2023

MATHEMATICS

Full Marks – 80

Time – 3 Hours

General Instructions :

(i) All questions are compulsory.

(ii) Figures in the margin indicate marks.

(iii) In question on construction, the drawing should be neat and exactly as per the given measurements.

(iv) Use of calculator is not allowed.

1. Choose the correct answer : $24\times1=24$

(a) The sum to be paid in cash under an instalment plan at the time of purchase of an article is called –

(i) cash price (ii) cash down payment

(iii) instalment (iv) principal

(b) Kunga covers 15 km in 3 hours, the distance covered by him in 8 hours is –

(i) 28 km (ii) 45 km

(iii) 40 km (iv) 120 km

(c) Mawii can do a piece of work in 10 days which Liani can do in 15 days. The time taken by them, working together on it, is –

(i) 6 days (ii) $\frac{1}{6}$ days

(iii) 30 days (iv) 25 days

(d) The expression $\frac{x^2+7x+12}{x^2-6x-27}$ in its lowest term is –

(i) $\frac{x+4}{x-3}$ (ii) $\frac{x-4}{x+3}$

(iii) $\frac{x+4}{x-9}$ (iv) $\frac{x-4}{x+9}$

(e) The zeroes of the polynomial $x^2 - 2x - 8$ are –

(i) 4 and –2 (ii) – 4 and 2

(iii) 4 and 2 (iv) – 4 and –2

(f) The quadratic equation $2x^2 - 3x + 5 = 0$ has –

(i) two equal real roots (ii) no real roots

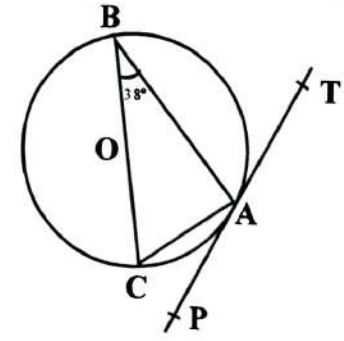
(iii) two unequal real roots (iv) None of these

(g) The 10^{th} term of the AP : 17, 14, 11 ..... is –

(i) –10 (ii) 10

(iii) –7 (iv) – 40

(h) In the given figure, BC is a diameter of the circle with centre O and PAT is the tangent at A. If $\angle ABC = 38°$, then $\angle BAT$ is equal to –



(i) 42° (ii) 48°

(iii) 52° (iv) 55°

(i) From a point Q, the length of tangent to a circle is $2\sqrt{7}$ cm and the distance of Q from the centre is 8 cm. The radius of a circle is –

(i) 6 cm (ii) 11 cm

(iii) 15 cm (iv) 34 cm

(j) The lengths of the diagonals of a rhombus are 16 cm and 12 cm. Then the length of the side of the rhombus is –

(i) 10 cm (ii) 9 cm

(iii) 8 cm (iv) 20 cm

(k) If AB is a diameter of a circle with centre (2, –3) and point A is (3,–10), what are the coordinates of point B ?

(i) (–1, 4) (ii) (4, 1)

(iii) (1, 4) (iv) (–4, 1)

(l) The distance between the points A(–5, 8) and B (0, –4) is –

(i) 13 units (ii) 12 units

(iii) $\sqrt{14}$ units (iv) $\sqrt{119}$ units

(m) A point P divides the join of A (5, –2) and B (9, 6) in the ratio 3 : 1, the coordinates of P are –

(i) (4, 8) (ii) (8, 4)

(iii) (–8, 3) (iv) (4, 3)

(n) The value of $\sec^2 50° - \cot^2 40°$ is equal to –

(i) 0 (ii) 1

(iii) 2 (iv) 10

(o) $(1+\sin\theta)(1-\sin\theta)\sec^2\theta$ is equal to –

(i) 0 (ii) 1

(iii) –1 (iv) $\cos^2\theta$

(p) 2 cubes each of 5cm edge are joined end to end, the volume of resulting cuboid is –

(i) 125 cm^3 (ii) 150 cm^3

(iii) 200 cm^3 (iv) 250 cm^3

(q) If an area of a circle is numerically equal to its circumference, then the radius of the circle is –

(i) 2 units (ii) 3 units

(iii) 4 units (iv) 5 units

(r) The length of the longest rod that can be placed in a room 12 m long, 9 m broad and 8 m high is –

(i) 13.2 m (ii) 15 m

(iii) 17 m (iv) 18.2 m

(s) The number of balls each having radius 3 cm that can be made from a solid sphere of radius 6 cm is –

(i) 8 (ii) 12

(iii) 24 (iv) 27

(t) Formula for finding curved surface area of a right circular cylinder is –

(i) $2\pi rh$ (ii) $2\pi r^2h$

(iii) πr^2h (iv) $2hr\,(h + r)$

(u) A card is drawn at random from a well shuffled deck of 52 cards. Then the probability that the card king is drawn is –

(i) $\frac{1}{13}$ (ii) $\frac{1}{26}$

(iii) $\frac{2}{13}$ (iv) $\frac{3}{26}$

(v) If A = { a, b, c, d}, how many subsets are there in the power set of A ?

(i) 32 (ii) 24

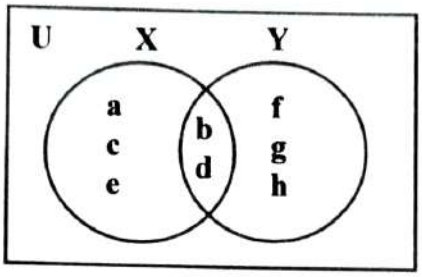
(iii) 16 (iv) 8

(w) In a group of 80 people, 37 like coffee, 52 like tea and 19 like both. The number of people who like neither coffee nor tea is –

(i) 10 (ii) 61

(iii) 70 (iv) 71

(x) Use the Venn diagram shown in figure to find the set X – Y.



(i) {a, c, e} (ii) {a, b, d}

(iii) {f, g, h} (iv) {c, b, d}

2. Answer the following questions : $10 \times 2 = 20$

(a) If the remainder when $x^3 + 2x^2 + kx + 3$ is divided by $(x - 3)$ is 21, find the value of k.

(b) Savanah travelled a distance of 61 km in 9 hrs. She travelled partly on foot at 4 km/hr and partly on bicycle at 9 km / hr. What is the distance travelled on foot ?

(c) Find the roots of the quadratic equation $4x^2 + 4\sqrt{3x} + 3 = 0$.

(d) Find the sum of the first hundred even natural numbers which are divisible by 5.

(e) ABC is a triangle in which $\angle A$ is bisected by AD meeting BC at D such that BD = DC. Prove that $\triangle ABC$ is isosceles.

(f) A tangent PQ at a point P of a circle of radius 8 cm meets a line through the centre O at a point Q such that OQ = 17 cm. Find the length of PQ.

(g) In a right isosceles $\triangle$ ABC, right angled at C, the vertices of A and B are $(-1, 2)$and $(3, 2)$ respectively. Find the 'x' coordinate of the third vertex, i.e., C.

(h) Prove that :

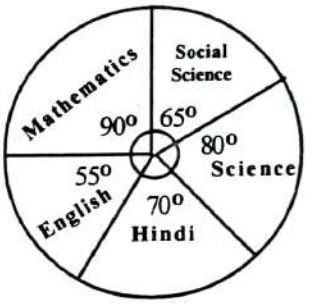
$\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\ldots \tan 89^\circ = 1$

(i) Volume of a cube is 8 cm^3. If the breadth and height of a cuboid are 1 cm each, find the length of the cuboid if its volume is the same with that of the cube.

(j) The pie-chart gives the marks scored in an examination by a student. If the total marks scored by the student is 540, answer the following questions –

(i) In which subject did the student score 120 marks ?

(ii) How many marks did the student score in English ?



3. A loan has to be paid back in two equal annual instalments. If the rate of interest is 15 % per annum, compounded annually and each instalment is Rs 2645, find the loan and total interest charged. 3

4. Find the HCF and LCM of the polynomials $6(x^2 + 6x + 8)$ and $2x^2 - 32$. 3

5. (a) Using ruler and compass only, construct a $\triangle$ABC in which base BC = 6 cm. $\angle A = 60^\circ$ and median through A is 4.5 cm long. (Steps of construction not required) 3

OR

(b) Using ruler and compass only, construct a $\triangle$ABC in which AB = 4 cm, BC = 5cm and AC = 6 cm. Now, construct a triangle similar to $\triangle$ABC such that each of its sides is $\frac{3}{4}$ the corresponding side of $\triangle$ABC. (Steps of construction not required) 3

6. (a) Find the ratio in which the y-axis divides the line segment joining the points A (5, – 6) and B (–1, – 4). Also find the points of intersection. 3

OR

(b) Prove that the points A (0, 0), B (0,5), C (6, 5) and D (6, 0) are vertices of a rectangle. 3

7. Prove that : $\frac{\cos^2\theta}{(1-\tan\theta)}+\frac{\sin^3\theta}{(\sin\theta-\cos\theta)}=1+\sin\theta\cos\theta$ 3

8. The angles of depression of two ships from the top of a lighthouse are 45° and 30° towards east. If the ships are 200 m apart, find the height of the lighthouse. Take ($\sqrt{3}=1.73$). 3

9. (a) A container, opened from the top and made up of a metal sheet, is in the form of a frustrum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of the milk which can completely fill the container at the rate of Rs. 20 per litre. (Take $\pi = 3.14$) 3

OR

(b) A solid is in the shape of a cone surmounted on a hemisphere, the radius of each of them is 3.5 cm and the total height of solid is 9.5 cm. Find the volume of the solid. 3

10. Solve the following system of linear equations graphically : 5

$2x + 3y = 8$ and $x - 2y + 3 = 0$

Also find the points where the lines meet x-axis.

11. (a) A line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points. Prove that the other two sides are divided in the same ratio. Also, in triangle ABC, D and E are points on the sides AB and AC respectively such that DE || BC. If AD = 2 cm, DB = 3 cm and AE = 4 cm, find EC. 3+2=5

OR

(b) Prove that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle. Also in the given figure, A, B and C are three points on the circle with centre O such that $\angle AOB = 90°$ and $\angle AOC = 110°$. Find $\angle BAC$. 5

12. Find the median from the following data : 3×2=5

Class Interval	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Frequency	10	12	15	7	6

Also, find the mean using empirical formula if mode is 22.4.