Time : 2½ Hours **MATHEMATICS**

Subject Code

H	7	5	4

Total No. Of Questions : 30 **(Printed Pages : 7)** **Maximum Marks : 80**

INSTRUCTION : (*i*) *All* questions are compulsory.

(*ii*) The question paper contains of **30** questions divided into five section A, B, C, D and E.

(*iii*) Section A contains **7** questions of **1** mark each, which are multiple choice type questions. Section B contains **7** questions of **2** marks each, Section C contains **7** questions of **3** marks each, Section D contains **7** questions of **4** marks each and Section E contains **2** questions of **5** marks each.

(*iv*) There is no overall choice in the paper. However internal choice is provided in **2** questions of **3** marks each, **2** questions of **4** marks each and **2** questions of **5** marks each. In questions with choices only one of the choices is to be attempted.

(*v*) Use of calculator is not allowed.

SECTION – A

Question numbers **1** to **7** carry **1** mark **each**. In **each** question, **four** options are provided out of which **one** is correct. Write the correct option.

1. If $2A^T - B + X = 0$ where $A = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}$ and A^T is the transpose of A then X = ______

   - $\begin{bmatrix} 8 & 1 \\ 0 & -1 \end{bmatrix}$
   - $\begin{bmatrix} 8 & 1 \\ 0 & 1 \end{bmatrix}$
   - $\begin{bmatrix} 8 & -1 \\ 0 & -1 \end{bmatrix}$
   - $\begin{bmatrix} -8 & -1 \\ 0 & -1 \end{bmatrix}$

2. If $|\bar{a}| = \sqrt{3}$, $|\bar{b}| = 2$, and $|\bar{a} \bullet \bar{b}| = 3$ then $|\bar{a} \times \bar{b}|$ is ________

   - 2
   - 3
   - $\sqrt{3}$
   - $\sqrt{2}$

3. The value of 'a' if $\int_0^1 \frac{1}{\sqrt{a^2 - x^2}} dx = \frac{\pi}{4}$ is

   - 2
   - 4
   - $\sqrt{2}$
   - 1

4. If $\int \frac{x e^x}{(x+1)^2} dx = e^x f(x) + c$, then f(x) = ________

   - $\frac{1}{x+1}$
   - $\frac{1}{(x+1)^2}$
   - $(x + 1)$
   - $(x + 1)^2$

5. The function f(x) = log (cos x) is strictly decreasing in the interval

   - $\left(\frac{\pi}{2}, \pi\right)$
   - $\left(\pi, \frac{3\pi}{2}\right)$
   - $\left(\frac{3\pi}{2}, 2\pi\right)$
   - $\left(\frac{5\pi}{2}, 3\pi\right)$

6. The total number of one-one functions defined from the set A = {1, 2, 3} to itself are

- 6
- 3
- 2
- 1

7. The position vector of the point where the line $\bar{r} = \bar{a} + \lambda\bar{b}$ meets the plane $\bar{r} \cdot \bar{n} = 0$ is

- $\left(\dfrac{\bar{a} \bullet \bar{n}}{\bar{b}}\right)$
- $-\left(\dfrac{\bar{a} \bullet \bar{n}}{\bar{b} \bullet \bar{n}}\right)$
- $\bar{a} - \left(\dfrac{\bar{a} \bullet \bar{n}}{\bar{b} \bullet \bar{n}}\right)\bar{b}$
- $\bar{a} + \left(\dfrac{\bar{a} \bullet \bar{n}}{\bar{b} \bullet \bar{n}}\right)\bar{b}$

SECTION – B

Question numbers 8 to 14 carry 2 marks each.

8. Find the value of ‘x’ and ‘y’ if

$$\begin{bmatrix} 3x & 2 \\ 5y & x \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 15 & 18 \\ -4 & 12 \end{bmatrix} + \begin{bmatrix} -10 & -10 \\ 8 & 0 \end{bmatrix}$$

9. Prove that $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$ where $x \in (-\infty, -1) \cup [1, \infty)$.

10. Find the derivative of f(x) where $f(x) = 3^{2x+5}$ using first principle.

11. If $\text{Tan}^{-1}a + \text{Tan}^{-1}b + \text{Tan}^{-1}c = \dfrac{\pi}{2}$, find the value of ‘ab + bc + ca’.

12. Find the general solution of the differential equation $\dfrac{dy}{dx} + Py = Q$ where P and Q are functions of ‘x’.

13. If $\bar{p}, \bar{q}, \bar{r}$ are three vectors of magnitude 2, 3 and 5 respectively and $\bar{p} + \bar{q} + \bar{r} = \bar{0}$, then find ‘$\bar{p} \cdot \bar{q} + \bar{q} \cdot \bar{r} + \bar{r} \cdot \bar{p}$’.

14. If A = (3, –4, 5) and B = (1, 2, –1) are two points on a line. Find the Cartesian equation of plane which is the perpendicular bisector of segment AB.

SECTION – C

Question numbers **15** to **21** carry **3** marks **each**.

15. Find inverse of the matrix $A = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ by adjoint method.

16. For three vectors $\bar{a}$, $\bar{b}$ and $\bar{c}$ if $[\bar{b}\ \bar{c}\ \bar{a}] = 5$ then find $[\bar{c} - 4\bar{a}\ \ 2\bar{a} - \bar{b}\ \ 3\bar{b} - 2\bar{c}]$.

17. Let $A = Q \times Q$ where Q is the set of rational numbers, define a binary operation $*$ on A by $(x, y) * (p, r) = (xp, xr + y)$. Prove that $*$ is not commutative but associative. Also find $(1, 2) * (3, 4)$.

18. In a certain university, there are 400 Indians, 200 Russians and 100 Americans pursuing their Master's degree. The probabilities of choosing mathematics for their Master's degree are $\frac{3}{5}, \frac{4}{5}, \frac{2}{5}$ respectively.

    A student persuing Master's in mathematics is selected from the university. Find the probability that the student is an Indian.

19. Evaluate $\int_0^{\pi} \log\left(\frac{2\sin x}{1+\cos x}\right)dx$.

20. Find the equation of line passing through the points (2, 1, 5) and (1, 3, 7). Find the angle between this line and the plane making intercepts 2, –3, 4 on the coordinate axes.

    OR

    Find the shortest distance between the lines
    $\bar{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\bar{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\,(2\hat{i} + \hat{j} + 2\hat{k})$.

21. Let X denote the number of vowels in word selected at random from the sentence "MEAN, MEDIAN, STANDARD DEVIATION ARE SOME OF THE TERMS IN STATISTICS". Find the probability distribution and expected value of 'X'.

OR

Assume that on an average one telephone number out of 3 called between 2.00 p.m. to 3.00 p.m. on week days is busy. If six randomly selected telephone numbers are called, find the probability that (i) atleast 5 of them will be busy (ii) none will be busy.

SECTION – D

Question numbers **22** to **28** carry **4** marks **each**.

22. Prove that $\int_0^{2a} f(x)\,dx = \int_0^{a} [f(x)+f(2a-x)]\,dx$. Hence find $\int_0^{2a} f(x)\,dx$ if $f(2a-x)=f(x)$.

23. Find the values of 'm' and 'n' if the function defined below is continuous at x = 0.

$$f(x) = \frac{e^{2mx}-e^{nx}}{3\sin x} \quad ; \; x<0$$

$$= 2\sin x + 3m\cos x + 2n \quad ; \; x = 0$$

$$= \frac{6x^2}{1-\cos 2x} \quad ; \; x>0$$

24. Find the area of the region bounded by the curve $y^2 = 4x$ and the line $2x + y = 4$ in the first quadrant using integration.

25. Solve the following LPP graphically :

Maximise $z = 19x + 38y$

Subject to the constraints

$5x + 2y \geq 10$

$x + y \leq 5$

$3x + 5y \geq 15$

$x, y \geq 0.$

26. Using properties of determinants, prove that

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$$

27. Solve the differential equation

$$y\,dx + x\log\left(\frac{y}{x}\right)dy - 2x\,dy = 0$$

OR

The tangent is drawn at (x, y) to the curve. The product of x-coordinate, square of y-coordinate of the point and the slope of tangent is 3 times the slope of line segment joining the point (x, y) and (–4, –3). Find the equation of curve given that it passes through the point (2, –2).

28. If $x = e^t(\sin t - \cos t)$, $y = e^t(\sin t + \cos t)$ find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$.

OR

Differentiate $(\tan x)^x + (x^2 + 1)^{\cot x}$ with respect to 'x'.

SECTION – E

Question numbers **29** to **30** carry **5** marks **each**.

29. Evaluate : $\int \frac{\tan^{-1}x}{(x+2)^2}\,dx$.

OR

Evaluate : $\int \cot^{-1}\left(\frac{1+x-x^2}{2x-1}\right)dx$.

30. The sum of the perimeters of a circle and the rectangle is 4 meters. The length of rectangle is 2 times its width. Show that sum of their areas is least when the radius of the circle is $\left(\frac{2}{3}\right)$ times the width of the rectangle.

OR

A water tank has the shape of an inverted right circular cone. The radius of its circular base is 10 m and vertical height is 30 m. Water is poured into it at a constant rate of 2 cubic meter per minute. Find at what rate is the water level rising and how fast is the surface area is increasing when the depth of the water is 5 m.