DR ACADEMY
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KCET EXAMINATION – 2021
SUBJECT : MATHEMATICS (VERSION – A3)

DATE :- 28-08-2021 **TIME : 02.30 PM TO 03.50 PM**

1. The equation of the line joining the points $(-3,4,11)$ and $(1,-2,7)$ is

a) $\frac{x+3}{2}=\frac{y-4}{3}=\frac{z-11}{4}$

b) $\frac{x+3}{-2}=\frac{y-4}{3}=\frac{z-11}{2}$

c) $\frac{x+3}{-2}=\frac{y+4}{3}=\frac{z+11}{4}$

d) $\frac{x+3}{2}=\frac{y+4}{-3}=\frac{z+11}{2}$

Ans. **b**

Sol. A(-3, 4, 11) B(1, -2, 7) Dr's of AB
(a, b, c)=1-(-3), (-2, -4) 7, -11
4, -6, -4
=-2, 3, 2

2. The angle between the lines whose direction cosines are $\left(\frac{\sqrt{3}}{4},\frac{1}{4},\frac{\sqrt{3}}{2}\right)$ and $\left(\frac{\sqrt{3}}{4},\frac{1}{4},\frac{-\sqrt{3}}{2}\right)$ is

a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{4}$

Ans. **c**

Sol. $\cos\theta=\left|\frac{\sqrt{3}}{4}\times\frac{\sqrt{3}}{4}+\frac{1}{4}\times\frac{1}{4}-\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}\right|=\left(\frac{-1}{2}\right),\ \theta=\frac{\pi}{3}$

3. If a plane meets the coordinate axes at A, B and C in such a way that the centroid of triangle ABC is at the point $(1,2,3)$ then the equation of the plane is

a) $\frac{x}{1}+\frac{y}{2}+\frac{z}{3}=1$ b) $\frac{x}{3}+\frac{y}{6}+\frac{z}{9}=1$

c) $\frac{x}{1}+\frac{y}{2}+\frac{z}{3}=\frac{1}{3}$ d) $\frac{x}{1}-\frac{y}{2}+\frac{z}{3}=-1$

Ans. **b**

Sol. (1, 2, 3)= $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$, a=3, b=6, c=9

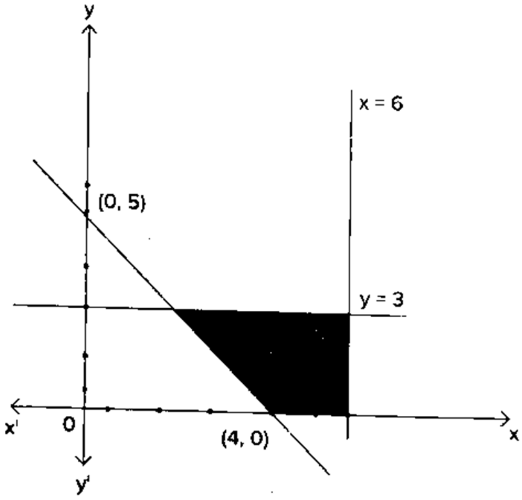
4. The area of the quadrilateral ABCD when $A(0,4,1)B(4,5,0)$ and $D(2,6,2)$ is equal to

a) 9 sq.units b) 18 sq.units
c) 27 sq.units d) 81 sq.units

Ans. **a**

Sol. $\frac{1}{2}\left(\overline{AC}\times\overline{BD}\right)=9\text{sq.units}$

5. The shaded region is the solution set of the inequalities



a) $5x+4y\geq 20, x\leq 6, y\geq 3, x\geq 0, y\geq 0$

b) $5x+4y\leq 20, x\leq 6, y\leq 3, x\geq 0, y\geq 0$

c) $5x+4y\geq 20, x\leq 6, y\leq 3, x\geq 0, y\geq 0$

d) $5x+4y\geq 20, x\geq 6, y\leq 3, x\geq 0, y\geq 0$

Ans. **c**

Sol. $x\leq 6,\ y\leq 3,\ 5x+4y\geq 20$

6. Given that A and B are two events such that $P(B)=\frac{3}{5}$, $P\left(\frac{A}{B}\right)=\frac{1}{2}$ and $P(A\cup B)=\frac{4}{5}$ then $P(A)=$

a) $\frac{3}{10}$ b) $\frac{1}{2}$ c) $\frac{1}{5}$ d) $\frac{3}{5}$

Ans. **b**

Sol. $\frac{1}{2}=\frac{P(A\cap B)}{\frac{3}{5}}\Rightarrow P(A\cap B)=\frac{3}{10}$

$\frac{4}{5}=\frac{3}{5}+P(A)-\frac{3}{10}\Rightarrow P(A)=\frac{1}{2}$

7. If A, B and C are three independent events such that $P(A)=P(B)=P(C)=P$ then P (at least two of A, B, C occur) =

a) P^3-3P b) $3P-2P^2$ c) $3P^2-2P^3$ d) $3P^2$

Ans. **c**

Sol. $P(A)=P(B)=P(C)=P$

$P(A).P(B).P(C)+3.P^2(1-p)$

$P^3+3p^2(1-P)=3P^2-2P^3$

8. Two dice are thrown. If it is known that the sum of numbers on the dice was less than 6 the probability of getting a sum as 3 is

a) $\frac{1}{18}$ b) $\frac{5}{18}$ c) $\frac{1}{5}$ d) $\frac{2}{5}$

Ans. c

Sol. (1, 1) (1, 2) (1, 3) (1, 4) (2, 1) (2, 2) (2, 3) (3, 1) (3, 2) (4, 1) P(B)= $\frac{10}{36}$

$n(A)=(1,2)(2,1)$ $P(A)=\frac{2}{36}$

$P\left(\frac{B}{A}\right)=\frac{\frac{2}{36}}{\frac{10}{36}}=\frac{2}{10}=\frac{1}{5}$

9. A car manufacturing factory has two plants X and Y. Plant X manufactures 70% of cars and plant Y manufactures 30% of cars. 80% of cars at plant X and 90% of cars at plant Y are rated as standard quality. A car is chosen at random and is found to be standard quality. The probability that it has come from plant X is

a) $\frac{56}{73}$ b) $\frac{56}{84}$ c) $\frac{56}{83}$ d) $\frac{56}{79}$

Ans. c

Sol. $=\frac{\frac{70}{100}\times\frac{80}{100}}{\frac{70}{100}\times\frac{80}{100}+\frac{90}{100}\times\frac{30}{100}}=\frac{56}{83}$

10. In a certain two 65% families own cell phones, 15000 families own scooter and 15% families own both. Taking into consideration that the families own at least one of the two, the total number of families in the town is

a) 20000 b) 30000 c) 40000 d) 50000

Ans. b

Sol. $x=\frac{65x}{100}+15000-\frac{15x}{100}$ =30,000

11. A and B are non-singleton sets and $n(A\times B)=35$. If $B\subset A$ then ${}^{n(A)}C_{n(B)}=$

a) 28 b) 35 c) 42 d) 21

Ans. d

Sol. $n(A\times B)=35=7\times 5,\ 7_{C_5}=7_{C_2}=21$

12. Domain of $f(x)=\frac{x}{1-|x|}$ is

a) $R-[-1,1]$ b) $(-\infty,1)$

c) $(-\infty,1)\cup(0,1)$ d) $R-\{-1,1\}$

Ans. d

Sol. $|x|\neq 1$

13. The value of $\cos 1200^0+\tan 1485^0$ is

a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) $-\frac{3}{2}$ d) $-\frac{1}{2}$

Ans. a

Sol. $\cos\left(3\times 360^0+120^0\right)+\tan\left(4\times 360^0+45^0\right)$

=1/2

14. The value of $\tan 1^0 \tan 2^0 \tan 3^0 .... \tan 89^0$ is

a) 0 b) 1 c) $\frac{1}{2}$ d) -1

Ans. b

Sol. $\tan\theta.\cot\theta=1$

15. If $\left(\frac{1+i}{1-i}\right)^x=1$ then

a) $x=4n+1;n\in N$ b) $x=2n+1;n\in N$

c) $x=2n;n\in N$ d) $x=4n;n\in N$

Ans. d

Sol. $\left(\frac{1+i}{1-i}\right)^x=1\Rightarrow i^x=1$

16. The cost and revenue functions of a product are given by $c(x)=20x+4000$ and $R(x)=60x+2000$ respectively where x is the number of items produced and sold. The value of x to earn profit is

a) >50 b) >60 c) >80 d) >40

Ans. a

Sol. $R(x)-c(x)>0$; $60x+2000-20x-4000>0$

$x>50$

17. A student has to answer 10 questions, choosing at least 4 from each of the parts A and B. If there are 6 questions in part A and 7 in part B, then the number of ways can the student choose 10 questions is

a) 256 b) 352 c) 266 d) 426

Ans. c

Sol. ${}^{13}C_{10}-{}^{6}C_3=286-20=266$

18. If the middle term of the A.P is 300 then the sum of its first 51 terms is

a) 15300 b) 14800 c) 16500 d) 14300

Ans. a

Sol. mid term is $T_{26}=300$

$T_1=300-25d$; $T_{51}=300+25d$

$S=\frac{51}{2}[300-25d+300+25d]$

$\frac{51}{2}[600]=15,300$

19. The equation of straight line which passes through the point $(a\cos^3\theta, a\sin^3\theta)$ and perpendicular to $x\sec\theta + y\operatorname{cosec}\theta = a$ is

a) $\frac{x}{a} + \frac{y}{a} = a\cos\theta$ b) $x\cos\theta - y\sin\theta = a\cos 2\theta$

c) $x\cos\theta + y\sin\theta = a\cos 2\theta$ d) $x\cos\theta - y\sin\theta = -a\cos 2\theta$

Ans. b

Sol. $\frac{x}{\sin\theta} - \frac{y}{\cos\theta} = \frac{a\cos^3\theta}{\sin\theta} - \frac{a\sin^3\theta}{\cos\theta}$

$\frac{x\cos\theta - y\sin\theta}{\sin\theta\cos\theta} = \frac{a(\cos 2\theta)}{\sin\theta\cos\theta}$

$x\cos\theta - y\sin\theta = a\cos 2\theta$

20. The mid points of the sides of triangle are (1, 5, -1) (0, 4, -2) and (2, 3, 4) then centroid of the triangle

a) (1, 4, 3) b) $\left(1, 4, \frac{1}{3}\right)$ c) (-1, 4, 3) d) $\left(\frac{1}{3}, 2, 4\right)$

Ans. b

Sol. $\left(\frac{1+0+2}{3}, \frac{5+4+3}{3}, \frac{-1-2+4}{3}\right)$

$\left(1, 4, \frac{1}{3}\right)$

21. Consider the following statements :

Statement 1 :

$\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a}$ is 1 (where $a+b+c \neq 0$)

Statement 2 : $\lim_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}$ is $\frac{1}{4}$

a) Only statement 2 is true

b) Only statement 1 is true

c) Both statements 1 and 2 are true

d) Both statements 1 and 2 are false

Ans. b

Sol. statement 1 is true

Statement 2 is false

$\left[\frac{a+b+c}{a+b+c} = 1\right]$

$\lim_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}$ is $-\frac{1}{4}$

22. If a and b are fixed non-zero constants, then the derivative of $\frac{a}{x^4} - \frac{b}{x^2} + \cos x$ is ma+nb-p where

a) $m = 4x^3;\ n = \frac{-2}{x^3};\ p = \sin x$

b) $m = \frac{-4}{x^5};\ n = \frac{2}{x^3};\ p = \sin x$

c) $m = \frac{-4}{x^5};\ n = \frac{-2}{x^3};\ p = -\sin x$

d) $m = 4x^3;\ n = \frac{2}{x^3};\ p = -\sin x$

Ans. b

Sol. $\frac{d}{dx}\left(\frac{a}{x^4} - \frac{b}{x^2} + \cos x\right) = \left(-\frac{4a}{x^5} + \frac{2b}{x^3} - \sin x\right)$

$= ma + nb - p$

$m = -\frac{4}{x^5}; n = \frac{+2}{x^3}; p = \sin x$

23. The Standard Deviation of the numbers 31, 32, 33……. 46, 47 is

a) $\sqrt{\frac{17}{12}}$ b) $\sqrt{\frac{47^2-1}{12}}$ c) $2\sqrt{6}$ d) $4\sqrt{3}$

Ans. c

Sol. $S.D. = \sqrt{\frac{n^2-1}{12}}\ (n = 17)$

$= \sqrt{\frac{17^2-1}{12}}$

$= 2\sqrt{6}$

24. If P(A)=0.59, P(B)=0.30 and $P(A \cap B)$=0.21 then $P(A' \cap B') =$

a) 0.11 b) 0.38 c) 0.32 d) 0.35

Ans. c

Sol. $P(A^1 \cap B^1) = 1 - P(A \cup B)$

$= 1 - [0.59 + 0.3 - 0.21]$

=0.32

25. f:R→R defined by f(x)=

$\begin{cases} 2x; x > 3 \\ x^2; 1 < x \leq 3 \text{ then } f(-2) + f(3) + f(4) \text{ is} \\ 3x; x \leq 1 \end{cases}$

a) 14 b) 9 c) 5 d) 11

Ans. d

Sol. $f(-2) + f(3) + f(4)$

$-6 + 9 + 8$

=11

26. Let A={x:x∈R ; x is not a positive integer)

Define f:A→R as f(x)= $\frac{2x}{x-1}$, then f is

a) injective but not surjective
b) surjective but not injective
c) bijective
d) neither injective nor surjective

Ans. a

Sol. $f'(x)=\frac{-2}{(x-1)^2}<0$

f is s.d.
f is one-one

$\frac{2x}{x-1}=y \Rightarrow x=\frac{y}{y-2} \notin \pi$ for $y=2$

f is not out

27. The function f(x)= $\sqrt{3}$ sin2x-cos2x+4 is one-one in the interval

a) $\left[\frac{-\pi}{6},\frac{\pi}{3}\right]$ b) $\left[\frac{\pi}{6},\frac{-\pi}{3}\right]$ c) $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ d) $\left[\frac{-\pi}{6},\frac{-\pi}{3}\right]$

Ans. a

Sol. $f=\sqrt{3}\sin 2x-\cos 2x+4=2\left[\sin\left(2x-\frac{\pi}{6}\right)\right]+4$

f is one-one

$-\frac{\pi}{2}\le 2x-\frac{\pi}{6}\le\frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{6}\le x\le\frac{\pi}{3}$

$\left[-\frac{\pi}{6},\frac{\pi}{3}\right]$

28. Domain of the function f(x)= $\frac{1}{\sqrt{[x^2]-[x]-6}}$

where [x] is greatest integer ≤ x is

a) $(-\infty,2)\cup[4,\infty]$ b) $(-\infty,-2)\cup[3,\infty]$
c) $[-\infty,-2]\cup[4,\infty]$ d) $[-\infty,2]\cup[3,\infty]$

Ans. a

Sol. $[x^2]-[x]-6>0$ $([x]-3)([x]+2)>0$

$[x]<-2,[x]>3\Rightarrow x\in(-\infty,-2)\cup(4,\infty)$

29. $\cos\left[\cot^{-1}\left(-\sqrt{3}\right)+\frac{\pi}{6}\right]=$

a) 0 b) 1 c) $\frac{1}{\sqrt{2}}$ d) -1

Ans. d

Sol. $\cos\left(\pi-\frac{\pi}{6}+\frac{\pi}{6}\right)=\cos\pi=-1$

30. $\tan^{-1}\left[\frac{1}{\sqrt{3}}\sin\frac{5\pi}{2}\right]\sin^{-1}\left[\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right]=$

a) 0 b) $\frac{\pi}{6}$ c) $\frac{\pi}{3}$ d) π

Ans. GRACE

Sol. $\left(\frac{\pi}{6}\right)^2$

31. If $A=\begin{bmatrix}1&-2&1\\2&1&3\end{bmatrix}B=\begin{bmatrix}2&1\\3&2\\1&1\end{bmatrix}$ then (AB)' is equal to

a) $\begin{bmatrix}-3&-2\\10&7\end{bmatrix}$ b) $\begin{bmatrix}-3&10\\-2&7\end{bmatrix}$ c) $\begin{bmatrix}-3&7\\10&2\end{bmatrix}$ d) $\begin{bmatrix}-3&7\\10&-2\end{bmatrix}$

Ans. b

Sol. $AB=\begin{pmatrix}-3&-2\\10&7\end{pmatrix}$

$(AB)^T=\begin{pmatrix}-3&10\\-2&7\end{pmatrix}$

32. Let M be 2 x 2 symmetric matrix with integer entries, then M is invertible if

a) the first column of M is the transpose of second row of M
b) the second row of M is the transpose of first column of M
c) M is diagonal matrix with non-zero entries in the principal diagonal
d) The product of entries in the principal diagonal of M is the product of entries in the other diagonal

Ans. c

Sol. $m=\begin{pmatrix}a&0\\0&a\end{pmatrix}$

m is invertible.

33. If A and B are matrices of order 3 and |A|=5, |B|=3 then |3AB| is

a) 425 b) 405 c) 565 d) 585

Ans. b

Sol. $|3AB|=3^3|AB|$

$=27\times3\times5$

$=405$

34. If A and B are invertible matrices then which of the following is not correct ?

a) adjA=|A|A^{-1} b) det $(A^{-1})=[\det(A)]^{-1}$
c) $(AB)^{-1}=B^{-1}A^{-1}$ d) $(A+B)^{-1}=B^{-1}+A^{-1}$

Ans. d

Sol. $(A+B)^{-1}=B^{-1}+A^{-1}$

35. If $f(x)=\begin{vmatrix} \cos x & 1 & 0 \\ 0 & 2\cos x & 3 \\ 0 & 1 & 2\cos x \end{vmatrix}$ then $\lim_{x\to\pi} f(x)=$

a) -1 b) 1 c) 0 d) 3

Ans. a

Sol. $f(x)=4\cos^3 x-3\cos x$

$=\cos 3x$

$\lim_{x\to\pi}\cos 3x=\cos 3\pi$

$=-1$

36. If $x^3-2x^2-9x+18=0$ and $A=\begin{vmatrix} 1 & 2 & 3 \\ 4 & x & 6 \\ 7 & 8 & 9 \end{vmatrix}$ then the maximum value of A is

a) 96 b) 36 c) 24 d) 120

Ans. a

Sol. $(x-2)(x^2-9)=0$

$x=2,3,-3$

$f(x)=|A|=-12x+60$

Max value at $x=-3$

$\therefore |A|=96$

37. At x=1, the function f(x)= $\begin{cases} x^3-1 & 1<x<\infty \\ x-1 & -\infty<x\le 1 \end{cases}$ is

a) continuous and differentiable
b) continuous and non-differentiable
c) discontinuous and differentiable
d) discontinuous and non-differentiable

Ans. b

Sol. $\lim_{x\to 1+} x^3-1=0$

$\lim_{x\to 1-}(x-1)=0$

F is continuous

$f'(x)=\begin{cases} 3x^2 & 1<x<\infty \\ 1 & -\infty<x< \end{cases}$

$f'(1^+)=3, f'(1^-)=1$

$\Rightarrow$ f is not differentiable

38. If $y=(\cos x^2)^2$, then $\frac{dy}{dx}$ is equal to

a) $-4x \sin 2x^2$ b) $-x \sin x^2$
c) $-2x \sin 2x^2$ d) $-x \cos 2x^2$

Ans. c

Sol. $\frac{dy}{dx}=2\cos x^2.(-\sin x^2)2x$

$=-2x\sin(2x^2)$

39. For constant a, $\frac{d}{dx}(x^x+x^a+a^x+a^a)$ is

a) $x^x(1+\log x)+ax^{a-1}$
b) $x^x(1+\log x)+ax^{a-1}+a^x\log a$
c) $x^x(1+\log x)+a^a(1+\log x)$
d) $x^x(1+\log x)+a^a(1+\log a)+ax^{a-1}$

Ans. b

Sol. $\frac{d}{dx}(x^x+x^a+a^x+a^a)$

40. Consider the following statements :
Statement 1 :

If $y=\log_{10}x+\log_e x$ then $\frac{dy}{dx}=\frac{\log_{10}e}{x}+\frac{1}{x}$

Statement 2 :

If $\frac{d}{dx}(\log_{10}x)=\frac{\log x}{\log 10}$ and $\frac{d}{dx}(\log_e x)=\frac{\log x}{\log e}$

a) Statement 1 is true ; Statement 2 is false
b) Statement 1 is false ; statement 2 is true
c) Both statements 1 and 2 are true
d) Both statements 1 and 2 are false

Ans. a

Sol. $x^x(1+\log x)+ax^{a-1}+a^x\log_e^a$

$y=\frac{\log x}{\log 10}+\log x$

$\frac{dy}{dx}=\frac{1}{x\log 10}+\frac{1}{x}$

41. If the parametric equation of curve is given by $x=\cos\theta+\log\tan\frac{\theta}{2}$ and $y=\sin\theta$, then the points for which $\frac{dy}{dx}=0$ are given by

a) $\theta=\frac{n\pi}{2}, n\in z$ b) $\theta=(2n+1)\frac{\pi}{2}, n\in z$
c) $\theta=(2n+1)\pi, n\in z$ d) $\theta=n\pi, n\in z$

Ans. d

Sol. $\frac{dx}{d\theta}=-\sin\theta+\frac{1}{\tan\left(\frac{\theta}{2}\right)}.\sec^2\left(\frac{\theta}{2}\right)\frac{1}{2}$

$=-\sin\theta+\frac{1}{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}=-\sin\theta+\frac{1}{\sin\theta}$

$=\frac{1-\sin^2\theta}{\sin\theta}; \frac{dx}{d\theta}=\frac{\cos^2\theta}{\sin\theta}; \frac{dy}{d\theta}=\cos\theta$

$\frac{dy}{dx}=0 ; \tan\theta=0$

$\theta=n\pi, n\in z$

42. If $y=(x-1)^2(x-2)^3(x-3)^5$ then $\frac{dy}{dx}$ at $x=4$ is equal to

a) 108 b) 54 c) 36 d) 516

Ans. d

Sol. $\log y = 2\log(x-1)+3\log(x-2)+5\log(x-3)$

$$\frac{dy}{dx}=(x-1)^2(x-2)^2(x-3)^5\left[\frac{2}{x-1}+\frac{3}{x-2}+\frac{5}{x-3}\right]$$

$$\left(\frac{dy}{dx}\right)_{x=4}=516$$

43. A particle starts form rest and its angular displacement (in radians) is given by $\theta=\frac{t^2}{20}+\frac{t}{5}$. If the angular velocity at the end of t = 4 is k, then the value of 5k is

a) 0.6 b) 5 c) 5k d) 3

Ans. d

Sol. $\frac{d\theta}{dt}=\frac{2t}{20}+\frac{1}{5}$

$=\frac{t}{10}+\frac{1}{5}$

$\left(\frac{d\theta}{dt}\right)_{\varepsilon=4}=\frac{4}{10}+\frac{1}{5}$

$k=\frac{3}{5}$

$5k=3$

44. If the parabola $y=\alpha x^2-6x+\beta$ passes through the point $(0,2)$ and has its tangent at $x=\frac{3}{2}$ parallel to x axis, then

a) $\alpha=2,\beta=-2$ b) $\alpha=-2,\beta=2$

c) $\alpha=2,\beta=2$ d) $\alpha=-2,\beta=-2$

Ans. c

Sol. $y=\alpha x^2-6x+\beta$ passes through $(0,2)$

$2=\beta$

$\frac{dy}{dx}=2\alpha x-6$

$\left(\frac{dy}{dx}\right)_{x=\frac{3}{2}}=0$

$2\alpha\left(\frac{3}{2}\right)-6=0$

$3\alpha=6$

$\alpha=2$

45. The function $f(x)=x^2-2x$ is strictly decreasing in the interval

a) $(-\infty,1)$ b) $(1,\infty)$ c) R d) $(-\infty,\infty)$

Ans. a

Sol. $f'(x)<0$; $2(x-1)<0$

$x<1$; $x\in(-\infty,\ 1)$

46. The maximum slope of the curve $y=-x^3+3x^2+2x-27$ is

a) 1 b) 23 c) 5 d) -23

Ans. c

Sol. Slope $m=\frac{dy}{dx}=-3x^2+6x+2$

$\frac{dm}{dx}=0$; $-6x+6=0$

$x=1$; $m=-3+6+2=5$

47. $\int\frac{x^3\sin\left(\tan^{-1}(x^4)\right)}{1+x^8}dx$ is equal to

a) $\frac{-\cos\left(\tan^{-1}(x^4)\right)}{4}+C$ b) $\frac{\cos\left(\tan^{-1}(x^4)\right)}{4}+C$

c) $\frac{-\cos\left(\tan^{-1}(x^3)\right)}{3}+C$ d) $\frac{\sin\left(\tan^{-1}(x^4)\right)}{4}+C$

Ans. a

Sol. $\text{Tan}^{-1}x^4=t$; $\frac{4x^3}{1+x^8}dx=dt$

$$I=\frac{1}{4}\int\sin t\,dt=\frac{-1}{4}\cos t+c=\frac{-1}{4}\cos\left(\text{Tan}^{-1}x^4\right)+c$$

48. The value of $\int\frac{x^2dx}{\sqrt{x^6+a^6}}$ is equal to

a) $\log\left|x^3+\sqrt{x^6+a^6}\right|+c$

b) $\log\left|x^3-\sqrt{x^6+a^6}\right|+c$

c) $\frac{1}{3}\log\left|x^3+\sqrt{x^6+a^6}\right|+c$

d) $\frac{1}{3}\log\left|x^3-\sqrt{x^6+a^6}\right|+c$

Ans. c

Sol. $x^3=t$ $3x^2dx=dt$

$$I=\frac{1}{3}\int\frac{1}{\sqrt{t^2+(a^3)^2}}dt=\frac{1}{3}\log\left[t+\sqrt{t^2+a^6}\right]$$

$$=\frac{1}{3}\log\left[x^3+\sqrt{x^6+a^6}\right]+c$$

49. The value of $\int \frac{xe^x dx}{(1+x)^2}$ is equal to

a) $e^x(1+x)+c$ b) $e^x(1+x^2)+c$

c) $e^x(1+x)^2+c$ d) $\frac{e^x}{1+x}+c$

Ans. d

Sol. $\int \frac{(x+1-1)e^x}{(1+x)^2}dx = \int e^x\left(\frac{1}{1+x}-\frac{1}{(1+x)^2}\right)dx$

$= \frac{e^x}{1+x}+c$

50. The value of $\int e^x\left[\frac{1+\sin x}{1+\cos x}\right]dx$ is equal to

a) $e^x \tan\frac{x}{2}+c$ b) $e^x \tan x + c$

c) $e^x(1+\cos x)+c$ d) $e^x(1+\sin x)+c$

Ans. a

Sol. $\int e^x\left(\frac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)dx$

$= \int e^x\left(\frac{1}{2}\sec^2\frac{x}{2}+\text{Tan}\frac{x}{2}\right)dx$

$= e^x\text{Tan}\frac{x}{2}+c$

51. If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x\,dx$ where n is positive integer then $I_{10}+I_8$ is equal to

a) 9 b) $\frac{1}{7}$ c) $\frac{1}{8}$ d) $\frac{1}{9}$

Ans. d

Sol. $I_n + I_{n-2} = \frac{1}{n-1}$

52. The value of $\int_0^{4042} \frac{\sqrt{x}\,dx}{\sqrt{x}+\sqrt{4042-x}}$ is equal to

a) 4042 b) 2021 c) 8084 d) 1010

Ans. b

Sol. $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}dx = \frac{b-a}{2}$

53. The area of the region bounded by $y=\sqrt{16-x^2}$ and x-axis is

a) 8 square units b) 20π square units

c) 16π square units d) 256π square units

Ans. a

Sol. $x^2+y^2=16$

$\frac{1}{2}\pi(4)^2 = 8\pi$

54. If the area of the Ellipse is $\frac{x^2}{25}+\frac{y^2}{\lambda^2}=1$ is 20π square units, then λ is

a) ±4 b) ±3 c) ±2 d) ±1

Ans. a

Sol. $\frac{x^2}{25}+\frac{y^2}{\lambda^2}=1$

$\pi ab = \pi.5.|\lambda| = 20\pi$

$|\lambda| = 4 \Rightarrow \lambda = \pm4$

55. Solution of Differential Equating xdy – ydx = 0 represents

a) A rectangular Hyperbola

b) Parabola whose vertex is at origin

c) Straight line passing through origin

d) A circle whose centre is origin

Ans. c

Sol. $xdy = ydx$

$\frac{dy}{dx}=\frac{y}{x}$

$m=\frac{y}{x}$

$y = mx$

56. The number of solutions of $\frac{dy}{dx}=\frac{y+1}{x-1}$ when $y(1)=2$ is

a) three b) one

c) infinite d) two

Ans. b

Sol. One solution

57. A vector $\vec{a}$ makes equal acute angles on the coordinate axis. Then the projection of vector $\vec{b} = 5\hat{i} + 7\hat{j} + \hat{k}$ on $\vec{a}$ is

a) $\frac{11}{15}$ b) $\frac{11}{\sqrt{3}}$ c) $\frac{4}{5}$ d) $\frac{3}{5\sqrt{3}}$

Ans. b

Sol. $\bar{a} = \hat{i} + \hat{j} + \hat{k}$

$$\frac{\bar{b}.\bar{a}}{|\bar{a}|} = \frac{5+7-1}{\sqrt{3}} = \frac{11}{\sqrt{3}}$$

58. The diagonals of a parallelogram are the vectors $3\hat{i} + 6\hat{j} - 2\hat{k}$ and $-\hat{i} - 2\hat{j} - 8\hat{k}$ then the length of the shorter side of parallelogram is

a) $2\sqrt{3}$ b) $\sqrt{14}$ c) $3\sqrt{5}$ d) $4\sqrt{3}$

Ans. GRACE

Sol. $\bar{a} = \frac{\bar{d}_1 + \bar{d}_2}{2} = \frac{2i + 4j - 10k}{2} = i + 2j - 5k$

$|\bar{a}| = \sqrt{30}$

$|\bar{b}| = \frac{\bar{d}_1 - \bar{d}_2}{2} = \frac{4i + 8j + 6k}{2} = 2i + 4j + 3k$

$|\bar{b}| = \frac{2}{\sqrt{4+16+9}} = \sqrt{29}$

59. If $\vec{a}.\vec{b} = 0$ and $\vec{a} + \vec{b}$ makes an angle 60° with $\vec{a}$ then

a) $|\vec{a}| = 2|\vec{b}|$ b) $2|\vec{a}| = |\vec{b}|$

c) $|\vec{a}| = \sqrt{3}|\vec{b}|$ d) $\sqrt{3}|\vec{a}| = |\vec{b}|$

Ans. d

Sol. $\cos 60 = \frac{(\bar{a} + \bar{b}).\bar{a}}{|\bar{a} + \bar{b}|\,|\bar{a}|} = \frac{|\bar{a}|^2 + 0}{\sqrt{|\bar{a}|^2 + |\bar{b}|^2}} =$

$\frac{1}{2} = \frac{|\bar{a}|}{\sqrt{|\bar{a}|^2 + |\bar{b}|^2}}$

$|\bar{a}|^2 + |\bar{b}|^2 = 4|\bar{a}|^2$

$|\bar{b}|^2 = 3|\bar{a}|^2$

$|\bar{b}| = \sqrt{3}|\bar{a}|$

60. If the area of the parallelogram with $\vec{a}$ and $\vec{b}$ as two adjacent sides is 15 sq. units then the area of the parallelogram having $3\vec{a} + 2\vec{b}$ and $\vec{a} + 3\vec{b}$ as two adjacent sides in sq. units is

a) 45 b) 75 c) 105 d) 120

Ans. c

Sol. $|\bar{a} \times \bar{b}| = 15$

$|(3\bar{a} + 2\bar{b}) \times (\bar{a} + 3\bar{b})| = |9(\bar{a} + \bar{b}) \times 2(\bar{b} + \bar{a})|$

$= |7(\bar{a} \times \bar{b})| = 7 \times 15 = 105$