MODEL TEST PAPER 2024-25

TIME ALLOWED: 3 hours **MATHEMATICS(10+2)** **MAX. MARKS:80**

Instructions:

1. All the questions are compulsory.
2. The question paper consists of 19 questions divided into 4 sections A,B,C and D.
3. Section A comprises of 2 questions:
   (i) Q.No.1 consists of 15 Multiple Choice Questions carrying 1 mark each.
   (ii) Q.No.2 consists of 5 Fill in the Blank type questions carrying 1 mark each.
4. Section B comprises of 7 questions of 2 marks each.
5. Section C comprises of 7 questions of 4 marks each.
6. Section D comprises of 3 questions of 6 marks each.
7. Internal choice has been provided in three questions of 2 marks, three questions of 4 marks and three questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
8. Use of calculator is not permitted.

SECTION – A

Q1 Choose the correct options in the following questions:

(i) Function $f: N \to N,\ f(x) = x^2 + 1$ is : 1
(A)one-one only (B)onto only (C)one-one and onto (D)neither one-one nor onto

(ii) Relation given by $R = \{(b,b),(g,g),(g,s),(s,g)\}$ in the set $A = \{b,g,s\}$ is : 1
(A)reflexive only (B)symmetric only (C)transitive only (D) equivalence relation

(iii) If $\begin{vmatrix} 1 & -x \\ 4 & -4 \end{vmatrix} = \begin{vmatrix} x & 8 \\ 4 & -3 \end{vmatrix}$, then value of x is: 1
(A)8 (B)-4 (C)3 (D)-8

(iv) If order of matrix A' is 2×3 and order of matrix B is 3×5 then order of matrix $B'A$ is: 1
(A)5×2 (B) 2×5 (C) 5×3 (D) 3×2

(v) If $y = \log x$, then y'' is equal to: 1
(A)1 (B) $\frac{1}{x}$ (C) $\frac{1}{x^2}$ (D) $-\frac{1}{x^2}$

(vi) Critical point of the function $f(x) = x^2 - 10x + 2$ is: 1
(A)$x = 4$ (B)$x = 6$ (C)$x = 5$ (D)$x = 2$

(vii) $\int e^{2\log x} dx$ is equal to: 1
(A)$x + c$ (B)$\frac{x^2}{2} + c$ (C)$\frac{x^3}{3} + c$ (D)$\frac{x^4}{4} + c$

(viii) $\int_{-1}^{1} x^{2024} dx$ is equal to: 1
(A)0 (B) $\frac{1}{2024}$ (C) $\frac{2}{2025}$ (D) $\frac{1}{2025}$

(ix) Degree of differential equation $\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right)^3 + 3\sin y = 0$ is: 1
(A)3 (B) 2 (C)1 (D) not defined

(x) $y = e^{2x}$ is solution of differential equation given by: 1
(A)$y'' - y' = 0$ (B) $y'' - 4y' = 0$ (C)$y'' + y' = 0$ (D) $y'' - 2y' = 0$

(xi) If $\vec{a}.\vec{b} = -\frac{1}{\sqrt{3}}|\vec{a} \times \vec{b}|$ then angle between vector $\vec{a}$ and vector $\vec{b}$ is : 1
(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{2\pi}{3}$

(xii) If $\vec{a}.\vec{b} = 0$ then angle between vectors $\vec{a}$ and $\vec{b}$ is : 1
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$

(xiii) Direction ratios of line given by $\frac{x-1}{3} = \frac{2y+6}{12} = \frac{1-z}{-7}$ are : 1
(A) $< 3,12,-7 >$ (B) $< 3,-6,7 >$ (C) $< 3,6,7 >$ (D)$< 3,6,-7 >$

(xiv) Common area for each constraint is called: 1
(A)infeasible region (B)feasible region (C)useless region (D)main region

(xv) If $P(A) = \frac{1}{2}$, $P(B) = \frac{3}{8}$ and $P(A \cap B) = \frac{1}{5}$ then $P(A/B)$ is equal to: 1
(A)$\frac{2}{5}$ (B) $\frac{8}{15}$ (C) $\frac{2}{3}$ (D) $\frac{5}{8}$

Q2 **Fill in the blanks:**

(i) $\cos^{-1}\left(\cos\frac{5\pi}{3}\right) =$ ________ 1

(ii) If $A = [a_{ij}]_{2\times3}$ such that $a_{ij} = -|i-j|$ then $a_{12} =$ ________ 1

(iii) $\int_{\pi/4}^{\pi/2} cotx\, dx =$ ________ 1

(iv) If a line makes angles $135^\circ, 90^\circ, 45^\circ$ with x,y,z axes respectively, then its direction cosines are ________ 1

(v) If A and B are independent events such that $P(B) = 0.3, P(A\cap B) = 0.12$, then $P(A) =$ ______ 1

SECTION – B

Q3 If the area of triangle is 3 square units with vertices(2,0), (0,0) and (1,k), then find k. 2

Q4 If $y = sin^{-1}\left(cos\,\frac{x}{2}\right)$, then find $\frac{dy}{dx}$ 2

Q5 Find the interval in which function $f(x) = x^2 + 4x + 7$ is decreasing. 2

OR

An edge of a variable cube is increasing at the rate of 4 cm/s. How fast is the volume of the cube increasing when the edge is 20cm long? 2

Q6 Evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$. 2

OR

Evaluate $\int \frac{1}{\sqrt{16-9x^2}} dx$ 2

Q7 Find the general solution of the differential equation $\frac{dy}{dx} = (4 + y^2)(1 + 3x^2)$ 2

Q8 Using integration find the area bounded by the parabola $y^2 = 8x$ straight lines $x = 2, x = 5$ in the first quadrant. 2

Q9 Find the value of λ if the vectors $\vec{a} = 2\hat{\imath} - \hat{\jmath} - \lambda\hat{k}$ and $\vec{b} = 5\hat{\imath} - \hat{\jmath} + 2\hat{k}$ are perpendicular to each other. 2

OR

Find the angle between the lines : $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-7}{-5}$ and $\frac{x+5}{3} = \frac{y-2}{2} = \frac{z-6}{4}$ 2

SECTION – C

Q10 Prove that $cos^{-1}\frac{12}{13} + sin^{-1}\frac{3}{5} = sin^{-1}\frac{56}{65}$ 4

Q11 If $2X + 3Y = \begin{bmatrix}5 & -6\\0 & 4\end{bmatrix}$ and $3X + 2Y = \begin{bmatrix}3 & -3\\7 & 1\end{bmatrix}$, then find X and Y 4

Q12 Differentiate $x^{\log x} + (log\, x)^x$ w.r.t. x 4

OR

If $x = 3\left(cos\,\theta + log\,tan\,\frac{\theta}{2}\right)$, $y = 5\,sin\,\theta$, then find $\frac{dy}{dx}$ 4

Q13 Evaluate $\int_0^4 (|x-2| + |x|)\, dx$ 4

OR

Evaluate $\int [\log(\log x) + \frac{1}{(\log x)^2}]\ dx$ 4

Q14 Find the particular solution of the differential equation $\frac{dy}{dx} + 2y\,tan\,x = sin\,x$; $y = 0$ when $x = \frac{\pi}{3}$ 4

Q15 Solve the following linear programming problem graphically:
Maximize and minimize $Z = 4x + 3y$ subject to the constraints 4

$x + y \le 10, \quad 5x + 2y \ge 10, \quad 3y \ge x, \quad x \ge 0, \quad y \ge 0$

Q16 Probability of solving a specific problem independently by A and B are 1/3 and 1/2 respectively. If both try to solve the problem independently, find the probability that: 4
(i)the problem is solved (ii)exactly one of them solves the problem

OR

An insurance company insures 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of accidents is 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a truck driver ? 4

SECTION – D

Q17(a) Express the matrix $A = \begin{bmatrix} 2 & -3 & 5 \\ 8 & -7 & 9 \\ -6 & 5 & -4 \end{bmatrix}$ as a sum of a symmetric matrix and a skew-symmetric matrix. 4

(b) If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ then find k so that $A^2 + 2I = kA$ 2

OR

Solve the following system of linear equations by matrix method: 6

$3x - 7y - 2z = 29$, $2x + 5y - 3z = -39$, $4x + 2z - 2y = 30$

Q18 Find the height of the cone of greatest volume that can be inscribed in a sphere of radius 30 cm. 6

OR

Solve $\int \frac{x^2}{x^4+1} dx$ 6

Q19(a) Find the projection of the vector $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ on the vector $\vec{b} = 6\hat{\imath} + \hat{\jmath} - 2\hat{k}$ 2

(b) Find any diagonal of the parallelogram whose adjacent sides are given by the vectors $\vec{a} = 5\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + 9\hat{\jmath} + 2\hat{k}$. Also find the area of the parallelogram. 4

OR

Find the shortest distance between the lines 6

$\frac{x-1}{-1} = y + 2 = \frac{3-z}{2}$ and $x - 1 = \frac{y+1}{2} = \frac{z+1}{-2}$