AMU

Engineering Entrance Exam

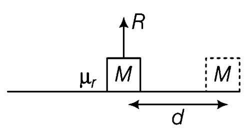
Solved Paper 2017

Physics

1. A particle has initial velocity $(2\hat{i} + 3\hat{j})$ and acceleration $(0.3\hat{i} + 0.2\hat{j})$. The magnitude of velocity after 10 seconds will be

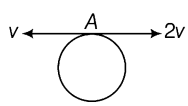
(a) $5\sqrt{2}$ units (b) $7\sqrt{2}$ units
(c) $9\sqrt{2}$ units (d) 9 units

2. If reaction is R and coefficient of friction μ_r, what is the work done against friction in moving a body by distance d?



(a) $\frac{\mu_r Rd}{4}$ (b) $2\mu_r, Rd$ (c) $\mu_r Rd$ (d) $\frac{\mu_r Rd}{2}$

3. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and $2v$ respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A?



(a) 4 (b) 3 (c) 2 (d) 1

4. A balloon has a mass of 10 gram in air. The air escapes from the balloon at a uniform rate with a velocity of 5 cm/s and the balloon shrinks completely in 2.5 s. The average force acting on the balloon will be

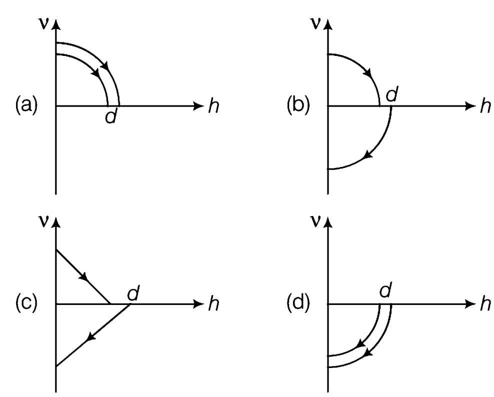
(a) 200 dyne (b) 20 dyne
(c) 20 newton (d) 2000 dyne

5. If a particle's position is given by $x = 4 - 12t + 3t^2$ where t is in the seconds and x in meters. What is its velocity at $t = 1\,s$? Whether the particle is moving in positive x direction or negative x direction?

(a) – 6 m/s, + x direction (b) – 6 m/s, – x direction
(c) 6 m/s, + x direction (d) 4 m/s, – x direction

6. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $\frac{d}{2}$.

Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as



7. If a particle of mass m is moving in a horizontal circle of radius r with a centripetal force $\left(-\frac{K}{r^2}\right)$, total energy is

(a) $-\frac{K}{2r}$ (b) $-\frac{K}{r}$ (c) $-\frac{2K}{r}$ (d) $-\frac{4K}{r}$

8. A thin circular ring of mass M and radius r is rotating about its axis at angular velocity ω. Two particles, each of mass m are attached gently to the ring at points which are at opposite ends of diameter of the ring. New angular velocity of the ring is

(a) $\frac{M\omega}{M+2m}$ (b) $\frac{\omega(M+2m)}{M}$

(c) ω (d) $\frac{\omega m}{(M+2m)}$

9. A body of mass m accelerates uniformly from rest to v_1 in the time t_1. The instantaneous power delivered to the body as a function of time

(a) $\frac{mv_1^2 t}{t_1}$ (b) $\frac{mv_1 t}{t_1}$ (c) $\frac{mv_1 t^2}{t_1}$ (d) $\frac{mv_1^2 t}{t_1^2}$

10. A satellite of mass M is revolving in circular orbit of radius r around the earth. Time of revolution of the satellite is

(a) $T \propto r^{\frac{1}{2}}$ (b) $T \propto r^{\frac{3}{2}}$ (c) $T \propto r^{-\frac{1}{2}}$ (d) $T \propto r^{-\frac{3}{2}}$

11. Two bodies of masses m and $4m$ are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is

(a) $-\frac{4Gm}{r}$ (b) $-\frac{6GM}{r}$ (c) $-\frac{9Gm}{r}$ (d) zero

12. If S is the stress and Y is Young's modulus of material of a wire, the energy stored in the wire per unit volume is

(a) $\frac{S}{2Y}$ (b) $\frac{2Y}{S}$ (c) $\frac{S^2}{2Y}$ (d) $2S^2Y$

13. The freezer in a refrigerator is located at the top section so that

(a) the entire chamber of the refrigerator is cooled quickly due to convection
(b) the motor is not heated
(c) the heat gained from the environment is high
(d) the heat gained from the environment is low

14. A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = kt^2$, ($k = 1$ m/s^2) where y is the vertical displacement. The time period now becomes T_2.
The ratio of $\frac{T_1^2}{T_2^2}$ ($g = 10$ m/s^2) is

(a) $\frac{5}{6}$ (b) $\frac{6}{5}$ (c) 1 (d) $\frac{4}{5}$

15. Two non-reactive monatomic ideal gases have their atomic masses in the ratio 2:3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature is 4:3. The ratio of their densities is

(a) 1: 4 (b) 1:2 (c) 6: 9 (d) 8 : 9

16. Two long conductors, separated by a distance d carry currents I_1 and I_2 in the same direction. They exert a force F on each other. Now the current in one of them is increased to two times and its direction is reversed. The distance is also increased to $3d$. The new value of force between them is

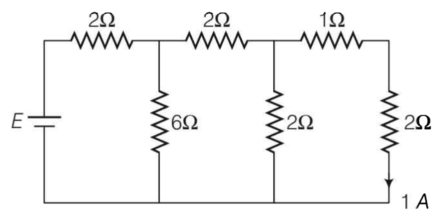
(a) $-2F$ (b) $\frac{F}{3}$

(c) $-\frac{2F}{3}$ (d) $-\frac{F}{3}$

17. Two identical cells are first connected in series and then in parallel. The ratio of power consumed by them is

(a) 1:1 (b) 1: 2 (c) 1: 3 (d) 1:4

18. A circuit is shown in the figure. The e.m.f. of the battery is

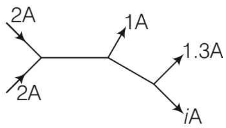


(a) 4 V (b) 6 V (c) 12 V (d) 8 V

19. At 600 Hz, an inductor and capacitor have equal reactances, the ratio of the capacitive reactance to the inductive reactance at 60 Hz will be

(a) 100: 1 (b) 200 :1
(c) 300: 1 (d) 400:1

20. The strength of current i in the given figure is



(a) 1.7 A (b) 1.3 A (c) 3.7 A (d) 1.0 A

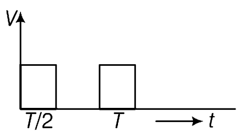
21. Substances, when placed in a magnetic field acquire feeble magnetisation in the direction opposite to that of the applied field are called

(a) diamagnetic substances
(b) ferromagnetic substances
(c) paramagnetic substances
(d) ferromagnetisation

22. 1 A current flows through an infinitely long straight wire. The magnetic field produced at a point 1 m away from it is

(a) 2×10^{-3} T (b) $\frac{2}{10}$ T
(c) 2×10^{-7} T (d) $2\pi\times10^{-6}$ T

23. The rms value of potential difference V_0 shown in figure is



(a) V_0 (b) $\frac{V_0}{\sqrt{2}}$ (c) $\frac{V_0}{2}$ (d) $\frac{V_0}{\sqrt{3}}$

24. On stretching a wire its length is increased by 0.2%, its resistance will

(a) increase by 0.1% (b) decrease by 0.1%
(c) increase by 0.2% (d) increase by 0.4%

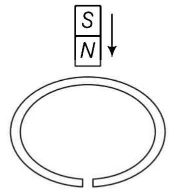
25. An electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then, the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A is the potential at $x = 2$ m, is given by

(a) $-80\,\hat{j}$ (b) $120\,\hat{j}$ (c) $-120\,\hat{j}$ (d) $80\,\hat{j}$

26. In a series LCR circuit with an alternating voltage source of frequency f, the current leads the voltage by 45°. The value of C is

(a) $\frac{1}{\pi f(2\pi fL - R)}$ (b) $\frac{1}{2\pi f(2\pi fL - R)}$
(c) $\frac{1}{\pi f(2\pi fL + R)}$ (d) $\frac{1}{2\pi f(2\pi fL + R)}$

27. A copper ring having a cut such that it does not form a complete loop is held horizontally and a bar magnet is dropped through the ring with its length along the axis of the ring. Then acceleration of the falling magnet is



(a) g (b) less than g
(c) more than g (d) zero

28. The electric field associated with an electromagnetic wave in vacuum is given by $\mathbf{E} = \hat{i}\,40\cos(kz - 6\times10^8 t)$, where E, z and t are Volt/m, metre and second respectively. The value of wave vector k is

(a) $2\ \text{m}^{-1}$ (b) $0.5\ \text{m}^{-1}$
(c) $6\ \text{m}^{-1}$ (d) $3\ \text{m}^{-1}$

29. The electromagnetic wave used in LASIK eye surgery is

(a) ultraviolet ray (b) X-ray
(c) microwave (d) radio wave

30. What is power dissipation in an a.c. circuit in which voltage and current are given by $V = 300\sin(\omega t + \pi/2)$, $I = 5\sin\omega t$?

(a) zero (b) 300 units
(c) 150 units (d) 75 units

31. The charging current for a capacitor at any instant is 0.78 A. The displacement current across the capacitor plates at that instant is

(a) $\frac{0.78}{\epsilon_0}$ A (b) $0.78\,\mu_0$ A
(c) $\frac{0.78}{2}$ A (d) 0.78 A

32. The bending of beam of light around corners of obstacles is called

(a) reflection (b) refraction
(c) diffraction (d) interference

33. Which of the following colour suffers maximum deviation in a prism?

(a) blue (b) yellow (c) green (d) orange

34. Light travels in a straight line because
(a) it is not absorbed by atmosphere
(b) its velocity is very high
(c) diffraction effect is negligible
(d) None of the above

35. Mirage is a phenomenon due to
(a) reflection of light
(b) refraction of light
(c) total internal reflection of light
(d) diffraction of light

36. The human eye has a lens which has a
(a) soft portion at its centre
(b) hard surface
(c) varying refractive index
(d) constant refractive index

37. There are four lenses L_1, L_2, L_3 and L_4 of focal lengths, 2, 4, 6 and 8 cm respectively. Two of these lenses form a telescope of length 10 cm and magnifying power 4. The objective and eye lenses are
(a) L_2, L_3 (b) L_1, L_4
(c) L_3, L_2 (d) L_4, L_1

38. The wavelength of 1 keV photon is 1.24×10^{-9} m. What is the frequency of 1 MeV photon?
(a) 1.24×10^{15} Hz (b) 2.4×10^{20} Hz
(c) 1.24×10^{18} Hz (d) 2.4×10^{23} Hz

39. The de-Broglie wavelength of a tennis ball of mass 60 g moving with a velocity of 10 m/s is Given: Planck's constant $= 6.63 \times 10^{-34}$ J s
(a) 10^{-16} m (b) 10^{-25} m
(c) 10^{-31} m (d) 10^{-33} m

40. What will be the ratio of de-Broglie wavelengths of proton and α-particle of the same energy?
(a) 2 : 1 (b) 1 : 2 (c) 4 :1 (d) 1 : 4

41. Density of nuclear matter is nearly
(a) 10^{26} kg/m^3 (b) 10^{24} kg/m^3
(c) 10^{17} kg/m^3 (d) 10^3 kg/m^3

42. Which of the series of hydrogen atom spectrum lies in the visible region of electromagnetic spectrum?
(a) Lyman series (b) Balmer series
(c) Pfund series (d) Brackett series

43. The mass-defect in a nuclear fusion reaction is 0.3 per cent. The amount of energy released in one kg of fusion reaction is
(a) 2.7×10^{23} J (b) 2.7×10^{19} J
(c) 2.7×10^{16} J (d) 2.7×10^{14} J

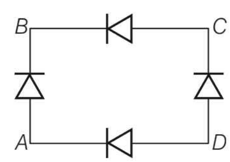
44. The relation between force acting on the electron and principle quantum number in hydrogen atom is
(a) $F \propto n^4$ (b) $F \propto n^2$ (c) $F \div \frac{1}{n^2}$ (d) $F \propto \frac{1}{n^4}$

45. A radioisotope has a half-life of 5 yr. The fraction of atoms of this material, that would decay in 15 yr would be
(a) 1 (b) $\frac{3}{4}$ (c) $\frac{7}{8}$ (d) $\frac{5}{8}$

46. The band gap in Ge and Si in eV respectively is
(a) 0.7, 1.1 (b) 1.1, 0.7 (c) 1.1, 0 (d) 0, 1.1

47. In a semiconductor crystal if the current flows due to breakage of crystal bonds, then the semiconductor is called
(a) donor
(b) acceptor
(c) intrinsic semiconductor
(d) extrinsic semiconductor

48. In adjoining figure, the input (A.C.) is across the terminals A and C and the output is across B and D. Then output is



(a) zero (b) the same as input
(c) half wave rectified (d) full wave rectified

49. A 100 m long antenna is mounted on a 500 m tall building. The complex can become a transmission tower for waves with λ
(a) $\approx$ 400 m (b) $\approx$ 25 m (c) $\approx$ 150 m (d) $\approx$2400 m

50. For an amplitude modulated wave, the maximum amplitude is found to be 12 V while the minimum amplitude is found to be 4 V. The modulation index μ will be
(a) 0.25 (b) 0.50 (c) 0.75 (d) 1.00

Chemistry

51. Aniline is insoluble in water and possesses a vapour pressure of 10.15 mm Hg at 373 K. It can be conveniently purified by

(a) sublimation (b) cystallisation
(c) steam distillation (d) simple distillation

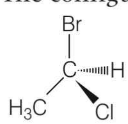
52. Which of the following is not a reaction intermediate?

(a) Carbenes (b) Nitrenes
(c) Electrophiles (d) Hydrophiles

53. IUPAC name of $(CH_3)_3C—CH=CH_2$ is

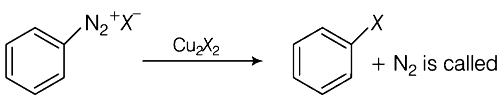
(a) 3, 3, 3-trimethyl-1-propene
(b) 1, 1, 1-trimethyl-2-propene
(c) 3, 3-dimethylbut-1-ene
(d) 2, 2-dimethyl-3-butene

54. The configuration of the compound



(a) *R* (b) *S* (c) *Z* (d) *E*

55. The reaction,



(a) Etard's reaction (b) Sandmeyer's reaction
(c) Wurtz-Fittig reaction (d) Perkin's reaction

56. What is the compound *'C'* in the following sequence of the reaction,

$$\text{Phenol} \xrightarrow{\text{NaOH}} A \xrightarrow[\text{4-7 atm}]{CO_2,\ 410\text{ K}} B \xrightarrow{\text{HCl}} C?$$

(a) Benzoic acid (b) Salicylic acid
(c) Benzaldehyde (d) Salicylaldehyde

57. The major product of nitration of benzoic acid is

(a) 3-nitrobenzoic acid (b) 4-nitrobenzoic acid
(c) 2-nitrobenzoic acid (d) 2, 4-dinitrobenzoic acid

58. Arrange the following carbanions in order of their decreasing stability

(i) $H_3C—C\equiv C^-$ (ii) $H—C\equiv C^-$
(iii) $H_3C—CH_2^-$

(a) i > ii > iii (b) ii > i > iii
(c) iii > ii > i (d) iii > i > ii

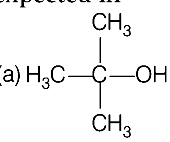
59. The following reaction,
$R—X + \text{NaI} \rightarrow R—\text{I} + \text{Na}X$ where, X = Cl, Br is known as

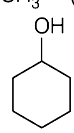
(a) Swarts reaction (b) Finkelstein reaction
(c) Sandmeyer's reaction (d) Wurtz-Fittig reaction

60. Which of the following will exhibit highest boiling point?

(a) $CH_3CH_2OCH_2CH_3$
(b) $CH_3CH_2CH_2CH_2CH_2OH$
(c) $CH_3CH_2CH_2CH(CH_3)OH$
(d) $CH_3CH_2C(CH_3)_2OH$

61. The fastest dehydration reaction could be expected in

(a) 

(b) $CH_3—CH_2—CH_2—OH$
(c) $CH_3—(CH_2)_4—OH$
(d) 

62. The correct arrangement of following in their decreasing order of basic strength is

(a) $NH_3 > C_6H_5NH_2 > (C_2H_5)_2NH > C_2H_5NH_2$
(b) $C_2H_5NH_2 > (C_2H_5)_2NH > C_6H_5NH_2 > NH_3$
(c) $(C_2H_5)_2NH > C_2H_5NH_2 > NH_3 > C_6H_5NH_2$
(d) $C_6H_5NH_2 > NH_3 > C_2H_5NH_2 > (C_2H_5)_2NH$

63. Which one of the following is known as animal starch?

(a) Amylose (b) Cellulose
(c) Glycogen (d) Amylopectin

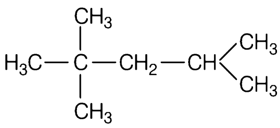
64. The polymer used as a substitute for wool in making commercial fibres is

(a) glyptal (b) novolac
(c) neoprene (d) polyacrylonitrile

65. The drug which was designed to prevent the interaction of histamine with the receptors present in the stomach wall

(a) cimetidine (b) nardil
(c) iproniazid (d) phenelzine

66. What is the IUPAC name for the following compound?



(a) 1, 3-pentamethylpropane
(b) 1, 1, 3, 3-tetramethylbutane
(c) 2, 4, 4-trimethylpentane
(d) 2, 2, 4-trimethylpentane

67. Which of the following will not give iodoform test?

(a) Isopropyl alcohol (b) Ethanol
(c) Ethanal (d) Benzyl alcohol

68. Which of the following has highest hydration energy?

(a) $MgCl_2$ (b) $CaCl_2$ (c) $BaCl_2$ (d) $SrCl_2$

69. The structure of BrF_3 is

(a) square planar (b) distorted tetrahedral
(c) angular (d) T-shaped

70. Sodium hydroxide is manufactured by

(a) Solvay process (b) Haber's process
(c) Castner-Kellner process (d) Evaporating process

71. Ziegler catalyst to manufacture polyethylene is made by the reaction

(a) $TiCl_4$ with $Al(CH_3)_3$ (b) $TiCl_4$ with CH_3Cl
(c) $TiCl_4$ with $(C_6H_5)_3CCl$ (d) $TiCl_4$ with $ZnCl_2$

72. Primary valency of Co in $[Co(en)_2Cl_2]$ Cl is

(a) + 1 (b) + 2 (c) + 3 (d) + 6

73. The oxidation number of Pt in $[Pt(C_2H_4)Cl_3]^-$ ion is

(a) + 1 (b) + 2 (c) + 3 (d) + 4

74. By the action of conc. H_2SO_4, phosphorus changes to

(a) phosphorus acid (b) orthophosphoric acid
(c) metaphosphoric acid (d) pyrophosphoric acid

75. Tear gas is

(a) CCl_3NO_2 (b) $COCl_2$
(c) $ClC_2H_4—S—C_2H_4Cl$ (d) $C_6H_5—S—Br$

76. The correct sequence of bond order is

(a) $O_2^+ > O_2^- > O_2$ (b) $O_2 > O_2^- > O_2^+$
(c) $O_2^+ > O_2 > O_2^-$ (d) $O_2^- > O_2^+ > O_2$

77. Inert pair effect is concerned with

(a) pair of *s*-electrons (b) pair of *p*-electrons
(c) pair of *d*-electrons (d) pair of *f*-electrons

78. Copper is extracted from sulphide ore using the method

(a) auto-reduction (b) carbon reduction
(c) CO reduction (d) biochemical reduction

79. The photochemical smog is

(a) formed due to oxidation of hydrocarbons trapped in flowing air in the presence of sunlight
(b) formed due to oxidation of smoke trapped in stagnant air mass in the presence of sunlight
(c) formed due to heterogeneous combination of smoke and fog in air in the presence of sunlight
(d) formed due to oxidation of hydrocarbons trapped in stagnant air mass in the presence of sunlight.

80. Correct formula for Wilkinson's catalyst is

(a) $[(Ph_3P)_3RhCl]$ (b) $[(Ph_3P)_2RhCl_2]$
(c) $[(Ph_3P)_3RuCl_3]$ (d) $[(Ph_3P)_2RuCl_2]$

81. Which of the following chelating ligands can be used to remove the excess of copper in the biosystems?

(a) D-pencillamine (b) Desferrioxamine B
(c) EDTA (d) Dimercaprol

82. Which of the following oxides of nitrogen is blue in nature?

(a) NO (b) N_2O_3 (c) N_2O_5 (d) NO_2

83. Arrange H_2O, H_2S and H_2Se in order of increasing acidity.

(a) $H_2O < H_2S < H_2Se$ (b) $H_2S < H_2O < H_2Se$
(c) $H_2Se < H_2S < H_2O$ (d) $H_2O < H_2Se < H_2S$

84. According to MOT, O_2 is paramagnetic in nature, what is the number of electrons in antibonding molecular orbitals?

(a) 8 (b) 7 (c) 9 (d) 6

85. What is the number of nitrogen-nitrogen bonds in N_2O_5?

(a) 1 (b) 0
(c) 1.5 (d) None of these

86. The half-life period of a radioactive element is 140 days. After 700 days, 1 g of the element will reduce to

(a) $\left(\frac{1}{2}\right)$g (b) $\left(\frac{1}{4}\right)$g (c) $\left(\frac{1}{8}\right)$g (d) $\left(\frac{1}{32}\right)$g

87. Which of the following gases has high Boyle's temperature?
(a) Ar (b) CO_2 (c) O_2 (d) He

88. Which of the following has the highest coagulating power for As_2S_3 colloid?
(a) PO_4^{3-} (b) SO_4^{2-} (c) Al^{3+} (d) Na^+

89. The rate of chemisorption of a gas
(a) decreases with increasing pressure
(b) increases with increasing pressure
(c) is independent of pressure
(d) is independent of temperature

90. For a buffer of a mixture of 0.12 mol L^{-1} CH_3COOH and 0.12 mol L^{-1} CH_3COONa, the buffer capacity is
(a) 1.38 (b) 0.130 (c) 0.06 (d) 0.60

91. Water + CH_3OH mixture shows positive deviation from ideal solution behaviour. 100 mL of water is mixed with 100 mL of CH_3OH. Then, total volume of the mixture will be
(a) 200 mL
(b) less than 200 mL because of additional H-bonding between H_2O and CH_3OH
(c) more than 200 mL because H-bonding within H_2O molecules vanishes
(d) more than 200 mL because H-bond between H_2O and CH_3OH is weaker than that between H_2O and H_2O.

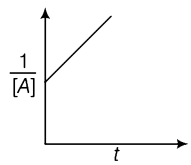
92. 0.02 molar solution of NaCl having degree of dissociation of 90% at 27° C has osmotic pressure equal to
(a) 0.94 bar (b) 9.4 bar
(c) 0.094 bar (d) 9.4×10^{-4} bar

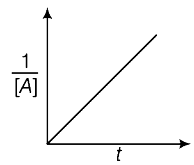
93. Benzene freezes at 5.6° C. Its value for K_f is 5.1. The value of ΔH_{fus} is
(a) 30.24 cal (b) 2358.72 cal
(c) 1179.36 cal (d) 15.12 cal

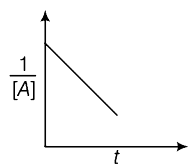
94. For the reaction, $S_{(rhombic)} + O_2 \rightarrow SO_2$, $\Delta H = -298$ kJ mol^{-1} at 25° C and 1 atm. Therefore, ΔE for the reaction should be
(a) -298 kJ mol^{-1}
(b) $-298 + 8.314 \times 298$ kJ mol^{-1}
(c) $-298 - 8.314 \times 298$ kJ mol^{-1}
(d) $-298 - 2 \times 8.314 \times 298$ kJ mol^{-1}

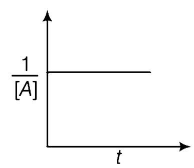
95. A better criterion for ideality of a gas than $\left(\frac{\partial U}{\partial V}\right)_T = 0$ is
(a) $\left(\frac{\partial H}{\partial p}\right)_T < 0$ (b) $\left(\frac{\partial H}{\partial p}\right)_T > 0$
(c) $\left(\frac{\partial H}{\partial p}\right)_T = 0$ (d) $\left(\frac{\partial H}{\partial p}\right)_T \neq 0$

96. For a second order reaction, ($2A \rightarrow$ Product), $\frac{1}{[A]}$ *vs t* is represented as

(a)


(b)


(c)


(d)


97. For $NH_4HS(s) \rightleftharpoons NH_3(g) + H_2S(g)$, the observed pressure for the reaction mixture in equilibrium is 1.12 atm at 106 °C. What is the value of K_p for the reaction?
(a) 0.56 atm^2 (b) 0.3136 atm^2
(c) 1.25 atm^2 (d) 1.12 atm^2

98. Calculation the maximum work done in expanding 16 g of oxygen at 300 K and occupying a volume of 5 dm^3 isothermally until volume becomes 25 dm^3.
(a) -2.01×10^3 J (b) $+2.01 \times 10^3$ J
(c) 2.01 J (d) 2.01×10^3 kJ

99. Number of electrons present in 3.6 mg of NH_4^+ are
(a) 1.20×10^{21} (b) 1.20×10^{20}
(c) 1.20×10^{22} (d) 2×10^{-3}

100. When a mixture of 10 moles of SO_2 and 16 moles of O_2 were passed over a catalyst, 8 moles of SO_3 were formed at equilibrium. The number of moles of SO_2 and O_2 remaining unreacted were
(a) 2, 12 (b) 12, 2 (c) 3, 10 (d) 10, 3

Mathematics

101. For any three sets A, B and C the set $(A\cup B\cup C)\cap(A\cap B'\cap C')'\cap C'$ is equal to

(a) $B\cap C'$ (b) $B'\cap C'$

(c) $B\cap C$ (d) $A\cap B\cap C$

102. Let $P=\{\theta;\ \sin\theta-\cos\theta=\sqrt{2}\cos\theta\}$ and $Q=\{\theta;\ \sin\theta+\cos\theta=\sqrt{2}\sin\theta\}$ be two sets. Then,

(a) $P\subset Q$ and $Q-P=\phi$ (b) $Q\not\subset P$

(c) $P\not\subset Q$ (d) $P=Q$

103. The function $f(x)=x-[x]$ where [ ] denotes the greatest integer function is

(a) continuous everywhere

(b) continuous at integer points only

(c) continuous at non-integer points only

(d) nowhere continuous

104. If $f(x)=xe^{x(1-x)}$, then $f(x)$ is

(a) increasing on $\left[-\frac{1}{2},1\right]$ (b) decreasing on R

(c) increasing on R (d) decreasing on $\left[-\frac{1}{2},1\right]$

105. The range of the function $f(x)=\dfrac{2+x}{2-x}$, $x\neq 2$ is

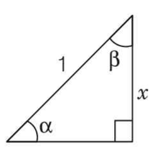
(a) R (b) $R-\{-1\}$ (c) $R-\{1\}$ (d) $R-\{2\}$

106. If $\sin(\alpha+\beta)=1$, $\sin(\alpha-\beta)=\frac{1}{2}$, $\alpha,\beta\in\left[0,\frac{\alpha}{2}\right]$ then the value of $\tan(\alpha+2\beta)\tan(2\alpha+\beta)$ is

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{1}{3}$ (d) 2

107. Using the information from the figure, $\cos^{-1}x$ is equal to



(a) $\frac{\pi}{2}+\cos x$ (b) $\frac{\pi}{2}+\sin x$

(c) $\frac{\pi}{2}-\sin^{-1}x$ (d) $\frac{\pi}{2}+\sin^{-1}x$

108. Let $f(x)$ be a function defined by

$$f(x)=\begin{cases}4x-5, & \text{if } x\le 2\\ x-\lambda, & \text{if } x>2\end{cases}\quad \text{if } \lim_{x\to 2} f(x) \text{ exists,}$$

then the value of λ is

(a) – 2 (b) – 1 (c) 0 (d) 1

109. The tangent of the angle between the lines whose intercepts on the axes are respectively $a,-b$ and $b,-a$ is

(a) $\pm\dfrac{2ab}{b^2-a^2}$ (b) $\pm\dfrac{ab}{2(b^2-a^2)}$

(c) $\pm\dfrac{b^2+a^2}{2ab}$ (d) $\pm\dfrac{b^2-a^2}{2ab}$

110. The equation of the circle whose radius is 5 and which touches the circle $x^2+y^2-2x-4y-20=0$ externally at the point $(5,5)$ is

(a) $(x-9)^2+(y-8)^2=5^2$

(b) $(x-5)^2+(y-5)^2=5^2$

(c) $(x-0)^2+(y-0)^2=5^2$

(d) None of the above

111. The variance of 20 observations in 5. If each observation is multiplied by 2, then the variance of the resulting observation is

(a) 10 (b) 20 (c) 30 (d) 40

112. If the sum of n terms of an AP is $nR+\frac{1}{2}n(n-1)T$, where R and T are constants, then the common difference is

(a) R (b) T (c) $R-T$ (d) $T-R$

113. A man repays a loan of ₹ 3250 by paying ₹ 20 in the first month and then increases the payment by ₹ 15 every month. How long will it take him to clear the loan?

(a) 20 months (b) 25 months

(c) 30 months (d) 35 months

114. If the two positive numbers whose difference is 12 and whose AM exceeds the GM by 2, then the numbers are

(a) 18, 6 (b) 16, 4

(c) 14, 2 (d) None of these

115. The coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio $1:7:42$, then the value of n is

(a) 55 (b) 54
(c) 56 (d) 66

116. The number of words with or without meaning which can be made using all the letters of the word AGAIN. If these words are written as in a dictionary, then the 50th word will be

(a) NAAGI (b) NAAIG (c) NAIAG (d) NAIGA

117. Coefficient of variation of two distributions are 60 and 70 and their standard deviations are 21 and 16 respectively. Then their AM's are

(a) 35 and 22.85 (b) 36 and 40
(c) 50 and 30 (d) 22 and 36

118. The equation of the normal to the curve $y=e^x$ at $(0,1)$ is

(a) $2x+y=1$ (b) $y-x=1$
(c) $x+y=1$ (d) None of these

119. The length 'x' of a rectangle is decreasing at the rate of 6 cm/min and the width y is increasing at the rate of 4 cm/min. When $x=8$ cm and $y=4$ cm, the rate of change of the area of the rectangle is

(a) 8 (b) 16
(c) 24 (d) 32

120. If A is an orthogonal matrix, then

(a) $|A|=0$ (b) $|A|=\pm 1$
(c) $|A|=\pm 2$ (d) $|A|=\pm\frac{1}{2}$

121. The area bounded by the curve $y=2x-x^2$ and then straight line $y=-x$ is given by

(a) $\frac{9}{2}$ (b) $\frac{43}{6}$
(c) $\frac{35}{6}$ (d) $\frac{23}{5}$

122. Let $g(x)$ be the inverse of an invertible function $f(x)$ which is differentiable at $x=c$, then $g'(f(c))$ equals

(a) $f'(c)$ (b) $\frac{1}{f'(c)}$
(c) $f(c)$ (d) $\frac{1}{f(c)}$

123. If A and B are two events associated to some experiment E such that $P(A)=0.5$, $P(B)=0.4$, $P(A\cap B)=0.3$, then $P\left(\frac{A^C}{B^C}\right)$ is equal to

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$
(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

124. If X follows a binomial distribution with parameters $n=100$, $p=\frac{1}{3}$, then $P(X=r)$ is maximum, when r is equal to

(a) 32 (b) 34 (c) 33 (d) 31

125. The most correct statement is

(a) Some optimal solution of a linear programming problem (LPP) is also a feasible solution of LPP
(b) Some optimal solution of a LPP is also a basic feasible solution of LPP
(c) No optimal solution of a LPP is a basic feasible solution of LPP
(d) No basic feasible solution is an optimal solution of LPP

126. The shortest distance between the lines $\vec{r}=(4\hat{i}-\hat{j})+\lambda(\hat{i}+2\hat{j}-3\hat{k})$ and $\vec{r}=(\hat{i}-\hat{j}+2\hat{k})+\mu(2\hat{i}+4\hat{j}-5\hat{k})$ is

(a) $\frac{2}{\sqrt{5}}$ (b) $\frac{2}{5}$
(c) $\frac{6}{\sqrt{5}}$ (d) $\frac{1}{\sqrt{5}}$

127. If the sum of two unit vectors is again a unit vector, then magnitude of their difference is

(a) 0 (b) 1
(c) $\sqrt{3}$ (d) 2

128. Let A and B be two points with position vectors $\vec{a}$ and $\vec{b}$ respectively and let C be a point dividing AB internally and the position vector of C on AB is $\vec{c}=\lambda\vec{a}+\mu\vec{b}$, then

(a) $\lambda+\mu=0$ (b) $\lambda+\mu=1$ (c) $\lambda+\mu<1$ (d) $\lambda+\mu>1$

129. The integrating factor of the D.E. $(x\log x)\frac{dy}{dx}+y=2\log x$ is

(a) $\log(\log x)$ (b) e^x
(c) $\log x$ (d) x

130. The function $y = c_1 \cos x + c_2 \sin x$ is a solution of the DE, where c_1 and c_2 are real numbers

(a) $\frac{d^2y}{dx^2} = y$ (b) $\frac{d^2y}{dx^2} + y = 0$

(c) $\frac{d^2y}{dx^2} + xy = 0$ (d) $\frac{d^2y}{dx^2} - xy = 0$

131. The general solution of differential equation $\frac{dy}{dx} + 2xy = 2e^{-x^2}$ is

(a) $y = 2xe^{-x}$ (b) $y = (2x + C)e^{-x^2}$

(c) $y = 2xe^{x}$ (d) $y = (2x + C)e^{x^2}$

132. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ equals

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

133. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$ is equal to

(a) $\cot x - \tan x + C$ (b) $\tan x + \cot x + C$

(c) $\tan x + \operatorname{cosec} x + C$ (d) $\tan x + \sec x + C$

134. The integral $\int e^x (1 + \tan x) \sec x \, dx$ equals

(a) $e^x \cot x + C$ (b) $e^x \tan x + C$

(c) $e^x \sec x + C$ (d) $e^x \cos x + C$

135. The area of the region bounded by the two parabolas $y = x^2$ and $y = x$ is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{1}{6}$

136. Differentiate $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0 < x < 1$ with respect to x.

(a) $\frac{-2}{1+x^2}$ (b) $\frac{2}{1-x^2}$ (c) $\frac{1}{1+x^2}$ (d) $\frac{1}{1-x^2}$

137. The total revenue in rupees received from the sale of 'x' units of a product is given by $R(x) = 5x^2 + 20x + 7$. The marginal revenue, when $x = 8$ is

(a) 60 (b) 100 (c) 360 (d) 487

138. If $y = e^{x + e^{x + e^{x + \dots \infty}}}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{y}{1-y}$ (b) $\frac{x}{1-y}$ (c) $\frac{x}{1-x}$ (d) $\frac{y}{1-x}$

139. The function $f(x) = \sqrt{|x| - x}$ is continuous for

(a) real numbers

(b) natural numbers

(c) rational numbers

(d) $[0, \infty)$

140. If $y = \sqrt{\frac{1-x}{1+x}}$, then $(1 - x^2)\frac{dy}{dx} + y$ is

(a) 1 (b) 0 (c) – 1 (d) 2

141. If the system of equations

$2x + ay + 6z = 8$

$x + 2y + bz = 5$

$x + y + 3z = 4$

has a unique solution then

(a) $a = 2$ or $b = 3$ (b) $a \neq 2$ or $b \neq 3$

(c) $a = 1, b = 5$ (d) $a = 0, b = 5$

142. If a, b, c are roots of the equation $x^3 + px + q = 0$, then the value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

(a) 1 (b) 2 (c) 0 (d) 3

143. If $y = \sin mx$, then the value of the determinant $\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$ where $y_n = \frac{d^n y}{dx^n}$ is

(a) m^9 (b) m^2

(c) m^3 (d) None of these

144. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = \lambda A$. Then, the value of λ is

(a) $\frac{1}{17}$ (b) $\frac{1}{18}$ (c) $\frac{1}{19}$ (d) $\frac{1}{21}$

145. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ and $A^2 = B$, then the value of α is

(a) $\alpha = \pm 1$ (b) $\alpha = 4$

(c) not possible (d) Both (a) and (b)

146. $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5}$ is equal to

(a) $\sin^{-1}\left(\frac{77}{85}\right)$ (b) $\tan^{-1}\left(\frac{77}{36}\right)$

(c) $\cos^{-1}\left(\frac{1}{36}\right)$ (d) Both (a) and (b)

147. If $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$, then the value of x is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) $-\frac{\sqrt{3}}{2}$

148. If $\cos^{-1}\sqrt{p} + \cos^{-1}\sqrt{1-p} + \cos^{-1}\sqrt{1-q} = \frac{3\pi}{4}$, then the value of q is

(a) $\frac{1}{\sqrt{2}}$ (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

149. The function $f(x) = \cos x$ is strictly decreasing on

(a) $[0, \pi]$ (b) $[0, \pi)$
(c) $(0, \pi]$ (d) $(0, \pi)$

150. If ${}^nP_4 = 20 \times {}^nP_2$. Then, the value of n is

(a) 18 (b) 13
(c) 7 (d) 4

Answers

Physics

1.	(a)	2.	(c)	3.	(c)	4.	(b)	5.	(b)	6.	(b)	7.	(a)	8.	(a)	9.	(d)	10.	(b)
11.	(c)	12.	(c)	13.	(a)	14.	(b)	15.	(d)	16.	(c)	17.	(a)	18.	(c)	19.	(a)	20.	(a)
21.	(a)	22.	(c)	23.	(b)	24.	(d)	25.	(*)	26.	(d)	27.	(a)	28.	(a)	29.	(a)	30.	(a)
31.	(d)	32.	(c)	33.	(a)	34.	(c)	35.	(c)	36.	(c)	37.	(d)	38.	(b)	39.	(d)	40.	(a)
41.	(c)	42.	(b)	43.	(d)	44.	(d)	45.	(c)	46.	(a)	47.	(c)	48.	(d)	49.	(a)	50.	(b)

Chemistry

51.	(c)	52.	(d)	53.	(c)	54.	(a)	55.	(b)	56.	(b)	57.	(a)	58.	(b)	59.	(b)	60.	(b)
61.	(a)	62.	(c)	63.	(c)	64.	(d)	65.	(a)	66.	(d)	67.	(d)	68.	(a)	69.	(d)	70.	(c)
71.	(a)	72.	(c)	73.	(b)	74.	(b)	75.	(a)	76.	(c)	77.	(a)	78.	(a)	79.	(d)	80.	(a)
81.	(a)	82.	(b)	83.	(a)	84.	(d)	85.	(b)	86.	(d)	87.	(b)	88.	(c)	89.	(b)	90.	(b)
91.	(d)	92.	(a)	93.	(b)	94.	(a)	95.	(c)	96.	(a)	97.	(b)	98.	(a)	99.	(a)	100.	(a)

Mathematics

101.	(a)	102.	(d)	103.	(c)	104.	(a)	105.	(b)	106.	(b)	107.	(c)	108.	(b)	109.	(d)	110.	(a)
111.	(b)	112.	(b)	113.	(a)	114.	(b)	115.	(a)	116.	(b)	117.	(a)	118.	(c)	119.	(a)	120.	(b)
121.	(a)	122.	(b)	123.	(c)	124.	(c)	125.	(b)	126.	(c)	127.	(c)	128.	(b)	129.	(c)	130.	(b)
131.	(b)	132.	(d)	133.	(b)	134.	(c)	135.	(d)	136.	(a)	137.	(b)	138.	(a)	139.	(a)	140.	(b)
141.	(b)	142.	(c)	143.	(d)	144.	(c)	145.	(c)	146.	(d)	147.	(a)	148.	(c)	149.	(d)	150.	(c)

Note : (*) None option is correct

Answer *with* Solutions

Physics

1. ***(a)*** Given, $\mathbf{u} = 2\hat{i} + 3\hat{j}, \mathbf{a} = 0.3\hat{i} + 0.2\hat{j}$

$$t = 10 \text{ s}$$

As $\mathbf{v} = \mathbf{u} + \mathbf{a}t = (2\hat{i} + 3\hat{j}) + (0.3\hat{i} + 0.2\hat{j}) \times 10$

$$= (2\hat{i} + 3\hat{j}) + (3\hat{i} + 2\hat{j}) = 5\hat{i} + 5\hat{j}$$

$\therefore$ Magnitude of velocity at $t = 10$ s is

$$v = |\mathbf{v}| = \sqrt{5^2 + 5^2} = 5\sqrt{2} \text{ units}$$

2. ***(c)*** Magnitude of frictional force acting on the block is given as

$f = \mu_r \times$ Reaction force $= \mu_r R$

Work done against friction force, $= f \cdot s = f\, s \cos\theta$

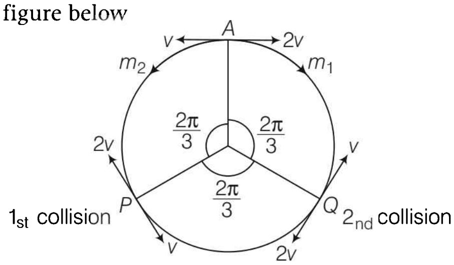
where, s is the displacement and θ is the angle between the frictional force and displacement.

Here, $\theta = 180°$, $s = d$

$\Rightarrow \quad W = \mu_R\, R\, d \cos 180° = -\mu_R R\, d$ [$\because \cos 180° = -1$]

where, negative sign implies that the displacement is in the opposite direction to that of the frictional force.

3. ***(c)*** The particle with velocity $2v$ and mass m_1 to P by AQP and will rotate $240°$ (or $4\pi/3$) while the particle with velocity v and mass m_2 reaches P from A by rotating $120°$ (or $2\pi/3$). At P, they make elastic collision, then m_2 moves back to Q by PAQ as it has now velocity $2v$ and m_1 come back from from P to Q to make collision once more. Now, m_1 has velocity $2v$ and m_2 has velocity v. So, the particles reach A after two collisions as shown in figure below



4. ***(b)*** According to Newton's second law of motion, $F =$ time rate of change of linear momentum

$$= \frac{dp}{dt} = \frac{d}{dt}(mv)$$

Since, here velocity (v) is constant.

$$\Rightarrow \quad F = v\frac{dm}{dt}$$

Here, $dm = 10$ g, $dt = 2.5$ s

$$\Rightarrow \quad F = \frac{5 \times 10}{2.5} = 20 \text{ g cm s} = 20 \text{ dyne}$$

5. ***(b)*** Given, position of the particle is

$$x = 4 - 12t + 3t^2$$

Differentiating both sides with respect to time (t), we get

$$\frac{dx}{dt} = 0 - 12 + 3(2t) = 6t - 12$$

$$\therefore \quad v(t) = 6t - 12$$

At $t = 1$ s, $v = 6 \times 1 - 12 = -6$ m/s

At $t = 0$ s, $x = 4$ m

At $t = 1$ s, $x = 4 - (12 \times 1) + (3 \times 1^2) = -5$ m

Hence, particle is moving along negative x direction.

6. ***(b)*** According to the kinematic equation of motion, $v^2 = u^2 \pm 2gh$

This equation is quadratic in terms of v and h.

Hence, the v-h graph would be a parabola.

As per the given condition, a ball is dropped vertically, implies that its initial velocity is downwards and therefore negative.

After collision, it reverses its original direction with smaller magnitude. Thus, the velocity is now upwards. Hence, only graph (b) satisfies these conditions.

7. ***(a)*** Given, centripetal force, $F = -\frac{K}{r^2}$

Also, $\frac{K}{r^2} = \frac{mv^2}{r}$ or $mv^2 = \frac{K}{r}$

$\therefore$ Kinetic energy, K.E. $= \frac{1}{2}mv^2 = \frac{K}{2r}$

For central forces, P.E. $= -2$K.E. $= -\frac{K}{r}$

Total energy, T.E. = P.E. + K.E. $= \frac{-K}{r} + \frac{K}{2r} = -\frac{K}{2r}$

8. ***(a)*** Since the two objects are attached gently to opposite ends of diameter the ring. So no external torque is applied to the system. Therefore, angular momentum remains constant.

i.e., $I_1\omega_1 = I_2\omega_2$

$$\therefore \quad \omega_2 = \frac{I_1\omega_1}{I_2} = \frac{Mr^2\omega}{(Mr^2 + 2mr^2)} = \frac{M\omega}{(M + 2m)}$$

9. ***(d)*** Acceleration of the body, $a = \frac{v_1 - 0}{t_1 - 0} = \frac{v_1}{t_1}$

[$\because$ Body was initially at rest]

Force on the body, $F = ma = m\frac{v_1}{t_1}$

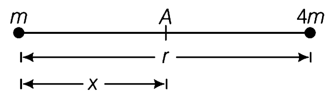
Instantaneous velocity of the body, $v = at = \frac{v_1}{t_1} t$

Instantaneous power $= Fv = m\frac{v_1}{t_1} \times \frac{v_1}{t_1} t = \frac{mv_1^2 t}{t_1^2}$

10. ***(b)*** According to Kepler's law of period, $T^2 \propto r^3$

$$\Rightarrow \quad T \propto r^{\left(\frac{3}{2}\right)}$$

11. ***(c)*** Let gravitational field at the point A in zero as shown in figure below



So, $\frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2}$ or $(r-x)^2 = (2x)^2$ or $r - x = 2x$

$$\Rightarrow \quad 3x = r \text{ (neglecting negative values)}$$

$$\therefore \quad x = \frac{r}{3}$$

Hence, gravitational potential at point A,

$$V = \frac{-GM}{x} - \frac{G(4m)}{(r-x)} = -\frac{3Gm}{r} - \frac{3G(4m)}{2r} = \frac{-9Gm}{r}$$

12. ***(c)*** Energy stored in the wire per unit volume

$$= \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$= \frac{1}{2} \times \text{Stress} \times \frac{\text{Stress}}{\text{Young's Modulus}} = \frac{S^2}{2Y}$$

13. ***(a)*** As the air gets cooled in the upper part of the refrigerator, it becomes denser and goes down. The warmer air of the lower part which is less denser moves up. Due to this, convection currents are set up. This quickly cools up the entire refrigerator from inside.

14. ***(b)*** Given, $y = kt^2$

Differentiating both sides of the above equation w.r.t. time 't', we get

$$\therefore \quad \frac{dy}{dt} = 2kt$$

Similarly, $\frac{d^2y}{dt^2} = 2k$

$$\Rightarrow \quad a = \frac{d^2y}{dt^2} = 2 \times 1 = 2\,\text{m s}^{-2}$$

For a pendulum, $T = 2\pi\sqrt{\frac{1}{g_{\text{eff}}}}$ or $T \propto \sqrt{\frac{1}{g_{eff}}}$

$$\therefore \quad \left(\frac{T_1}{T_2}\right)^2 = \frac{g+a}{g} = \frac{12}{10} = \frac{6}{5}$$

15. ***(d)*** According to an ideal gas equation, pressure $p = \frac{\rho RT}{M}$, where ρ is the density of the gas and M is the atomic mass of the gas.

$$\therefore \quad p_1 = \frac{\rho_1 RT}{M_1} \quad \text{...(i)}$$

and $$p_2 = \frac{\rho_2 RT}{M_2} \quad \text{...(ii)}$$

Divide (i) by (ii), we get

$$\frac{p_1}{p_2} = \frac{\rho_1}{\rho_2}\frac{M_2}{M_1} \text{ or } \frac{\rho_1}{\rho_2} = \frac{p_1}{p_2}\frac{M_1}{M_2}$$

Here, $\frac{\rho_1}{\rho_2} = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$

16. ***(c)*** Magnitude of magnetic force per unit length between parallel current carrying wires,

$$F = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{r}$$

Initially, $F = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{d}$

As both the wires carry current in same direction, this force is attractive.

Finally $i'_1 = 2i$ and $r = 3d$

So, $F' = \frac{\mu_0}{2\pi}\frac{(2i_1) i_2}{3d} = \frac{2}{3}F$

Since, the direction of current is reversed in one of the wire, hence F' would be repulsive in nature.

$$\therefore \quad F' = -\frac{2}{3}F$$

17. ***(a)*** Let emf of a cell $= \varepsilon$

and its internal resistance $= r$

Power consumed by any resistance R is given by

$$P = \frac{V^2}{R}$$

For series combination, $V = 2\varepsilon$ and $R = 2r$

$$\therefore \quad P_s = \frac{(2\varepsilon)^2}{2r} = \frac{2\varepsilon^2}{r}$$

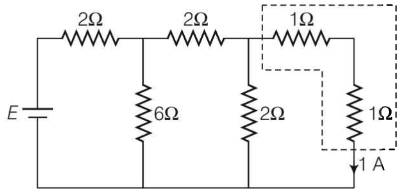
For parallel combination, $V = \varepsilon$ and $R = \frac{r}{2}$

$$\therefore \quad P_p = \frac{\varepsilon^2}{\frac{r}{2}} = \frac{2\varepsilon^2}{r}$$

Hence, $\frac{P_s}{P_p} = \frac{2\varepsilon^2}{r} \times \frac{r}{2\varepsilon^2} = \frac{1}{1}$

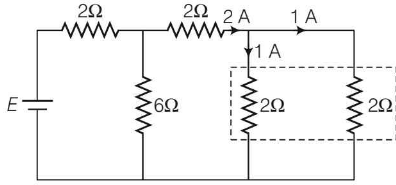
18. ***(c)*** Equivalent resistance and current in the circuit has been calculated in the figures given below

In series, $R_{\text{eq}} = 1 + 1 = 2\,\Omega$



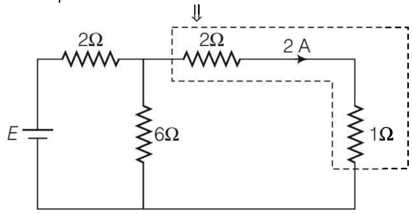
In parallel, $R_{eq} = \frac{2\times2}{2+2} = 1\Omega$

$\Downarrow$



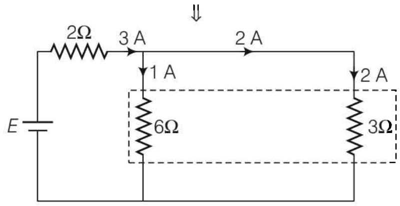
In series $R_{eq} = 2+1 = 3\ \Omega$

$\Downarrow$

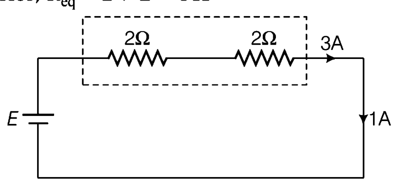


In series, $R_{eq} = \frac{6\times3}{6+3} = 2\,\Omega$

$\Downarrow$



In series, $R_{eq} = 2 + 2 = 4\,\Omega$



$\therefore$ emf, E = (Total current) $\times$ (Net equivalent resistance)

$= 3\times4 = 12\text{V}$

19. ***(a)*** At 600 Hz, $X_C = X_L$

$$\therefore \quad \frac{1}{\omega C} = \omega L \text{ or } \omega^2 = \frac{1}{LC} \quad \ldots\text{(i)}$$

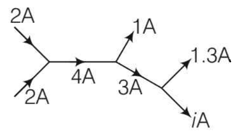
At 60 Hz, $\frac{X_C}{X_L} = \frac{1}{\omega' C(\omega' L)} = \frac{1}{\omega'^2 LC} = \frac{\omega^2}{\omega'^2} = \frac{\nu^2}{\nu'^2}$

(Using Eqs. (i))

$$= \frac{(600)^2}{(60)^2} = \frac{100}{1}$$

$\therefore \quad X_C : X_L = 100:1$

20. ***(a)*** According to kirchhoff' junction rule the algebric sum of the currents meeting at a point in an electrical circuit is always zero. Thus, one can redraw the given figure as shown below,



$\therefore \quad i = 3 - 1.3 = 1.7\text{ A}$

21. ***(a)*** Diamagnetic substances are those substances which have the tendency to move from stronger to weaker part of the magnetic field. Thus, when a bar of diamagnetic material is placed in an external magnetic field, the field lines are repelled or expelled and lines inside the material is reduced. This implies, it acquires feeble magnetisation in the direction opposite to that of the applied field.

22. ***(c)*** Magnetic field produced by straight current carrying conductor at a distance r is $B = \frac{\mu_0}{4\pi}\cdot\frac{2I}{r}$

Here, $I = 1$ A, $r = 1$ m, $B = ?$

Substituting these values, we get

$B = 10^{-7}\times\frac{2\times1}{1} = 2\times10^{-7}\text{ T}$

23. ***(b)*** Given, $V_{rms}^2 = \frac{1}{T}\int_0^T V^2\,dt = \frac{1}{T}\int_0^{T/2} V_0^2\,dt + \frac{1}{T}\int_{T/2}^{T} 0\,dt$

$$= \frac{V_0^2}{T}\times\frac{T}{2} = \frac{V_0^2}{2}$$

$\therefore$ According to the given figure,

$\therefore \quad V_{rms} = \frac{V_0}{\sqrt{2}}$

24. ***(d)*** As resistance, $R = \frac{\rho l}{A} = \frac{\rho l^2}{V}$ $\quad (\because V = Al)$

For given ρ and V, $\frac{dR}{R} = 2\frac{dl}{l}$

$\therefore \frac{dR}{R}\times100 = 2\frac{dl}{l}\times100 = 2\times0.2\,\% = 0.4\%$

Positive sign implies, that the resistance would increase.

25. ***(*)*** As we know, the relation between electric field and potential difference is given as, $\int_{V_O}^{V_A} dV = -\int_0^r \mathbf{E}\cdot d\mathbf{r}$

$$\Rightarrow V_A - V_O = -\int_0^{2m} 30x^2\,dx = -30\left[\frac{x^3}{3}\right]_0^2$$

$$\Rightarrow V_A - V_O = -30\times\left[\frac{2^3}{3}-\frac{0}{3}\right] = -30\times\frac{8}{3} = -80\text{ V}$$

Note Potential is a scalar quantity. So, it cannot be written as $-80\hat{j}$ as given in option.

26. *(d)* In a series LCR circuit, $\tan\phi = \frac{X_L - X_C}{R}$

Here, $\phi = 45°$

$$\Rightarrow \tan 45° = \frac{X_C - X_L}{R} \quad 1 = \frac{1}{R}\left(\frac{1}{2\pi fC} - 2\pi fL\right)$$

$$\Rightarrow \frac{1}{2\pi fC} = R + 2\pi fL$$

$$\therefore C = \frac{1}{2\pi f(R + 2\pi fL)}$$

27. *(a)* As the loop is incomplete so there will be no induced current in the loop. Hence, bar magnet will fall with acceleration due to gravity (g).

28. *(a)* Given, $\mathbf{E} = \hat{i}\ 40\cos(kz - 6\times10^8 t)$ ...(i)

Comparing eq. (i) with the standard equation which is $\mathbf{E} = \hat{i}E_0\cos(kz - \omega t)$, we get

$\omega = 6\times10^8$ rad s^{-1}, $v = c = 3\times10^8$ m s^{-1}

$$\therefore k = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{v} = \frac{\omega}{v} = \frac{6\times10^8}{3\times10^8} = 2\text{m}^{-1}$$

29. *(a)* LASIK eye surgery which is stands for Laser-Assisted in-situ keratomileusis used a laser beam of ultraviolet radiations. It is used to modify the shape of the retina such that the defected vision can be corrected.

30. *(a)* Given, $V = 300\sin\left(\omega t + \frac{\pi}{2}\right)$, $I = 5\sin\omega t$

Here, $\phi = \frac{\pi}{2}$, $I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}}$ and $V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{300}{\sqrt{2}}$

$$\therefore \text{Power, } P = I_{\text{rms}}V_{\text{rms}}\cos\phi = \frac{5}{\sqrt{2}}\cdot\frac{300}{\sqrt{2}}\cos\frac{\pi}{2} = 0$$

31. *(d)* Displacement current across the capacitor = Charging current = 0.78 A

32. *(c)* The phenomenon of bending of light around the corners of an obstacle/aperture of size comparable to the wavelength is called diffraction. Other waves, such as sound waves and water waves also have this property of spreading when passing through apertures or by sharp edges.

33. *(a)* Deviation through prism is given by $\delta = (\mu - 1)A$

As, μ is maximum for blue colour among the given colours.

$\therefore$ δ is maximum for blue colour.

34. *(c)* Light can only be diffracted when the obstacle or aperture size is comparable to the wavelength of the light. This means, aperture size has to be very very small. Since, in real life it is not possible. Thus, light is almost appears to travel in straight lines.

35. *(c)* Mirage is an optical illusion occuring in deserts and on roads on a hot summer day due to total internal reflection. Because optical density at different layers of air increases with height. As a result, light from tall object such as a tree passes through a medium whose refractive index decreases towards the ground. Thus, a ray of from such object successively bends away from the normal and undergoes total internal reflection, if the angle of incidence for the air near the ground exceeds the critical angle.

36. *(c)* The refractive index of the lens in human eye ranges from 1.406 at the center to about 1.386 in the outer layers, making it a varying refractive index lens.

37. *(d)* Length of telescope, $l = f_o + f_e$

Also, magnifying power, $m = \frac{f_o}{f_e}$

Given, $f_o + f_e = 10$ ...(i)

and $\frac{f_o}{f_e} = 4$ ...(ii)

Solving eq. (i) and eq. (ii), we get

$\Rightarrow$ $f_e = 2\text{ cm}$

Then, $f_0 = 2\times 4 = 8\text{ cm}$

As, objective has large focal length as compare to eye lens. Thus, required lenses are L_4 and L_1.

38. *(b)* Given, energy, $E = 1\text{ keV} = 1000\text{ eV}$

$\lambda = 1.24\times10^{-9}\text{ m} = 1.24\text{ nm}$

Energy of a photon, $E = \frac{hc}{\lambda}$

$$\Rightarrow hc = E\times\lambda = (1000\text{ eV})\times(1.24\text{ nm}) = 1240\text{ eV nm}$$

For, $E' = 1\text{ MeV} = 10^6\text{ eV}$

$$\lambda = \frac{h\cdot c}{E'} = \frac{1240\text{ eV nm}}{10^6\text{ eV}} = 1240\times10^{-15}\text{ m}$$

$$\therefore \text{Frequency, } \nu = \frac{c}{\lambda} = \frac{3\times10^8}{1240\times10^{-15}} = 2.4\times10^{20}\text{ Hz}$$

39. *(d)* de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}}{60\times10^{-3}\times10} = \frac{6.63}{6}\times10^{-33}$$

$$= 1.105\times10^{-33}\text{ m} \approx 10^{-33}\text{ m}$$

40. *(a)* de-Broglie wavelength, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m\text{KE}}}$

For particles of same energy, $\lambda \propto \frac{1}{\sqrt{m}}$

$$\therefore \quad \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha}{m_p}} = \sqrt{\frac{4m_p}{m_p}} = \frac{2}{1}$$

41. *(c)* As we know that,

density of the nucleus $= \frac{\text{mass of nucleus}}{\text{volume of nucleus}}$

Mass of the nucleus $= A \times 1.66 \times 10^{-27}$ kg,

where A is the mass number of nucleus.

Volume of the nucleus $= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (R_0 A^{1/3})^3$

$$= \frac{4}{3}\pi R_0^3 A$$

Thus, density $= \frac{A \times 1.66 \times 10^{-27}}{\left(\frac{4}{3}\pi R_0^3\right) A} = \frac{1.66 \times 10^{-27}}{\left(\frac{4}{3}\pi R_0^3\right)}$

which shows that the density is independent of mass number A. Using, $R_0 = 1.2 \times 10^{-15}$ m, we get

density $= 2.99 \times 10^{17}$ kg m^{-3}

42. *(b)* Wavelength of Balmer series is given by

$$= \frac{1}{\lambda} = R_H \left[\frac{1}{2^2} - \frac{1}{n_i^2}\right]$$

where, $n_i = 3, 4, 5, \dots \infty$

When transition of electron takes place from $n_i = 3$ to $n_f = 2$, wavelength of emitted photon is maximum, given by

$$\frac{1}{\lambda_{\max}} = R_H \left[\frac{1}{2^2} - \frac{1}{3^2}\right] \text{or } \lambda_{\max} = 6563\,\text{Å}$$

When transition of electron takes place from $n_i = \infty$ to $n_f = 2$, wavelength of emitted photon is minimum, given by

$$\frac{1}{\lambda_{\min}} = R_H \left[\frac{1}{2^2} - \frac{1}{\infty^2}\right] \text{or } \lambda_{\min} = 3646\,\text{Å}$$

The range of the wavelength thus calculated is equivalent to the wavelength range at visible region of spectrum.

∴ Balmer series lie in the visible region of electromagnetic spectrum.

43. *(d)* Given, mass defect in 1 kg of substance,

$\Delta m = 0.3\%$ of 1 kg $\Rightarrow \Delta m = \frac{0.3}{100} \times 1$ kg $= \frac{3}{1000}$ kg

Amount of energy released, $E = \Delta mc^2$

$$E = \frac{3}{1000} \times (3 \times 10^8)^2 = 2.7 \times 10^{14}\ \text{J}$$

44. *(d)* Coulomb force acting on a electron in a hydrogen atom is given by,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \quad \text{or } F \propto \frac{1}{r^2} \qquad \dots(\text{i})$$

Since $\quad r \propto n^2 \qquad \dots$(ii)

From relation (i) and (ii), we get

$$\therefore \quad F \propto \frac{1}{n^4}$$

45. *(c)* Let initial number of radioisotope $= N_0$

Given, $T_{1/2} = 5$ years, $t = 15$ years

Number of radioisotope decayed after time t

$$N = N_0 - N_0\left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}} = N_0 - N_0\left(\frac{1}{2}\right)^3 = \frac{7N_0}{8}$$

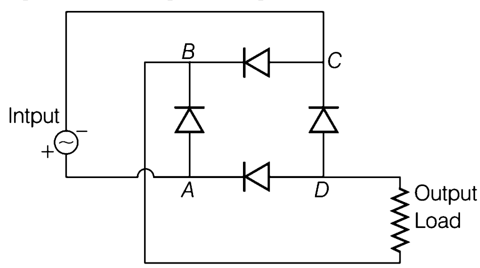
Required fraction, $\frac{N}{N_0} = \frac{7}{8}$

46. *(a)* The bandgap in Ge and Si is 0.7 eV and 1.1 eV, respectively.

47. *(c)* In a semiconductor crystal if the current flows due to breakage of crystal bonds, then the semiconductor is called intrinsic semiconductor.

However, if this current is flown due to the excess of holes or electrons, then that semiconductor is called extrinsic semiconductor.

48. *(d)* Considering the given conditions as per the question, the updated figure is shown below



This figure represents the circuit of full wave bridge rectifier.

49. *(a)* Length of antenna, $l = 100$ m,

As, $\quad l = \frac{\lambda}{4}$

or $\quad \lambda = 4\,l = 4 \times 100 = 400$ m

50. *(b)* Given, $V_{\max} = 12$ V, $V_{\min} = 4$ V, $\mu = ?$

Modulating index,

$$\mu = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}} = \frac{12 - 4}{12 + 4} = \frac{8}{16} = 0.50$$

Chemistry

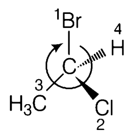
51. *(c)* If one of the substances in the mixture is water and other is a water insoluble substance, then then mixture will boil close to but below, 373 K and it is purified by steam distillation.

52. *(d)* Reaction intermediate is a molecular entity that is formed from the reactants and reacts further to give products of a chemical reaction e.g., carbenes, nitrenes, carbocations and other electrophiles. On the other hand, hydrophile is a molecular entity that is attracted towards water molecule and tends to be dissolved by water.

53. *(c)* $\overset{4}{C}H_3-\overset{3}{C}(CH_3)_2-\overset{2}{C}H=\overset{1}{C}H_2$

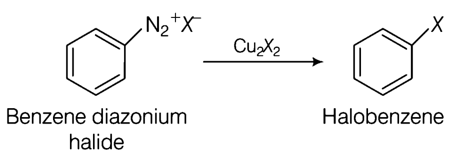
IUPAC name: 3, 3-dimethylbut-1-ene

54. *(a)* The given compound is



In the *wedge-dash* formula, if lowest priority group or atom is below the plane and direction of rotation is clockwise, the configuration is '*R*'.

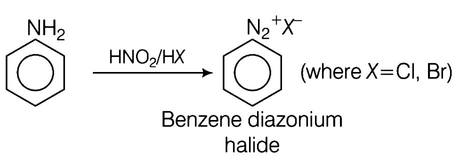
55. *(b)* The given reaction,



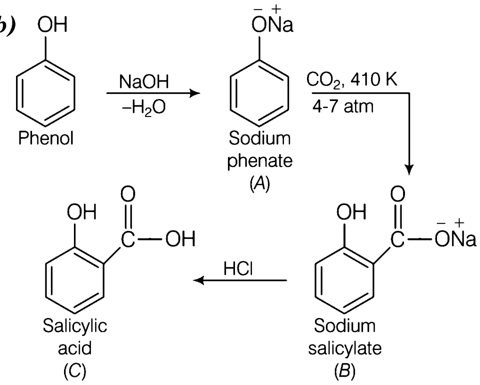
is called Sandmeyer's reaction.

Benzene diazonium halide is prepared by the action of $NaNO_2$ + H*X* on aniline.

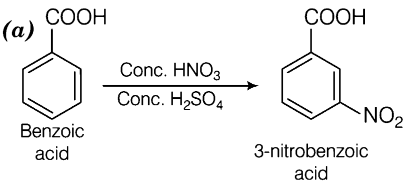
(where X = Cl, Br)



56. *(b)*



57. *(a)*



—COOH group is *m*-directing group, as it decreases the electron density at *ortho* and *para* positions, as a result *meta* position becomes comperetivly electron rich. Hence, an electrophile —(NO_2^+) attack on *meta* position.

58. *(b)* (i) $H_3C \rightarrow\!\!\rightarrow \underset{sp}{C} \equiv C^-$

More electronegative (50% *s*-character)

+ *I*-effect of —CH_3 group destabilises the carbanion.

(ii) $H \rightarrow \underset{sp}{C} \equiv C$

More electronegative (50% *s*-character)

No + *I*-effect to destabilise the carbanion.

(ii) $H_3C - \underset{sp}{C}H_2$

Less electronegative (25% *s*-character)

More the *s*-character of charged carbons, greater will be the stability. Hence, the correct decreasing order of stability is (ii) > (i) > (iii).

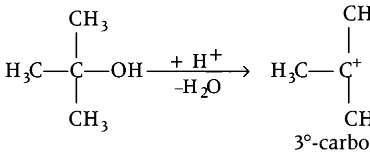
59. *(b)* The given reaction is an example of Finkelstein reaction, in which more reactive halide-ion will displace the less reactive halogen atom from alkyl-halide, i.e.

$R—X + NaI \longrightarrow R—I + NaX$

where, (X = Cl, Br)

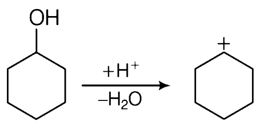
60. ***(b)*** Ethers have lesser boiling point than alcohols. In alcohols, the boiling point decreases with increase in branching in carbon chains due to decrease in van der Waals' forces with decrease in surface area. Hence, $CH_3CH_2CH_2CH_2CH_2OH$ has the highest boiling point.

61. ***(a)*** More stable the carbocation formed, faster is the rate of dehydration.



$$CH_3—CH_2—CH_2—OH \xrightarrow[-H_2O]{+H^+} CH_3—CH_2—\overset{+}{C}H_2$$
1°-carbocation

$$CH_3—(CH_2)_4—OH \xrightarrow[-H_2O]{+H^+} CH_3—(CH_2)_3—\overset{+}{C}H_2$$
1°-carbocation



2°-carbocation

Therefore, compound in option (a) will undergo the fastest dehydration reaction.

62. ***(c)*** Aliphatic amines are more basic in comparison to aryl amines due to the presence of alkyl group(s) which exhibit $+I$ effect. In aryl amines, the lone pair on N-atom is delocalised over the benzene ring. Hence, the correct decreasing order of basic strength is

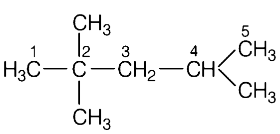
$$(C_2H_5)_2NH > C_2H_5NH_2 > NH_3 > C_6H_5NH_2$$

63. ***(c)*** Glycogen is also known as animal-starch, it act as a storage material for animals.

64. ***(d)*** Poly-acrylonitrile is used as a substitute for wool.

65. ***(a)*** Drug, cimetidine is used to prevent the interaction of histamine with the receptors, present in stomach wall.

66. ***(d)*** The given compound is

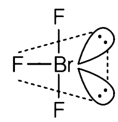


2, 2, 4– trimethylpentane

67. ***(d)*** Aldehydes and ketones having at least one —CH_3 group linked to the carbonyl carbon atom or $CH_2CH(OH)$ given iodoform test. On the other hand, benzyl alcohol ($PhCH_2OH$) does not give iodoform test.

68. ***(a)*** As size of metal ion decreases, possibility of water molecules to surround the metal ion increases and hence, hydration energy increases. Therefore, $MgCl_2$ has highest hydration energy.

69. ***(d)*** The species those central atom has sum of orbitals having sigma (σ) bonds and lone pair of electrons equal to 5 can show sp^3d-hybridisation. If species contain three sigma bonds and two lone pair of electrons (by the central atom), it has T-shape, thus, the structure of BrF_3 is T-shape.



(T-shape)

70. ***(c)*** Sodium hydroxide (NaOH) is prepared by Castner-Kellner process using mercury as cathode and carbon as anode.

71. ***(a)*** *Ziegler-Natta* catalyst is trimethyl aluminium and titanium tetrachloride, i.e. $Al(CH_3)_3$ with $TiCl_4$.

It is used to manufacture high-density polyethylene.

72. ***(c)*** Primary valency is the oxidation state of the central metal ion.

Let, oxidation state of Co in $[Co(en)_2Cl_2]$ Cl be x.

$$x + 2(0) + 2(-1) - 1 = 0$$

$\therefore$ $x = +3$

$\therefore$ Primary valency of Co $= +3$

73. ***(b)*** The given complex ion is $[Pt(C_2H_4)Cl_3]^-$

Let, the oxidation number of Pt be x.

$$x + (0) + 3(-1) = -1 \Rightarrow x = +2$$

74. ***(b)*** $P_4 + 10H_2SO_4 \longrightarrow \underset{\text{(Orthophosphoric acid)}}{4H_3PO_4} + 10SO_2 + 4H_2O$

H_2SO_4 (conc.) oxidises Pu to its oxo-acid having maximum possible oxidation-state.

75. ***(a)*** Tear gas is CCl_3NO_3 (chloropicrin).

76. *(c)*

Species	Total no. of electrons	Bond order (B.O.)
O_2^+	15	2.5
O_2	16	2.0
O_2^-	17	1.5

Hence, the correct bond order is $O_2^+ > O_2 > O_2^-$

Because,

(i) Bond-order of O_2 = Number of bonds between two oxygen atoms, i.e. (two)

(ii) An increase of electron antibonding molecular-orbital decreases the B.O, while decrease of electron in antibonding molecular orbital increases the B.O.

77. *(a)* Two *s*-electrons develop more and more tendency to remain together as we go down the group.

78. *(a)* Copper is extracted from sulphide ore by auto-reduction method. The reaction occurs as follows:

(i) $2Cu_2S + 3O_2 \longrightarrow 2Cu_2O + 2SO_2 \uparrow$

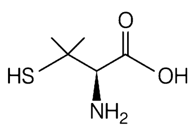
(ii) $2Cu_2O + Cu_2S \longrightarrow 6Cu + SO_2 \uparrow$

79. *(d)* Photochemical smog is formed due to oxidation of hydrocarbons trapped in stagnant air mass in the presence of sun-light. It happens because of high-concentration of oxidising agents, thus is also called oxidising smog.

80. *(a)* Correct formula for Wilkinson-catalyst is $(Ph_3P)_3Rh \cdot Cl$. It is a red-brown coloured solid. It is used as a catalyst for hydrogenation of alkenes etc.

81. *(a)* D-pencillamine is the chelating ligand that can be used to remove excess of copper in the bio-system.

Its structure is also follows :



It is also used in Wilson disease.

82. *(b)* N_2O_3 is an oxide of nitrogen. It is blue in nature. It is formed by the reaction between NO and N_2O_4 at about 250 K.

$$2NO + N_2O_4 \xrightarrow{250\text{ K}} 2N_2O_3$$

It is acidic in nature.

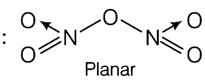
83. *(a)* The acidic character of the hydrides of group 16 elements increases down the group i.e., $H_2O > H_2S < H_2Se$ as E—H bond strength decreases down the group, thus, release of H^+ becomes easier.

84. *(d)* Molecular orbital electronic configuration of $O_2 = \sigma 1s^2 \sigma^* 1s^2 \sigma 2\sigma^2 s^* 2s^2 \sigma 2p_z^2 \pi 2p_x^2$

$= \pi 2p_y^2 \pi^* 2p_x^1 = \pi^* 2p_y^1$

Hence, the total number of electrons in antibonding molecular orbitals of O_2 is 6.

85. *(b)* N_2O_5 :



There is no N—N bond present in N_2O_5. i.e. = 0

86. *(d)* Given: $t_{1/2} = 140$ days

$\because$ Radioactive reaction follows first order kinetics.

$$\therefore k = \frac{0.693}{t_{1/2}} = \frac{0.693}{140\text{ days}} = 4.95 \times 10^{-3}\text{ days}^{-1}$$

For 1st order reaction,

$$k = \frac{2.303}{t}\log\frac{[A]_0}{[A]_t}$$

$$\log\frac{1}{[A]_t} = \frac{kt}{2.303} = \frac{4.95 \times 10^{-3} \times 700}{2.303}$$

$$-\log[A]_t = 1.504 \Rightarrow [A]_t = \left(\frac{1}{32}\right)\text{g}$$

Alternatively Given, Half life $(t_{1/2}) = 140$ days

Total time (T) = 700 days

$\because$ $T = n \times t_{1/2}$

$\therefore$ $n = \frac{T}{t_{1/2}} = \frac{700}{140} = 5$

(where n = no. of half-life's) and

elements reduce to $(a - x) = \frac{1}{2^n} = \frac{1}{2^5} = \frac{1}{32}$ g

87. *(b)* The temperature at which real gases behave like an ideal gas over a wide range of pressure is called Boyle's-temperature. It depends on the nature of gas. Among the given options, CO_2 has high value for Boyle's temperature.

88. *(c)* Since, As_2S_3 colloid is a negatively charged sol. Coagulating power increases with increase in positive charge or decrease in size of the cation. Hence, Al^{3+} has the highest coagulating power for As_2S_3 colloid.

89. ***(b)*** Rate of chemisorption of a gas increases with the increase in pressure because more molecules are adsorbed on the surface at higher pressure. This can also understood by the following relation:

$$\log \frac{x}{m} = \log k + \frac{1}{n} \log p$$

where, $\frac{x}{m}$ = mass adsorbed over per unit surface of adsorbent.

n and k are constants

p = pressure,

i.e. $\frac{x}{m} \propto p$

90. ***(b)*** Buffer capacity $(\beta) = \frac{\Delta C_a}{\Delta pH}$

where, C_a = concentration of acid.

For acidic buffer,

concentration of acid = concentration of salt

$\therefore \Delta C_a = 0.12 \text{ mol L}^{-1}$

$pH = -\log [H^+] = -\log(0.12) = 0.92$

Buffer capacity $(\beta) = \frac{0.12}{0.92} = 0.130$

91. ***(d)*** For (+)ve deviation,

$\Delta H > 0$, attractive forces between the molecules becomes weaker, and $\Delta V > 0$,

Thus, on mixing 100 mL of water with 100 mL of CH_3OH, the total volume increases, i.e becomes more than 200 mL (100+100) due to weaker attractive force between H_2O and CH_3OH molecules.

92. ***(a)*** Given: $T = 27 + 273 \text{ K} = 300 \text{ K}$

$$C = 0.02 \text{ M}, \ \pi = ?$$

$$i = (n-1)\alpha + 1$$

$$= (2-1)0.9 + 1 = 1.9$$

$$\pi = iCRT$$

$$\pi = 1.9 \times 0.02 \times 0.08314 \times 300 = 0.94 \text{ bar}$$

93. ***(b)*** Given :

$$T_f = 5.6 + 273 \text{ K} = 278.6 \text{ K}$$

$$K_f = 5.1, \ \Delta_{fus}H = ?$$

$$\Delta H_{fus} = \frac{R \times M \times T_f^2}{1000 \times K_f}$$

$$= \frac{1.987 \times 78 \times (278.6)^2}{1000 \times 5.1}$$

$$= 2358.76 \text{ cal}$$

94. ***(a)*** Given : $\Delta H = -298 \text{ kJ mol}^{-1}$

$T = 25 + 273 = 298 \text{ K}, P = 1 \text{ atm}, \Delta E = ?$

$$\Delta H = \Delta E + \Delta n_g RT$$

or $\Delta E = \Delta H$ $(\because \Delta n_g = 0)$

$$\Delta E = -298$$

$$\Delta E = -298 \text{ kJ mol}^{-1}$$

95. ***(c)*** For an ideal gas, $\left(\frac{\partial H}{\partial p}\right)_T = 0$ is the correct relation, which shows the ideality of a gas.

96. ***(a)*** For second order reaction,

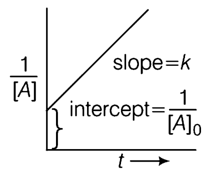
$$k = \frac{1}{t}\left[\frac{1}{[A]} - \frac{1}{[A]_0}\right]$$

$$\Rightarrow \quad \frac{1}{[A]} = kt + \frac{1}{[A]_0}$$

Comparing with

$$y = mx + c,$$

We get this graph.



97. ***(b)***

	$NH_4HS(s) \rightleftharpoons$	$NH_3(g)$ +	$H_2S(g)$
Initial moles	1	0	0
Eqm. moles	$1-x$	x	x

$$p_{Total} = 1.12 \text{ atm}$$

$$K_p = p_{NH_3} \times p_{H_2S}$$

$$p_{NH_3} = p_{Total} \times x_{NH_3}$$

$$= 1.12 \times \frac{x}{2x} = 0.56$$

$$p_{H_2S} = p_{Total} \times x_{H_2S}$$

$$= 1.12 \times \frac{x}{2x} = 0.56$$

$\therefore \quad K_p = 0.56 \times 0.56 = 0.3136 \text{ atm}^2$

98. ***(a)*** Word done for isothermal expansion,

$$w = -2.303 \, nRT \log \frac{V_f}{V_i}$$

$$= -2.303 \times \frac{16}{32} \times 8.314 \times 300 \log \left(\frac{25}{5}\right)$$

$$= -2007.49 \text{ J}$$

$$= -2.01 \times 10^3 \text{ J}$$

99. (a) Given, Mass of $NH_4^+ (w) = 3.6$ mg

$$= 3.6 \times 10^{-3} \text{ g}$$

Molar mass of $NH_4^+ (M) = 18$ g

$\because \frac{w}{M} = \frac{N}{N_A}$, where N = Number of NH_4^+ ion

$$\therefore N = \frac{w \times N_A}{M} = \frac{3.6 \times 10^{-3} \times 6.02 \times 10^{23}}{18}$$

$$= 1.20 \times 10^{20}$$

$\therefore$ No. of electron in one $NH_4^+ = 10(7 + 4 + 1)$

$\therefore$ Total no. of electrons $= 10 \times 1.2 \times 10^{20} = 1.2 \times 10^{21}$

100. ***(a)***

	$2SO_2(g)$	$+ O_2(g) \rightleftharpoons$	$2SO_3(g)$
	2 moles	1 mole	2 moles
Given:	10 moles	16 moles	8 moles

Here, SO_2 is the limiting reagent, which decides the formation of SO_3.

Now, 8 moles of SO_3 is formed means only 8 moles of SO_2 is consumed and 2 moles of SO_2 remains unreacted.

Also, 4 moles of O_2 is consumed hence, the number of moles of O_2 remains unreacted $= 16 - 4 = 12$ moles

Mathematics

101. ***(a)*** We have,

$(A \cup B \cup C) \cap (A \cap B' \cap C)' \cap C'$

$= (A \cup B \cup C) \cap (A' \cup B \cup C) \cap C'$

[by using distributed law]

$= [(A \cup B \cup C) \cap A'] \cup [(A \cup B \cup C) \cap (B \cup C)]$

$= [(B \cup C) \cup (B \cup C)] \cap C'$

$= (B \cup C) \cap C'$

$= (B \cap C') \cup (C \cap C')$

$= (B \cap C')$ $\quad [\because C \cap C' = \phi$ and $(B \cap C') \cup \phi = B \cap C']$

102. ***(d)*** For set P,

$$\sin\theta - \cos\theta = \sqrt{2}\cos\theta$$

$\Rightarrow \quad \tan\theta - 1 = \sqrt{2}$

[dividing both sides by $\cos\theta$]

$\Rightarrow \quad \tan\theta = \sqrt{2} + 1 \quad$ ...(i)

For set Q,

$$\sin\theta + \cos\theta = \sqrt{2}\sin\theta$$

$\Rightarrow \quad 1 + \cot\theta = \sqrt{2}$

[dividing both sides by $\sin\theta$]

$\Rightarrow \quad \cot\theta = \sqrt{2} - 1$

$\Rightarrow \quad \tan\theta = \sqrt{2} + 1 \quad$ ...(ii)

Hence, $P = Q$ [from Eqs. (i) and (ii)]

103. ***(c)*** The function $f(x) = x - [x]$ is discontinuous at all integral points.

For this, let c be an integer.

Then, $[c - h] = c - 1$,

$[c + h] = c$ and $[c] = c$

Now, LHL $= \lim_{x \to c} f(x) = \lim_{x \to c} (x - [x])$

$= \lim_{h \to 0} \{(c - h) - [c - h]\}$

$= \lim_{h \to 0} (c - h) - (c - 1)$

$= \lim_{h \to 0} (c - h - c + 1) = 1$

Also, RHL $= \lim_{x \to c^+} f(x) = \lim_{x \to c^+} (x - [x])$

$= \lim_{h \to 0} \{(c + h) - [c + h]\}$

$= \lim_{h \to 0} (c + h - c) = 0$

$\therefore$ LHL $\neq$ RHL

Thus, $f(x)$ is discontinuous at all integers.

Hence, $f(x)$ is continuous at non-integer points only.

104. ***(a)*** We have,

$$f(x) = xe^{x(1-x)}$$

$\Rightarrow \quad f'(x) = e^{x(1-x)} + xe^{x(1-x)} \cdot (1 - 2x)$

$\Rightarrow \quad f'(x) = e^{x(1-x)} [1 + x - 2x^2]$

We put $f'(x) = 0 \quad (\because e^{x(1-x)} \neq 0)$

$\Rightarrow \quad 1 + x - 2x^2 = 0$

$\Rightarrow \quad (1 - x)(2x + 1) = 0$

$\Rightarrow \quad x = 1$ or $x = \frac{-1}{2}$

For $\quad x \in \left[-\frac{1}{2}, 1\right]$, $f'(x) > 0$

Hence, $f(x)$ is increasing on $\left[\frac{-1}{2}, 1\right]$

105. ***(b)*** We have,

$$f(x) = y = \frac{2 + x}{2 - x}$$

$\Rightarrow \quad 2y - xy = 2 + x$

$\Rightarrow \quad 2y - 2 = x + xy$

$\Rightarrow \quad 2y - 2 = x(1 + y)$

$\Rightarrow \quad x = \frac{2y - 2}{1 + y}$

Since, $x = \frac{2y-2}{y+1}$ is defined if $y + 1 \neq 0$ i.e. $y \neq -1$

$\therefore$ Range of $f(x) = R - \{-1\}$

106. ***(b)*** Given, $\sin(\alpha + \beta) = 1$

$$\Rightarrow \quad \alpha + \beta = \frac{\pi}{2} \quad \ldots(i)$$

$$\sin(\alpha - \beta) = \frac{1}{2}$$

$$\Rightarrow \quad \alpha - \beta = \frac{\pi}{6} \quad \ldots(ii)$$

On solving Eqs. (i) and (ii), we get

$$\alpha = \frac{\pi}{3}, \beta = \frac{\pi}{6}$$

Now, $\tan(\alpha + 2\beta)\tan(2\alpha + \beta)$

$$= \tan\left[\frac{\pi}{3} + 2\left(\frac{\pi}{6}\right)\right]\tan\left[\frac{2\pi}{3} + \frac{\pi}{6}\right]$$

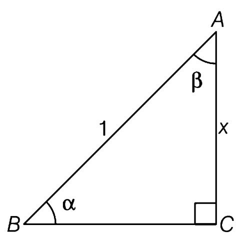
$$= \tan\left(\frac{2\pi}{3}\right)\tan\left(\frac{5\pi}{6}\right) = (-\sqrt{3})\left(\frac{-1}{\sqrt{3}}\right) = 1$$

107. ***(c)*** From the given figure,

$$BC = \sqrt{AB^2 - AC^2} = \sqrt{1 - x^2}$$

$$\therefore \quad \cos\alpha = \sqrt{1 - x^2}, \sin\alpha = x \quad \ldots(i)$$

$$\text{and} \quad \cos\beta = x, \sin\beta = \sqrt{1 - x^2} \quad \ldots(ii)$$



Now, from Eqs. (i) and (ii), we have

$$\sin^{-1} x = \alpha, \cos^{-1} x = \beta$$

$$\because \quad \alpha + \beta = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \Rightarrow \cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

108. ***(b)*** Since, $\lim_{x \to 2} f(x)$ exists

$$\therefore \quad \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2) \quad \ldots(i)$$

Now,

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2} (4x - 5) = 8 - 5 = 3 \quad \ldots(ii)$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2} (x - \lambda) = 2 - \lambda \quad \ldots(iii)$$

From Eqs. (i), (ii) and (iii), we get

$$3 = 2 - \lambda \Rightarrow \lambda = -1$$

109. ***(d)*** The equations of lines whose intercepts on the axes are respectively $a, -b$ and $b, -a$ are given by

$$\frac{x}{a} - \frac{y}{b} = 1 \quad \ldots(i)$$

$$\frac{x}{b} - \frac{y}{a} = 1 \quad \ldots(ii)$$

Slope of line (i) is given by $m_1 = \frac{b}{a}$

Slope of line (ii) is given by $m_2 = \frac{a}{b}$

Now, angle between lines (i) and (ii) is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{b}{a} - \frac{a}{b}}{1 + \frac{b}{a} \times \frac{a}{b}}\right| = \left|\frac{b^2 - a^2}{2ab}\right|$$

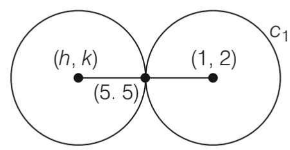
$$\therefore \quad \tan\theta = \pm\left(\frac{b^2 - a^2}{2ab}\right)$$

110. ***(a)*** Let equation of circle be

$(x - h)^2 + (y - k)^2 = 5^2$, where 5 is the radius of the circle.

Also, the radius of given circle

$c_1 : x^2 + y^2 - 2x - 4y - 20 = 0$ is given by $\sqrt{1 + 4 + 20} = 5$ units



$\because$ Two circles touch each other externally.

$\therefore (5, 5)$ is the mid-point of line segment joining (h, k) and $(1, 2)$

$$\Rightarrow \quad 5 = \frac{h + 1}{2}, 5 = \frac{k + 2}{2}$$

$$\Rightarrow \quad h = 9, k = 8$$

$\therefore$ Equation of circle becomes

$$(x - 9)^2 + (y - 8)^2 = 5^2$$

111. ***(b)*** Since each observation is multiplied by 2.

$\therefore$ Variance of resulting observation is $2^2 \times 5 = 20$

112. ***(b)*** Given, sum of n terms of an AP

$$\therefore \quad S_n = nR + \frac{1}{2}n(n - 1)T$$

$$\Rightarrow \quad S_1 = a_1 = R + \frac{1}{2}(1)(0)T = R \quad \ldots(i)$$

$$\Rightarrow \quad S_2 = a_1 + a_2 = 2R + \frac{1}{2}(2)(2 - 1)T$$

$$= 2R + T = R + (R + T) \quad \ldots(ii)$$

Now, $S_2 - S_1 = a_2 = R + T$

$\therefore$ Common difference,

$$d = a_2 - a_1 = R + T - R = T$$

113. *(a)* Here, $a = 20$, common difference $(d) = 15$

Let man clears the loan in n months.

According to the question, $S_n = 3250$

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 3250$

$\Rightarrow \frac{n}{2}[40 + (n-1)15] = 3250$

$\Rightarrow \frac{n}{2}[40 + 15n - 15] = 3250$

$\Rightarrow n(15n + 25) = 2 \times 3250$

$\Rightarrow 3n^2 + 5n - 1300 = 0$

$\Rightarrow (3n + 65)(n - 20) = 0$

$\therefore n = 20 \left(\because n \neq \frac{-65}{3}\right)$

Hence, it will take 20 months for man to clear the loan.

114. *(b)* Let two number be a and b; $(a > b)$

According to the question,

$a - b = 12$ ...(i)

and $\frac{a+b}{2} - \sqrt{ab} = 2$

$\Rightarrow a + b - 2\sqrt{ab} = 4$ ...(ii)

$\Rightarrow a + b = 4 + 2\sqrt{ab}$

$\Rightarrow (a + b)^2 = (4 + 2\sqrt{ab})^2$

[Squaring both sides]

$\Rightarrow (a - b)^2 + 4ab = 16 + 4ab + 16\sqrt{ab}$

$[\because (x + y)^2 = (x - y)^2 + 4xy]$

$\Rightarrow (12)^2 = 16 + 16\sqrt{ab}$ [from Eq. (i)]

$\Rightarrow \sqrt{ab} = 8$

On substituting $\sqrt{ab} = 8$ in Eq. (ii), we get

$a + b - 16 = 4$ or $a + b = 20$ ...(iii)

On solving Eqs. (i) and (iii), we get $a = 16$, $b = 4$

115. *(a)* Suppose the three consecutive terms in the expansion of $(1 + x)^n$ are $(r-1)$ th, rth and $(r+1)$th terms.

Now,

$$T_{r-1} = {}^nC_{r-2}(x)^{r-2}$$

$$T_r = {}^nC_{r-1}(x)^{r-1}$$

$$T_{r+1} = {}^nC_r\,(x)^r$$

$\therefore$ Their coefficients are ${}^nC_{r-2}, {}^nC_{r-1}, {}^nC_r$

Since, the coefficients are in the ratio $1:7:42$, so we have,

$$\frac{{}^nC_{r-2}}{{}^nC_{r-1}} = \frac{1}{7}$$

$\Rightarrow \frac{r-1}{n-r+2} = \frac{1}{7}$

$\Rightarrow 7r - 7 = n - r + 2$

$\Rightarrow n - 8r + 9 = 0$ ...(i)

and $\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{7}{42} = \frac{1}{6}$

$\Rightarrow \frac{r}{n-r+1} = \frac{1}{6}$

$\Rightarrow 6r = n - r + 1$

$\Rightarrow n - 7r + 1 = 0$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$r = 8 \text{ and } n = 55$$

116. *(b)* First we arrange the words starting with A, we fix A at extreme left position

and remaining 4 letters can be arranged in $4! = 24$ ways.

Now, placing G at first position, letters A, A, I, N can be arranged in $\frac{4!}{2!} = 12$ ways.

Similarly, words starting with I are $\frac{4!}{2!} = 12$ ways.

$\therefore$ Total number of words obtained

$$= 24 + 12 + 12 = 48$$

The 49th word is NAAGI.

$\therefore$ 50th word will be NAAIG.

117. *(a)* Given, coefficient of variation of 1st distribution, $C.V_1 = 60$, $\sigma_1 = 21$

and coefficient of variation of 2nd distribution;

$$C.V_2 = 70\ \sigma_2 = 16$$

Let $\bar{x}_1$ and $\bar{x}_2$ be means of 1st and 2nd distribution respectively.

Then, $C.V_1 = \frac{\sigma_1}{\bar{x}_1} \times 100$

$\Rightarrow 60 = \frac{21}{\bar{x}_1} \times 100$

or $\bar{x}_1 = \frac{21}{60} \times 100 = 35$

Also, $C.V_2 = \frac{\sigma_2}{\bar{x}_2} \times 100$ or $\bar{x}_2 = \frac{16}{70} \times 100 = 22.85$

118. *(c)* Given curve is $y = e^x \Rightarrow \frac{dy}{dx} = e^x$

Slope of normal $= \frac{-1}{e^x}$

$\therefore$ Slope of normal at point (0, 1) $= \frac{-1}{e^0} = -1$

The equation of normal at (0, 1) is given by

$$(y-1) = -1\,(x-0)$$

$$\Rightarrow \quad x + y = 1$$

119. ***(a)*** We have,

$$\frac{dx}{dt} = -6, \frac{dy}{dt} = 4$$

Area of rectangle, $A = xy$

$$\Rightarrow \quad \frac{dA}{dt} = x\frac{dy}{dt} + y \cdot \frac{dx}{dt}$$

$$\Rightarrow \quad \frac{dA}{dt} = x(4) + (-6)y$$

$$\Rightarrow \quad \frac{dA}{dt} = 4x - 6y$$

$\because$ Rate of change of area (A) of rectangle i.e. $\frac{dA}{dt}$

$$\therefore \left(\frac{dA}{dt}\right)_{(8,4)} = 4(8) - 6(4) = 8$$

120. ***(b)*** The determinant of an orthogonal matrix is 1 or −1 i.e. $|A| = \pm 1$.

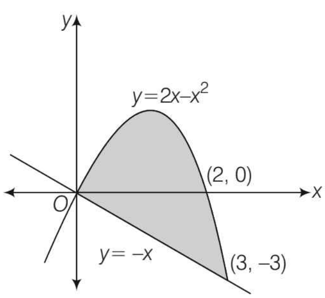
121. ***(a)*** Given curves are

$$y = 2x - x^2 \qquad \ldots(i)$$

$$y = -x \qquad \ldots(ii)$$

On solving Eqs. (i) and (ii) we get,

$$-x = 2x - x^2$$



$$\Rightarrow \quad x^2 - 3x = 0 \Rightarrow x(x-3) = 0$$

$$\Rightarrow \quad x = 0 \text{ or } x = 3$$

The area (A) bounded by the curve is the shaded region shown in the figure.

$$\text{Now, } A = \int_0^3 [(2x - x^2) - (-x)]\,dx$$

$$= \int_0^3 (-x^2 + 3x)\,dx$$

$$= \left|\frac{-x^3}{3} + \frac{3x^2}{2}\right|_0^3 = \frac{-1}{3}(3)^3 + \frac{3}{2}(3)^2$$

$$= -9 + \frac{27}{2} = \frac{9}{2} \text{ sq units}$$

122. ***(b)*** Since, $g(x)$ is the inverse of an invertible function $f(x)$.

$$\therefore \quad g[f(x)] = x \qquad \ldots(i)$$

On differentiating Eq. (i) both sides w.r.t. 'x', we get

$$g'[f(x)]f'(x) = 1$$

$$\Rightarrow \quad g'[f(x)] = \frac{1}{f'(x)}$$

$$\therefore \quad g'[f(c)] = \frac{1}{f'(c)}$$

123. ***(c)*** Given, $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.3$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = 0.5 + 0.4 - 0.3 = 0.6$

$\therefore P(A \cup B)^C = 1 - 0.6 = 0.4 \qquad \ldots(i)$

$$\text{Now, } P(A^C / B^C) = \frac{P(A^C \cap B^C)}{P(B^C)}$$

$$= \frac{P(A \cup B)^C}{P(B^C)} = \frac{0.4}{1 - 0.4} \text{ [from Eq.(i)]}$$

$$= \frac{2}{3}$$

124. ***(c)*** $P(X = r)$ will be maximum when r is mode.

Here, $(n+1)p = (100+1)\frac{1}{3} = \frac{101}{3} = 33.67$

Which is not an integer.

Hence, unique mode is it's integral part i.e. 33.

$\therefore P(X = r)$ will be maximum when $r = 33$.

125. ***(b)*** Some optimal solution of a LPP is also a basic feasible solution of LPP.

126. ***(c)*** $\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k}) \qquad \ldots(i)$

and $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + u(2\hat{i} + 4\hat{j} - 5\hat{k}) \qquad \ldots(ii)$

Shortest distance between lines (i) and (ii)

$$= \frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}$$

Here, $\vec{a}_1 = 4\hat{i} - \hat{j}$; $\vec{b}_1 = \hat{i} + 2\hat{k} - 3\hat{k}$

and $\vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k}$; $\vec{b}_2 = 2\hat{i} + 4\hat{j} - 5\hat{k}$

Now, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}$

$= \hat{i}\,(-10+12) - \hat{j}\,(-5+6) + \hat{k}\,(4-4) = 2\hat{i} - \hat{j}$

$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{4+1} = \sqrt{5}$

Also, $(\vec{a}_2 - \vec{a}_1) = (-3\hat{i} + 2\hat{k})$

$\therefore$ Required shortest distance $= \left| \dfrac{(-3\hat{i} + 2\hat{k}) \cdot (2\hat{i} - \hat{j})}{\sqrt{5}} \right|$

$= \left| \dfrac{-6}{\sqrt{5}} \right| = \dfrac{6}{\sqrt{5}}$ units

127. ***(c)*** Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that

$$|\vec{a} + \vec{b}| = 1,\ |\vec{a}| = 1,\ |\vec{b}| = 1$$

Now, $|\vec{a} + \vec{b}| = 1$ [squaring both sides]

$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = 1$

$\Rightarrow \quad 1 + 1 + 2\vec{a} \cdot \vec{b} = 1$

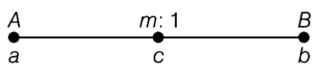
$\Rightarrow \quad \vec{a} \cdot \vec{b} = \dfrac{-1}{2}$

Now, $|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$

$= 1 + 1 - 2\left(\dfrac{-1}{2}\right) = 3$

$\therefore$ Magnitude of their difference, $|\vec{a} - \vec{b}| = \sqrt{3}$

128. ***(b)*** Let C divides AB internally in the ratio $m:1$, $m > 0$



Then, using section formula we have

$$\vec{c} = \frac{m\vec{b} + 1 \cdot (\vec{a})}{m+1}$$

$$\lambda \vec{a}\, t\mu \vec{b} = \frac{m\vec{b}}{m+1} + \frac{1}{m+1}\vec{a}$$

[given, $\vec{c} = \lambda \vec{a} + \mu \vec{b}$]

On comparing both sides, we get

$$\mu = \frac{m}{m+1},\ \lambda = \frac{1}{m+1}$$

$\therefore \quad \lambda + \mu = \dfrac{1}{m+1} + \dfrac{m}{m+1} = \dfrac{m+1}{m+1} = 1$

129. ***(c)*** Given D.E. is $(x \log x) \dfrac{dy}{dx} + y = 2 \log x$

$$\frac{dy}{dx} + \frac{1}{(x \log x)} y = \frac{2 \log x}{x \log x} = \frac{2}{x}$$

[dividing each term by $x \log x$]

It is a linear D.E. of the form

$$\frac{dy}{dx} + Py = Q$$

Here, $P = \dfrac{1}{x \log x},\ Q = \dfrac{2}{x}$

$\therefore$ Integrating factor $= e^{\int P dx}$

$$= e^{\int \frac{1}{x \log x} dx} = e^{\int \frac{dt}{t}}$$

$\left[\because t = \log x \Rightarrow dt = \dfrac{1}{x} dx\right]$

$$= e^{\log_e t} = t = \log x$$

130. ***(b)*** Given, $y = c_1 \cos x + c_2 \sin x$

Here, number of parameters = 2

On differentiating the given equation two times w.r.t. 'x', we get

$$\frac{dy}{dx} = c_1(-\sin x) + c_2(\cos x)$$

$\Rightarrow \quad \dfrac{d^2y}{dx^2} = -c_1 \cos x - c_2 \sin x$

$\Rightarrow \quad \dfrac{d^2y}{dx^2} = -y$ or $\dfrac{d^2y}{dx^2} + y = 0$

131. ***(b)*** We have,

$$\frac{dy}{dx} + (2x)\,y = 2e^{-x^2}$$

which is the linear D.E. of the form $\dfrac{dy}{dx} + Py = Q$

Here, $P = 2x,\ Q = 2e^{-x^2}$

Now, integrating factor I.F. $= e^{\int P dx}$

$$= e^{\int 2x dx} = e^{x^2}$$

$\therefore$ Solution is given by

$$y \times (e^{x^2}) = \int (2e^{-x^2} \times e^{x^2})\, dx + C$$

$\Rightarrow \quad ye^{x^2} = \int 2 dx + C$

$\Rightarrow \quad ye^{x^2} = 2x + C$

$$y = (2x + C)\, e^{-x^2}$$

132. *(d)* $\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = \left| \tan^{-1} x \right|_1^{\sqrt{3}}$

$$= \tan^{-1}(\sqrt{3}) - \tan^{-1}(1)$$

$$= \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

133. *(b)* $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$

$$= \int \left(\frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} \right) dx$$

$$= \int (\sec^2 x - \text{cosec}^2 x)$$

$$= \tan x + \cot x + C$$

134. *(c)* $\int e^x (1 + \tan x) \sec x \, dx$

$$= \int e^x (\sec x + \sec x \tan x) \, dx$$

$$= \int e^x [f(x) + f'(x)] \, dx,$$

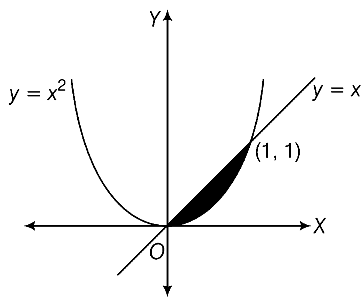
where $f(x) = \sec x$

$$= e^x f(x) + C = e^x \sec x + C$$

135. *(d)* Given curves are

$$y = x^2 \quad \ldots(i)$$

and $$y = x \quad \ldots(ii)$$



On solving Eqs. (i) and (ii), we get

$$x(x-1) = 0$$

$\Rightarrow \quad x \neq 0$ or $x = 1$

$\therefore$ Required area $= \int_0^1 (x - x^2) \, dx$

$$= \left| \frac{x^2}{2} - \frac{x^3}{3} \right|_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

136. *(a)* Let $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, where $0 < x < 1$

Put $\quad x = \tan\theta$

$\Rightarrow \quad y = \sin^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$\Rightarrow \quad y = \sin^{-1}(\cos 2\theta)$

$\Rightarrow \quad y = \sin^{-1}\left[\sin\left(\frac{\pi}{2} - 2\theta\right)\right]$

$\Rightarrow \quad y = \frac{\pi}{2} - 2\theta$

$[\because 0 < x < 1 \Rightarrow 0 < \tan\theta < 1 \Rightarrow 0 < \theta < \frac{\pi}{4} \Rightarrow 0 < 2\theta < \frac{\pi}{2}$

$\Rightarrow 0 < \frac{\pi}{2} - 2\theta < \frac{\pi}{2}]$

$\Rightarrow \quad y = \frac{\pi}{2} - 2\tan^{-1} x$

$[\because x = \tan\theta \Rightarrow \theta = \tan^{-1} x]$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{2}{1+x^2}$

137. *(b)* Given total revenue $R(x) = 5x^2 + 20x + 7$

Now, marginal revenue $= \frac{dR(x)}{dx} = \frac{d}{dx}(5x^2 + 20x + 7)$

$\Rightarrow \quad \frac{dR(x)}{dx} = 10x + 20$

when $x = 8$, then

$$\left.\frac{dR(x)}{dx}\right|_{x=8} = 10 \times 8 + 20 = 80 + 20 = 100$$

138. *(a)* We have, $y = e^{x + e^{x + e^{x + \ldots\infty}}}$

$\Rightarrow \quad y = e^{x+y} \quad \ldots(i)$

On taking log both sides of Eq. (i), we get

$$\log y = x + y \quad \ldots(ii)$$

On differentiating Eq. (ii) w.r.t. 'x', we get

$$\frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx}\left(\frac{1}{y} - 1\right) = 1$$

$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{1-y}$

139. *(a)* $f(x) = \sqrt{|x| - x}$ is contintious for all real values.

140. *(b)* We have, $y = \sqrt{\frac{1-x}{1+x}}$

$\Rightarrow \quad y^2 = \frac{1-x}{1+x} \quad \ldots(i)$

$\Rightarrow \quad y^2 = \frac{(1-x)(1-x)}{(1+x)(1-x)}$ [rationalising]

$\Rightarrow \quad y^2 = \frac{(1-x)^2}{1-x^2} \Rightarrow (1-x^2)y^2 = (1-x)^2$

On differentiating both sides w.r.t. 'x', we get

$$(1-x^2)\times 2y\frac{dy}{dx}+y^2\times(-2x)=2(1-x)\times(-1)$$

$\Rightarrow \quad y(1-x^2)\frac{dy}{dx}-xy^2=x-1$

$\Rightarrow \quad (1-x^2)\frac{dy}{dx}-xy=\frac{x-1}{y}$

$\Rightarrow \quad (1-x^2)\frac{dy}{dx}=\frac{x-1}{y}+xy$

$\Rightarrow \quad (1-x^2)\frac{dy}{dx}+y=\frac{x-1}{y}+xy+y$

$\Rightarrow \quad (1-x^2)\frac{dy}{dx}+y=\frac{(x-1)+(x+1)\,y^2}{y}$

$\Rightarrow \quad \dfrac{(x-1)+(x+1)\times\left(\dfrac{1-x}{x+1}\right)}{y}$

[from Eq. (i)]

$$=\frac{x-1+1-x}{y}=0$$

$\therefore \quad (1-x^2)\frac{dy}{dx}+y=0$

141. ***(b)*** For the system of equation to have unique solution,

Determinant of coefficient matrix $\neq 0$

i.e $\quad \begin{vmatrix} 2 & a & 6 \\ 1 & 2 & b \\ 1 & 1 & 3 \end{vmatrix}\neq 0$

$\Rightarrow \quad 2(6-b)-a(3-b)+6(1-2)\neq 0$

$\Rightarrow \quad 12-2b-3a+ab-6\neq 0$

$\Rightarrow \quad 3a+2b-6-ab\neq 0$

$\Rightarrow \quad (a-2)(3-b)\neq 0$

$\Rightarrow \quad a\neq 2 \text{ or } b\neq 3$

142. ***(c)*** Let $\Delta=\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

On applying $C_1\to C_1+C_2+C_3$, we get

$$\Delta=\begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix}$$

$$=(a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

[taking common $(a+b+c)$ from c_1]

On applying $R_2\to R_2-R_1$ and $R_3\to R_3-R_1$

$$=(a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix}$$

$$=(a+b+c)[-(b-c)]^2-(a-c)(a-b)]$$

$$=-(a+b+c)[(b-c)^2+(a-c)(a-b)]$$

$\Rightarrow \quad \Delta=-(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

Since, a, b and c are roots of $x^3+px+q=0$

$\therefore \quad a+b+c=0$

Hence, $\quad \Delta=0$

143. ***(d)*** We have $y=\sin mx$

$\therefore\ y_1=m\cos mx,\ y_2=-m^2\sin mx$

$y_3=-m^3\cos mx,\ y_4=m^4\sin mx$

$y_5=m^5\cos mx,\ y_6=-m^6\sin mx$

$y_7=-m^7\cos mx,\ y_8=m^8\sin mx$

Let $\Delta=\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$

On substituting the values, we get

$$\Delta=\begin{vmatrix} \sin mx & m\cos mx & -m^2\sin mx \\ -m^3\cos mx & m^4\sin mx & m^5\cos mx \\ -m^6\sin mx & -m^7\cos mx & m^8\sin mx \end{vmatrix}$$

$$\Delta=-m^6\begin{vmatrix} \sin mx & m\cos mx & -m^2\sin mx \\ -m^3\cos mx & m^4\sin mx & m^5\cos mx \\ \sin mx & m\cos mx & -m^2\sin mx \end{vmatrix}$$

$=0 \quad$ [$\because$ row 1 and row 3 are identical]

144. ***(c)*** Given, $A=\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$

$\therefore \quad |A|=-4-15=-19$

adj $A=\begin{bmatrix} -2 & -5 \\ -3 & 2 \end{bmatrix}^T=\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$

Hence, $A^{-1}=\frac{1}{|A|}\text{ adj }(A)=\frac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$

Now, given $A^{-1}=\lambda A$

$\Rightarrow \quad \frac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}=\lambda\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} \frac{2}{19} & \frac{3}{19} \\ \frac{5}{19} & \frac{-2}{19} \end{bmatrix}=\begin{bmatrix} 2\lambda & 3\lambda \\ 5\lambda & -2\lambda \end{bmatrix}$

On comparing, we get $\lambda=\frac{1}{19}$

145. *(c)* We have,

$$A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}; B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

Now, given $\quad A^2 = B$

$$\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

On comparing both sides, we get

$\Rightarrow \quad \alpha^2 = 1$ and $\alpha + 1 = 5$

$\Rightarrow \quad \alpha = \pm 1$ and $\alpha = 4$.

$\because$ Both the values cannot occur simultaneously.

$\therefore$ No value of α is possible for which $A^2 = B$.

146. *(d)* We have, $\sin^{-1}\left(\frac{8}{17}\right) + \sin^{-1}\left(\frac{3}{5}\right)$

$$= \sin^{-1}\left\{\frac{8}{17}\sqrt{1 - \left(\frac{3}{5}\right)^2} + \frac{3}{5}\sqrt{1 - \left(\frac{8}{17}\right)^2}\right\}$$

$[\because \sin^{-1} x + \sin^{-1} y = \sin^{-1}\{x\sqrt{1 - y^2} + y\sqrt{1 - x^2}\}]$

$$= \sin^{-1}\left\{\left(\frac{8}{17} \times \frac{4}{5}\right) + \left(\frac{3}{5} \times \frac{15}{17}\right)\right\}$$

$$= \sin^{-1}\left\{\frac{32}{85} + \frac{45}{85}\right\} = \sin^{-1}\left(\frac{77}{85}\right)$$

$$= \tan^{-1}\frac{\frac{77}{85}}{\sqrt{1 - \left(\frac{77}{85}\right)^2}} \qquad \left[\because \sin^{-1} x = \tan^{-1}\frac{x}{\sqrt{1 - x^2}}\right]$$

$$= \tan^{-1}\left(\frac{77}{36}\right)$$

147. *(a)* Given, $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6} \qquad \ldots(\text{i})$

Also, $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \qquad \ldots(\text{ii})$

On solving, Eqs. (i) and (ii), we get

$\sin^{-1} x = \frac{\pi}{3}$ and $\cos^{-1} x = \frac{\pi}{6}$

$\therefore \quad x = \frac{\sqrt{3}}{2}$

148. *(c)* We have,

$$\cos^{-1}\sqrt{p} + \cos^{-1}\sqrt{1 - p} + \cos^{-1}\sqrt{1 - q} = \frac{3\pi}{4}$$

$$\Rightarrow \cos^{-1}\sqrt{p} = \cos^{-1}\sqrt{1 - p} = \cos^{-1}\sqrt{1 - q} = \frac{\pi}{4}$$

$$\Rightarrow \cos^{-1}\sqrt{1 - q} = \frac{\pi}{4} \Rightarrow \sqrt{1 - q} = \sqrt{\frac{1}{2}}$$

$$\Rightarrow 1 - q = \frac{1}{2} \text{ or } q = \frac{1}{2}$$

149. *(d)* Let $f(x) = \cos x$

$\Rightarrow f'(x) = -\sin x$

For strictly decreasing, $f'(x) < 0 \Rightarrow -\sin x < 0$

$\Rightarrow \sin x > 0$, which is true for all $x \in (0, \pi)$

150. *(c)* Given, ${}^nP_4 = 20 \times {}^nP_2$

$$\Rightarrow \quad \frac{n!}{(n - 4)!} = 20 \times \frac{n!}{(n - 2)!}$$

$$\Rightarrow \quad \frac{1}{(n - 4)!} = \frac{20}{(n - 2)(n - 3)(n - 4)!}$$

$\Rightarrow \quad (n - 2)(n - 3) = 20$

$\Rightarrow \quad n^2 - 5n - 14 = 0$

$\Rightarrow \quad (n + 2)(n - 7) = 0$

$\Rightarrow \quad n = 7 \qquad (\because n = -2$ is not possible$)$