)

Hint : $V = -\frac{GM}{r} + \frac{A}{r^2}$

$$[A] = \frac{[GM]}{[r]}\left[r^2\right] = [GM]\,[r]$$

now, we know, $\frac{GM}{r}$ gives dimension of c^2

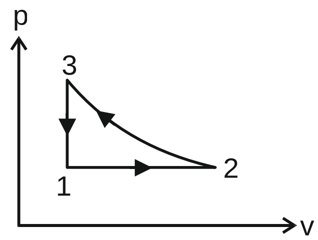
$$\frac{[GM]}{[r]} = \left[c^2\right] \Rightarrow [r] = \frac{[GM]}{\left[c^2\right]}$$

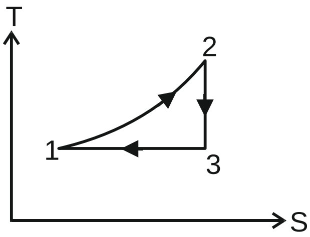
$$\Rightarrow [A] = \frac{[GM][GM]}{\left[c^2\right]}$$

$$[A] = \frac{G^2M^2}{c^2}$$

Category 3 (Q36 to 40)

(Carry 2 marks each. One or more options are correct. No negative marks)

36. 



A cyclic process is shown in p – v diagram and T – S diagram. Which of the following statement(s) is/are true?

(A) $1 \rightarrow 2$: Isobaric, $2 \rightarrow 3$: Isothermal

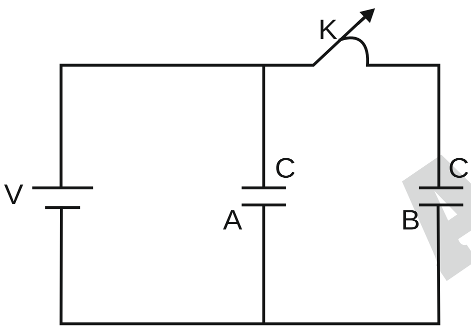
(B) $3 \rightarrow 1$: Isochoric, $2 \rightarrow 3$: adiabatic

(C) Work done by the system in the complete cyclic process in non-zero

(D) The heat absorbed by the system in the complete cyclic process in non-zero

Ans : (B, C, D)

Hint :

37. 

The figure shows two identical parallel plate capacitors A and B of capacitances C connected to a battery. The key K is initially closed. The switch is now opened and the free spaces between the plates of the capacitors are filled with a dielectric constant 3. Then which of the following statement (s) is/are true?

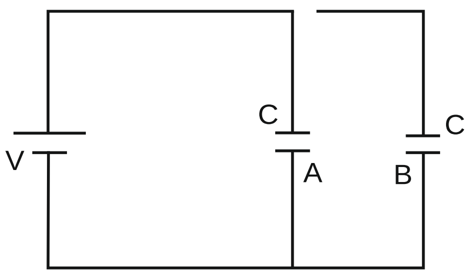
(A) Which the switch is closed, total energy stored in the two capacitors is CV^2

(B) When the switch is opened, no charge is stored in the capacitor B

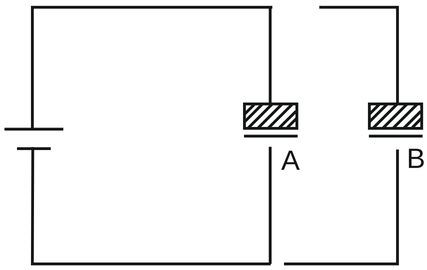
(C) When the switch is opened, energy stored in the capacitor B is $\frac{3}{2}CV^2$

(D) When the switch is opened, total energy stored in two capacitors is $\frac{5}{2}CV^2$

Ans : (A, D)

Hint : 

$$U = \frac{1}{2} \times 2CV^2$$
$$= CV^2$$



$$U_A = \frac{1}{2}(KC)V^2 = \frac{3}{2}CV^2$$

$$U_B = \frac{q^2}{2KC} = \frac{[CV]^2}{2KC} = \frac{CV^2}{2K} = \frac{1}{6}CV^2$$

Total energy when switch is open

$$U = \frac{1}{2}KCV^2 + \frac{1}{6}CV^2$$

$$= \frac{3}{2}CV^2 + \frac{1}{6}CV^2$$

$$= \frac{10}{6}CV^2 \quad = \frac{5}{3}CV^2$$

38. A charged particle of charge q and mass m is placed at a distance 2R from the centre of a vertical cylindrical region of radius R where magnetic field varies as $\vec{B} = (4t^2 - 2t + 6)\hat{k}$ where t is time. Then which of the following statement(s) is/are true?

(A) Induced electric field lines form closed loops

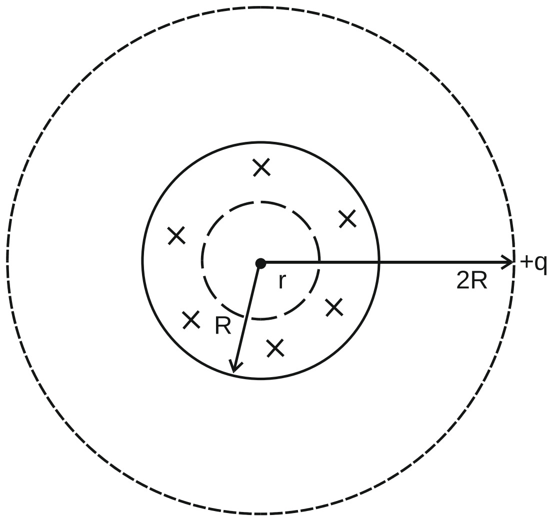
(B) Electric field varies linearly with r if r < R, where r is the radial distance from the centerline of the cylinder

(C) The charged particle will move in clockwise direction when viewed from top

(D) Acceleration of the charged particle is $\frac{7q}{2m}$ when t = 2 sec

Ans : (A, B)

Hint :



r < R

$$E \times 2\pi r = \frac{d\phi}{dt} = \frac{d}{dt}(4t^2 - 2t + 6) \times \pi r^2$$

$E \times 2\pi r = (8t - 2)\pi r^2$

$E = \frac{(8t-2)r}{2}$

E = (4t – 1)r

$E \propto r$

For r > R

$E \times 2\pi.(2R) = \frac{d}{dt}[4t^2 - 2t + 6] \times \pi[R]^2$

$E.4\pi R = [8t - 2]\pi R^2$

$E = \frac{[8t-2]R}{4}$ at t = 2, $E = \frac{14}{4}R = \frac{7R}{2}$

acceleration = $\frac{Eq}{m} = \frac{7Rq}{2m} = \frac{7qR}{2m}$

39. A uniform magnetic field B exists in a region. An electron of charge q and mass m moving with velocity v enters the region in a direction perpendicular to the magnetic field. Considering Bohr angular momentum quantization, which of the following statement(s) is/are true?

(A) The radius of n^{th} orbit $r_n \propto \sqrt{n}$

(B) The maximum velocity of the electron is $\frac{\sqrt{qB\hbar}}{m}$

(C) Energy of the n^{th} level $E_n \propto n$

(D) Transition frequency ω between two successive levels is independent of n

Ans : (A, B, C, D)

Hint : $r = \frac{mv}{qB}$

$mvr = \frac{nh}{2\pi}$

$mv = \frac{nh}{2\pi r}$

$r = \frac{nh}{2\pi rqB}$

$r^2 = \frac{nh}{2\pi qB} \Rightarrow r = \sqrt{\frac{nh}{2\pi qB}}$

$r \propto \sqrt{n}$

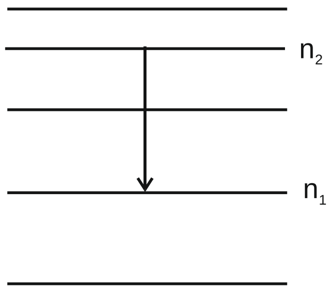
$v = \frac{qBr}{m} = \frac{qB}{m}\sqrt{\frac{nh}{2\pi qB}}$

$v_{min} = \sqrt{\frac{q^2B^2}{m^2} \times \frac{nh}{2\pi qB}} = \sqrt{\frac{nqBh}{2\pi m^2}} = \frac{1}{m}\sqrt{qB\hbar}$

$E = \frac{1}{2}mv^2$

$= \frac{1}{2}m\left[\frac{nqBh}{2\pi m^2}\right]$

$E \propto n$



$E_2 - E_1 = (n_2 - n_1)\frac{qBh}{4\pi m}$

$hf = (n_2 - n_1)\frac{qBh}{4\pi m}$

$n_2 - n_1 = 1$ for successive levels

40. A train is moving along the tracks at a constant speed u. A girl on the train throws a ball of mass m straight ahead along the direction of motion of the train with speed v with respect to herself. Then

(A) Kinetic energy of the ball as measured by the girl on the train is $mv^2/2$

(B) Work done by the girl in throwing the ball is $mv^2/2$

(C) Work done by the train is mvu

(D) The gain in kinetic energy of the ball as measured by a person standing by the rail track is $mv^2/2$

Ans : (A, B, C)

Hint : w.r.t. the girl $E_k = \frac{1}{2}mv^2$

$\therefore W = \Delta E_k = \frac{1}{2}mv^2$

Work by the train = $\left\{\frac{1}{2}(v+u)^2 - \frac{1}{2}mu^2\right\} - \frac{1}{2}mv^2$

$= \frac{1}{2}m\left(v^2 + u^2 + 2vu\right) - \frac{1}{2}m\left(v^2 + u^2\right)$

= mvu

Gain in E_k = $\frac{1}{2}m\left(v+u\right)^2 - \frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mvu$

measured from rail track

CHEMISTRY

CATEGORY - 1 (Q 41 to 70)

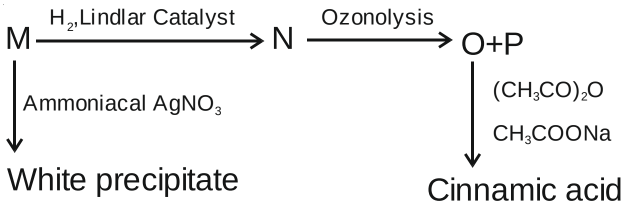
(Carry 1 mark each. Only one option is correct. Negative marks –¼)

41. The correct order of boiling points of N-ethylethanamine (I), ethoxyethane (II) and butan-2-ol (III) is

(A) III < II < I (B) II < III < I (C) II < I < III (D) III < I < II

Ans : (C)

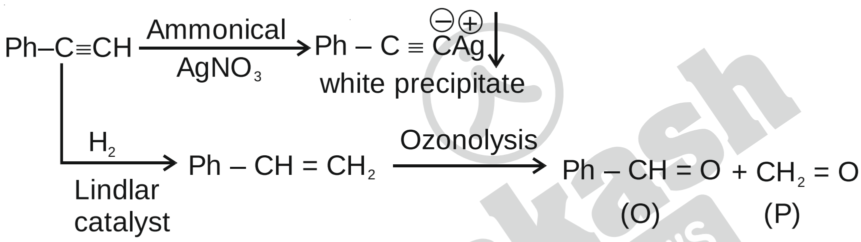
Hint : Butan-2-ol shows stronger H-bonding than N-ethylenthanamine. Ethoxyethane involves dipole association, weaker than H-bonding

42. 

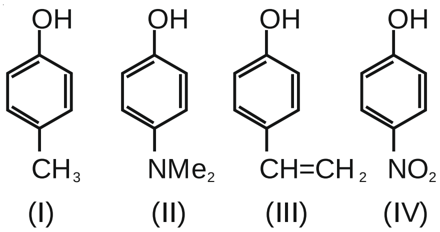
Structure of M is,

(A) $Ph-C\equiv CH$ (B) $Ph-C\equiv C-CH_3$ (C) $H_3C-C\equiv CH$ (D) $H_3C-C\equiv C-CH_3$

Ans : (A)

Hint : 

Benzaldehyde (O) gives Cinnamic acid (Ph – CH = CH – COOH) on reaction with $(CH_3CO)_2O$ and CH_3COONa known as Perkin's condensation

43. 

The correct order of acidity of above compounds is

(A) II > IV > I > III (B) III > IV > II > I (C) IV > II > III > I (D) IV > III > I > II

Ans : (D)

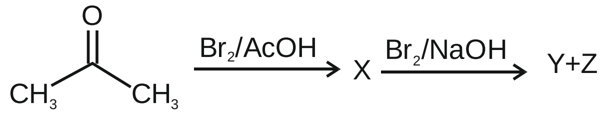
Hint : – NO_2 shows strong –R

–CH = CH_2 shows –R

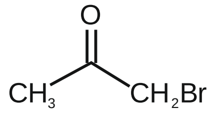
–CH_3 shows hyperconjugation

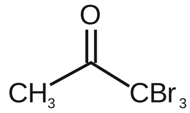
– NMe_2 shows +R

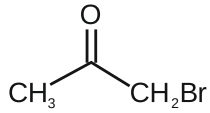
Donating groups lowers acidity while withdrawing groups raise acidity

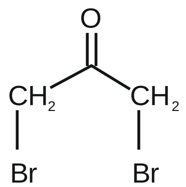
44. 

The correct option for the above reaction is

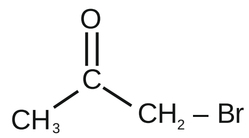
(A) X=  Y = $CHBr_3$ Z = CH_3CO_2Na

(B) X=  Y = $CHBr_3$ Z = CH_3CO_2Na

(C) X=  Y = $CHBr_3$ Z = $\underset{Br}{\underset{|}{CH_2}}CO_2Na$

(D) X=  Y = $CHBr_3$ Z = $\underset{Br}{\underset{|}{CH_2}}CO_2Na$

Ans : (A)

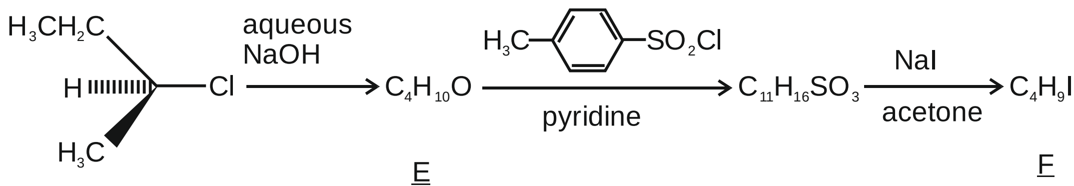
Hint : X : 

In acidic medium, only monohalogenation takes place.

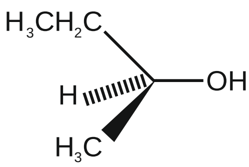
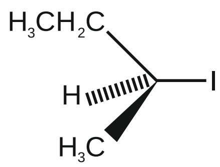
Y : $CHBr_3$

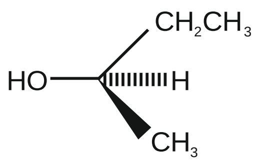
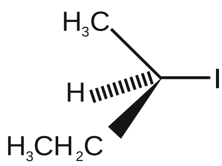
Z : CH_3COONa

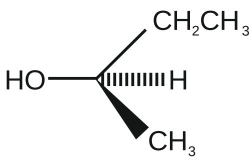
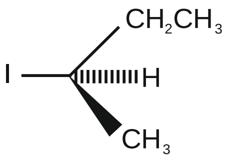
Second step is bromoform reaction that takes place here on $CH_3COCH_2 - Br$

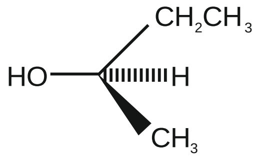
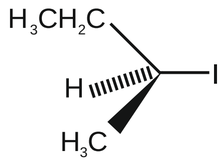
45. 

If all the nucleophilic substitution reactions at saturated carbon atoms in the above sequence of reactions follow S_N2 mechanism, then E and F will be respectively,

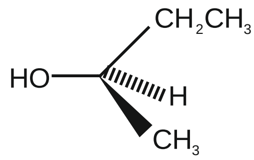
(A)  and 

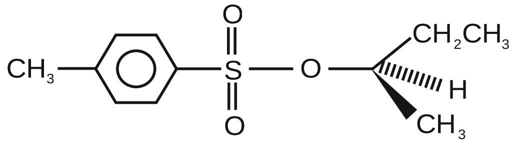
(B)  and 

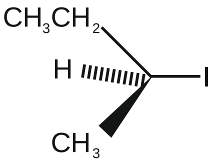
(C)  and 

(D)  and 

Ans : (D)

Hint : E :  (Product of S_N^2)

$C_{11}H_{16}SO_3$: 

F :  (Product of S_N^2)

46. Two base balls (masses: m_1 = 100 g, and m_2 = 50 g) are thrown. Both of them move with uniform velocity, but the velocity of m_2 is 1.5 times that of m_1. The ratio of de Broglie wavelengths $\lambda(m_1) : \lambda(m_2)$ is given by

(A) 4 : 3 (B) 3 : 4 (C) 2 : 1 (D) 1 : 2

Ans : (B)

Hint : $\frac{\lambda_1}{\lambda_2} = \frac{m_2V_2}{m_1V_1} = \frac{50\times1.5V_1}{100\times V_1} = \frac{1.5}{2} = \frac{3}{4}$

47. What is the edge length of the unit cell of a body centred cubic crystal of an element whose atomic radius is 75 pm?

(A) 170 pm (B) 175 pm (C) 178 pm (D) 173.2 pm

Ans : (D)

Hint : In BCC, 4r = $\sqrt{3}a$

$\therefore a = \frac{4r}{\sqrt{3}} = \frac{4\times75}{\sqrt{3}} = \frac{300}{\sqrt{3}} = \sqrt{3} \times 100$ pm =173.2 pm

48. The root mean square (rms) speed of X_2 gas is x m/s at a given temperature. When the temperature is doubled, the X_2 molecules dissociated completely into atoms. The root mean square speed of the sample of gas then becomes (in m/s)

(A) x/2 (B) x (C) 2x (D) 4x

Ans : (C)

Hint : $C_{rms} = \sqrt{\frac{3RT}{M}}$

$T_1 = T$ T_2=2T

$M_1 = M$ $M_2 = M/2$

$C_1 = X$ $C_2 = ?$

$$\frac{C_1}{C_2} = \sqrt{\frac{T_1}{M_1}\times\frac{M_2}{T_2}} = \sqrt{\frac{T}{M}\times\frac{M/2}{2T}} = \frac{1}{2}$$

$\therefore \frac{x}{C_2} = \frac{1}{2}$, Hence $\boxed{C_2 = 2x\ m/s}$

49. Arrange the following in order of increasing mass

I. 1 mole of N_2

II. 0.5 mole of O_3

III. 3.011×10^{23} molecules of O_2

IV. 0.5 gram atom of O_2

(A) IV < III < II < I (B) IV < I < III < II (C) III < II < IV < I (D) I < III < II < IV

Ans : (A)

Hint : 1mole N_2 = 28g

0.5 mole O_3 = 24g

3.011×10^{23} molecules of O_2 = ½ mole O_2 = 1 mole O = 16 g

0.5 g atom O_2 = ½ mole of atoms of O = 8g

50. Which of the following would give a linear plot?

(A) k vs T (B) k vs 1/T (C) ln k vs T (D) ln k vs 1/T

(k is the rate constant of an elementary reaction and T is temp. in absolute scale)

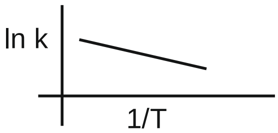
Ans : (D)

Hint : Arrhenius equation gives us

$$k = Ae^{-E_a/RT}$$

$$\ln k = \ln A - \frac{E_a}{R}\left(\frac{1}{T}\right)$$

$$y = c - mx$$



51. The equivalent conductance of NaCl, HCl and CH_3COONa at infinite dilution are 126.45, 426.16 and 91 $ohm^{-1}cm^2eq^{-1}$ respectively at 25°C. The equivalent conductance of acetic acid (at infinite dilution) would be

(A) 461.61 ohm^{-1} cm^2 eq^{-1}

(B) 390.71 ohm^{-1} cm^2 eq^{-1}

(C) cannot be determined from the given data

(D) 208.71 ohm^{-1} cm^2 eq^{-1}

Ans : (B)

Hint : According to Kohlrausch's law

$$\wedge^0_{CH_3COOH} = \wedge^0_{CH_3COONa} + \wedge^0_{HCl} - \wedge^0_{NaCl}$$

$$\wedge^0_{CH_3COOH} = (91 + 426.16 - 126.45)\,ohm^{-1}cm^2eq^{-1}$$

$$\wedge^0_{CH_3COOH} = 390.71\,ohm^{-1}cm^2eq^{-1}$$

52. For the reaction $A + B \rightarrow C$, we have the following data :

Initial concentration of A (in molarity)	Initial concentration of B (in molarity)	Rate (initial) (Relevant unit)
1	10	100
1	1	1
10	1	10

The order of the reaction with respect to A and B are

(A) Not possible to tell with the given data

(B) First order with respect to both A and B

(C) First order with respect to A and second order with respect to B

(D) Second order with respect to A and first order with respect to B.

Ans : (C)

Hint : Let us assume $R = K[A]^x[B]^y$

Where x and y are orders wrt A and B respectively

$\therefore$ We can write from given data

$100 = k(1)^x (10)^y$ ____ (1)

$1 = k(1)^x (1)^y$ ____ (2)

$10 = k(10)^x(1)^y$ ____ (3)

$2 \div 1$ gives

$$\frac{1}{100} = \frac{k(1)^x (1)^y}{k(1)^x (10)^y}, \quad \frac{1}{100} = \left(\frac{1}{10}\right)y$$

$\boxed{y = 2}$

$3 \div 1$ gives $\frac{10}{1} = \frac{K(10)^x (1)^y}{k(1)^x (1)^y}$

$10 = (10)^x$ $\boxed{x = 1}$

So reaction is 2nd order w.r.t B but 1st order w.r.t A.

53. The equivalent weight of KIO_3 in the given reaction is (M = molecular mass) :

$2Cr(OH)_3 + 4OH^- + KIO_3 \rightarrow 2CrO_4^{2-} + 5H_2O + KI$

(A) M (B) M/2 (C) M/6 (D) M/8

Ans : (C)

Hint : $Cr(OH)_3 + 4\bar{O}H + K\overset{+5}{I}O_3 \longrightarrow 2CrO_4^{-2} + 5H_2O + K\overset{-1}{I}$

Change in oxidation state of iodine = 6

$\therefore$ Equivalent weight of KIO_3 = M/6

54. At STP, the dissociation reaction of water is $H_2O \rightleftharpoons H^+ (aq.) + OH^- (aq.)$, and the pH of water is 7.0. The change of standard free energy ($\Delta G°$) for the above dissociation process is given by

(A) 20301 cal/mol (B) 19091 cal/mol (C) 20096 cal/mol (D) 21301 cal/mol

Ans : (B)

Hint : $\Delta G^\circ = -2.303 \ RT\log K_w$

$= -2.303 \times 1.987 \times 298 \log 10^{-14}$

$= +2.303 \times 1.987 \times 298 \times 14$ cal/mol $[K_w = [H^+][OH^-] = 10^{-7} \times 10^{-7} = 10^{-14}$ as pH = 7]

= 19091.3 cal/mol = 19091 cal/mol

55. Na_2CO_3 is prepared by Solvay process but K_2CO_3 cannot be prepared by the same because

(A) K_2CO_3 is highly soluble in H_2O (B) $KHCO_3$ is sparingly soluble

(C) $KHCO_3$ is appreciably soluble (D) $KHCO_3$ decomposes

Ans : (C)

Hint : $(NH_4)HCO_3 + KCl \longrightarrow KHCO_3(aq) + NH_4Cl(aq)$

$KHCO_3$ being appreciably soluble cant be isolated from reaction medium easily.

56. If in case of a radio isotope the value of half-life ($T_{1/2}$) and decay constant (λ) are identical in magnitude, then their value should be

(A) 0.693/2 (B) $(0.693)^{1/2}$ (C) $(0.693)^2$ (D) 0.693

Ans : (B)

Hint : For a radio decay $T_{1/2} = \frac{0.693}{\lambda}$

If $T_{1/2} = \lambda = x$ then $x = \frac{0.693}{x}$

$\Rightarrow x^2 = 0.693$, $\Rightarrow x = T_{1/2} = \lambda = (0.693)^{1/2}$

57. Suppose a gaseous mixture of He, Ne, Ar and Kr is treated with photons of the frequency appropriate to ionize Ar. What ion(s) will be present in the mixture ?

(A) Ar^+ (B) $Ar^+ + Kr^+$ (C) $Ar^+ + He^+ + Ne^+$ (D) $He^+ + Ar^+ + Kr^+$

Ans : (B)

Hint : He > Ne > Ar > Kr > Xe > Rn (Order of Ionization energy)

Energy of photon is sufficient to ionize Ar, hence Kr will also ionize.

Therefore mixture contains Ar^+ and Kr^+

58. A solution containing 4g of polymer in 4.0 litre solution at 27°C shows an osmotic pressure of 3.0×10^{-4} atm. The molar mass of the polymer in g/mol is

(A) 820000 (B) 82000 (C) 8200 (D) 820

Ans : (B)

Hint : $\pi = iC(M)RT$

$3.0 \times 10^{-4} = 1 \times C(M) \times 0.0821 \times 300$

$\therefore C(M) = 1.22 \times 10^{-5}$, $\text{Molarity} = \frac{\text{no. of moles}}{\text{vol. of solution(L)}}$

$1.22 \times 10^{-5} = \frac{4/M}{4}$. Hence M = 81967 $\approx$ 82000 g/mol

59. The molecular shapes of SF_4, CF_4 and XeF_4 are

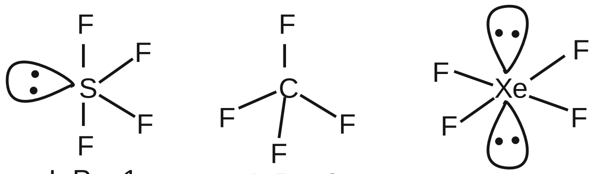
(A) the same with 2, 0 and 1 lone pairs of electrons on the central atoms, respectively.

(B) the same with 1, 1 and 1 lone pairs of electrons on the central atoms, respectively

(C) different with 0, 1 and 2 lone pairs of electrons on the central atoms, respectively

(D) different with 1, 0 and 2 lone pairs of electrons on the central atoms, respectively

Ans : (D)

Hint : 

L.P = 1 L.P = 0 L.P = 2

60. The species in which nitrogen atom is in a state of sp hybridisation is

(A) NO_3^- (B) NO_2 (C) NO_2^+ (D) NO_2^-

Ans : (C)

Hint : Steric Number in $NO_2^+ = \frac{5+0-1}{2} = 2$

$\therefore$ sp hybridization

61. The correct statement about the magnetic properties of $[Fe(CN)_6]^{3-}$ and $[FeF_6]^{3-}$ is

(A) Both are paramagnetic

(B) Both are diamagnetic

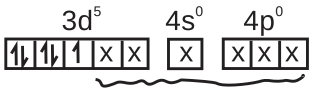
(C) $[Fe(CN)_6]^{3-}$ is diamagnetic, $[FeF_6]^{3-}$ is paramagnetic

(D) $[Fe(CN)_6]^{3-}$ is paramagnetic, $[FeF_6]^{3-}$ is diamagnetic

Ans : (A)

Hint : $\overset{+3}{Fe} = [Ar]3d^5 4s^0$

For $[Fe(CN)_6]^{3-}$

Fe^{+3} = 

d^2sp^3 hybridization

Pairing of e^- takes place as CN^- is strong field ligand but has one unpaired electron thus paramagnetic.

For $[FeF_6]^{-3}$

As F^- is weak field ligand, so no pairing of electron, thus it has five unpaired electron. Therefore paramagnetic.

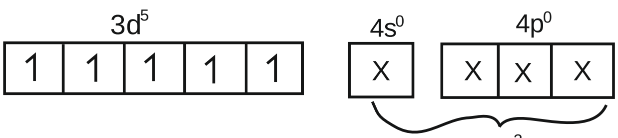
62. The calculated spin-only magnetic moment values in BM for $[FeCl_4]^-$ and $[Fe(CN)_6]^{3-}$ are

(A) 5.9 BM, 1.732 BM (B) 4.89 BM, 1.732 BM (C) 3.87 BM, 1.732 BM (D) 1.732 BM, 2.82 BM

Ans : (A)

Hint : Fe^{+3} = [Ar] $3d^5 4s^0$

$[FeCl_4]^-$

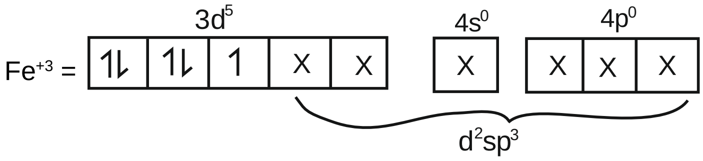
Fe^{+3} = 

sp^3

No pairing as Cl^- is weak field ligand, hence have five unpaired electron (n = 5).

$\therefore \quad \mu = \sqrt{n(n+2)}$ B.M = $\sqrt{5(5+2)}$ B.M = 5.9 B.M

$\left[Fe(CN)_6\right]^{-3}$



Pairing takes place as CN^- is strong field ligand but has one unpaired electron (n=1)

$\therefore \ \mu = \sqrt{n(n+2)}$ B.M = $\sqrt{1(1+2)}$ = 1.732 B.M

63. BrF_3 self ionises as following

(A) $2BrF_3 \rightleftharpoons BrF^+ + BrF_5^-$

(B) $2BrF_3 \rightleftharpoons BrF_2^+ + BrF_4^-$

(C) $2BrF_3 \rightleftharpoons BrF_4^+ + BrF_2^-$

(D) $2BrF_3 \rightleftharpoons BrF_3^+ + BrF_3^-$

Ans : (B)

Hint : $2BrF_3 \rightleftharpoons BrF_2^+ + BrF_4^-$ (Relatively more stable structures.)

64. $4f^2$ electronic configuration is found in

(A) Pr (B) Pr^{3+} (C) Nd^{3+} (D) Pm^{3+}

Ans : (B)

Hint : Pr(59) = [Xe] $4f^3\ 6s^2$

$\therefore Pr^{+3}$ = [Xe] $4f^2$

65. Which of the following statements is incorrect ?

(A) $\left[VF_6\right]^{3-}$ is paramagnetic with 2 unpaired electrons.

(B) $\left[CuCl_4\right]^{2-}$ is paramagnetic with 1 unpaired electron.

(C) $\left[Co(NH_3)_6\right]^{3+}$ is diamagnetic.

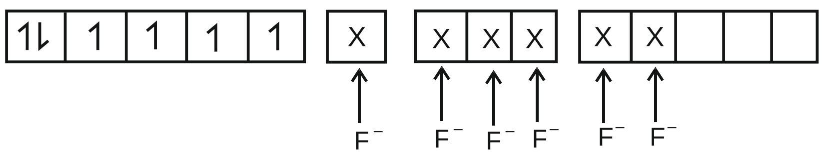
(D) $\left[CoF_6\right]^{3-}$ is paramagnetic with 2 unpaired elecstrons.

Ans : (D)

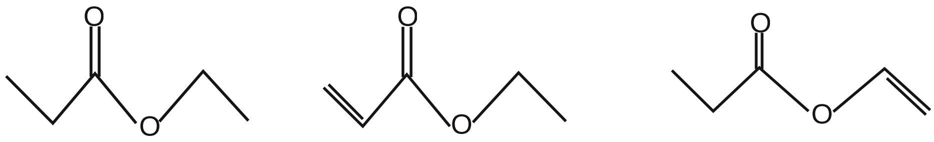
Hint : $[\overset{III}{Co}F_6]^{3-}$ As F^- is weak field ligand.

Oxidation Number of Co = +3

Co^{3+} ($4s^0\ 3d^6$)



Number of unpaired e^- = 4

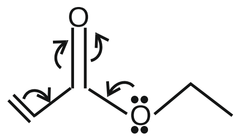
66. 

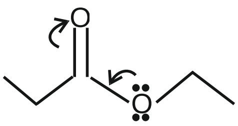
I II III

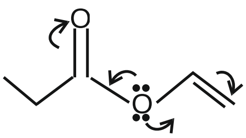
The correct order of C = O bond length in ethyl propanoate (I), ethyl propenoate (II) and ethenyl propanoate (III) is

(A) I > II > III (B) III > II > I (C) I > III > II (D) II > I > III

Ans : (D)

Hint : (II) 

(I) 

(III) 

C = O bond length

II > I > III

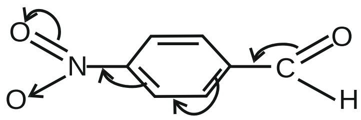
>C = O bond has the most single bond character in compound II and the least single bond character in compound III.

67. Select the molecule in which all the atoms may lie on a single plane is

(A) 4-Nitrobenzaldehyde (B) 4-Methoxybenzaldehyde

(C) 4-Methylnitrobenzene (D) 4-Nitroacetophenone

Ans : (A)

Hint : 

All atoms other than Hydrogen are sp^2 hybridised.

68. The IUPAC name of $CH_3CH = C(CHO) – CH_2 – CH_3$ is :

$$CH_3CH = \underset{\displaystyle CHO}{\underset{|}{C}} – CH_2 – CH_3$$

(A) 3-Formyl-2-pentene (B) 2-Ethylbut-2-enal (C) 3-Ethylbut-3enal (D) 2-Ethylcrotonaldehyde

Ans : (B)

Hint : $\overset{4}{C}H_3-\overset{3}{C}H=\overset{2}{C}(\overset{1}{C}HO)-CH_2-CH_3$

2-Ethylbut-2-enal

69. The correct stability order of the following carbocations is

$H_2\overset{\oplus}{C}-CH=CH-CH_3$ (I)

$\overset{\oplus}{C}H_2-CH=CH-BMe_2$ (II)

$H_2\overset{\oplus}{C}-CH=CH-NMe_2$ (III)

$H_2\overset{\oplus}{C}-CH=CH-OMe$ (IV)

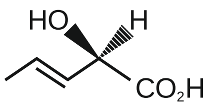
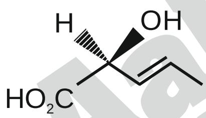
(A) II > I > III > IV (B) III > I > II > IV (C) III > IV > I > II (D) IV > III > II > I

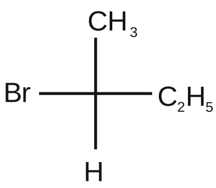
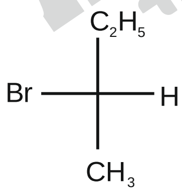
Ans : (C)

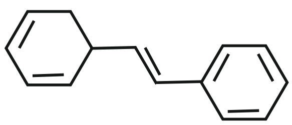
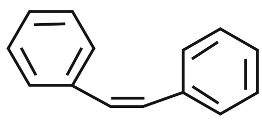
Hint : III > IV > I > II

+ R effect : $-\ddot{N}Me_2 > -\ddot{O}CH_3$

$-BMe_2$ can show –R effect

70. 1.  and 

2.  and 

3.  And 

The relationship between the pair of compounds shown above are respectively,

(A) enantiomer, diastereomer, diastereomer

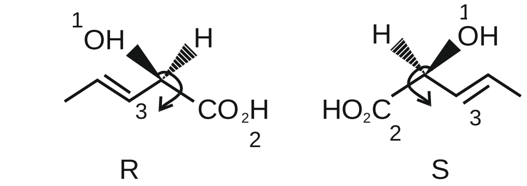
(B) enantiomer, enantiomer, diastereomer

(C) enantiomer, homomer (identical), diastereomer

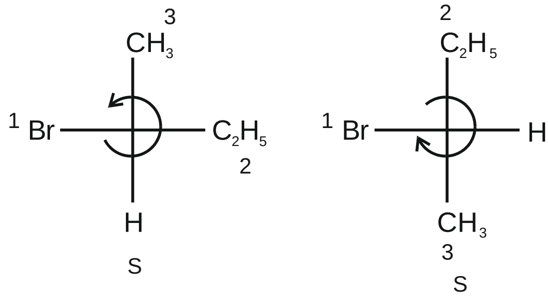
(D) homomer (identical), diastereomer, geometrical isomer

Ans : (C)

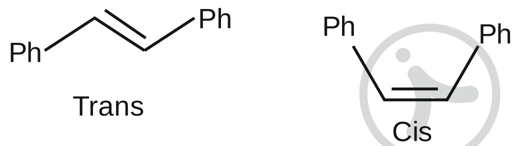
Hint :



(Enantiomers)



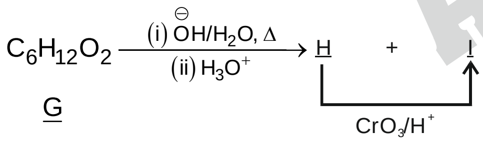
(Identical)



(Diastereomers)

Category 2 (Q71 to Q 75)

(Carry 2 marks each. Only one option is correct. Negative marks :– ½)

71. 

‘G’ in the above sequence of reactions is

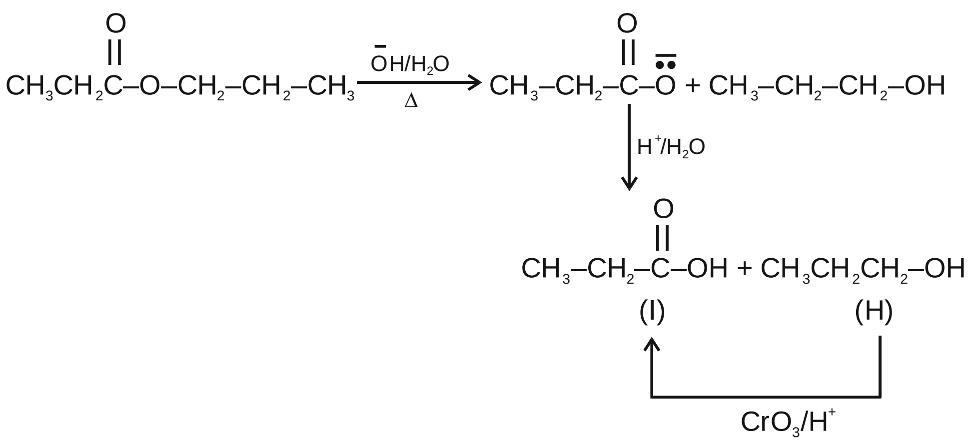
(A) $(CH_3)_2CHCOOCH_2CH_3$

(B) $CH_3CH_2CH_2COOCH_2CH_3$

(C) $CH_3CH_2COOCH_2CH_2CH_3$

(D) $CH_3CH_2COOCH(CH_3)_2$

Ans : (C)

Hint : 

72. Case – 1 : An ideal gas of molecular weight M at temperature T.

Case – 2 : Another ideal gas of molecular weight 2M at temperature T/2

Identify the correct statement in context of above two cases.

(A) Average kinetic energy and avereage speed will be the same in the two cases

(B) Both the averages are halved

(C) Both the averages are doubled

(D) Only average speed is halved in the second case

Ans : (B)

Hint : As temperature is halved, average KE is halved.

Average speed $(C) \propto \sqrt{\frac{T}{M}}$

In case – I, $(C) \propto \sqrt{\frac{T}{M}}$

In case – II $(C) \propto \sqrt{\frac{T}{2 \times 2M}} = \frac{1}{2}\sqrt{\frac{T}{M}}$

So average speed is also halved.

73. 63 g of a compound (Mol. Wt. = 126) was dissolved in 500 g distilled water. The density of the resultant solution as 1.126 g/ml. The molarity of the solution is

(A) 1.25 M (B) 1.0 M (C) 0.75 M (D) 1.1 M

Ans : (B)

Hint : Mass of compound (solute) = 63 g

Mole of compound $= \frac{63}{126} = \frac{1}{2}$ mole

Mass of solution = Mass of solute + Mass of solvent

= 63 + 500

= 563 g

Volume of solution = $\frac{\text{Mass}}{\text{Density}} = \frac{563}{1.126}$ ml

Molarity $= \frac{\text{mole of compound}}{\text{volume of solution}(\text{in}L)}$

$$= \frac{1/2 \times 1000}{563/1.126}$$

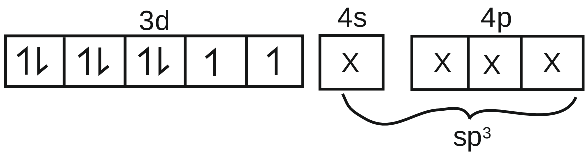
$$= \frac{1.126 \times 1000}{2 \times 563} = 1$$

74. Nickel combines with a uninegative monodentate ligand (X^-) to form a paramagnetic complex $[NiX_4]^{2-}$. The hybridisation involved and number of unpaired electrons present in the complex are respectively.

(A) sp^3, two (B) dsp^2, zero (C) dsp^2, one (D) sp^3, one

Ans : (A)

Hint : Ni^{2+} (d^8)



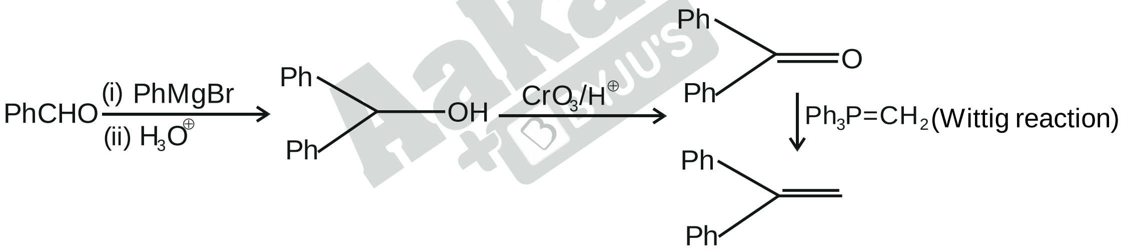
it should be paramagnetic with 2 unpaired electrons

75. $$\underline{\underline{L}} \xrightarrow[\text{(ii) } H_3O^{\oplus}]{\text{(i) PhMgBr}} \underline{\underline{M}} \xrightarrow{CrO_3/H^{\oplus}} \underline{\underline{N}} \xrightarrow{Ph_3P=CH_2} Ph_2C=CH_2$$

"$\underline{\underline{L}}$" in the above sequence of reaction is/are (where $L \neq M \neq N$)

(A) Benzaldehyde (B) Methyl benzoate (C) Benzoyl chloride (D) Benzonitrile

Ans : (A)

Hint : 

Category 3 (Q76 to Q80)

(Carry 2 marks each. One or more options are correct. No negative marks)

76. The correct set(s) of reactions to synthesize benzoic acid starting from benzene is/are

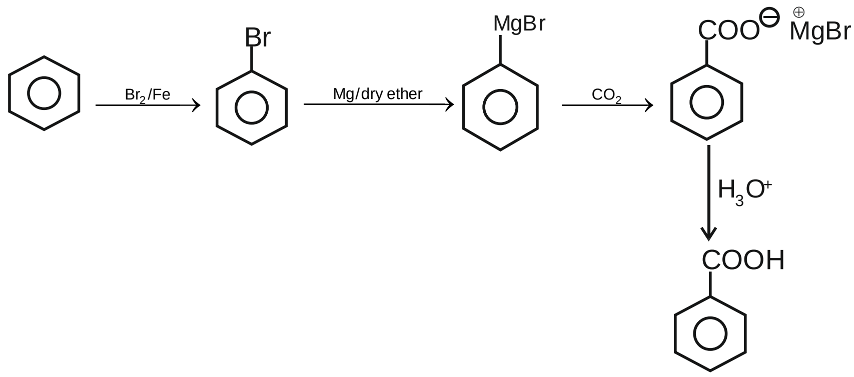
(A) (i) Br_2 / Fe (ii) Mg / dry ether (iii) CO_2 (iv) $H_3O^{\oplus}$

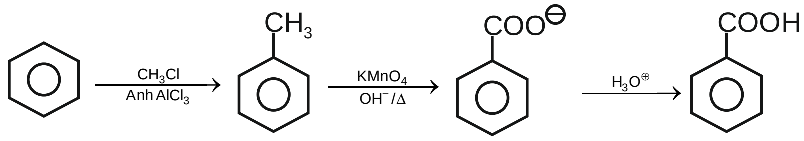
(B) (i) Br_2 / Fe (ii) NH_3, 25°C
(iii) $NaNO_2$, dil. HCl, 0° to 5°C (iv) CuCN / KCN
(v) dil. HCl, Δ

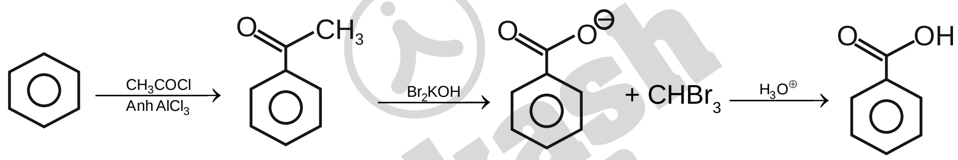
(C) (i) CH_3Cl, Anhydrous $AlCl_3$ (ii) $KMnO_4$ | $OH^{\ominus}$, Δ
(iii) $H_3O^{\oplus}$

(D) (i) CH_3COCl, Anhydrous $AlCl_3$ (ii) Br_2, NaOH, (iii) $H_3O^{\oplus}$

Ans : (A,C,D)

Hint : (A) 

(C) 

(D) 

77. Which statement(s) is/are applicable above critical temperature ?

(A) A gas cannot be liquified

(B) Surface tension of a liquid is very high

(C) A liq. phase cannot be distinguished from a gas phase

(D) Density changes continuously with P or V

Ans : (A)

Hint : Gas cannot be liquified above critical temperature (fact.)

78. Which of the following mixtures act(s) as buffer solution ?

(A) $NaOH + CH_3COOH$ (1 : 1 mole ratio)

(B) $NH_4OH + HCl$ (2 : 1 mole ratio)

(C) $CH_3COOH + NaOH$ (2 : 1 more ratio)

(D) $CH_3COOH + NaOH$ (1 : 2 mole ratio)

Ans : (B,C)

Hint : (B) $NH_4OH + HCl(2:1\,\text{mole ratio}) \rightarrow NH_4Cl + NH_4OH$

1 : 1

Basic buffer

(C) $CH_3COOH + NaOH(2:1\,\text{mole ratio}) \rightarrow CH_3COONa + CH_3COOH$

1 : 1

Acidic buffer

79. An electron in the 5d orbital can be represented by the following (n, 1, m_0) values

(A) (5, 2, 1) (B) (5, 1, –1) (C) (5, 0, 1) (D) (5, 2, –1)

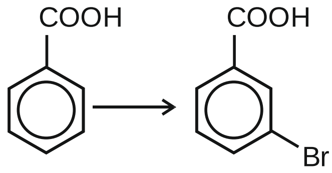
Ans : (A,D)

Hint : 5d $\therefore$ n = 5 l = 2

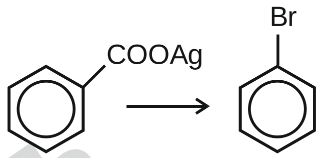
and m can be –2 to 2

80. The conversions(s) that can be carried out by bromine in carbon tetrachloride solvent is/are

(A) $PhCH=CHCH_3 \rightarrow PhCHBrCHBrCH_3$

(B) 

(C) $CH_3CH_2COOH \rightarrow CH_3CHBrCOOH$

(D) 

Ans : (A,D)

Hint : (A) Addition Reaction

(D) Borodine Hunsdiecker reaction

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