Total No. of Printed Pages—15

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MATHEMATICS

(FOR CANDIDATES WITH INTERNAL ASSESSMENT)

Full Marks : 80
Pass Marks : 24

(FOR CANDIDATES WITHOUT INTERNAL ASSESSMENT)

Full Marks : 100
Pass Marks : 30

Time : 3 hours

(FOR ALL CATEGORIES OF CANDIDATES)

General Instructions :

(i) This Question Paper comprises of 32 questions divided into six Sections A, B, C, D, E and F.

(ii) Marks allocated to every question are indicated against each.

(iii) Question Nos. **1** to **30** (Section—A to Section—E) are to be answered by all candidates.

(iv) Question Nos. **31** and **32** of Section—F are to be answered by **Candidates without Internal Assessment**.

(v) In question on construction, the drawing should be neat and exactly as per the given measurements.

(vi) Questions, which are meant for Visually Handicapped (Blind) Students, should be answered by them only.

(vii) Use of Calculator/Mobile Phone is not permitted.

SECTION—A

(*Marks* : 8)

(*Question Nos.* ***1*** *to* ***8*** *carry 1 mark each*)

1. Write the exponent of 2 in the prime factorization of 288. 1

2. What is the 2nd term of the sequence $a_n = \frac{n}{n+1}$? 1

3. Check whether the equation $x + \frac{1}{x} = 2,\ x \neq 0$ is a quadratic equation or not. 1

4. Express $\cos 65° + \tan 65°$ in terms of angles between 0° and 30°. 1

5. State AA-similarity criterion. 1

6. Find the distance between two parallel tangents of a circle of radius 6 cm. 1

7. Write the formula of the total surface area of a right circular cylinder whose radius of the base is r units and its height is h units. 1

8. Define modal class. 1

SECTION—B

(*Marks* : 14)

(*Question Nos.* ***9*** *to* ***15*** *carry 2 marks each*)

9. Find the value of k so that $8k+4$, $6k-2$ and $2k-7$ are the three consecutive terms of an A.P. 2

10. Prove that

$$\cos 48^\circ \operatorname{cosec} 42^\circ + \sec 48^\circ \sin 42^\circ = 2$$ 2

11. Find the value of $x (0^\circ < x < 90^\circ)$ in

$$\cos x = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$$ 2

Or

If $x = a\cos^3\theta$ and $y = b\sin^3\theta$, then prove that

$$\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$$ 2

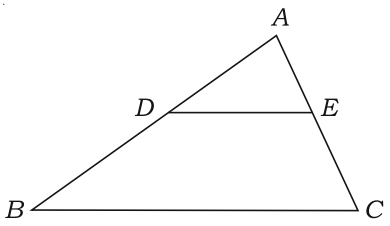
12. Find the distance between the pair of points (a, 0) and (0, b). 2

Or

Find the coordinates of the midpoint of the line segment joining the points $P(7, 0)$ and $Q(-5, 4)$. 2

13. Find the third vertex of a triangle *ABC*, if two of its vertices are *B* (–3, 1) and *C*(0, –2), and its centroid is at the origin. 2

14.



In the above figure, *D* and *E* are the points on the sides *AB* and *AC* respectively of ΔABC. Determine whether *DE* || *BC* or not if *AD* = 5·7 cm, *DB* = 9·5 cm, *AE* = 4·8 cm and *EC* = 8 cm. 2

[For Visually Handicapped (Blind) Students only, instead of Question No. 14 given above]

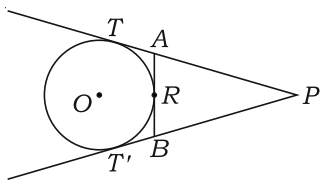
14. *(a)* Two geometric figures having the same shape but different sizes are said to be _____ figures.

(Fill in the blank) 1

(b) If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

(State whether True or False) 1

15.



In the above figure, *PT* and *PT'* are tangents from *P* to the circle with centre *O*. *R* is a point on the circle. Prove that *PA* + *AR* = *PB* + *BR*. 2

[For Visually Handicapped (Blind) Students only, instead of Question No. 15 given in Page No. 4]

15. *(a)* Define a tangent. 1

(b) How many tangents can be drawn through a point lying on the circle? 1

SECTION—C

(*Marks* : 24)

(*Question Nos.* ***16*** *to* ***23*** *carry 3 marks each*)

16. Using ruler and compass only, construct a triangle ABC such that each of its sides is $\frac{2}{3}$rd of the corresponding side of ΔABC. It is given that $AB = 4$ cm, $BC = 5$ cm and $AC = 6$ cm. (Only traces of constructions are required.) 3

[For Visually Handicapped (Blind) Students only, instead of Question No. 16 given above]

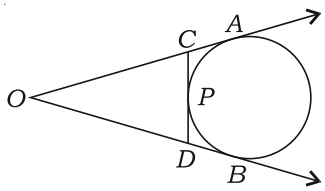
16. *(a)* Define an equilateral triangle. 1

(b) When are two triangles said to be equiangular? 1

(c) The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

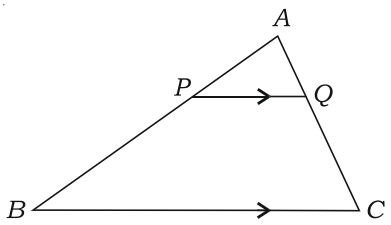
(State whether True or False) 1

17. In the figure below, *OA* and *OB* are tangents to the circle drawn from an external point *O*. *CD* is a third tangent touching the circle at *P*. If $OB = 10$ cm and $CP = 2$ cm, then find the length of *OC*. 3



Or

In the figure below, $PQ \parallel BC$ such that $AQ = \frac{1}{4}AC$. If $AB = 6$ cm, find *AP*. 3



[For Visually Handicapped (Blind) Students only, instead of Question No. 17 given above]

17. *(a)* Define a circle. 1

(b) Can a triangle have two right angles? 1

(c) If two triangles are congruent, then their areas are _____.

(Fill in the blank) 1

18. A race track is in the form of a ring whose inner circumference is 352 m and the outer circumference is 396 m. Find the width of the track. (Use $\pi = \frac{22}{7}$) 3

Or

Find the area of the minor sector of a circle of radius 4 cm and of angle 30°. (Take $\pi = 3{\cdot}14$) 3

19. Find the smallest number which when divided by 35, 56 and 91 leaves remainder of 7 in each case. 3

Or

Using Euclid's division algorithm, find the HCF of 105 and 120. 3

20. Find a quadratic polynomial whose zeroes are $3+\sqrt{5}$ and $3-\sqrt{5}$. 3

21. The 9th term of an A.P. is equal to 6 times its second term. If its 5th term is 22, find the A.P. 3

Or

Find the sum of the first 100 natural numbers. 3

22. Prove that

$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$$ 3

Or

If $\sin(A+B)=1$ and $\cos(A-B)=1$, $0^\circ < A+B \le 90^\circ$, $A > B$, find A and B. 3

23. A bag contains 5 red, 8 white and 7 black balls. A ball is drawn at random from the bag. Find the probability that the drawn ball is—

(a) red or white;

(b) neither black nor white. 3

SECTION—D

(*Marks* : 16)

(*Question Nos.* ***24*** *to* ***27*** *carry 4 marks each*)

24. The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction. 4

Or

Find two consecutive natural numbers whose product is 20. 4

25. An angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 9·5 m away from the wall. Find the length of the ladder. 4

Or

The angle of elevation of the top of a hill from the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If the tower is 50 m high, then what is the height of the hill? 4

[For Visually Handicapped (Blind) Students only, instead of Question No. 25 given in Page No. 8]

25. *(a)* Evaluate $\frac{\cos 17^\circ}{\sin 73^\circ}$. 1

(b) If $\cos\theta = 1$, then $\theta =$ _____.

(Fill in the blank) 1

(c) If $\sec\theta + \tan\theta = m$ and $\sec\theta - \tan\theta = n$, then prove that $mn = 1$. 2

26. Find the ratio in which the point $P(m, 6)$ divides the line segment joining the points $A(-1, 3)$ and $B(2, 8)$. Also, find the value of m. 4

Or

Find the value of p for which the points $A(-1, 3)$, $B(2, p)$ and $C(5, -1)$ are collinear. 4

27. Prove that in a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle. 4

[For Visually Handicapped (Blind) Students only, instead of Question No. 27 given above]

27. *(a)* What is the area of an equilateral triangle having side a units? 2

(b) The greatest side of a right-angled triangle is called hypotenuse.

(State whether True or False) 1

(c) All equilateral triangles are _____.

(Fill in the blank) 1

SECTION—E

(*Marks* : 18)

(*Question Nos.* ***28*** *to* ***30*** *carry 6 marks each*)

28. Solve the following system of linear equations graphically :

$$4x - y - 4 = 0$$

$$3x + 2y - 14 = 0$$

Find the vertices of the triangle formed by the lines and the y-axis. (Plot at least three points for each graph.) 6

[For Visually Handicapped (Blind) Students only, instead of Question No. 28 given above]

28. Solve the following system of linear equations : 6

$$2x + y = 7$$

$$4x - 3y + 1 = 0$$

29. The dimensions of a metallic cuboid are 100 cm × 80 cm × 64 cm. It is melted and recast into a cube. Find the total surface area of the cube. 6

Or

The volume of a hemisphere is $2425\frac{1}{2}$ cm^3. Find its curved surface area. (Use $\pi = \frac{22}{7}$) 6

30. Find the mean of the following data : 6

Class Interval	0–100	100–200	200–300	300–400	400–500
Frequency	6	9	15	12	8

Or

Find the mode of the following frequency distribution : 6

Age (in years)	0–5	5–10	10–15	15–20	20–25	25–30	30–35
Number of Patients	6	11	18	24	17	13	5

SECTION—F

(*Marks* : 20)

[For Candidates without Internal Assessment (WIM)]

31. Answer the following as directed (any *eight*) : 1×8=8

(a) Every composite number can be expressed as a product of

(A) coprimes

(B) primes

(C) twin primes

(D) None of the above

(Choose the correct option)

(b) The number of zeroes of a polynomial of degree n is

(A) equal to n

(B) greater than n

(C) less than n

(D) less than or equal to n

(Choose the correct option)

(c) A quadratic equation $ax^2 + bx + c = 0,\ a \neq 0$ has real roots if

(A) discriminant = 0

(B) discriminant > 0

(C) discriminant ≥ 0

(D) discriminant ≤ 0

(Choose the correct option)

(d) The total surface area of a hemisphere of radius r units is

(A) πr^2 sq. units

(B) $2\pi r^2$ sq. units

(C) $3\pi r^2$ sq. units

(D) $4\pi r^2$ sq. units

(Choose the correct option)

(e) The degree of a constant polynomial is _____.

(Fill in the blank)

(f) Abscissa of any point on the y-axis is zero.

(State whether True or False)

(g) The different numbers in a sequence are called terms.

(State whether True or False)

(h) $\tan^2 \theta - \sec^2 \theta = 1$

(State whether True or False)

(i) Two angles are said to be complementary if their sum is _____.

(Fill in the blank)

(j) The number $\frac{343}{2\times7\times5^2}$ has terminating decimal representation.

(State whether True or False)

(k) Write the first term of 2^n.

(l) Write the maximum value of $\cos\theta$.

(m) The lengths of two tangents drawn from an external point to a circle are _____.

(Fill in the blank)

(n) Ogives are cumulative frequency curves.

(State whether True or False)

32. Answer any *six* from the following : 2×6=12

(a) Find the prime factorization of 2025.

(b) Without performing long division method, state whether $\frac{23}{8}$ has a terminating decimal expansion or a non-terminating repeating decimal expansion.

(c) Find the zeroes of the polynomial $x^2 - 2x - 3$.

(d) Solve the quadratic equation

$$\left(x - \frac{1}{2}\right)^2 = 4$$

(e) Write the first two terms of the sequence

$$t_n = \frac{3^n}{2^n + 1}$$

(f) Determine whether the following given sides of a triangle form a right triangle or not :

$a = 3$ cm, $b = 4$ cm and $c = 7$ cm

(g) If $A = 30°$, verify that

$$\cos 2A = 1 - 2\sin^2 A$$

(h) Eliminate 'θ' for the equations :

$x = a\sec\theta$ and $y = b\tan\theta$

(i) If $P(E) = 0{\cdot}05$, what is the probability of 'not E'?

(j) State whether the following pair of linear equations has a unique solution, no solution or infinitely many solutions :

$$3x - 5y = 7$$
$$6x - 10y = 3$$

(k) Find the coordinates of the centroid of the triangle whose vertices are (–2, 3), (2, –1) and (4, 0).

(l) Write the discriminant of the quadratic equation

$$x^2 + x - 2 = 0$$

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