1. If A = [(x, y) : x2 + y2 = 25] and B = [(x, y) : x2 + 9y2 = 144], then A ∩ B contains ________ points.

Solution:
A = Set of all values (x, y) : x2 + y2 = 25 = 52
B = [x2 / 144] + [y2 / 16] = 1
i.e., [x2 / (12)2] + [y2 / (4)2] = 1.
A ∩ B consists of four points.

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2. In a college of 300 students, every student reads 5 newspapers, and every newspaper is read by 60 students. The number of newspapers is _________.

Solution:
Let the number of newspapers be x.
If every student reads one newspaper, the number of students would be x (60) = 60x
Since every student reads 5 newspapers, the number of students = [x * 60] / [5] = 300
x = 25
Consider the system of equations x + y + z = 1, 2x + 3y + 2z = 1, 2x + 3y + (a2 – 1)z = a + 1 then
(a) System has a unique solution for |a| = √3
(b) System is inconsistence for |a| = √3
(c) System is inconsistence for a = 4
(d) System is inconsistence for a = 3
Answer: (b)
Solution:
Given a system of linear equations:
x + y + z = 1 ….(1)
2x + 3y + 2z = 1 ….(2)
2x + 3y + (a2 – 1)z = a + 1 …..(3)
D=|11123223a2−1|
= 1[3(a2 – 1) – (3)(2)] – 1[2(a2 – 1) – (2)(2)] + 1[2(3) – 2(3)]
= 3a2 – 3 – 6 – 2a2 + 2 + 4 + 0
= a2 – 3
Consider D ≠ 0
So, a2 – 3 ≠ 0
⇒ a2 ≠ 3
⇒ |a| ≠ ±√3
That means the given linear equation system is inconsistent for |a| = ±√3.
So option (b) is correct.

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3. If the system of linear equations are

2x + 2ay + az = 0
2x + 3by + bz = 0 and
2x + 4cy + cz = 0,
where a, b, c ∈ R are non-zero and distinct; has a non-zero solution, then
(a) a + b + c = 0

(b) 1/a, 1/b, 1/c are in A.P.
(c) a, b, c are in A.P.
(d) a, b, c are in G.P.
Answer: (b)
Solution:
Given a system of linear equations
2x + 2ay + az = 0
2x + 3by + bz = 0 and
2x + 4cy + cz = 0,
As the given system of equations has a non-zero solution, we have D = 0.

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0$$

$$R_2 \to R_2 - R_1 \text{ and } R_3 \to R_3 - R_1$$

$$\begin{vmatrix} 2 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 4c-2a & c-a \end{vmatrix} = 0$$

⇒ (3b − 2a)(c − a) − (4c − 2a)(b − a) = 0
⇒ 3bc − 2ac − 3ab + 2a2 − [4bc − 4ac − 2ab + 2a2] = 0
⇒ −bc + 2ac − ab = 0
⇒ ab + bc = 2ac
⇒ 1/c + 1/a = 2/b
Which shows that 1/a, 1/b, and 1/c are in A.P.
If (1 + i) (1 + 2i) (1 + 3i) ….. (1 + ni) = a + ib, then what is 2 * 5 * 10….(1 + n2) is equal to?
Solution:
We have (1 + i) (1 + 2i) (1 + 3i) ….. (1 + ni) = a + ib …..(i)
(1 − i) (1 − 2i) (1 − 3i) ….. (1 − ni) = a − ib …..(ii)
Multiplying (i) and (ii),
we get 2 * 5 * 10 ….. (1 + n2) = a2 + b2

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4. If z is a complex number, then the minimum value of |z| + |z − 1| is _______.

Solution:
First, note that |−z|=|z| and |z1 + z2| ≤ |z1| + |z2|
Now |z| + |z − 1| = |z| + |1 − z| ≥ |z + (1 − z)|
= |1|
= 1
Hence, the minimum value of |z| + |z − 1| is 1.
A letter lock consists of three rings, each marked with ten different letters. In how many ways is it possible to make an unsuccessful attempt to open the lock?
Solution:

Two rings may have the same letter at a time, but the same ring cannot have two letters at a time. Therefore, we must proceed ring-wise. Each of the three rings can have any one of the 10 different letters in 10 ways.
Therefore, the total number of attempts = 10 × 10 × 10 = 1000.
But out of these 1000 attempts, only one attempt is successful.
Therefore, the required number of unsuccessful attempts = 1000 – 1 = 999.

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5. The total number of positive integral solutions for x, y, z such that x* y * z = 24, is ________.

Solution:
We have,
x* y * z = 24
x* y * z = 23 × 31
The number of ways of distributing ‘n’ identical balls into ‘r’ different boxes is (n + r − 1)C(r − 1)
Here we have to group 4 numbers into three groups
Number of integral positive solutions
= (3 + 3 − 1)C(3 − 1) × (1+ 3 − 1)C(3 − 1)
= 5C2 × 3C2
= 30

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6. The projection of any line on coordinate axes be, respectively 3, 4, 5 then its length is ______.

Solution:
Let the line segment be AB, then as given AB cos α = 3, AB cos β = 4, AB cos γ = 5
⇒ AB2 (cos2α + cos2β + cos2γ) = 32 + 42 + 52
AB = √[9 + 16 + 25] = 5√2,
where α, β and γ are the angles made by the line with the axes.

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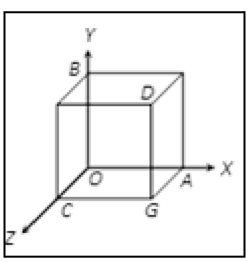
7. The angle between two diagonals of a cube will be _____.

Solution:



Let the cube be of side ‘a’ and O (0, 0, 0), D (a, a, a), B (0, a, 0), G (a, 0, a).
Then the equation of OD and BG are x/a = y/a = z/a and x/a = [y − a]/[−a] = z/a, respectively.
Hence, the angle between OD and BG is

$$cos^{-1}\frac{a^2-a^2+a^2}{\sqrt{3a^2}\times\sqrt{3a^2}} = cos^{-1}\left(\frac{1}{3}\right)$$

If any four numbers are selected, and they are multiplied, then the probability that the last digit will be 1, 3, 5 or 7 is ___________.

Solution:
The total number of digits in any number at the unit's place is 10.
Therefore, n (S) = 10
If the last digit is 1, 3, 5 or 7, then it is necessary that the last digit in each number must be 1, 3, 5 or 7.
Therefore, n (A) = 4
P (A) = 4 / 10 = 2 / 5
Hence, the required probability is (2 / 5)4 = 16 / 625.

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8. Forty teams play a tournament. Each team plays with every other team just once. Each game results in a win for one team. If each team has a 50% chance of winning each game, the probability that at the end of the tournament, every team has won a different number of games is _______.

Solution:
Team totals must be 0, 1, 2, 39.
Let the teams be T1, T2,……., T40, so that T1 loses to T1 for i < j. In other words, this order uniquely determines the result of every game. There are 40! such orders and 780 games, so 2780 possible outcomes for the games.
Hence, the probability is 40! / 2780.
The equation of 2x2 + 3y2 − 8x − 18y + 35 = k represents a ________.
Given equation, 2x2 + 3y2 − 8x − 18y + 35 – k = 0
Compare with ax2 + by2 + 2hxy + 2gx + 2fy + c = 0,we get
a = 2, b = 3, h = 0, g = −4, f = −9, c = 35 − k
Δ = abc + 2fgh − af2 − bg2 − ch2 = 6 (35 − k) + 0 − 162 − 48 − 0
Δ = 210 − 6k − 210 = −6k;
Δ = 0, if k = 0
So, that given equation is a point if k = 0.

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9. The locus of the midpoint of the line segment joining the focus to a moving point on the parabola y2 = 4ax is another parabola with the directrix ___________.

Solution:
Let the midpoint be (h, k) and the moving point be (at2, 2at).
Let focus is (a, 0).
So h = [at2 + a] /2, k = [2at + 0] / 2
=> t = k/a
So 2h = a(k/a)2 + a
= (k2/a) + a
=> k2 = 2ah – a2
=> k2 = 2a(h – a/2)
Replace (h, k) by (x, y)
y2 = 2a(x – a/2)
Directrix is (x − a/2) = -a/2
=> x = 0

So x = 0 is the directrix.

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10. If p is true and q is false, then which of the following statements is not true?

A) $p \vee q$

B) $p \Rightarrow q$

C) $p \wedge (\sim q)$

D) $q \Rightarrow p$

Solution:

When p is true, and q is false, then;

$p \vee q$ is true

$q \Rightarrow p$ is true

$p \wedge (\sim q)$ is true. (Therefore, both p and ~q are true)

Here, $p \Rightarrow q$ is not true as a true statement cannot imply a false statement.
