2019
MATHEMATICS

Total marks : 80   Time : 3 hours

General Instructions:
i)   Approximately 15 minutes is allotted to read the question paper and revise the answers.
ii)   The question paper consists of 22 questions.
iii)   All questions are compulsory.
iv)   Internal choice has been provided in some questions.
v)   Marks allocated to every question are indicated against it.
N.B: Check that all pages of the question paper is complete as indicated on the top left side.

Section - A

1. **Choose the correct answer from the given alternatives.**

    (a) A quadratic polynomial whose zeros are −5 and 1 is   **1**
    (i) $x^2 + 4x - 5$ (ii) $x^2 + 4x + 5$ (iii) $x^2 - 4x + 5$ (iv) $x^2 - 4x - 5$

    (b) If a pair of linear equations is inconsistent, then the lines representing them will be   **1**
    (i) parallel   (ii) coincident
    (iii) intersecting at a point   (iv) perpendicular

    (c) The nature of the roots of the equation $4x^2 + 4\sqrt{3}\,x + 3 = 0$ are   **1**
    (i) real and distinct.   (ii) real and equal.
    (iii) not real.   (iv) unreal and distinct.

    (d) If the sum of first five terms of an A.P. is 30 and the sum of first four terms is 20 then the 5th term of the A.P. is   **1**
    (i) 6   (ii) 10   (iii) 12   (iv) 30

    (e) $\cot 10° \cot 15° \cot 60° \cot 75° \cot 80° =$ **1**

    (i) −1   (ii) $\dfrac{-1}{\sqrt{3}}$ (iii) $\dfrac{1}{\sqrt{3}}$ (iv) 1

    (f) The midpoint of the join of the points (−3, −5) and (5, 9) is   **1**
    (i) (1, −2)   (ii) (−1, −2)   (iii) (−1, 2)   (iv) (1, 2)

(g) Two concentric circles have radii 10 cm and 6 cm. The length of the chord of the larger circle which is a tangent to the smaller circle is   **1**
(i) 8 cm   (ii) 12 cm   (iii) 16 cm   (iv) 20 cm

(h) The perimeter of a semicircle of diameter 14 cm is   **1**
(i) 14 cm   (ii) 22 cm   (iii) 29 cm   (iv) 36 cm

(i) A circus tent is cylindrical in shape with hemispherical top. If the height of the cylindrical part is half of its radius (r) then the surface area of the tent will be   **1**
(i) $4\pi r^2$ (ii) $3\pi\, r^2$ (iii) $2\pi\, r^2$ (iv) $\pi\, r^2$

(j) A dice is thrown once. The probability of getting a number less than 3 is   **1**
(i) $\dfrac{1}{3}$ (ii) $\dfrac{1}{4}$ (iii) $\dfrac{1}{6}$ (iv) 1
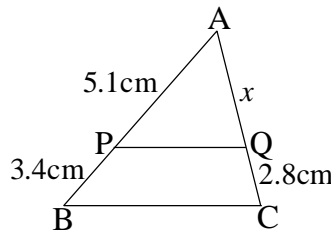
Section - B

2. Given that HCF of two numbers is 97 and their LCM is 13871. If one of the numbers is 1067, find the other number.   **2**

3. Find the roots of the quadratic equation $2x^2 - x + \dfrac{1}{8} = 0$ by factorization.   **2**

4. Find the point on the x-axis which is equidistant from (0, −2) and (4, 2).   **2**

5. In the adjoining figure, if PQ ∥ BC, find x.   **2**

6. The length of a minute hand of a clock is 14 cm. Find the area swept by the minute hand in 10 minutes [Use $\pi = \dfrac{22}{7}$].   **2**

Section - C

7. If the zeros of the polynomial $ax^2 + bx + c$ are in the ratio 3 : 4, then prove that $12b^2 = 49ac$.   **3**

8. **a.** Solve the system of equations $2x + 3y = 17$ and $3x - 2y = 6$ using the cross-multiplication method.

<div align="center">**Or**</div> **3**

    b. Solve the quadratic equation $2x^2 + x - 4 = 0$ by the method of completing the square.

9. **a.** The first and the last terms of an A.P. are 8 and 350 respectively. If its common difference is 9, how many terms are there and what is their sum?

<div align="center">**Or**</div> **3**

    b. If the 8th term of an A.P. is zero, prove that its 38th term is triple of its 18th term.

10. If $\cot\theta = \sqrt{3}$, prove that : $7\sin^2\theta + 3\cos^2\theta = 4$ [Proof is to be explained using a right angled triangle]. **3**

11. **a.** Prove that: $\left(1 + \dfrac{1}{\tan^2\theta}\right) + \left(1 + \dfrac{1}{\cot^2\theta}\right) = \dfrac{1}{\sin^2\theta - \sin^4\theta}$

<div align="center">**Or**</div> **3**

    b. Evaluate : $\dfrac{\cos\,67°}{\sin\,23°} + \dfrac{\cos 55°\, \mathrm{cosec}\, 35°}{\tan 5°\tan 25°\tan 45°\tan 65°\tan 85°}$

12. A(1,1), B(6,1) and C(10,4) are three vertices of a rhombus ABCD. Find the area of the rhombus. Also, if DM $\perp$ AB, find the height of the rhombus. **3**

13. **a.** Draw an isosceles triangle with base 5 cm and altitude 6 cm. Construct a ΔPQR similar to the given isosceles triangle with scale factor 5 : 3. (Traces of construction only is required.)

<div align="center">**Or**</div> **3**

    b. Draw a line segment AB of length 9cm. Draw two circles of radii 2.5cm and 3.5cm with A and B as centre respectively. Construct a pair of tangents to each circle from the centre of the other circle. (Traces of construction only is required.)

14. **a.** A wire bent in the form of a square encloses an area of 1936 sq. cm. If the same wire is bent into a circle, find the area enclosed by it.

<div align="center">**Or**</div> **3**

    b. A sector of arc length 22 cm and central angle 60° is cut off from a circular metal disc. Find the perimeter of the remaining portion of the disc.

15. Calculate the median marks obtained by the students of Class X in a class test: **3**

Marks	40-49	50-59	60-69	70-79	80-89	90-99
No. of students	5	10	20	30	20	15

16. A bag contains 6 red balls, 8 white balls, 5 green balls and 3 black balls. One ball is drawn at random from the bag. Find the probability that the balls drawn is: (i) white, (ii) red or black, (iii) neither white nor black.   **3**

Section - D

17. **a.** Solve the following pair of equations by reducing them to a pair of linear equations:

$$\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4},$$

$$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$$

Or   **5**

b. A train covered a certain distance at a uniform speed. If the train would have been 10 km/hr faster, it would have taken 2 hours less than the scheduled time. And if the train were slower by 10 km/hr, it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

18. **a.** An aeroplane flying horizontally at a height of 2500 m above the ground is observed at an elevation of 60°. After 15 seconds, the elevation is observed to be 30°. Find the speed of the aeroplane in km per hour. [Use $\sqrt{3} = 1.73$]
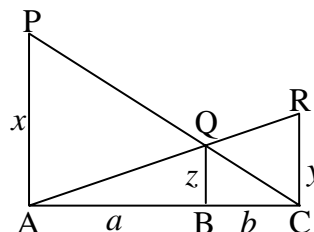
Or   **5**

b. A ladder leans against a wall making an angle of 45° with the ground. It reaches $\left(\sqrt{3} - \sqrt{2}\right)$m higher when the angle with the ground is made 60°. How long is the ladder?

19. **a.** In the adjoining figure, PA, QB and RC are perpendicular to AC such that PA = x, RC = y, QB = z, AB = a and BC = b.

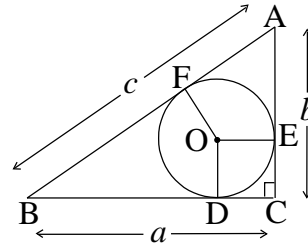Prove that: $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{z}$

Or   **5**

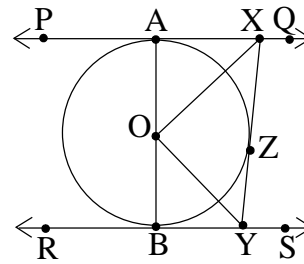**b.** State and prove Pythagoras theorem.

20. **a.** In the figure, a, b, c are the sides of a right triangle where c is the hypotenuse. Prove that the radius r of the circle which touches the sides of the triangle is given by, $r = \dfrac{a+b-c}{2}$.

Or   **5**

b. In the adjoining figure, PQ, RS and XY are tangent at A, B and Z respectively. AOB is the diameter of the circle. If $\angle AXO = 25°$, find $\angle BOX$.

21. **a.** A bucket is in the form of a frustrum of a cone with capacity of 12308.8 cm^3 of water. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of the metal sheet used in making it. [Use $\pi = 3.14$]

Or   **5**

b. A conical cavity of height 10 cm and diameter 14 cm is carved out of a solid cylinder of same height and diameter. Find the volume and total surface area of the remaining solid. [Use $\sqrt{149} = 12.21$]

22. **a.** Calculate the mean of the following data using Step-deviation method: [Take assumed mean, $a = 35$]

Marks	Number of persons
More than 0	100
More than 10	90
More than 20	75
More than 30	50
More than 40	25
More than 50	15
More than 60	5

Or   **5**

b. The following frequency distribution table shows the heights of 150 students of a school:

Height (in cm)	130-135	135-140	140-145	145-150	150-155	155-160
No. of students	22	18	15	32	54	9

Draw a 'more than' type cumulative frequency curve to represent the above data.

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