Total No. of Printed Pages—11

HS/XII/A. Sc. Com/M/NC/20

2020

MATHEMATICS

(New Course)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

General Instructions :

(i) All questions are compulsory.

(ii) This question paper contains 36 questions divided into four Sections A, B, C and D. Section—A comprises of 20 questions of *1* mark each, Section—B comprises of 6 questions of *2* marks each, Section—C comprises of 6 questions of *4* marks each and Section—D comprises of 4 questions of *6* marks each.

(iii) There is no overall choice. However, internal choice has been provided in 7 questions of Section—A, 2 questions of Section—B, 2 questions of Section—C and 1 question of Section—D. You have to attempt only one of the alternatives in all such questions.

(iv) Use of calculator is not permitted.

SECTION—A

1. Show that the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is symmetric. 1

2. Find the Cartesian equation of the plane

$$\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$$ 1

3. Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$. 1

4. Construct a 2×2 matrix $A = [a_{ij}]$, whose elements are given by

$$a_{ij} = \frac{(i + j)^2}{2}$$ 1

5. Find the values of x, y, z if

$$\begin{bmatrix} x + y + z \\ x + z \\ y + z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$ 1

Or

If

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

then find $2A - B$. 1

6. If a line has direction ratios -18, 12, -4, then determine its direction cosines. 1

7. If $P(A) = 0{\cdot}8$, $P(B) = 0{\cdot}5$ and $P(B|A) = 0{\cdot}4$, find $P(A \cap B)$. 1

8. If A and B are two independent events with $P(A) = 0{\cdot}3$ and $P(B) = 0{\cdot}4$, find $P(A \cup B)$. 1

9. Show that the function

$$f(x) = \begin{cases} x & \text{if } x \leq 1 \\ 5 & \text{if } x > 1 \end{cases}$$

is not continuous at $x = 1$. 1

Or

Find the minimum value of $f(x) = (2x - 1)^2 + 3$. 1

10. Differentiate $y = \sin(x^2 + 5)$ with respect to x. 1

Or

Show that the function $f(x) = e^{2x}$ is increasing on $\mathbb{R}$. 1

11. Find the rate of change of the area of a circle with respect to its radius r, when $r = 4$ cm. 1

12. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 4$. 1

13. Find $g \circ f$ and $f \circ g$, if $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are given by $f(x) = \cos x$ and $g(x) = 3x^2$. 1

14. Find the order and degree of the differential equation

$$\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$$ 1

Or

Find $\int_0^{\pi/2} \cos^2 x\, dx$. 1

15. Find the unit vector in the direction of the vector

$$\vec{a} = \hat{j} + \hat{i} + 2\hat{k}$$ 1

Or

Find the vector joining the points $P(2, 3, 0)$ and $Q(-1, -2, -4)$ directed from P to Q. 1

Choose the correct answer :

16. Which of the given values of x and y make the following pair of matrices equal? 1

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} \text{ and } \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

(a) $x = -\frac{1}{3}$, $y = 7$

(b) $x = -\frac{2}{3}$, $y = 7$

(c) Not possible to find

(d) $x = -\frac{1}{3}$, $y = -\frac{2}{3}$

17. If

$$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$$

then x is equal to

(a) 6

(b) ± 6

(c) -6

(d) 0 1

Or

Which of the following is correct? 1

(a) Determinant is a square matrix

(b) Determinant is a number associated to a matrix

(c) Determinant is a number associated to a square matrix

(d) None of the above

18. The antiderivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ equals

(a) $\frac{1}{3}x^{1/3} + 2x^{1/2} + c$

(b) $\frac{2}{3}x^{2/3} + \frac{1}{2}x^{2} + c$

(c) $\frac{2}{3}x^{3/2} + 2x^{1/2} + c$

(d) $\frac{3}{2}x^{3/2} + \frac{1}{2}x^{1/2} + c$ 1

Or

$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$ is equal to

(a) $\tan x + \cot x + c$

(b) $\tan x + \operatorname{cosec} x + c$

(c) $-\tan x + \cot x + c$

(d) $\tan x + \sec x + c$ 1

19. The value of $\hat{i}\cdot(\hat{j}\times\hat{k}) + \hat{j}\cdot(\hat{i}\times\hat{k}) + \hat{k}\cdot(\hat{i}\times\hat{j})$ is

(a) 0

(b) −1

(c) 1

(d) 3 1

20. The maximum value of $Z = 3x + 4y$ subject to the constraints $x + y \leq 4$, $x \geq 0$, $y \geq 0$ is

(a) 8

(b) 16

(c) 10

(d) 4 1

SECTION—B

21. Find the value of

$$\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$$ 2

Or

Show that

$$\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{77}{36}$$ 2

22. Find $\int \frac{1}{1-\tan x}\,dx$. 2

23. Verify that $y = x\sin x$ is a solution of the differential equation $xy' = y + x\sqrt{x^2 - y^2}$ $(x \neq 0$ and $x > y$ or $x < -y)$. 2

24. If $y = (\tan^{-1} x)^2$, show that

$$(x^2+1)^2 y_2 + 2x(x^2+1)y_1 = 2$$ 2

25. If x and y are connected parametrically by the equations

$$x = a(\theta - \sin\theta) \text{ and } y = a(1+\cos\theta)$$

find $\frac{dy}{dx}$ without eliminating the parameter. 2

26. Find the general solution of

$$y \log y\, dx - x\, dy = 0$$ 2

Or

Find the general solution of

$$e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$$ 2

SECTION—C

27. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$ is an equivalence relation. 4

28. Find the value of k so that the function

$$f(x) = \begin{cases} \dfrac{1 - \cos kx}{x \sin x} & \text{if } x \neq 0 \\ \dfrac{1}{2} & \text{if } x = 0 \end{cases}$$

is continuous at $x = 0$. 4

29. Find the equation of all lines having slope 2 and being tangent to the curve $y + \dfrac{2}{x - 3} = 0$. 4

Or

Show that $y = \log(1 + x) - \dfrac{2x}{2 + x}$, $x > -1$ is an increasing function of x throughout its domain. 4

30. Find $\int \frac{(x^2+1)e^x}{(x+1)^2}dx$. 4

Or

Find the area of the region bounded by the two parabolas $y = x^2$ and $y^2 = x$. 4

31. Let

$$\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$$
$$\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$$
$$\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$$

Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c} \cdot \vec{d} = 15$. 4

32. Solve the following graphically : 4

Minimize $Z = 3x + 5y$

subject to the constraints

$$x + 3y \geq 3$$
$$x + y \geq 2$$
$$x,\ y \geq 0$$

SECTION—D

33. The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method. 6

Or

Obtain the inverse of the following matrix using elementary operations : 6

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

34. Evaluate : 6

$$\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$

35. Find the vector equation of the plane passing through the intersection of the planes

$$\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 7$$
$$\vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) = 9$$

through the point (2, 1, 3). 6

36. State Bayes' theorem on probability. Use it to solve the following :

A laboratory blood test is 99% effective in detecting a certain disease when it is in fact present. However, the test also yields a false positive result for 0·5% of the healthy persons tested (i.e., if a healthy person is tested, then, with probability 0·005, the test would imply he has the disease). If 0·1% of the population actually has the disease, then what is the probability that a person has the disease given that his test result is positive? 6

★★★