Time : 3 Hours

MATHEMATICS

Subject Code

H	4	7	5	4

Total No. Of Questions : 36 **(Printed Pages : 10)** **Maximum Marks : 80**

INSTRUCTIONS :

(*i*) The question paper consists of **36** questions.

(*ii*) *All* questions are compulsory.

(*iii*) Question numbers **1** to **8** are multiple choice type questions of *one* mark each.

(*iv*) Question numbers **9** to **16** are very short answer type questions of *one* mark each.

(*v*) Question numbers **17** to **22** are short answer type-I questions of *two* marks each.

(*vi*) Question numbers **23** to **28** are short answer type-II questions of *three* marks each.

(*vii*) Question numbers **29** to **34** are long answer type-I questions of *four* marks each.

(*viii*) Question numbers **35** to **36** are long answer type-II questions of *five* marks each.

(*ix*) There is no overall choice. However an internal choice has been provided in two questions of **4** marks each and 2 questions of **5** marks each.

(*x*) Use of calculator is not permitted.

(*xi*) Log tables will be supplied on request.

(*xii*) Graph should be drawn on the answer paper only.

1. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + xy = \left(\frac{dy}{dx}\right)^4$ is ....

- 1
- 2
- 3
- 4

2. If $f : \mathrm{R} \rightarrow \mathrm{R}$ and $g : \mathrm{R} \rightarrow \mathrm{R}$ are the two real functions defined by $f(x) = 3x^2 + 1$ and $g(x) = 1 - x$, then $(gof)(-2)$ is ....

- 12
- 28
- –12
- –28

3. The value of $\left[\hat{i} + \hat{j} \quad 2\hat{j} \quad 3\hat{k}\right]$ is ....

- 0
- 6
- 3
- 5

4. If $\sin\left(\tan^{-1}(x)+\cot^{-1}\left(\frac{1}{3}\right)\right)=1$, then the value of x is ....

- 3
- –3
- $\frac{\pi}{2}$
- $\frac{1}{3}$

5. If R is a relation in the set $A = \{3\}$ as $R = \{(x, y) / x, y \in A$ and $x^2 + y + 4$ is a perfect square$\}$, then R is ....

- reflexive, symmetric but not transitive
- reflexive but neither symmetric nor transitive
- an equivalence relation
- symmetric but neither reflexive nor transitive

6. If $\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} + X = \begin{bmatrix} 3 & 1 \\ 5 & 0 \end{bmatrix}$, then the matrix X is ....

- $\begin{bmatrix} 6 & 5 \\ 4 & 4 \end{bmatrix}$
- $\begin{bmatrix} 0 & 5 \\ 4 & 2 \end{bmatrix}$
- $\begin{bmatrix} 0 & 5 \\ 4 & -2 \end{bmatrix}$
- $\begin{bmatrix} 0 & 5 \\ -4 & -2 \end{bmatrix}$

7. The value of $\int_{1}^{3} |x-3|\, dx$ is ....

- 1
- –2
- 2
- 0

8. The value of $\int_{-1}^{1} x^4 \sin^3 x\, dx$ is ....

- 1
- 0
- –1
- 2

9. Using determinants, show that the points (1, 3), (2, 2) and (0, 4) are collinear.

10. Find the slope of tangent to the curve $2y = 3 - x^3$ at the point (1, 1).

11. Find the distance between the two planes :

$$x + y + 3z = 4 \text{ and } 2x + 2y + 6z = 10.$$

12. Find the area of parallelogram whose adjacent sides are given by the vectors $\bar{a} = \hat{i} + \hat{j}$ and $\bar{b} = 2\hat{i} + 3\hat{k}$.

13. Find the principal value of $\text{cosec}^{-1}(-2)$.

14. If

$$A = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}_{3 \times 1} \quad \text{and} \quad B = \begin{bmatrix} 2 & 0 & 1 \end{bmatrix}_{1 \times 3}$$

then find the matrix (AB)', where (AB)' is the transpose of matrix (AB).

15. The random variable X has the following probability distribution :

X	**P(X)**
0	K
1	2K
2	3K
3	4K

Find $P(X < 2)$.

16. If $y = e^x + y^2$, then find $\frac{dy}{dx}$.

17. Find the angle between the following pairs of lines :

$$\bar{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k}) \text{ and}$$

$$\bar{r} = 7\hat{i} - 6\hat{k} + \mu(2\hat{i} + 2\hat{j} + \hat{k}).$$

18. If E and F are the events of a sample space S, such that $P(F) \neq 0$, then prove that :

$$P(E'/F) = 1 - P(E/F),$$

where E' is the complement of event E.

19. Form the differential equation of family of curves represented by $A(y + A)^2 = x^3$, by eliminating arbitrary constant A.

20. Let * be a binary operation defined on set A = {1, 2, 3, 6, 12} as $a * b$ = H.C.F $\{a, b\}$. Prepare composition table for the binary operation *. Also, compute 3* (6 * 12).

21. Prove that :

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x - y}{1 + xy}\right), \quad xy > -1.$$

22. If A(1, –2, 3) and B(–1, –4, 3) are the given points and $\bar{d} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ is a given vector, then find the scalar projection of vector $\overline{AB}$ on $\bar{d}$.

23. Using properties of determinants, prove that :

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3.$$

24. Find the general solution of the differential equation :

$$x^2 \frac{dy}{dx} = x^2 + xy - 2y^2.$$

25. One third of the students in a class are boys and the rest are girls. It is known that the probability of a girl getting first class is 0.32 and that a boy getting first class is 0.22. If a student chosen at random gets first class marks in the subject, what is the probability that the chosen student is a boy ?

26. If

$$y = (\sqrt{x})^{\sin 2x} + (\log 3x)^{\sqrt{x+1}},$$

find $\frac{dy}{dx}$.

27. Find the equation of the plane passing through the line of intersection of planes $\overline{r}\,.\,(2\hat{i} + \hat{j} - \hat{k}) = 3$ and $\overline{r}\,.\,(5\hat{i} - 3\hat{j} + 4\hat{k}) = -9$ and parallel to the line

$$\overline{r} = (\hat{i} + 3\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 4\hat{j} + 5\hat{k}).$$

28. Find :

$$\int \frac{x+5}{\sqrt{x^2 + 3x - 7}}\, dx.$$

29. Prove that :

$$\int_0^{2a} f(x)\,dx = \int_0^{a} [f(x) + f(2a-x)]\,dx$$

Hence, show that :

$$\int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx, \quad \text{if} \quad f(2a-x) = f(x)$$
$$= 0 \qquad , \quad \text{if} \quad f(2a-x) = -f(x).$$

30. Using integration, find the area of the smaller region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and the straight line $\frac{x}{4} + \frac{y}{3} = 1$.

Or

Using integration, find the area of the smaller region enclosed between parabola $y^2 = 16x$ and the line $x - y + 3 = 0$.

31. Solve the following Linear Programming Problem graphically :

Minimize $Z = 5x + 7y$

Subject to constraints :

$2x + y \geq 8$

$x + 2y \geq 10$

$2x + 3y \leq 24$

$x \geq 0,\ y \geq 0.$

32. If $x = \sin t$, $y = \sin(pt)$ prove that :

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$$

where t is a parameter.

Or

If $y = Ae^{-kx}\cos(px+4)$, prove that :

$$\frac{d^2y}{dx^2} + 2k\frac{dy}{dx} + (p^2+k^2)y = 0.$$

33. If the function $f(x)$ defined by :

$$f(x) = \begin{cases} \dfrac{1+\sin x}{A\cos^2 x} & ; \quad -\pi \le x < \dfrac{-\pi}{2} \\ = 3\sin 2x + B & ; \quad \dfrac{-\pi}{2} \le x \le 0 \\ = \dfrac{e^{5x} - e^{3x}}{x} & ; \quad 0 < x \le \pi \end{cases}$$

is continuous on $[-\pi, \pi]$, then find the values of A and B.

34. Find the inverse of matrix $A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$.

Hence, solve the system of equations :

$$x + 2y + 5z = 10$$

$$x - y - z = -2$$

$$2x + 3y - z = -11.$$

35. The perimeter of an isosceles triangle is 200 cm. If its base is changing at the rate 5 cm/sec, then find the rate at which the altitude is changing when the base is 40 cm.

Or

Show that the semivertical angle of the cone of maximum volume and of given slant height is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

36. Find :

$$\int \frac{2\cos^2 x + \cos x}{(\sin x - 2)\,(\sin^2 x + 3)}\, dx.$$

Or

Find :

$$\int \cos^{-1}\left(\sqrt{x}\right) dx.$$