1. Two spherical stars A and B have densities ρA and ρB, respectively. A and B have the same radius, and their masses MA and MB are related by MB = 2MA. Due to an interaction process, star A loses some of its mass, so that its radius is halved, while its spherical shape is retained, and its density remains ρA. The entire mass lost by A is deposited as a thick spherical shell on B with the density of the shell being ρA. If vA and vB are the escape velocities from A and B after the interaction process, the ratio

$$\frac{v_B}{v_A} = \sqrt{\frac{10n}{15^{1/3}}}.$$

Answer -

$$v_A = \sqrt{\frac{2GM_A}{8\times\left(\frac{R}{2}\right)}} = \frac{v_0}{2}$$

For B,

$$\frac{4}{3}\pi\left(r^3 - R^3\right) = \frac{4}{3}\pi R^3 \times \frac{7}{8}$$

$$\Rightarrow r = \left(\frac{15}{8}\right)^{\frac{1}{3}} R$$

$$\therefore \sqrt{\frac{2G\times\left(2M_A+\frac{7}{8}M_A\right)}{\frac{(15)^{\frac{1}{3}}R}{2}}}$$

$$= v_0 \times \sqrt{\frac{23\times 2}{8\times(15)^{\frac{1}{3}}}}$$

$$\therefore \frac{v_B}{v_A} = \sqrt{\frac{23}{(15)^{\frac{1}{3}}}} = \sqrt{\frac{2.30\times 10}{(15)^{\frac{1}{3}}}}$$

2. The minimum kinetic energy needed by an alpha particle to cause the nuclear reaction

$${}^{16}_{7}N + {}^{4}_{2}He \rightarrow {}^{1}_{1}H + {}^{19}_{8}O$$

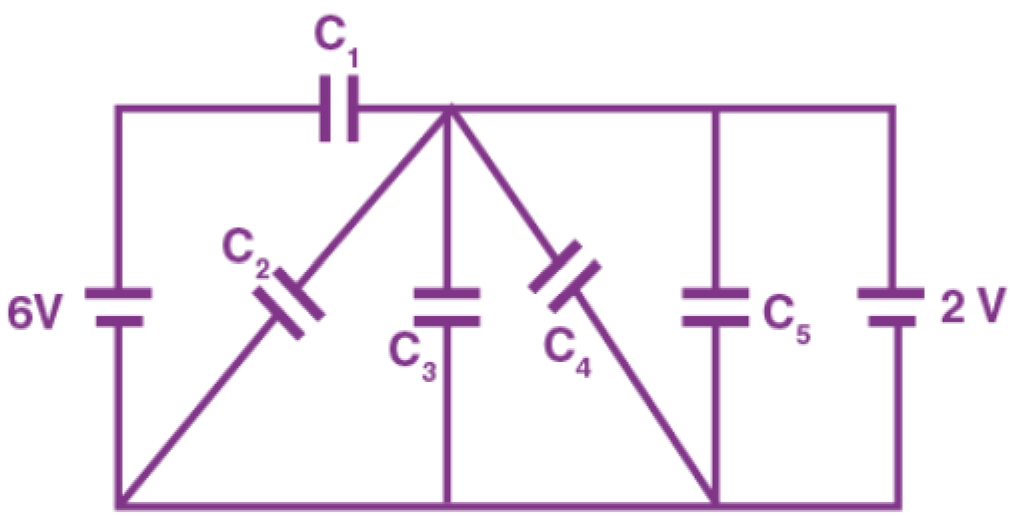
in a laboratory frame is n (in MeV). Assume that
716N is at rest in the laboratory frame.

The masses of ${}^{16}_{7}N,\ {}^{4}_{2}He,\ {}^{1}_{1}H$ and ${}^{19}_{8}O$

can be taken to be 16.006 u, 4.003 u, 1.008 u and 19.003 u, respectively, where 1 u = 930 MeVc–2. The value of n is _______.

Answer (2.33)
Sol. Q = (mN + mHe – mH – mO) × c2
= (16.006 + 4.003 – 1.008 – 19.003) × 930 MeV
= – 1.86 MeV
= 1.86 MeV energy absorbed.
And,
12×m×4m5m×v2=max loss in kinetic energy
⇒12mv2=54×Q

3. In the following circuit C1 = 12 μF, C2 = C3 = 4 μF and C4 = C5 = 2 μF. The charge stored in C3 is _____ μC.



Answer (8.00)

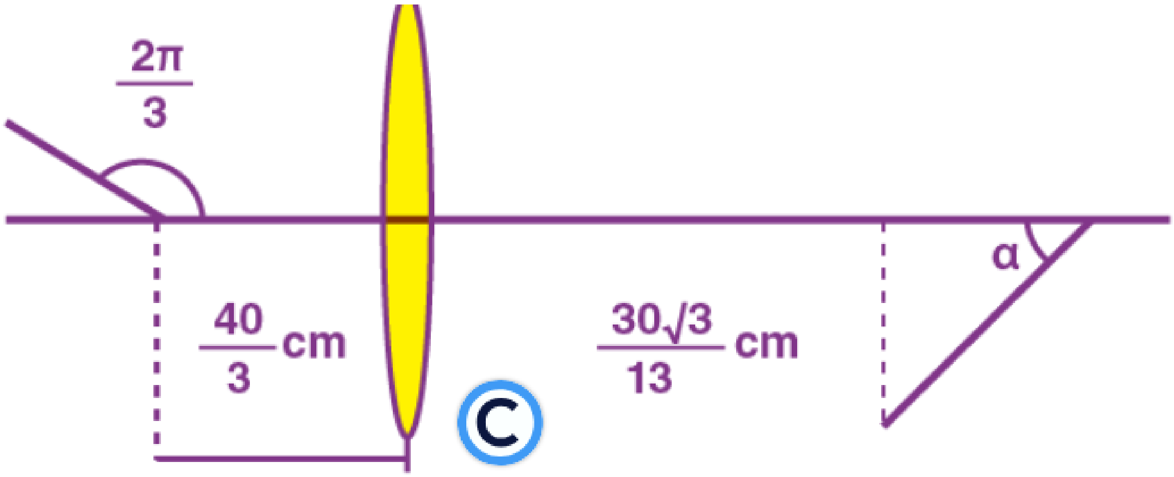
Sol. From the circuit given,

The potential difference across C3 is 2 V (constant)

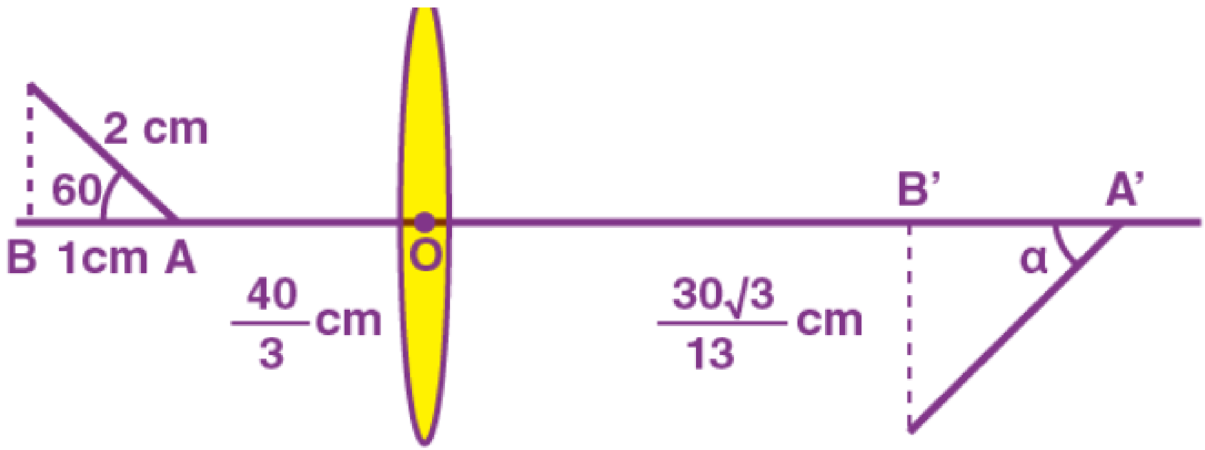
∴ Q3 = 2 × 4 μC

= 8 μC

4. A rod of length 2 cm makes an angle 2π/3 rad with the principal axis of a thin convex lens. The lens has a focal length of 10 cm and is placed at a distance of 40/3 cm from the object as shown in the figure. The height of the image is (30√3)/13 cm and the angle made by it concerning the principal axis is α rad. The value of α is π/n rad, where n is ________.



Answer -



$$OA' = \frac{\frac{40}{3}\times 10}{\frac{43}{3}-10} = 40\ cm$$

$$OB' = \frac{\frac{43}{3}\times 10}{\frac{43}{3}-10} = \frac{430}{13}\ cm$$

$$\therefore A'B' = 40 - \frac{430}{13} = \frac{90}{13}\ cm$$

$$\therefore \tan\alpha = \frac{30\sqrt{3}}{13\times\left(\frac{90}{13}\right)} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \alpha = \frac{\pi}{6}$$

$\therefore$ n = 6.00

5. At time t = 0, a disk of radius 1 m starts to roll without slipping on a horizontal plane with an angular acceleration of $\alpha = \frac{2}{3}$ rad s-2. A small stone is stuck to the disk. At t = 0, it is at the contact point of the disk and the plane. Later, at time $t = \sqrt{\pi}$ s the stone detaches itself and flies

off tangentially from the disk. The maximum height (in m) reached by the stone measured from the plane is -

$$\frac{1}{2} + \frac{x}{10}.$$

The value of x is ______. [Take g = 10 ms–2.]

Answer (00.52)

Sol. The angle rotated by disc in t = √π s is

Answer (00.52)

Sol. The angle rotated by disc in t = √π s is

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\Rightarrow \theta = \frac{1}{2} \times \frac{2}{3}\left(\sqrt{\pi}\right)^2$$
$$= \frac{\pi}{3}\ rad$$

and the angular velocity of disc is

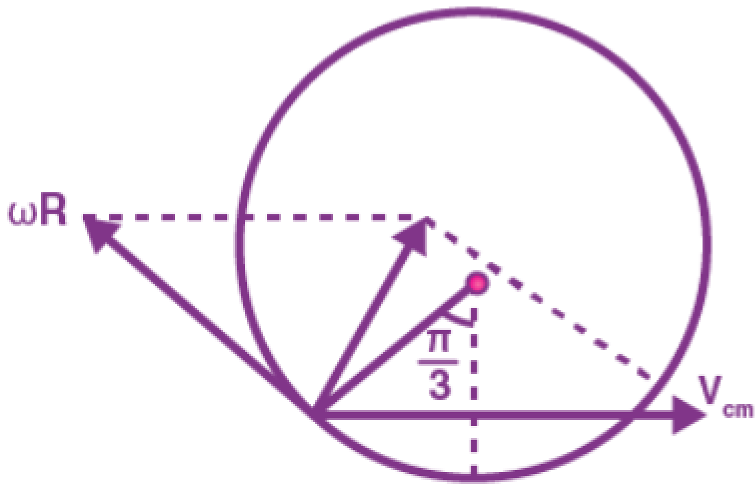
$\omega = \omega_0 + \alpha t$

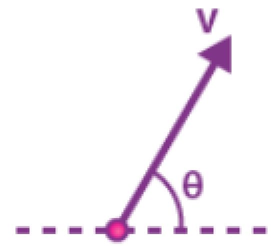
$$= \frac{2\sqrt{\pi}}{3}\text{rad/s}$$

and

$$v_{cm} = \omega R = \frac{2\sqrt{\pi}}{3} \times 1$$
$$= \frac{2\sqrt{\pi}}{3} m/s$$

So, at the moment it detaches the situation is


ωR
π/3
Vcm


v
θ

$$v=\sqrt{(\omega R)^2+v_{\text{cm}}^2+2(\omega R)v_{\text{cm}}\cos 120^\circ}$$
$$=v_{cm}=\frac{2\sqrt{\pi}}{3}m/s$$
and

$$\tan\theta=\frac{\omega R\sin 120^\circ}{v_{cm}+\omega R\cos 120^\circ}$$
$$\Rightarrow \tan\theta=\sqrt{3}$$
$$\Rightarrow \theta=\frac{\pi}{3}\ rad$$
So,

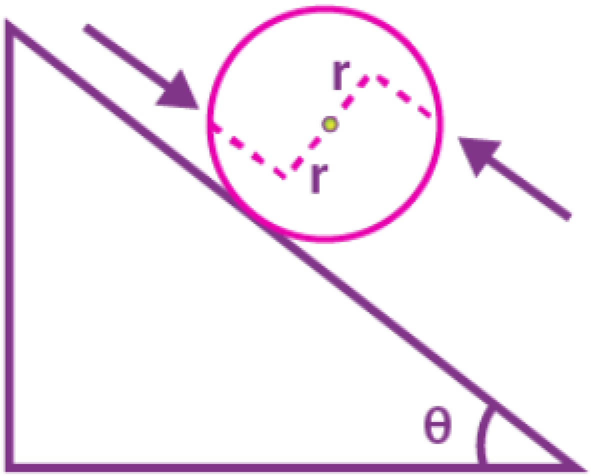
$$H_{\max}=\frac{u^2\sin^2\theta}{2g}$$
$$=\frac{\left(\frac{2\sqrt{\pi}}{3}\right)^2\times\sin^2 60^\circ}{2\times10}$$
$$=\frac{4\pi\times3}{9\times2\times10\times4}$$
$$=\frac{\pi}{60}\ m$$
So, height from ground will be

$$R(1-\cos 60^\circ)+\frac{\pi}{60}=\frac{1}{2}+\frac{x}{10}$$
$$\Rightarrow x=\frac{\pi}{6}=0.52$$

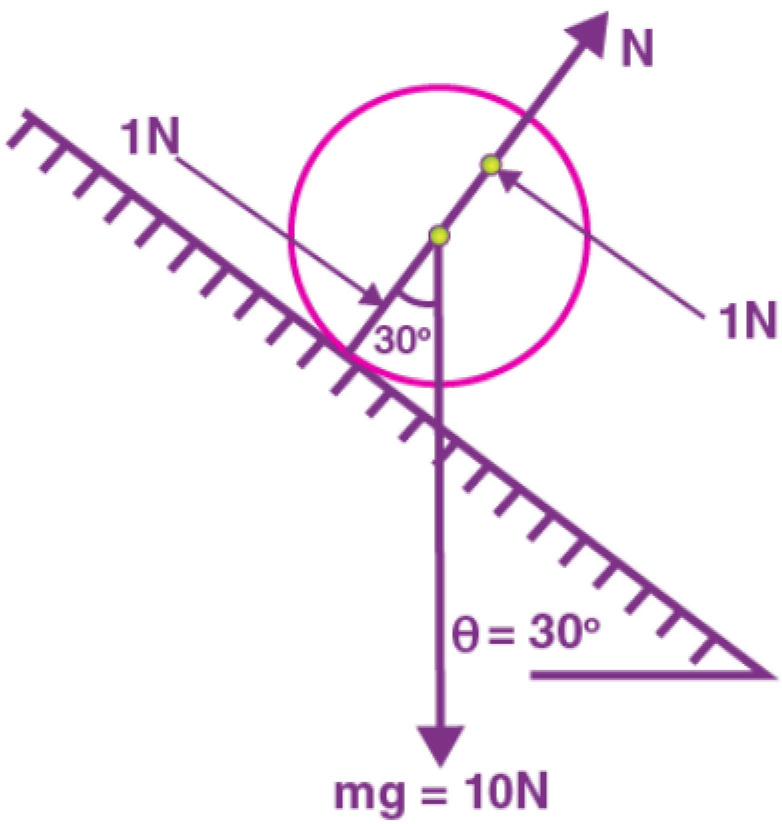
6. A solid sphere of mass 1 kg and radius 1 m rolls without slipping on a fixed inclined plane with an angle of inclination θ = 30° from the horizontal. Two forces of magnitude 1 N each, parallel to the incline, act on the sphere, both at a distance r = 0.5 m from the centre of the sphere, as shown in the figure. The acceleration of the sphere down the plane is _______ ms–2. (Take g = 10 ms–2.)

Answer (02.86)

7. Consider an LC circuit, with inductance L = 0.1 H and capacitance C = 10–3 F, kept on a plane. The area of the circuit is 1 m2. It is placed in a constant magnetic field of strength B0, which is perpendicular to the plane of the circuit. At time t = 0, the magnetic field strength starts increasing linearly as B = B0 + βt with β = 0.04 Ts–1. The maximum magnitude of the current in the circuit is ______ mA.



Answer (02.86)



Taking torque about contact point.

$$\vec{\tau} = mgR \sin 30^{\otimes} + 1 \times 1^{\odot}$$
taking $\otimes$ is positive
$$5–1 = \frac{7}{5} mR^2\alpha$$
$$\Rightarrow \alpha = \frac{20}{7} rad/s^2$$
So,

$$a_{cm} = \alpha R = \frac{20}{7} m/s^2$$
$$a_{cm} = 2.86 \text{ m/s}^2$$

8. A projectile is fired from horizontal ground with speed v and projection angle θ. When the acceleration due to gravity is g, the range of the projectile is d. If at the highest point in its trajectory, the projectile enters a different region where the effective acceleration due to gravity is g’= g/0.81, then the new range is d′ = nd. The value of n is _____.
Answer (00.95)
Sol.
$$d = \frac{u^2 \sin 2\theta}{g}$$
$$H = \frac{u^2 \sin^2\theta}{2g}$$
So, after entering the new region, the time taken by the projectile to reach the ground

$$t = \sqrt{\frac{2H}{g'}}$$
$$= \sqrt{\frac{2u^2 \sin^2 \theta \times 0.81}{2g \times g}}$$
$$= \frac{0.94 \sin \theta}{g}$$
So, horizontal displacement done by the projectile in the new region is

$$x = \frac{0.9u \sin \theta}{g} \times u \cos \theta$$
$$= 0.9 \frac{u^2 \sin 2\theta}{2g}$$
So,

$$d' = \frac{d}{2} + x = 0.95d$$

9. A medium having dielectric constant K >1 fills the space between the plates of a parallel plate capacitor. The plates have a large area, and the distance between them is d. The capacitor is connected to a battery of voltage V, as shown in Figure (a). Now, both the plates are moved by a distance of d/2 from their original positions, as shown in Figure (b).

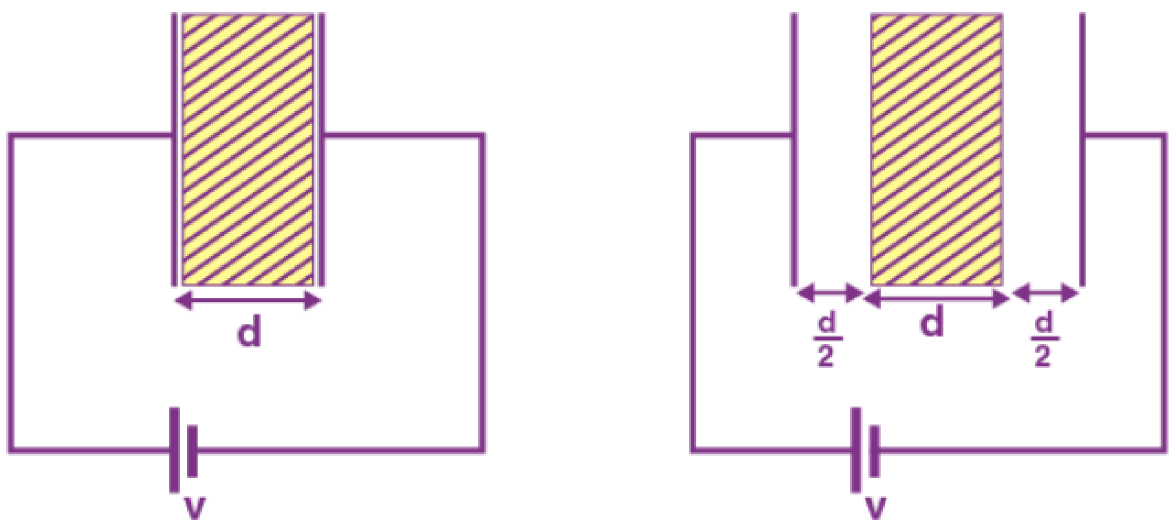


Figure (a) Figure (b)

In the process of going from the configuration depicted in Figure (a) to that in Figure (b), which of the following statement(s) is(are) correct?

(A) The electric field inside the dielectric material is reduced by a factor of $2K$.

(B) The capacitance is decreased by a factor of $1/(K + 1)$.

(C) The voltage between the capacitor plates is increased by a factor of $(K + 1)$.

(D) The work done in the process **DOES NOT** depend on the presence of the dielectric material

Answer (B)

Sol.

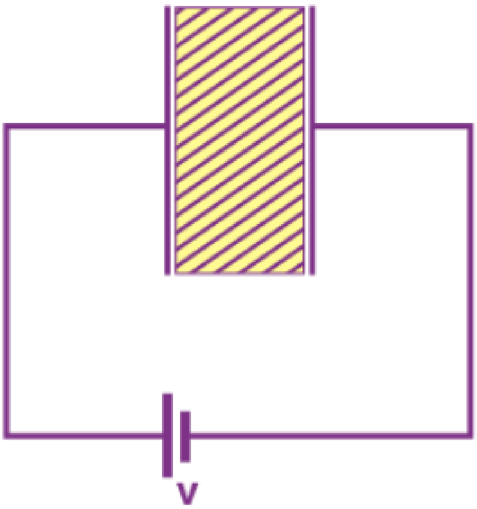


Figure (a)

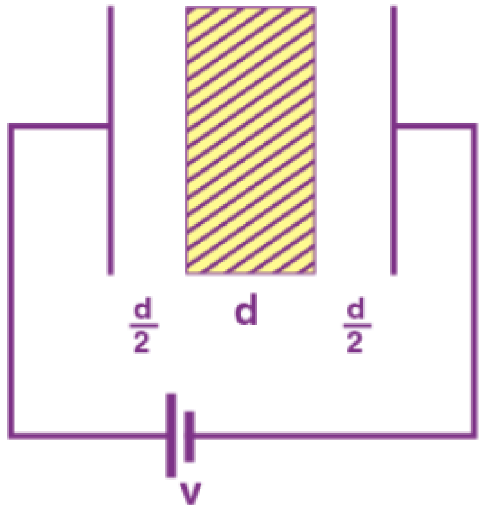


Figure (b)

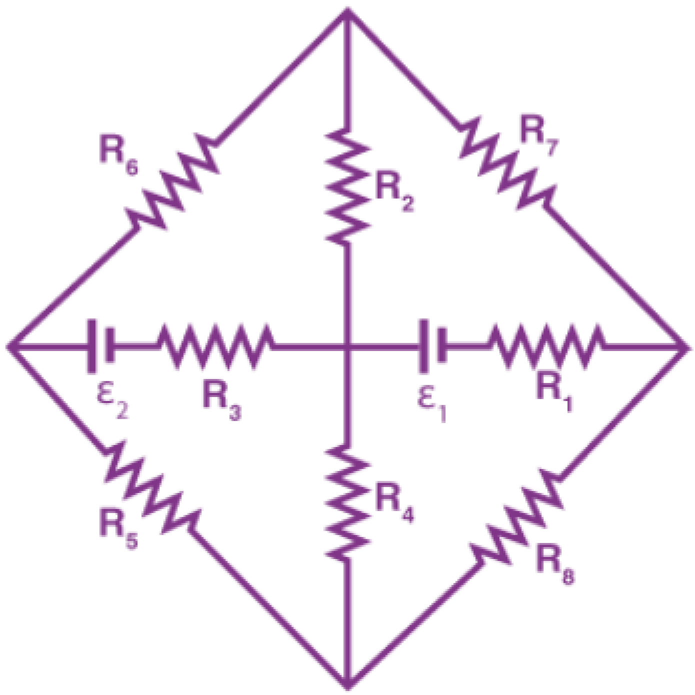
$C_a = \frac{K\varepsilon_0 A}{d}$ $q_a = \frac{K\varepsilon_0 A}{d}V$ $E_a = \frac{q_a}{KA\varepsilon_0} = \frac{K\varepsilon_0 AV}{dK\varepsilon_0 A}$ $= \frac{V}{d}$	$C_b = \frac{\varepsilon_0 A}{d+(\frac{d}{K})}$ $= \frac{\varepsilon_0 AK}{d(K+1)}$ $q_b = \frac{\varepsilon_0 AKV}{d(K+1)}$ $(E_b)_{\text{dielectric}} = \frac{E_{air}}{K}$ $= \frac{q_b}{KA\varepsilon_0}$ $= \frac{\varepsilon_0 AKV}{d(K+1)(KA\varepsilon_0)}$ $= \frac{V}{d(K+1)}$

Capacitance decreases by a factor of 1/(K + 1)

Work done in the process = Uf – Ui

$$= \frac{1}{2}(C_f - C_i)V^2$$
$$= \frac{1}{2}\left(\frac{\varepsilon_0 AK}{d(K+1)} - \frac{K\varepsilon_0 A}{d}\right)V^2$$
$$= \frac{1}{2}V^2\frac{\varepsilon_0 AK}{d}\left(\frac{1}{K+1} - 1\right)$$
$$= \frac{1}{2}\frac{\varepsilon_0 AKV^2}{d}\frac{1-K-1}{K+1}$$
$$= \frac{1}{2}\frac{\varepsilon_0 AV^2}{d}\left(\frac{-K^2}{K+1}\right)$$

10. The figure shows a circuit having eight resistances of 1 Ω each, labelled R1 to R8, and two ideal batteries with voltages ε1 = 12 V and ε2 = 6 V.



Which of the following statement(s) is(are) correct?

(A) The magnitude of current flowing through R1 is 7.2 A.

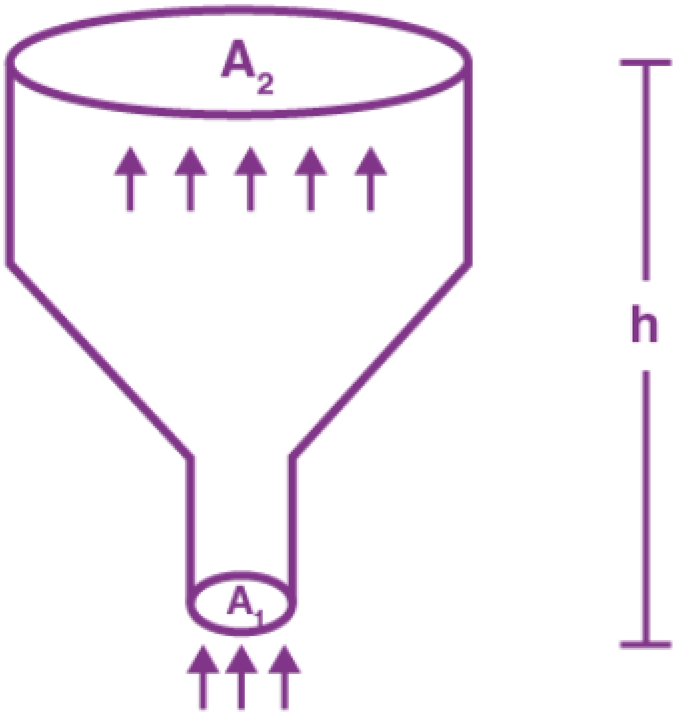
(B) The magnitude of current flowing through R2 is 1.2 A.

(C) The magnitude of current flowing through R3 is 4.8 A.

(D) The magnitude of current flowing through R5 is 2.4 A.

Answer (A, B, C, D)

11. An ideal gas of density ρ = 0.2 kg m–3 enters a chimney of height h at the rate of α = 0.8 kg s–1 from its lower end, and escapes through the upper end as shown in the figure. The cross-sectional area of the lower end is A1 = 0.1 m2 and the upper end is A2 = 0.4 m2. The pressure and the temperature of the gas at the lower end are 600 Pa and 300 K, respectively, while its temperature at the upper end is 150 K. The chimney is heat insulated so that the gas undergoes adiabatic expansion. Take g = 10 ms–2 and the ratio of specific heats of the gas γ = 2. Ignore atmospheric pressure.



Which of the following statement(s) is(are) correct?

(A) The pressure of the gas at the upper end of the chimney is 300 Pa.

(B) The velocity of the gas at the lower end of the chimney is 40 ms–1 and at the upper end is 20 ms–1.

(C) The height of the chimney is 590 m.

(D) The density of the gas at the upper end is 0.05 kg m–3 .

Answer (B)

Sol.

$$\frac{\rho_1}{\rho_2} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma - 1}}$$

$$\Rightarrow \rho_2 = (0.2)\left(\frac{150}{300}\right)^{\frac{1}{1}} = 0.1\mathrm{kg/m^3}$$

Rate of mass flow = 0.8 kg/s

⇒ Volume flow rate at top = 8 m3/s

$$\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} \Rightarrow P_2 = 600\left(\frac{1}{2}\right)^{\gamma}$$

Velocity of gas at lower end

$$= \frac{V_1}{A_1} = \frac{0.8}{0.2\times0.1} = 40\text{m/s}$$

Velocity of gas at upper end

$$= \frac{0.8\times2}{0.2\times0.4} = 20\text{m/s}$$

By applying energy conservation

$$\frac{1}{2}\rho_1 v_1^2 + P_1 = \frac{1}{2}\rho_2 v_2^2 + P_2 + \rho_2 g h_2$$
$$\Rightarrow \frac{1}{2}(0.2)(1600) + 600 = \frac{1}{2}(0.1)(400) + 150 + (0.1)(10)h_2$$
$$\Rightarrow h_2 = 590\text{ m}$$