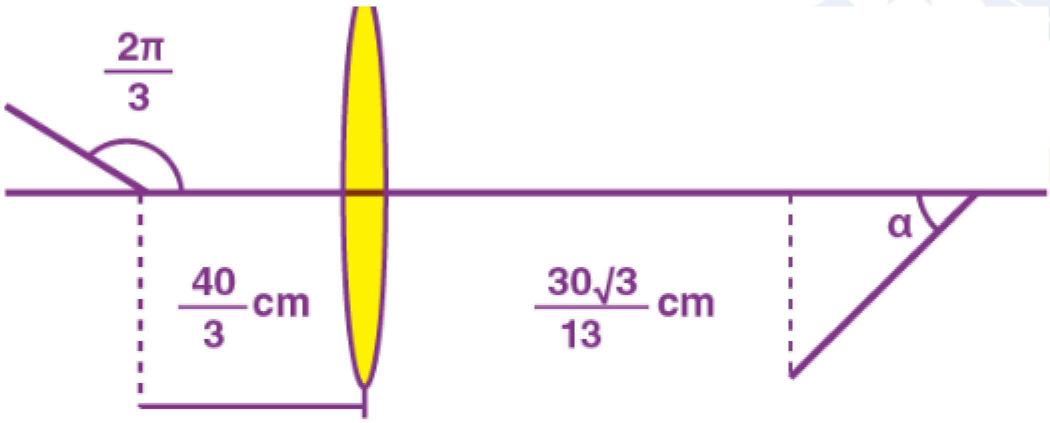
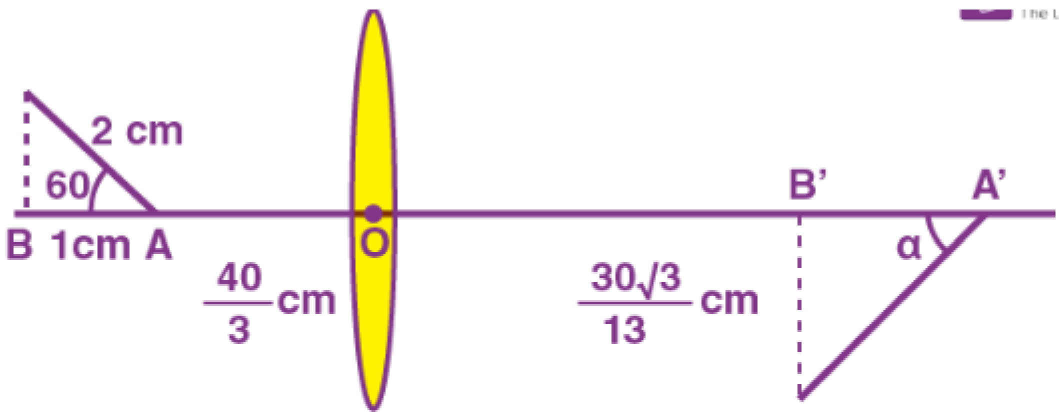
1. A rod of length 2 cm makes an angle 2π/3 rad with the principal axis of a thin convex lens. The lens has a focal length of 10 cm and is placed at a distance of 40/3 cm from the object as shown in the figure. The height of the image is (30√3)/13 cm and the angle made by it concerning the principal axis is α rad. The value of α is π/n rad, where n is _______. Answer (6.00)





$$OA' = \frac{\frac{40}{3}\times 10}{\frac{43}{3}-10} = 40\ cm$$

$$OB' = \frac{\frac{43}{3}\times 10}{\frac{43}{3}-10} = \frac{430}{13}\ cm$$

$$\therefore A'B' = 40 - \frac{430}{13} = \frac{90}{13}\ cm$$

$$\therefore \tan\alpha = \frac{30\sqrt{3}}{13\times\left(\frac{90}{13}\right)} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \alpha = \frac{\pi}{6}$$

∴ n = 6.00

At time t = 0, a disk of radius 1 m starts to roll without slipping on a horizontal plane with an angular acceleration of α = ⅔ rad s-2. A small stone is stuck to the disk. At t = 0, it is at the contact point of the disk and the plane. Later, at time t = √π s the stone detaches itself and flies off tangentially from the disk. The maximum height (in m) reached by the stone measured from the plane is

$\frac{1}{2} + \frac{x}{10}$.

The value of x is _____. [Take g = 10 ms–2.]

Answer (00.52)

Sol. The angle rotated by the disc in t = √π s is

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\Rightarrow \theta = \frac{1}{2} \times \frac{2}{3}\left(\sqrt{\pi}\right)^2$$
$$= \frac{\pi}{3}\ rad$$

and the angular velocity of the disc is

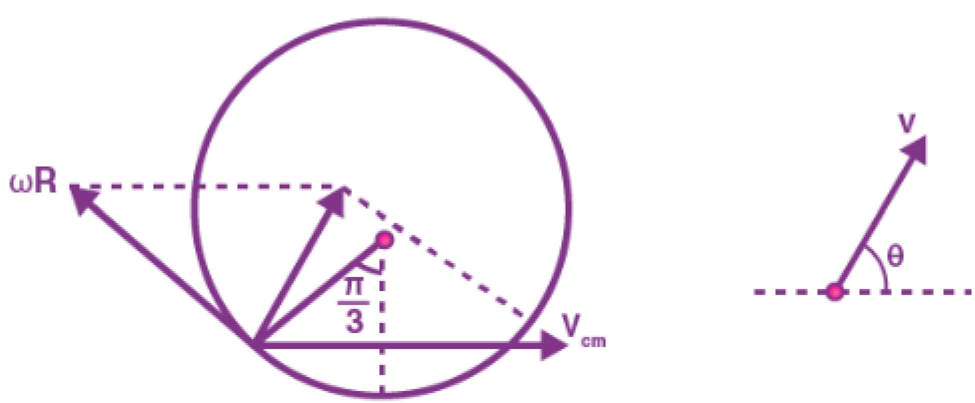
$$\omega = \omega_0 + \alpha t$$

$$= \frac{2\sqrt{\pi}}{3} \mathrm{rad/s}$$

and

$$v_{cm} = \omega R = \frac{2\sqrt{\pi}}{3} \times 1$$
$$= \frac{2\sqrt{\pi}}{3} m/s$$

So, at the moment it detaches the situation is



$$v=\sqrt{(\omega R)^2+v_{\mathrm{cm}}^2+2(\omega R)v_{\mathrm{cm}}\cos 120^\circ}$$
$$=v_{cm}=\frac{2\sqrt{\pi}}{3}m/s$$
and

$$\tan\theta=\frac{\omega R\sin 120^\circ}{v_{cm}+\omega R\cos 120^\circ}$$
$$\Rightarrow \tan\theta=\sqrt{3}$$
$$\Rightarrow \theta=\frac{\pi}{3}\ rad$$
So,

$$H_{\max}=\frac{u^2\sin^2\theta}{2g}$$
$$=\frac{\left(\frac{2\sqrt{\pi}}{3}\right)^2\times\sin^2 60^\circ}{2\times10}$$
$$=\frac{4\pi\times3}{9\times2\times10\times4}$$
$$=\frac{\pi}{60}\ m$$
So, height from ground will be

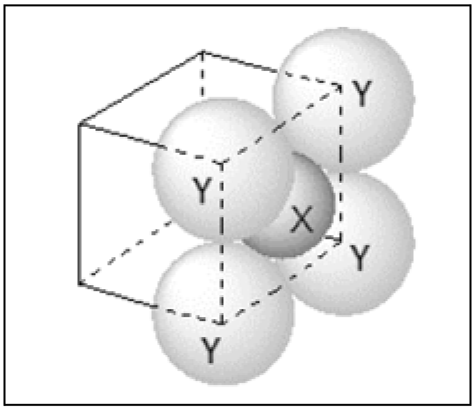
$$R(1-\cos 60^\circ)+\frac{\pi}{60}=\frac{1}{2}+\frac{x}{10}$$
$$\Rightarrow x=\frac{\pi}{6}=0.52$$

---

2. The calculated spin-only magnetic moments of $[Cr(NH_3)_6]^{3+}$ and $[CuF_6]^{3-}$ in BM, respectively, are (Atomic numbers of Cr and Cu are 24 and 29, respectively).
a. 3.87 and 2.84
b. 4.90 and 1.73
c. 3.87 and 1.73
d. 4.90 and 2.84
Answer: (a)
$[Cr(NH_3)_6]^{3+}$ = Cr^{3+}
Cr^{3+} = $3d^3\ 4s^0$
It has 3 unpaired electrons
μ = n √(n+2) BM
μ = 3 √(3+2) BM
μ = 3.87 BM
$[CuF_6]^{3-}$ = Cu^{+3}
Cu^{+3} = $3d^8\ 4s^0$
It has 2 unpaired electrons
μ = 2 √(2+2) BM
= 2.84 BM

---

3. For the given close-packed structure of a salt made of cation X and anion Y shown below (ions of only one face are shown for clarity), the packing fraction is approximately (packing fraction = packing efficiency / 100)



a. 0.74
b. 0.63
c. 0.52
d. 0.48

Answer: (b)
a = edge length of unit cell
2ry = a
2 (rx – ry) = √2a
2rx + a = √2a
2rx = a (√2a – 1)
rx = 0.207 a
Packing fraction = 3 × vol. of x + vol. of y / vol. of unit cell
3×43×πrx3+43+π×ry3a3
4×π×(0.207a)3+43×π×(0.5a)3a3

---

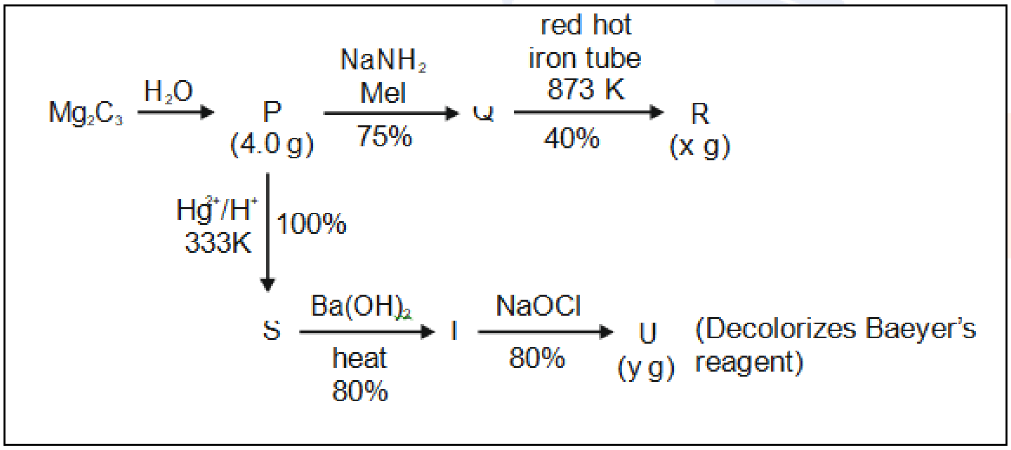
4. The calculated spin-only magnetic moments of [Cr(NH3)6]3+ and [CuF6]3– in BM, respectively, are (Atomic numbers of Cr and Cu are 24 and 29, respectively).

a. 3.87 and 2.84
b. 4.90 and 1.73
c. 3.87 and 1.73
d. 4.90 and 2.84

Answer: (a)
[Cr(NH3)6]3+ = Cr3+
Cr3+ = 3d3 4s0
It has 3 unpaired electrons
μ = n √(n+2) BM

μ = 3 √(3+2) BM
μ = 3.87 BM
[CuF6]3- = Cu+3
Cu+3 = 3d8 4s0
It has 2 unpaired electrons
μ = 2 √(2+2) BM
= 2.84 BM

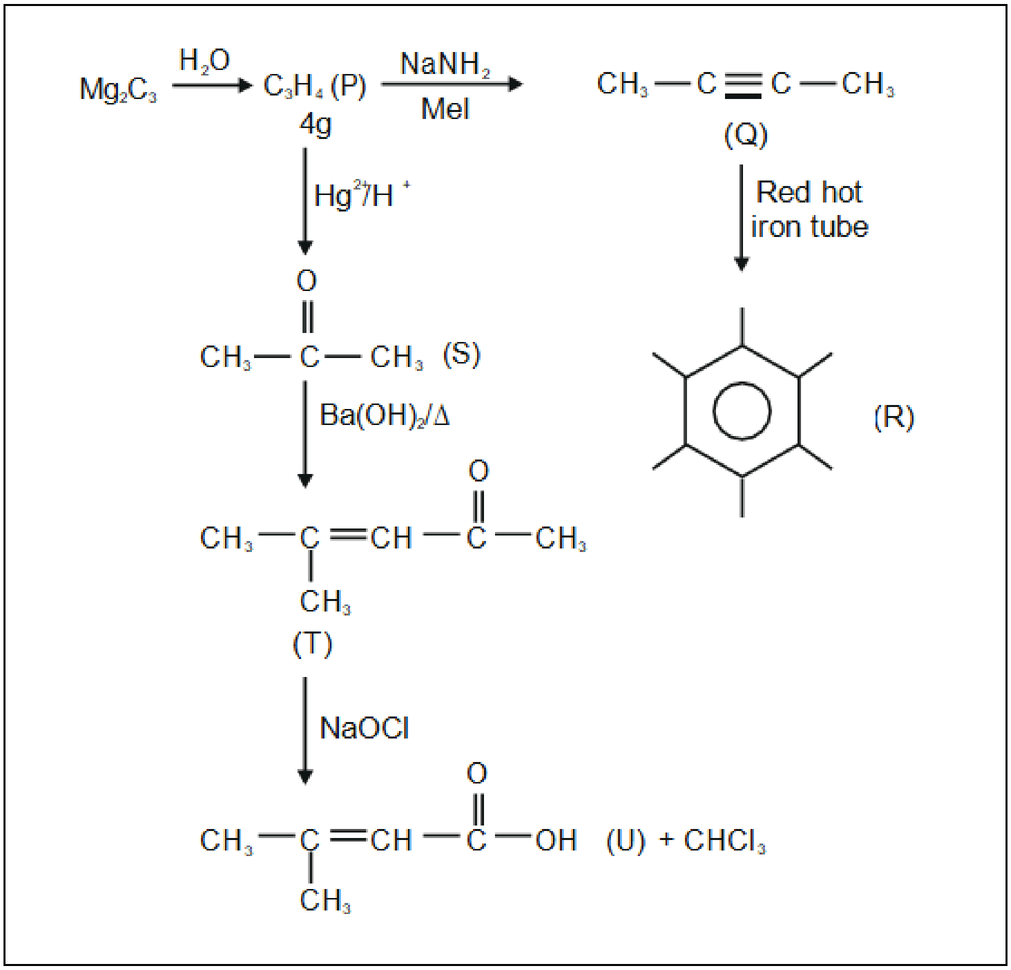
Question Branch for Questions 5 and 6:
For the following reaction scheme, percentage yields are given along the arrow:



x g and y g are the masses of R and U, respectively. (Use: Molar mass (in g mol–1) of H, C and O as 1, 12 and 16, respectively)
Question 5. The value of x is______.

Answer: (1.62)
Question 6. The value of y is ______.

Answer: (3.20)
Solution for both Questions 5 and 6



4 g of C3H4 = 0.1 mol
From 0.1 mol of P, 0.01 mol of R will be produced
⇒ 1.62 g of R is produced
From 0.1 mol of P, 0.032 mol of U is produced
= 3.2 g of U is produced

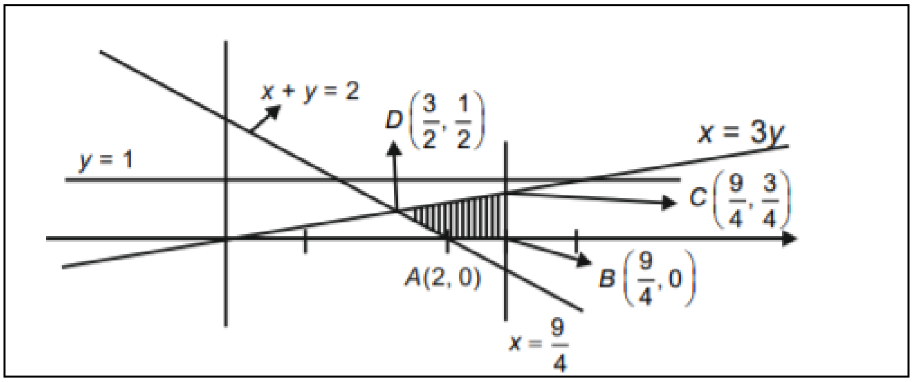
---

7. Consider a triangle Δ whose two sides lie on the x-axis and the line x + y + 1 = 0. If the orthocenter of Δ is (1, 1), then the equation of the circle passing through the vertices of the triangle Δ is;
a. x2 + y2 – 3x + y = 0
b. x2 + y2 + x + 3y = 0
c. x2 + y2 + 2y – 1 = 0
d. x2 + y2 + x + y = 0
Answer: b
As we know mirror image of the orthocenter lies on the circumcircle.
Image of (1, 1) in the x-axis is (1, -1)
The image of (1, 1) in x + y + 1 = 0 is (-2, -2).
∴ The required circle will be passing through both (1, -1) and (-2, -2).

∴ Only x2 + y2 + x + 3y = 0 satisfy both.

---

8. The area of the region {(x, y): 0 ≤ x ≤ 9/4, 0 ≤ y ≤ 1, x ≥ 3y, x + y ≥ 2}
a. 11/32
b. 35/96
c. 37/96
d. 13/32
Answer: a
A rough sketch of the required region is;



∴ The required area is an area of ΔACD + Area of ΔABC
i.e (¼) + (3/32) = 11/32 sq.units.

Question Stem for Questions 9 and 10.
The boiling point of water in a 0.1 molal silver nitrate solution (solution A) is x ºC. To this solution A, an equal volume of 0.1 molal aqueous barium chloride solution is added to make a new solution B. The difference in the boiling points of water in the two solutions A and B is y × 10-2 ºC.
(Assume: Densities of the solutions A and B are the same as that of water and the soluble salts dissociate completely. Use: Molal elevation constant (Ebullioscopic constant), Kb = 0.5 K kg mol-1; Boiling point of pure water as 100ºC.)
Question 9. The value of x is ______.

Answer: (100.1)

Question 10. The value of |y| is ______.

Answer: (2.5)
Given molality of AgNO3 solution is 0.1 molal (solution-A)
ΔTb = ikb m
AgNO3 → Ag+ + NO3 –
van’t Hoff factor (i) for AgNO3 = 2
ΔTb = 2 × 0.5 × 0.1
(Ts – Tº) = 0.1
(Ts)A = 100.1ºC, so x = 100.1

Now solution – A of equal volume is mixed with 0.1 molal $BaCl_2$ solution to get solution-B.
$AgNO_3$ reacts with $BaCl_2$ to form AgCl(s).
0.1 mole of $AgNO_3$ present in 1000 gram solvent or 1017 gram or 1017 mL solution,
milli moles of $AgNO_3$ in V ml 0.1 molal solution is nearly 0.1 V. Similarly in $BaCl_2$.
$2AgNO_3(aq) + BaCl_2(aq) \rightarrow 2AgCl(s) + Ba(NO_3)_2\ (aq)$

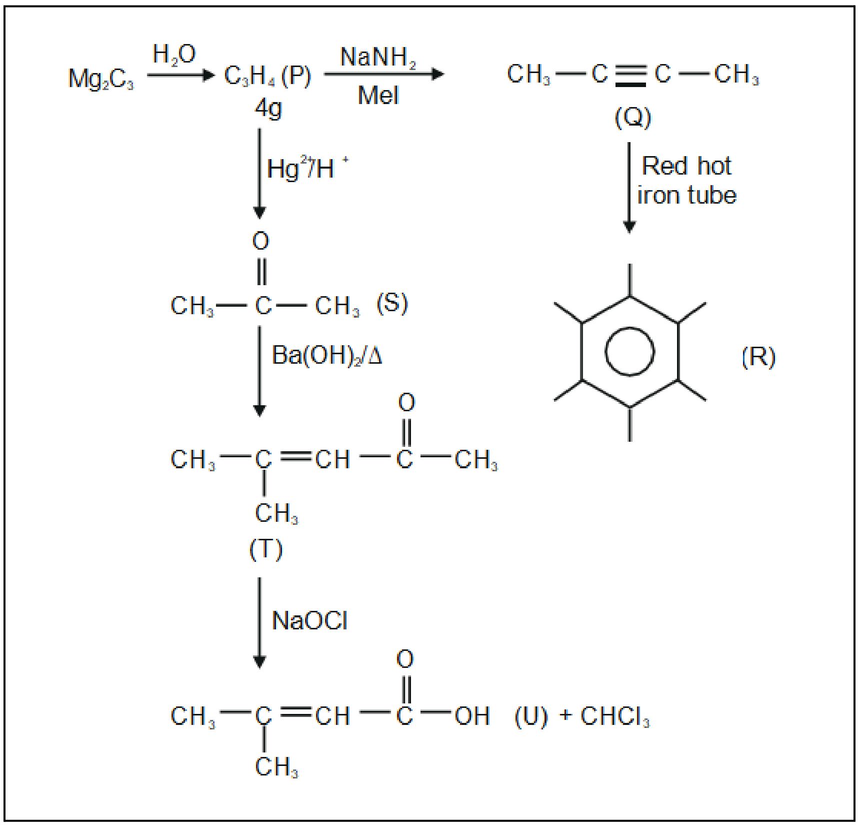
0.1 V	0.1 V	0	0
0	0.05 V	0.1 V	0.05 V

$$\Delta T_b = \left[\frac{0.05V \times 3}{2V} + \frac{0.05V \times 3}{2V}\right] \times 0.5 = 0.075$$

$$(T_s)_B = 100.075°C$$

$$(T_s)_A - (T_s)_B = 100.1 - 100.075 = 0.025°C$$
$$= 2.5 \times 10^{-2}\ °C$$

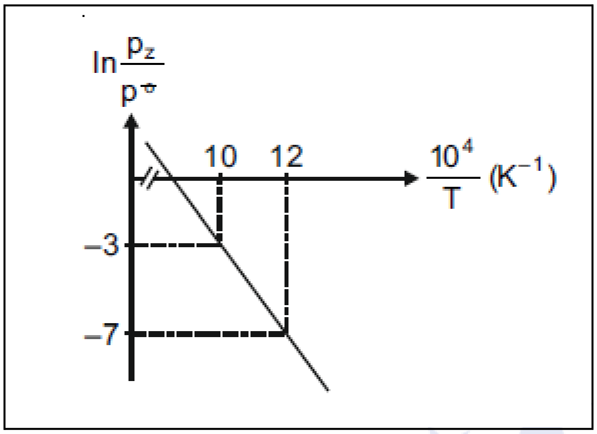
So x = 100.1 and |y| = 2.5



4 g of C_3H_4 = 0.1 mol
From 0.1 mol of P, 0.01 mol of R will be produced
⇒ 1.62 g of R is produced

From 0.1 mol of P, 0.032 mol of U is produced
= 3.2 g of U is produced

Question statement for Questions 7 and 8.
For the reaction, X(s) ⇌ Y(s) + Z(g), the plot of
Inpzp∅
Versus 104 / T is given below (in solid line), where pz is the pressure (in bar) of the gas Z at temperature T and
p∅
= 1 bar.



(Given,
d(InK)d(1T)=−ΔH∅R
, where the equilibrium constant
K=pzp∅
and the gas constant, R = 8.314 J K–1 mol–1)
Question 7. The value of standard enthalpy,
ΔH∅
(in kJ mol–1) for the given reaction is _______.
Answer: (166.28)

$$X(s) \rightleftharpoons Y(s) + Z(g)$$

$$\text{Given } K = \frac{p_z}{p^{\ominus}}$$

$$\ln K = \ln A - \frac{\Delta H^o}{RT}$$

$$\Rightarrow \ln \frac{p_z}{p^{\ominus}} = \ln A - \frac{\Delta H}{RT}$$

$$\text{Slope of } \ln \frac{p_z}{p^{\ominus}} \text{ vs } \frac{1}{T} \text{ is } \frac{d\left[\ln\left(\frac{p_z}{p^{\ominus}}\right)\right]}{d\left(\frac{1}{T}\right)} = \frac{-\Delta H^o}{R}$$

$$\text{From the graph, we have } \frac{-\Delta H^o}{R} = -2 \times 10^4$$

⇒ ΔH° = 2 × 104 × 8.314 J

ΔH° = 166.28 kJ mol-1

Question 8. The value of

ΔS∅

(in J K–1 mol–1) for the given reaction, at 1000 K is ____.

Answer: (141.34)

–RTln K = ΔG° = ΔH° – TΔS°

lnk = H° / RT + S° / R

ΔS° / R = 17

ΔS° = 17R

= 141.338 J K-1

Question Stem for Questions 9 and 10.

The boiling point of water in a 0.1 molal silver nitrate solution (solution A) is x °C. To this solution A, an equal volume of 0.1 molal aqueous barium chloride solution is added to make a new solution B. The difference in the boiling points of water in the two solutions A and B is y × 10-2 °C.

(Assume: Densities of the solutions A and B are the same as that of water and the soluble salts dissociate completely. Use Molal elevation constant (Ebullioscopic constant), Kb = 0.5 K kg mol-1; Boiling point of pure water as 100°C.)

Question 9. The value of x is ______.

Answer: (100.1)

Question 10. The value of |y| is ______.

Answer: (2.5)

Given molality of AgNO3 solution is 0.1 molal (solution-A)

ΔTb = ikb m

$AgNO_3 \rightarrow Ag^+ + NO_3^-$
van't Hoff factor (i) for $AgNO_3$ = 2
$\Delta T_b = 2 \times 0.5 \times 0.1$
$(T_s - T^o) = 0.1$
$(T_s)_A = 100.1^oC$, so x = 100.1
Now solution – A of equal volume is mixed with 0.1 molal $BaCl_2$ solution to get solution-B.
$AgNO_3$ reacts with $BaCl_2$ to form AgCl(s).
0.1 mole of $AgNO_3$ present in 1000 gram solvent or 1017 gram or 1017 mL solution,
milli moles of $AgNO_3$ in V ml 0.1 molal solution is nearly 0.1 V. Similarly in $BaCl_2$.
$2AgNO_3(aq) + BaCl_2(aq) \rightarrow 2AgCl(s) + Ba(NO_3)_2\ (aq)$

0.1 V	0.1 V	0	0
0	0.05 V	0.1 V	0.05 V

$$\Delta T_b = \left[\frac{0.05V \times 3}{2V} + \frac{0.05V \times 3}{2V}\right] \times 0.5 = 0.075$$

$$(T_s)_B = 100.075^oC$$

$(T_s)_A - (T_s)_B = 100.1 - 100.075 = 0.025^oC$
$= 2.5 \times 10^{-2}\ ^oC$

So x = 100.1 and |y| = 2.5

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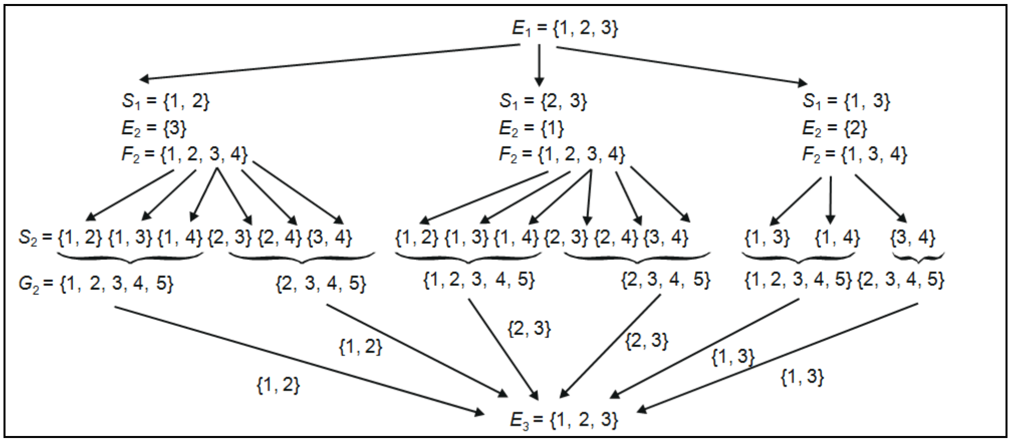
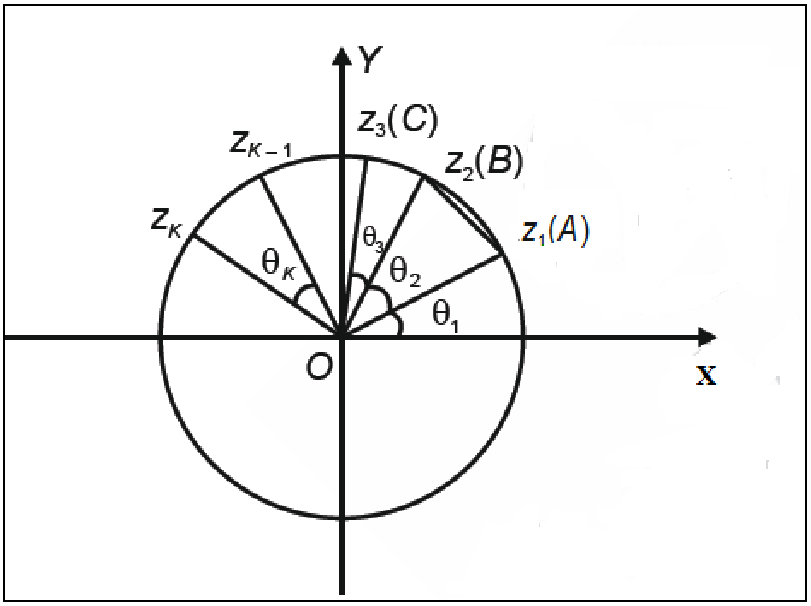
11. Consider three sets $E_1 = \{1, 2, 3\}$, $F_1 = \{1, 3, 4\}$ and $G_1 = \{2, 3, 4, 5\}$. Two elements are chosen at random, without replacement, from the set E_1, and let S_1 denote the set of these chosen elements. Let $E_2 = E_1 - S_1$ and $F_2 = F_1 \cup S_1$. Now two elements are chosen at random, without replacement, from the set F_2 and let S_2 denote the set of these chosen elements.
Let $G_2 = G_1 \cup S_2$. Finally, two elements are chosen at random, without replacement from the set G_2 and S_3 denote the set of these chosen elements. Let $E_3 = E_2 \cup S_3$. Given that $E_1 = E_3$, let p be the conditional probability of the event $S_1 = \{1, 2\}$. Then the value of p is;
a. 1/5
b. 3/5
c. 1/2
d. 2/5
Answer: a
We will follow the tree diagram,



P(E1 = E3) = ⅓ [(½ × 1/10) + (½ × 0) + (½ × 1/10) + (½ × ⅙) + (⅔ × 1/10) + (⅓ × 0)]
= ⅓ (¼)
Required probability = ⅓ (½ × 1/10)/(⅓ × ¼)
= 1/5
12. Let θ1, θ2, …., θ10 be positive valued angles (in radian) such that θ1+ θ2+ ….+ θ10 = 2π.
Define the complex numbers z1 =
eiθ1
, zk =
zk−1eiθk
for k = 2, 3, …, 10, where i = √-1. Consider the statement P and Q given below:
P: |z2 – z1| + |z3 – z2| + …. +|z10 – z9| + |z1 – z10| ≤ 2π
Q: |z22 – z12| + |z32 – z22| + …. +|z102 – z92| + |z12 – z102| ≤ 4π
Then,
a. P is TRUE and Q is FALSE
b. Q is TRUE and P is FALSE
c. Both P and Q are TRUE
d. Both P and Q are FALSE
Answer: c



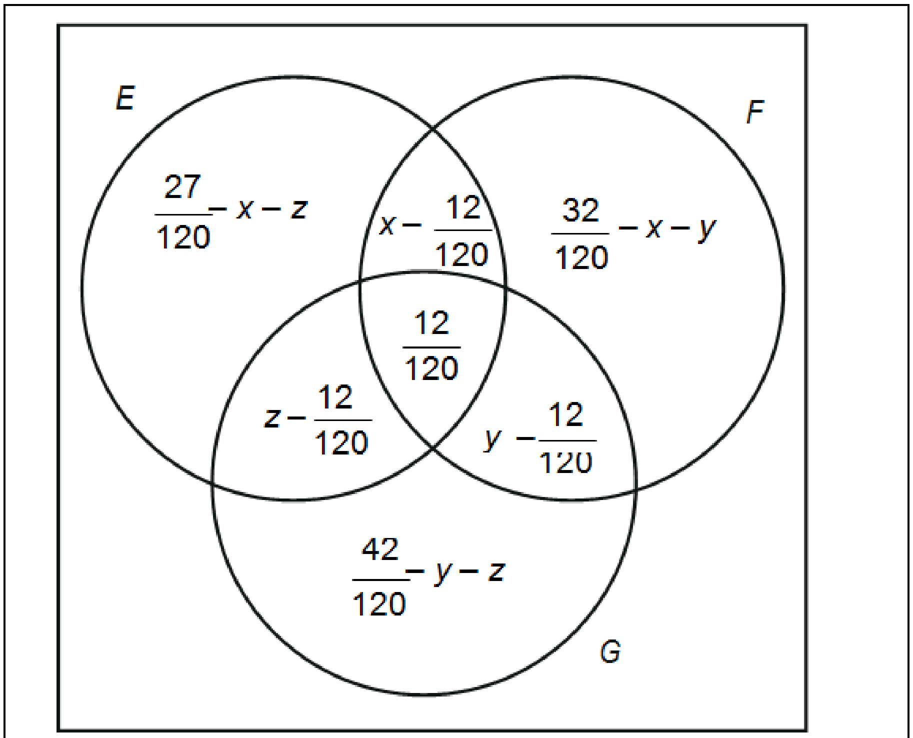
$|z_2 - z_1|$ = length of line AB ≤ length of arc AB
$|z_3 - z_2|$ = length of line BC ≤ length of arc BC
∴ The sum of the length of these 10 lines ≤ Sum of the length of arcs (i.e. 2π)
(As $\theta_1 + \theta_2 + \ldots + \theta_{10} = 2\pi$)
∴ $|z_2 - z_1| + |z_3 - z_2| + \ldots + |z_1 - z_{10}| \leq 2\pi$
And $|z_k^2 - z_{k-1}^2| = |z_k - z_{k-1}|\,|z_k + z_{k-1}|$
As we know $|z_k + z_{k-1}| \leq |z_k| + |z_{k-1}| \leq 2$
$|z_2^2 - z_1^2| + |z_3^2 - z_2^2| + \ldots + |z_1^2 - z_{10}^2| \leq 2(|z_2 - z_1| + |z_3 - z_2| + \ldots + |z_1 - z_{10}|) \leq 2(2\pi)$
∴ Both P and Q are true.

---

12. Let E, F and G be three events having probabilities $P(E) = 1/8$, $P(F) = 1/6$ and $P(G) = 1/4$, and $P(E \cap F \cap G) = 1/10$. For any event H, if H^c denotes its complement, then which of the following statements is(are) TRUE?

a. $P(E \cap F \cap G^c) \leq 1/40$
b. $P(E^c \cap F \cap G) \leq 1/15$
c. $P(E \cup F \cup G) \leq 13/24$
d. $P(E^c \cap F^c \cap G^c) \leq 5/12$

Answer: (a, b, c)
Let $P(E \cap F) = x$, $P(F \cap G) = y$ and $P(E \cap G) = z$
Clearly $x, y, z \geq 1/10$



Since $x + z \leq 27/120$

$\Rightarrow$ $x, z \leq 15/120$

$x+y \leq 32/120$

$\Rightarrow$ $x, y \leq 20/120$

And $y + z \leq 42/120 \Rightarrow y, z \leq 30/120$

Now $P(E \cap F \cap G^c) = x - 12/120 \leq 3/120 = 1/40$

$P(E^c \cap F \cap G) = y - 12/120 \leq 80/120 = 1/15$

$P(E \cup F \cup G) \leq P(E) + P(F) + P(G) = 13/24$

And $P(E^c \cap F^c \cap G^c) = 1 - P(E \cup F \cup G) \geq 11/24 \geq 5/12$