Solved Paper 2015
AMU
Engineering Entrance Exam

Physics

1. An oil drop of n excess electrons is held stationary under a constant electric field E in Millikan's oil drop experiment. The density of oil is ρ. The radius of the drop is

(a) $\left[\frac{3neE}{2\pi\rho g}\right]^{1/2}$ (b) $\left[\frac{3n\rho g}{2\pi eE}\right]^{1/2}$

(c) $\left[\frac{3nEe}{4\pi\rho g}\right]^{1/3}$ (d) $\left[\frac{3n\rho g}{4\pi eE}\right]^{1/3}$

2. The potential at a point x (measured in μm) due to some charges situated on the x-axis is given by

$$V(x) = \frac{20}{x^2 - 4} \text{ volt}$$

The electric field E at $x = 4\,\mu$m is given by

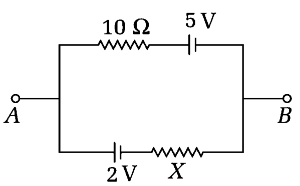
(a) $\left(\frac{10}{9}\right)$ volt/μm and in the +ve x-direction

(b) $\left(\frac{5}{3}\right)$ volt/μm and in the –ve x-direction

(c) $\left(\frac{5}{3}\right)$ volt/μm and in the +ve x-direction

(d) $\left(\frac{20}{9}\right)$ volt/μm and in the –ve x-direction

3. In the following circuit, if potential difference between A and B is $V_{AB} = 4$V, then the value of X will be



(a) 5 Ω
(b) 10 Ω
(c) 15 Ω
(d) 20 Ω

4. A wire when connected to 220 V mains supply has power dissipation P_1. Now the wire is cut into two equal pieces which are connected in parallel to the same supply. Power dissipation in this case is P_2. Then, $P_2 : P_1$ is

(a) 1 (b) 4 (c) 2 (d) 3

5. The resistance of the series combination of two resistors is S. When they are joined in parallel the total resistance is P. If $S = nP$, then the minimum possible value of n is

(a) 4 (b) 3 (c) 2 (d) 1

6. A triangular loop of side l carries a current i. It is placed in a magnetic field B, such that the plane of the loop is in the direction of B. The torque on the loop is

(a) iBl (b) i^2Bl (c) $\frac{\sqrt{3}}{4}Bil^2$ (d) infinity

7. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B. It is then bent into a circular loop of n turns. The magnetic field at the centre of the coil will be

(a) nB (b) n^2B (c) $2nB$ (d) $2n^2B$

8. A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60°. The torque needed to maintain the needle in this position will be

(a) $\sqrt{3}W$ (b) W (c) $\left(\frac{\sqrt{3}}{2}\right)W$ (d) $2W$

9. The self inductance of the motor of an electric fan is 10H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of

(a) 1 μF (b) 2 μF
(c) 3 μF (d) 8 μF

10. An alternating current is given by $i = i_1 \cos \omega t + i_2 \sin \omega t$. The rms current is given by

(a) $\frac{i_1 + i_2}{2}$ (b) $\frac{|i_1 + i_2|}{2}$ (c) $\frac{\sqrt{i_1^2 + i_2^2}}{2}$ (d) $\frac{\sqrt{i_1^2 + i_2^2}}{\sqrt{2}}$

11. If the wavelengths of light used in an optical instrument are $\lambda_1 = 4000 Å$ and $\lambda_2 = 5000 Å$, then ratio of their respective resolving powers (corresponding to λ_1 to λ_2) is

(a) 16 : 25 (b) 9 : 1 (c) 4 : 5 (d) 5 : 4

12. Refractive index of glass is 1.520 for red light and 1.525 for blue light. Let D_1 and D_2 be angles of minimum deviation for red and blue light respectively in a prism of this glass. Then,

(a) $D_1 > D_2$
(b) $D_1 < D_2$
(c) $D_1 = D_2$
(d) D_1 can be less than or greater than depending upon angle of prism

13. The maximum number of possible interference maxima for slit separation equal to twice the wavelength in Young's double slit experiment is

(a) infinite (b) five (c) three (d) zero

14. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately

(a) 540 nm (b) 400 nm (c) 310 nm (d) 220 nm

15. A proton when accelerated through a potential difference of V volts has a wave length λ associated with it. An α-particle in order to have the same wavelength λ, must be accelerated through a potential difference (in volts)

(a) $2V$ (b) V (c) $\frac{V}{4}$ (d) $\frac{V}{8}$

16. Frequency of the series limit of Balmer series of hydrogen atom in terms of Rydberg constant R and speed of light c is

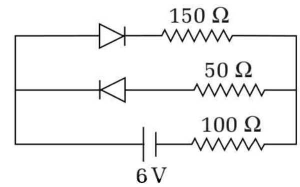
(a) Rc (b) $4Rc$ (c) $\frac{4}{Rc}$ (d) $\frac{Rc}{4}$

17. In the following nuclear reaction, how many α and β-particles are emitted?

$$_{92}U^{238} \rightarrow {}_{82}Pb^{206}$$

(a) $8\alpha, 6\beta$ (b) $6\alpha, 10\beta$
(c) $8\alpha, 8\beta$ (d) $12\alpha, 6\beta$

18. The circuit shown in figure contains two diodes, each with a forward resistance of 50Ω and with infinite reverse resistance. If the battery voltage is 6V, the current through 100Ω resistance is



(a) zero (b) 0.02 A
(c) 0.03 A (d) 0.036 A

19. Which of the following statements is not true for Zener diode?

(a) It is used as a voltage regulator
(b) It is fabricated by heavily doping both p and n sides of the junctions
(c) It depletion region is very thin
(d) The electric field of the junction is very low

20. For an amplitude modulated wave the maximum amplitude is found to be 10V while the minimum amplitude is found to be 2V, the modulation index is

(a) 5 (b) 0.2 (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

21. The dimension of magnetic field in M, L, T and C (coulomb) is given as

(a) $[MLT^{-1}C^{-1}]$ (b) $[MT^{-2}C^{-1}]$
(c) $[MT^{-2}C^{-2}]$ (d) $[MT^{-1}C^{-1}]$

22. A particle located at $x = 0$ at time $t = 0$, starts moving the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$, where α is dimensionless constant. The displacement of the particle varies with time as

(a) t^3 (b) t^2 (c) t (d) $t^{1/2}$

23. From a building two balls A and B are thrown such that A is thrown upwards and B downwards with the same speed (both vertically). If v_A and v_B are their respective velocities on reaching the ground, then

(a) $v_B > v_A$
(b) $v_A = v_B$
(c) $v_A > v_B$
(d) their velocities depend on their masses

24. A shell fired from a gun at sea level rises to a maximum height of 5 km when fired at a ship 20 km away. The muzzle velocity should be

(a) 7 m/s (b) 14 m/s (c) 28 m/s (d) 56 m/s

25. Two racing cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that each makes a complete circle in the same time t. The ratio of the angular speeds of the first to the second car is

(a) $r_1 : r_2$ (b) $m_1 : m_2$
(c) 1 : 1 (d) $m_1 m_2 : r_1 r_2$

26. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force P is applied at the free end of the rope, the force exerted by the rope on the block is

(a) $\frac{Pm}{(M+m)}$ (b) $\frac{Pm}{(M-m)}$
(c) P (d) $\frac{PM}{(M+m)}$

27. A man fires a bullet of mass 200 g at a speed of 5 m/s. The gun is of 1 kg mass. By what velocity the gun rebounds backward?

(a) 1 m/s (b) 0.01 m/s
(c) 0.1 m/s (d) 10 m/s

28. A man weighs 80 kg. He stands on a weighing scale in a lift which is moving upwards with a uniform acceleration of $5\,m/s^2$. What would be the reading on the scale? ($g = 10\,m/s^2$)

(a) Zero (b) 400 N
(c) 800 N (d) 1200 N

29. A wire of length 100 cm is connected to a cell of emf 2 V and negligible internal resistance. The resistance of the wire is $3\,\Omega$, the additional resistance required to produce a potential difference of 1 mV/cm on the wire is

(a) $297\,\Omega$ (b) $60\,\Omega$
(c) $57\,\Omega$ (d) $35\,\Omega$

30. The only force acting on a 2.0 kg body as it moves along a positive x-axis has an x-component $F_x = -6x$ with x in metres. The velocity at $x = 3.0$ m is 8.0 m/s. The velocity of the body at $x = 4.0$ m is

(a) 6.6 m/s (b) 46.6 m/s
(c) 60 m/s (d) 96.6 m/s

31. A quarter horse power motor runs at a speed of 600 rpm. Assuming 40% efficiency, the work done by the motor in the one rotation will be

(a) 7.46 J (b) 74.6 J
(c) 7400 J (d) 7.46 erg

32. A rocket is launched vertically upward from the surface of the earth with an initial velocity of 10 km/s. If the radius of the earth is 6400 km and atmospheric resistance is negligible. Find the distance above the surface of the earth that the rocket will go.

(a) 2.5×10^4 km (b) 3.0×10^4 km
(c) 4.0×10^3 km (d) 3.0×10^3 km

33. Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved so as to keep the centre of mass at the same position?

(a) d (b) $\frac{m_2}{m_1}d$
(c) $\frac{m_1}{m_1+m_2}d$ (d) $\frac{m_1}{m_2}d$

34. A body of mass M while falling vertically downwards under gravity breaks into two parts a body B of mass $\frac{1}{3}M$ and body C of mass $\frac{2}{3}M$. The centre of mass of bodies B and C taken together shifts compared to that of body A towards

(a) body C
(b) body B
(c) does not shift
(d) depends on height of breaking

35. A meter stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. The mass of the meter stick is

(a) 13 g (b) 33 g (c) 66 g (d) 77 g

36. A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then new angular momentum is

(a) $\frac{L}{4}$ (b) $2L$ (c) $4L$ (d) $\frac{L}{2}$

37. Suppose the gravitational force varies inversely as the nth power of distance. Then, the time period of a planet in circular orbit of radius R around the sun will be proportional to

(a) $R^{\left(\frac{n+1}{2}\right)}$ (b) $R^{\left(\frac{n-1}{2}\right)}$ (c) R^n (d) $R^{\left(\frac{n-2}{2}\right)}$

38. A geo-stationary satellite is in an orbit of radius 36000 km. Approximately what would be the time period of a spy satellite orbiting a few hundred kilometres above the surface of the earth? (Earth radius = 6400 km)

(a) 1 h (b) 2 h
(c) 4 h (d) 8 h

39. If S is the stress and Y is the Young's modulus of material of a wire, the energy stored in the wire per unit volume is

(a) $\frac{2Y}{S}$ (b) $\frac{S}{2Y}$
(c) $2S^2Y$ (d) $\frac{S^2}{2Y}$

40. Two identical cylindrical vessels with their bases at the same level, each contains a liquid of density 1.3×10^3 kg/m^3. The area of each base is $4.00\ \text{cm}^2$, but in one vessel, the liquid height is 0.854 m and in the other it is 1.560 m. Find the work done by the gravitational force in equalizing the levels when the two vessels are connected.

(a) 0.0635 J (b) 0.635 J (c) 6.35 J (d) 63.5 J

41. A wire extends by 1 mm when a force is applied. Double the force is applied to another wire of the same material and length but half the radius of cross-section. The elongation of the wire in mm will be

(a) 8 (b) 4 (c) 2 (d) 1

42. The energy density $\frac{u}{V}$ of an ideal gas is related to its pressure p as

(a) $\frac{u}{V} = 3p$ (b) $\frac{u}{V} = \frac{3}{2}p$
(c) $\frac{u}{V} = \frac{1}{3}p$ (d) $\frac{u}{V} = \frac{2}{3}p$

43. 1kg of a diatomic gas is at a pressure of 8×10^4 N/m^2. The density of the gas is $4\ \text{kg/m}^3$, what is the energy of the gas due to its thermal motion?

(a) 3×10^4 J (b) 5×10^4 J
(c) 6×10^4 J (d) 7×10^4 J

44. A refrigerator is to maintain eatables at 9°C. If room temperature is 36°C, then the coefficient of performance is

(a) 8.6 (b) 10.4
(c) 11.2 (d) 12.5

45. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is the difference of the temperature of the body and surroundings, and n is equal to

(a) four (b) three (c) two (d) one

46. Two spheres of the same material have radii 1 m and 4 m and temperatures 4000 K and 2000 K respectively., The ratio of energy radiated per second by the first sphere to the second is

(a) 1 : 1 (b) 16 : 1
(c) 4 : 1 (d) 1 : 9

47. A whistle producing sound waves of frequency 9500 Hz and above is approaching a stationary person with speed v m/s. The velocity of sound in air is 300 m/s. If the person can hear frequencies up to maximum of 10000 Hz, the maximum value of v up to which he can hear the whistle is

(a) 30 m/s (b) $15\sqrt{2}$ m/s
(c) $\frac{15}{\sqrt{2}}$ m/s (d) 15 m/s

48. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos nt$ metre. The time at which the maximum speed first occurs is

(a) 0.25 s (b) 0.50 s
(c) 0.75 s (d) 0.125 s

49. Two concentric spherical shells of radii r_1 and r_2 have similar charges and equal surface charge densities (σ). What is the potential at the common centre?

(a) $\frac{\sigma}{\varepsilon_0}(r_1 + r_2)$ (b) $\frac{\sigma}{\varepsilon_0}(r_1 - r_2)$
(c) $\frac{\sigma}{\varepsilon_0} \cdot \frac{r_1^2}{r_2}$ (d) $\frac{\sigma}{\varepsilon_0} \cdot \frac{r_2^2}{r_1}$

50. The electric field in a region is given by $\mathbf{E} = \left(\frac{A}{x^3}\right)\hat{\mathbf{i}}$. An expression for the potential in the region assuming the potential at infinity to be zero is

(a) $\frac{A}{2x^3}$ (b) $\frac{2x^3}{A}$
(c) $\frac{A^2}{2x}$ (d) $\frac{A}{2x^2}$

Chemistry

1. What is the minimum volume of water required to dissolve 1 g of calcium sulphate at 298K? K_{sp} for $CaSO_4$ is 9.0×10^{-6}.
(Molar mass of $CaSO_4$ = 136 g mol^{-1})

(a) 2.45 L (b) 4.08 L
(c) 4.90 L (d) 3.00 L

2. What will be the value $[OH]^{-2}$ in the 0.1M solution of ammonium hydroxide having $K_b = 1.8 \times 10^{-5}$?

(a) 1.8×10^{-6} (b) 1.8×10^{-5}
(c) 1.8×10^{-4} (d) 1.8×10^{-3}

3. One mole of $N_2O_4(g)$ at 300K is kept in a closed vessel at 1 atm pressure. It is heated to 600K when 20% by mass of $N_2O_4(g)$ decomposes to $NO_2(g)$. The resultant pressure is

(a) 1.2 atm (b) 2.4 atm (c) 2.0 atm (d) 1.0 atm

4. A boy after swimming comes out from a pool covered with a film of water weighing 80 g. How much heat must be supplied to evaporate this water? (ΔH_v° = 40.79 kJ mol^{-1})

(a) 1.61×10^2 kJ (b) 1.71×10^2 kJ
(c) 1.81×10^2 kJ (d) 1.91×10^2 kJ

5. Which of the following is planar?

(a) XeO_4 (b) XeO_3F
(c) XeO_2F_2 (d) XeF_4

6. Schrodinger wave equation, for a particle in a one dimension box is

(a) $\frac{\delta^2\psi}{\delta x^2} + \frac{2m}{h}(E - \infty)\psi = 0$

(b) $\frac{\delta^2\psi}{\delta x^2} + \frac{8\pi^2 m}{h^2}(E - V) = 0$

(c) $\frac{\delta^2\psi}{\delta x^2} + \frac{2m}{h}(E - V)\varphi = 0$

(d) $\frac{\delta^2\psi}{\delta x^2} + \frac{8\pi^2 m}{h^2}(E - \infty) = 0$

7. The number of Cl^- ions in 100 mL of 0.001 M HCl solution is

(a) 6.022×10^{23}
(b) 6.022×10^{20}
(c) 6.022×10^{19}
(d) 6.022×10^{24}

8. The energy associated with first orbital of hydrogen atom is

(a) -8.72×10^{-18} J (b) -8.80×10^{-18} J
(c) -87.0×10^{-18} J (d) -8.82×10^{-19} J

9. The energy of one mole of protons of radiation whose frequency is 5×10^{14} Hz is

(a) 192.51 kJ mol^{-1} (b) 199.51 kJ mol^{-1}
(c) 195.45 kJ mol^{-1} (d) 170.00 kJ mol^{-1}

10. The number of unshared valence electron pairs in XeF_2 is

(a) 2 (b) 4
(c) 3 (d) 1

11. Mustard gas is

(a) $COCl_2$
(b) CCl_3NO_3
(c) $CHCl_2NO_2$
(d) $[ClCH_2—CH_2SCH_2\ CH_2Cl]$

12. Amorphous solids are characterised by property as

(a) isotropic (b) anisotropic
(c) sharp melting point (d) true solid

13. The species having no S—S bond is

(a) $S_2O_4^{2-}$ (b) $S_2O_3^{2-}$ (c) $S_2O_7^{2-}$ (d) $S_2O_5^{2-}$

14. Television picture tubes are

(a) cathode-ray tubes (b) α-particle tube
(c) γ-rays tube (d) X-ray tube

15. Density of 3M NaCl solution is 1.25 g/cc. The molality of solution is

(a) 2.79 molal (b) 0.279 molal
(c) 1.279 molal (d) 3.85 molal

16. Molecular structure of XeO_3 and $XeOF_4$ respectively are

(a) trigonal planar and octahedral
(b) pyramidal and square pyramidal
(c) pyramidal and trigonal bipyramidal
(d) both have imperfect tetrahedral shape

17. Which of the following defects are present in AgBr and ZnS crystal systems?

(a) Frenkel and Schottky
(b) Schottky and Frenkel
(c) Frenkel and Frenkel
(d) Schottky and Schottky

18. The hybridisation of atomic orbital of nitrogen in NO_2^+, NO_3^- and NH_4^+ are respectively

(a) sp, sp^2, sp^3 (b) sp^2, sp^3, sp
(c) sp^2, sp, sp^3 (d) sp^2d, sp^3, sp

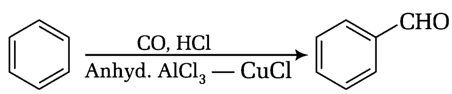
19. The correct order of increasing polarising power of the cations in the following is $AlCl_3, MgCl_2, NaCl$

(a) $AlCl_3 < MgCl_2 < NaCl$
(b) $MgCl_2 < NaCl < AlCl_3$
(c) $NaCl < MgCl_2 < AlCl_3$
(d) $NaCl < AlCl_3 < MgCl_2$

20. Which of the following oxides of nitrogen is not an air pollutant?

(a) NO_2 (b) N_2O (c) N_2O_5 (d) NO

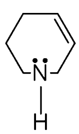
21. The chemical reaction,

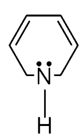


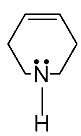
is known as

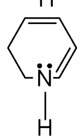
(a) Gattermann reaction
(b) Tishchenko reaction
(c) Gattermann-Koch reaction
(d) Frankland reaction

22. Which one of the following is an aromatic compound?

(a) 

(b) 

(c) 

(d) 

23. The number of possible alcohol isomers for $C_4H_{10}O$ is

(a) 4 (b) 3 (c) 2 (d) 5

24. Intramolecular hydrogen bonding is found in

(a) *o*-nitrophenol (b) *m*-nitrophenol
(c) *p*-nitrophenol (d) phenol

25. The heat produced on combustion of methane is approximately

(a) 890 kJ per g (b) 74.2 kJ per g
(c) 55.6 kJ per g (d) 49.5 kJ per g

26. Maximum number of σ-bonds that may be present in an isomer of C_4H_8 are

(a) 10 (b) 11 (c) 12 (d) 13

27. Tick the statement which is not true.

(a) Boiling point of ethanol is greater than ethoxyethane due to H-bonding
(b) Ethoxyethane is soluble in water due to H-bonding
(c) Ethanol is soluble in water due to H-bonding
(d) Ethoxyethane has nearly same boiling point as that of propane

28. Which one of the following statements is not true about vitamins?

(a) They were known as vitamins in their early discovery
(b) Vitamin C and D are water soluble
(c) Deficiency of vitamin B_6 is responsible for convulsions
(d) Thiamine is one from the class vitamin B

29. Tick the statement which is not true about carboxylic acids?

(a) Higher carboxylic acids are odourless
(b) Benzoic acid is insoluble in water
(c) Acetic acid exists as dimer in vapour phase
(d) Carboxylic acids show higher boiling point than alcohols of comparable molecular mass

30. Which one of the following alcohols is known as wood's spirit?

(a) Methanol (b) Ethanol (c) Propanol (d) Butanol

31. Which one of the following is an essential amino acid?

(a) Valine (b) Serine (c) Cystein (d) Proline

32. Polymer with low degree of polymerisation is known as

(a) higher polymer (b) oligomer
(c) macro molecule (d) copolymer

33. The reaction, $R\text{—}NH_2 + CHCl_3 + 3KOH \xrightarrow{\text{Heat}} R\text{—}NC + 3KCl + 3H_2O$

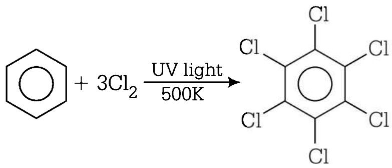
is known as

(a) Gabriel phthalimide synthesis
(b) Hofmann reaction
(c) Carbylamine reaction
(d) Leibermann nitroso reaction

34. Schiff's base is formed by the reaction of aldehyde with

(a) amine (b) alcohol
(c) phenol (d) carboxylic acid

35. The chemical reaction,



is

(a) a substitution reaction
(b) an addition reaction
(c) an elimination reaction
(d) a rearrangement reaction

36. $:CCl_2$ is

(a) an electrophile (b) a free radical
(c) a nucleophile (d) None of these

37. DIBAL-H is

(a) AlH $(i\text{-Bu})_2$ (b) $Al(OC_2H_5)_3$
(c) $Al[(CH_3)_2CHO]_3$ (d) $AlCl_3$

38. Which of the following is not correct?

(a) Chemical adsorption is reversible in nature
(b) Physical adsorption is reversible in nature
(c) ΔH is small in physical adsorption
(d) ΔH is large in chemical adsorption

39. What is the mole fraction of the solute in 2.5 molal aqueous solution?

(a) 0.043 (b) 0.053
(c) 0.063 (d) 0.073

40. A solution containing 2.44g of a solute dissolved in 75g of water boiled at 100.413°C. What will be the molar mass of the solute (K_b for water = 0.52 K kg mol^{-1})?

(a) 40.96 g mol^{-1}
(b) 20.48 g mol^{-1}
(c) 81.92 g mol^{-1}
(d) None of the above

41. In the lead-acid battery during charging, the cathode reaction is

(a) formation of PbO_2
(b) formation of $PbSO_4$
(c) reduction of Pb^{2+} to Pb
(d) decomposition of Pb at the anode

42. Which concentration plot is linear for a first order reaction?

(a) [*A*] *versus* time
(b) In [*A*] *versus* time
(c) log [*A*] *versus* 1/time
(d) Square root of [*A*] *versus* time

43. For the reaction,
$H_2F_2(g) \longrightarrow H_2(g) + F_2(g); \Delta E = -14.2$ k is cal/mole at 25°C. The change in enthalpy of the reaction is

(a) 13.6 kJ/mol (b) – 13.6 kJ/mol
(c) 1.36 kJ/mol (d) – 56.87 kJ/mol

44. In the reversible reaction, $2NO_2 \underset{k_2}{\overset{k_1}{\rightleftharpoons}} N_2O_4$, the rate of disappearance of NO_2 is equal to

(a) $\frac{2k_1}{k_2}[NO_2]^2$
(b) $2k_1[NO_2] - 2k_2[N_2O_4]$
(c) $2k_1[NO_2]^2 - 2k_2[N_2O_4]$
(d) $(2k_1 - k_2)[NO_2]$

45. A metal oxide has the empirical formula $M_{0.96}O_{1.00}$. What will be the percentage of M^{2+} ions in the crystal?

(a) 90.67 (b) 91.67 (c) 8.33 (d) 9.33

46. 2 g of benzoic acid (C_6H_5COOH) is dissolved in 25 g of benzene. The observed molar mass of benzoic acid is found to be 241.98. What is the percentage association of acid if it forms in solution?

(a) 85.2% (b) 89.2% (c) 95.2% (d) 99.2%

47. 0.6 mL of acetic acid is dissolved in 1 L of water. The value of van't Hoff factor is 1.04. What will be the degree of dissociation of the acetic acid?

(a) 0.01 (b) 0.02 (c) 0.03 (d) 0.04

48. What pressure of H_2 would be required to make emf of the hydrogen electrode zero in water at 25°C?

(a) 10^{-7} atm (b) 10^{-14} atm
(c) 1 atm (d) 0.5 atm

49. The root mean square velocity of an ideal gas at constant pressure varies with density (d) as

(a) d^2 (b) d (c) $\sqrt{d}$ (d) $\frac{1}{\sqrt{d}}$

50. How many grams of potassium dichromate are required to oxidise 20.0 g of Fe^{2+} in $FeSO_4$ to Fe^{3+}, if the reaction is carried out in an acidic medium? Molar masses of $K_2Cr_2O_7$ and $FeSO_4$ are 294 and 152 respectively.

(a) 6.45 g (b) 7.45 g
(c) 8.45 g (d) 9.45 g

Mathematics

1. Let $A = [a_{ij}]_{m \times n}$ be a matrix such that $a_{ij} = 1$, $\forall\ i, j$. Then,

(a) rank $(A) > 1$ (b) rank $(A) = 1$
(c) rank $(A) = m$ (d) rank $(A) = n$

2. The equation $\sqrt{(x-2)^2 + y^2} + \sqrt{(x+2)^2 + y^2} = 4$ represents

(a) circle (b) a parabola
(c) pair of straight lines (d) an ellipse

3. $\lim_{n \to \infty} \frac{(n!)^{1/n}}{n}$ equals

(a) e (b) e^{-1}
(c) 1 (d) None of these

4. If $\int_0^\infty \frac{x^2}{(x^2+a^2)(x^2+b^2)(x^2+c^2)} dx = \frac{\pi}{2(a+b)(b+c)(c+a)}$, then $\int_0^\infty \frac{dx}{(x^2+4)(x^2+9)}$ is equal to

(a) $\frac{\pi}{60}$ (b) $\frac{\pi}{20}$ (c) $\frac{\pi}{40}$ (d) $\frac{\pi}{80}$

5. For all real x, $4^{\sin^2 x} + 4^{\cos^2 x}$ is

(a) ≥ 4 (b) > 4
(c) < 4 (d) None of these

6. The length of the perpendicular drawn from (1, 2, 3) to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ is

(a) 4 units (b) 5 units (c) 6 units (d) 7 units

7. The number of equivalence relations on the set {1, 2, 3} containing (1, 2) and (2, 1) is

(a) 3 (b) 1
(c) 2 (d) None of these

8. Let $A = \left\{x \in R;\ x \geq \frac{1}{2}\right\}$ and $B = \left\{x \in R;\ x \geq \frac{3}{4}\right\}$. If $f : A \to B$ is defined as $f(x) = x^2 - x + 1$, then the solution set of the equation $f(x) = f^{-1}(x)$ is

(a) {1} (b) {2}
(c) $\left\{\frac{1}{2}\right\}$ (d) None of these

9. The complex number $z = x + iy$ satisfy the equation $\left|\frac{z-5i}{z+5i}\right| = 1$ lies on

(a) the X-axis
(b) the straight line $y = 5$
(c) a circle passing through origin
(d) None of the above

10. The equation of the curve satisfying the differential equation $y(x+y^3)dx = x(y^3 - x)dy$ and passing through the point (1, 1) is

(a) $y^3 - 2x + 3x^2y = 0$ (b) $y^3 + 2x + 3x^2y = 0$
(c) $y^3 + 2x - 3x^2y = 0$ (d) None of the above

11. If $\lim_{x \to a} \frac{a^x - x^a}{x^x - a^a} = -1$, then a is equal to

(a) 0 (b) e
(c) 2 (d) None of the above

12. Let $f(x) = \sin x, g(x) = x^2$ and $h(x) = \log_e x$. If $G(x) = hogof(x)$, then $G''(x)$ is equal to

(a) $2\operatorname{cosec}^2 x \cot x$ (b) $-2\operatorname{cosec}^2 x \cot x$
(c) $2\operatorname{cosec}^2 x$ (d) $-2\operatorname{cosec}^2 x$

13. For any integer n, the integral $\int_0^\pi e^{\cos^2 x} \cos^3 (2n+1)x\, dx$ has the value

(a) 1 (b) π (c) 2π (d) None of these

14. The area defined by $1 \leq |x-2| + |y+1| \leq 2$ is (in sq units)

(a) 2 (b) 4 (c) 6 (d) None of these

15. If $C_0, C_1, C_2, \ldots, C_{15}$ are the binomial coefficients in the expansion of $(1+x)^{15}$, then $\frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_2} + \ldots + \frac{15C_{15}}{C_{14}}$ is equal to

(a) 32 (b) 64
(c) 128 (d) None of these

16. Let the function f, g and h be defined from R to R such that $f(x) = x^2 - 1$, $g(x) = \sqrt{x^2+1}$ and $h(x) = \begin{cases} 0, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$, then $(ho(fog))(x)$ is equal to

(a) x (b) x^2
(c) 0 (d) None of these

17. If AB is a diameter of a circle and C is any point on the circumference of the circle, then

(a) the perimeter of ΔABC is maximum, when it is isosceles
(b) the area of ΔABC is minimum, when it is isosceles
(c) the area of ΔABC is maximum, when it is isosceles
(d) None of the above

18. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors $(1, a, a^2)$, $(1, b, b^2)$ and $(1, c, c^2)$ are non-coplanar, then the product abc equals

(a) 2 (b) – 1 (c) 1 (d) 0

19. If $\mathbf{a} = 4\hat{\mathbf{i}} + 6\hat{\mathbf{j}}$ and $\mathbf{b} = 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$, then the vector form of component of $\mathbf{a}$ along $\mathbf{b}$ is

(a) $\frac{18}{10\sqrt{3}}(3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ (b) $\frac{18}{25}(3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$
(c) $\frac{18}{\sqrt{3}}(3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ (d) $\frac{18}{25}(4\hat{\mathbf{i}} + 6\hat{\mathbf{j}})$

20. Let $f(x) = \sin x - \tan x$, $x \in (0, \pi / 2)$, then tangent drawn to the curve $y = f(x)$ at any point will

(a) lie above the curve (b) lie below the curve
(c) Nothing can be said (d) be parallel to a fixed line

21. If $[x]$ denotes the greatest integer $\leq x$, then the $\lim_{x \to 0} |x|^{[\cos x]}$ is equal to

(a) 0 (b) 1
(c) – 1 (d) Does not exist

22. The differential equation of all conics whose centre lies as origin, is of order

(a) 2 (b) 3
(c) 4 (d) None of the above

23. Let $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ be three vectors such that $\mathbf{a} \times \mathbf{b} = \mathbf{c}$ and $\mathbf{c} \times \mathbf{a} = \mathbf{b}$, then

(a) $\mathbf{a} \cdot \mathbf{b} = |\mathbf{c}|^2$ (b) $\mathbf{c} \cdot \mathbf{a} = |\mathbf{b}|^2$ (c) $\mathbf{b} \cdot \mathbf{c} = |\mathbf{a}|^2$ (d) $\mathbf{a} || \mathbf{b} \times \mathbf{c}$

24. If $\cos^{-1} x > \sin^{-1} x$, then x belongs to the interval

(a) $(-\infty, 0)$ (b) $(-1, 0)$ (c) $\left[0, \frac{1}{\sqrt{2}}\right)$ (d) $\left[-1, \frac{1}{\sqrt{2}}\right)$

25. The number of non-zero diagonal matrices of order 4 satisfying $A^2 = A$ is

(a) 2 (b) 4 (c) 16 (d) 15

26. The number of ways in which 52 cards can be divided into 4 sets, three of them having 17 cards each and the fourth one having just one card, is

(a) $\frac{52!}{(17!)^3}$ (b) $\frac{52!}{(17!)^3 3!}$ (c) $\frac{51!}{(17!)^3}$ (d) $\frac{51!}{(17!)^3 3!}$

27. If R is a relation on a finite set having n elements, then the number of relations on A is

(a) 2^n (b) 2^{n^2} (c) n^2 (d) n^n

28. If the area bounded by the curve $y = f(x)$, X-axis and the abscissae $x = 1$ and $x = b$ is $(b - 1)\sin(3b + 4)$, then $f(x)$ is

(a) $(x - 1)\cos(3x + 4)$
(b) $\sin(3x + 4)$
(c) $\sin(3x + 4) + 3(x - 1)\cos(3x + 4)$
(d) None of the above

29. If the angles of a triangle are in the ratio 4 :1:1, then the ratio of} the longest side and perimeter is

(a) $\sqrt{3} : 2 + \sqrt{3}$ (b) $1 : 6$ (c) $1 : 2 + \sqrt{3}$ (d) $2 : 6$

30. If $|z - 1| + |z + 3| = 8$, then the range of value of $|z - 4|$ is

(a) (0, 8) (b) [0, 8] (c) [1, 9] (d) [5, 9]

31. If a, b and c are the roots of equation $x^3 - px^2 + qx - r = 0$, then the value of $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$ is

(a) $\frac{p^2 - 2qr}{r^2}$ (b) $\frac{q^2 - 2pr}{r^2}$
(c) $\frac{r^2 - 2pq}{q^2}$ (d) $\frac{r^2 - 2pq}{p^2}$

32. If $\int f(x)\, dx = F(x)$, then $\int x^3 f(x^2)\, dx$ is equal to

(a) $\frac{1}{2}\{x^2F(x^2) - \int F(x^2)\, dx^2\}$
(b) $\frac{1}{2}\{x^2F(x^2) - \int F(x^2)\, dx\}$
(c) $\frac{1}{2}\left\{x^2F(x) - \frac{1}{2}\int F(x^2)\, dx\right\}$
(d) None of the above

33. In three dimensional space, $x^2 - 5x + 6 = 0$ represents

(a) two points (b) two parallel planes
(c) two parallel lines (d) a pair of non-parallel lines

34. If $f(x)=\log_x(\log x)$, then $\frac{d}{dx}(f(x))$ at $x=e$ is

(a) $\frac{1}{e}$ (b) e

(c) $e+1$ (d) None of the above

35. Let $f(x)=\begin{cases}\frac{x^3+x^2-16x+20}{(x-2)^2}, & \text{if } x\neq 2\\ b, & \text{if } x=2\end{cases}$.

If $f(x)$ is continuous for all x, then b is equal to

(a) 7 (b) 3 (c) 2 (d) 5

36. The locus of the middle points of the chords of the circle $x^2+y^2=a^2$, which subtend a right angle at the centre, is

(a) $x^2+y^2=\frac{a^2}{2}$ (b) $x^2+y^2=2a^2$

(c) $x^2+y^2=\frac{a^2}{4}$ (d) None of the above

37. An integrating factor of the differential equation $y\log y\frac{dx}{dy}+x-\log y=0$ is

(a) $\log(\log y)$ (b) $\log y$

(c) $\frac{1}{\log y}$ (d) $\frac{1}{\log(\log y)}$

38. A fair die is rolled. Consider the events $A=\{1,3,5\}$, $B=\{2,3\}$ and $C=\{2,3,4,5\}$. Then, the conditional probability $P(A\cup B/C)$ is

(a) $\frac{1}{4}$ (b) $\frac{5}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

39. India plays two matches each with West Indies and Australia. In any match, the probabilities of India getting 0, 1 and 2 points are 0.45, 0.05 and 0.50, respectively. Assuming that the outcomes are independent, the probability of India getting atleast 7 points is

(a) 0.0875 (b) $\frac{1}{16}$

(c) 0.1125 (d) None of these

40. The region represented by the inequation system $x, y\geq 0$, $y\leq 6$, $x+y\leq 3$, is

(a) unbounded in first quadrant

(b) unbounded in first and second quadrants

(c) bounded in first quadrant

(d) None of the above

41. If x^2+px+1 is a factor of ax^3+bx+c, then

(a) $a^2+c^2=ab$ (b) $a^2+c^2=-ab$

(c) $a^2-c^2=ab$ (d) None of these

42. The set of values of x for which $\frac{\tan 3x-\tan 2x}{1+\tan 3x\tan 2x}=1$, is

(a) ϕ (b) $\left\{\frac{\pi}{4}\right\}$

(c) $\{n\pi+\pi/4, n=1,2,3,\ldots\}$

(d) $\{2n\pi+\pi/4, n=1,2,3,\ldots\}$

43. If $y=a\log x+bx^2+x$ has its extremum values at $x=-1$ and $x=2$, then

(a) $a=2, b=-1$ (b) $a=2, b=\frac{1}{2}$

(c) $a=-2, b=\frac{1}{2}$ (d) $a=2, b=-\frac{1}{2}$

44. The angle through which the axes must be rotated without translation, in anti-clockwise sense, so that the expression $ax^2+2hxy+by^2+2gx+2fy+c=0$ does not contain the mixed product xy, is given by

(a) $\tan^{-1}\left(\frac{2h}{a-b}\right)$ (b) $\frac{1}{2}\tan^{-1}\left(\frac{2h}{b-a}\right)$

(c) $\frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right)$ (d) $\frac{1}{2}\tan^{-1}\left(\frac{h}{a-b}\right)$

45. If $\cos A+\cos C=4\sin^2\frac{1}{2}B$, then the sides a, b and c of the triangle are in

(a) AP (b) GP (c) HP (d) None of the above

46. The two consecutive terms in the expansion of $(3+2x)^{74}$ whose coefficients are equal, are

(a) 30th and 31st terms (b) 31st and 32nd terms

(c) 29th and 30th terms (d) None of these

47. If $g(f(x))=|\sin x|$, $f(g(x))=(\sin\sqrt{x})^2$, then

(a) $f(x)=\sin x$, $g(x)=|x|$

(b) $f(x)=x^2$, $g(x)=\sin\sqrt{x}$

(c) f and g cannot be determined

(d) $f(x)=\sin^2 x$, $g(x)=\sqrt{x}$

48. Let $*$ be a binary operation on the set $Q*$ of all positive rational numbers defined by $a*b=\frac{ab}{100}$ for all $a,b\in Q*$. The inverse of 0.1 under operation $*$ is

(a) 10^5 (b) 10^6 (c) 10^4 (d) None of these

49. If a, b, c, d and p are distinct real numbers such that $(a^2+b^2+c^2+d^2)p^2 - 2(ab+bc+cd)p + (b^2+c^2+d^2) \leq 0$, then a, b, c and d are in

(a) AP (b) HP (c) $ab = cd$ (d) GP

50. If the position vectors of three points are $\mathbf{a} - 2\mathbf{b} + 3\mathbf{c}$, $2\mathbf{a} + 3\mathbf{b} - 4\mathbf{c}$ and $-7\mathbf{b} + 10\mathbf{c}$, then the three points are

(a) collinear (b) non-collinear
(c) coplanar (d) None of the above

Answers

Physics

1. (c)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (c)	**7.** (b)	**8.** (a)	**9.** (a)	**10.** (c)
11. (d)	**12.** (b)	**13.** (c)	**14.** (c)	**15.** (d)	**16.** (d)	**17.** (a)	**18.** (b)	**19.** (d)	**20.** (c)
21. (d)	**22.** (b)	**23.** (b)	**24.** (*)	**25.** (c)	**26.** (d)	**27.** (a)	**28.** (d)	**29.** (c)	**30.** (a)
31. (a)	**32.** (a)	**33.** (a)	**34.** (c)	**35.** (c)	**36.** (a)	**37.** (a)	**38.** (b)	**39.** (d)	**40.** (b)
41. (a)	**42.** (b)	**43.** (b)	**44.** (b)	**45.** (d)	**46.** (a)	**47.** (d)	**48.** (b)	**49.** (a)	**50.** (d)

Chemistry

1. (a)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (a)	**7.** (c)	**8.** (*)	**9.** (b)	**10.** (c)
11. (d)	**12.** (a)	**13.** (c)	**14.** (a)	**15.** (d)	**16.** (b)	**17.** (c)	**18.** (a)	**19.** (c)	**20.** (b)
21. (c)	**22.** (b)	**23.** (a)	**24.** (a)	**25.** (c)	**26.** (b)	**27.** (b)	**28.** (b)	**29.** (a)	**30.** (a)
31. (a)	**32.** (d)	**33.** (c)	**34.** (a)	**35.** (b)	**36.** (a)	**37.** (a)	**38.** (a)	**39.** (a)	**40.** (a)
41. (a)	**42.** (b)	**43.** (d)	**44.** (c)	**45.** (b)	**46.** (d)	**47.** (d)	**48.** (c)	**49.** (d)	**50.** (a)

Mathematics

1. (b)	**2.** (c)	**3.** (b)	**4.** (a)	**5.** (a)	**6.** (d)	**7.** (c)	**8.** (a)	**9.** (a)	**10.** (d)
11. (d)	**12.** (d)	**13.** (d)	**14.** (c)	**15.** (d)	**16.** (b)	**17.** (c)	**18.** (b)	**19.** (b)	**20.** (a)
21. (b)	**22.** (c)	**23.** (d)	**24.** (d)	**25.** (d)	**26.** (b)	**27.** (b)	**28.** (c)	**29.** (a)	**30.** (c)
31. (b)	**32.** (a)	**33.** (b)	**34.** (a)	**35.** (a)	**36.** (a)	**37.** (b)	**38.** (d)	**39.** (a)	**40.** (c)
41. (c)	**42.** (a)	**43.** (d)	**44.** (c)	**45.** (a)	**46.** (a)	**47.** (d)	**48.** (a)	**49.** (d)	**50.** (a)

(*) None option is correct.

Solutions

Physics

1. In Millikan's oil drop experiment, the charged oil drop remains suspended (in equilibrium) when downward weight of drop is balanced by upward electrostatic force and charge on drop $q = ne$, i.e.

$$qE = mg \Rightarrow neE = mg$$

If r is radius of oil drop, then mass $m = \frac{4}{3}\pi r^3 \rho$

$$neE = \frac{4}{3}\pi r^3 \rho g$$

$$r = \left[\frac{3neE}{4\pi\rho g}\right]^{1/3}$$

2. Given, $V(x) = \frac{20}{(x^2 - 4)}$ volt

We know that

$$E = -\frac{dV}{dx}$$

$$= -\frac{d}{dx}\left[\frac{20}{(x^2-4)}\right]$$

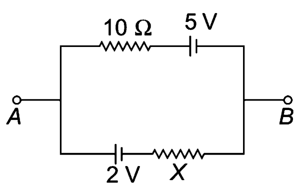
$$= \frac{20 \times (2x)}{(x^2-4)^2} \text{ volt}$$

At $x = 4\ \mu\text{m}$

$$v = \frac{20 \times (2 \times 4)}{[(4)^2 - 4]} = \frac{10}{9}\ \text{V}/\mu\text{m}$$

Since, V decreases as x increases, it follows from $E = -\frac{dV}{dx}$ that $\mathbf{E}$ is along the +ve x-direction.

3.



In the circuit $10\ \Omega$ and $X\ \Omega$ resistance are in series combination. Thus, $R = 10 + X$

From the circuit diagram,

Current, $i = \frac{E}{R}$

$$= \frac{5-2}{10+X} = \frac{3}{10+X}$$

We know that

$$V_{AB} = E + ir$$

[∵ because current flows from positive terminal to negative terminal]

Here, $V_{AB} = 4\text{ V}$

$$4 = 2 + \frac{3}{10+X} \times X$$

$$2 = \frac{3X}{10+X}$$

$$2(10 + X) = 3X$$

$$20 + 2X = 3X$$

$$X = 20\ \Omega$$

4. Let the resistance of whole wire $= R$

We know that

$$P_1 = \frac{V^2}{R} \quad \ldots\text{(i)}$$

Now after cutting the wire into two equal pieces the resistance of each piece $= \frac{R}{2}$

Now, equivalent resistance

$$= \frac{\frac{R}{2} \times \frac{R}{2}}{\frac{R}{2} + \frac{R}{2}} = \frac{R}{4}$$

Power dissipation becomes

$$P_2 = \frac{V^2}{\left(\frac{R}{4}\right)}$$

$$P_2 = \frac{4V^2}{R}$$

$$P_2 = 4P_1 \quad \text{[from Eq. (i)]}$$

$$\frac{P_2}{P_1} = 4$$

5. Given $R_1 = R_2 = R$

Now, in the series combination,

Equivalent resistance

$$S = R_1 + R_2 = 2R \quad \ldots\text{(i)}$$

In parallel combination,

Equivalent resistance, $P = \frac{R_1 \times R_2}{R_1 + R_2}$

$$= \frac{R \times R}{R + R} = \frac{R^2}{2R}$$

$$= \frac{R}{2} \quad \ldots\text{(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{S}{P} = 2R \times \frac{2}{R}$$

$$S = 4P$$

So, $n = 4$

6. Given

Side of triangles loop = l

Current = i

Magnetic field = B

The torque acting on a loop is given by

$$\tau = p_m B \sin\theta \quad \ldots(i)$$

Here, $\theta = 90°$, $p_m = iA$

$$p_m = i\left(\frac{1}{2}l\left(\frac{\sqrt{3}}{2}\right)l\right)$$

$$p_m = \frac{\sqrt{3}}{4}l^2 i \quad \ldots(ii)$$

From the Eqs. (i) and (ii), we get

$$\tau = \frac{\sqrt{3}}{4}l^2 iB$$

7. The magnetic field at the centre of circular coil is

$$B = \frac{\mu_0 i}{2r}$$

where, r = radius of circle = $\frac{l}{2\pi}$

$$\therefore \quad B = \frac{\mu_0 i}{2} \times \frac{2\pi}{l} \quad [\because l = 2\pi r]$$

$$\Rightarrow \quad B = \frac{\mu_0 i\pi}{l} \quad \ldots(i)$$

when wire of length l bents into a circular loop of n turns, then

$$l = n \times 2\pi r' \Rightarrow r' = \frac{l}{n \times 2\pi}$$

Thus, new magnetic field

$$B' = \frac{\mu_0 ni}{2r'} = \frac{\mu_0 ni}{2} \times \frac{n \times 2\pi}{l}$$

$$= \frac{\mu_0 i\pi}{l} \times n^2$$

From Eq. (i),

$$B' = n^2 B$$

8. Given, $\theta = 60°$

We know that

$$\tau = MB\sin\theta \quad \ldots(i)$$

$$W = MB(1 - \cos\theta)$$

$$= MB(1 - \cos 60°)$$

$$W = \frac{MB}{2} \quad \ldots(ii)$$

From Eq. (i),

$$\tau = MB\sin 60°$$

$$\tau = \frac{\sqrt{3}MB}{2}$$

From Eq. (ii),

$$\tau = \sqrt{3}W$$

9. Given, self-inductance $(L) = 10$ H

Frequency $(f) = 50$ Hz

Now, we know that

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$\sqrt{LC} = \frac{1}{2\pi f}$$

$$LC = \frac{1}{4\pi^2 f^2}$$

$$C = \frac{1}{4\pi^2 f^2 L}$$

$$C = \frac{1}{4 \times \pi^2 \times (50)^2 \times 10}$$

$$C = 1\,\mu\text{F}$$

10. $i_0 = \sqrt{i_1^2 + i_2^2}$

Since, i_1 and i_2 are mutually perpendicular.

$$i_{\text{rms}} = \frac{i_0}{\sqrt{2}} = \frac{\sqrt{i_1^2 + i_2^2}}{\sqrt{2}}$$

$$= \sqrt{\frac{i_1^2 + i_2^2}{2}}$$

11. Given, $\lambda_1 = 4000$ Å

and $\lambda_2 = 5000$ Å

We know that,

Resolving power of instrument $\propto \frac{1}{\lambda}$

$$\frac{(\text{RP})_1}{(\text{RP})_2} = \frac{\lambda_2}{\lambda_1}$$

$$= \frac{5000}{4000}$$

$$\frac{(\text{RP})_1}{(\text{RP})_2} = \frac{5}{4}$$

12. The refractive index of a prism is given by

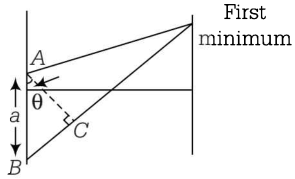
$$\mu = \frac{\sin\frac{1}{2}(A + D)}{\sin\left(\frac{A}{2}\right)} \quad \ldots(i)$$

where, A = angle of prism and

D = angle of minimum deviation

It follows from Eq. (i) that greater the refractive index, the greater is the angle of minimum deviation. Since, the refractive index for the blue light greater than that of red light, the angle of minimum deviation (D_2) for blue light will be greater than that (i.e. angle D_1) for red light.

13.



According the figure, the direction θ along which we have the nth interference maximum is given by

$$d\sin\theta = n_2$$

$$n = \frac{d\sin\theta}{\lambda}$$

If $d = 2\lambda$, we have

$$n = 2\sin\theta$$

Since, the maximum value of θ is 90°, $n_{max} = 2\sin 90° = 2$. Thus there are two interference maxima in addition to the central maximum (which corresponds to $\theta = 0$). Hence, the maximum of possible interference maxima is three.

14. Given,

Work function $\phi = 4.0 \text{ eV} = 4.0 \times 1.6 \times 10^{-19}\text{V}$

We know that

$$\phi = \frac{hc}{\lambda_{max}}$$

$$\lambda_{max} = \frac{hc}{\phi}$$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{4.0 \times 1.6 \times 10^{-19}}$$

$$= 310 \text{ nm}$$

15. For a charged particle

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

Since, $\lambda_p = \lambda_\alpha$ we have

$$\therefore \quad \frac{h}{\sqrt{2m_p q_p V}} = \frac{h}{\sqrt{2m_\alpha q_\alpha V_\alpha}}$$

$$V_\alpha = \frac{m_p q_p V}{m_\alpha q_\alpha}$$

$$m_p q_p V = m_\alpha q_\alpha V_\alpha$$

$$= \frac{m_p q_p V}{4m_p \cdot 2q_p} = \frac{V}{8}$$

16. Frequency of the series limit of Balmer series of hydrogen atom in terms of Rydberg constant R and speed of light c is $\frac{1}{\lambda} = \frac{RC}{4}$.

17. ${}_{92}U^{238} \rightarrow {}_{82}Pb^{206}$

Number of α-particles is given by

$$238 = 4n + 206$$

$$4n = 238 - 206$$

$$n = \frac{32}{4}$$

$$n = 8$$

Number of β-particles is given by

$$92 = 2n - m + 82$$

$$92 = 2 \times 8 - m + 82$$

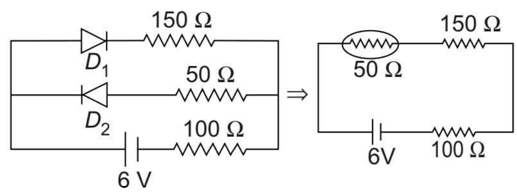
$$92 = 16 - m + 82$$

$$92 = 98 - m$$

$$m = 98 - 92$$

$$m = 6$$

18. As per give circuit diode D_1 is forward biased and offers a resistance of 50 Ω. Diode D_2 is reverse biased and as it corresponding resistance is infinite no current flow through it. The equivalent circuit is as shown in figure.



$$I = \frac{6}{(50 + 150 + 100)} = \frac{6}{300} = 0.02 \text{ A}$$

19. The electric field of the junction of Zener diode is not very low.

20. Given, $E_{max} = 10$ V

$E_{min} = 2$ V

Now, we know that modulation index

$$(m_a) = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}$$

$$= \frac{10 - 2}{10 + 2} = \frac{8}{12} = \frac{2}{3}$$

21. We know that,

$$F = q(v \times B)$$

$$B = \frac{[F]}{[q][v]}$$

$$= \frac{[MLT^{-2}]}{[C][LT^{-1}]} = [MT^{-1}C^{-1}]$$

22. Given $v = \alpha\sqrt{x}$

$$\Rightarrow \quad \frac{dx}{dt} = \alpha\sqrt{x} \Rightarrow \frac{dx}{\sqrt{x}} = \alpha\, dt$$

Integrating both sides, we get

$$\int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha\, dt$$

$$x^{1/2} = \frac{\alpha t}{2}$$

$$x = \frac{\alpha^2 t^2}{4}$$

$$x \propto t^2$$

23. As there is no external force acting on the body, all the internal forces are conservative. So, we can apply the principle of conservation of energy.

From conservation of energy,

Potential energy at height h = KE at ground

[$\because$ KE at height = PE at ground = 0]

Therefore, at height h, PE of ball A

$$\text{PE} = m_A g h$$

$$\text{KE at ground} = \frac{1}{2} m_A v_A^2$$

$$v_A = \sqrt{2gh}$$

Similarly, $v_B = \sqrt{2gh}$

Therefore, $v_A = v_B$

24. $H = 5000 \text{ m} = \dfrac{u_y^2}{2g} = \dfrac{u_y^2}{2 \times 10}$

$$u_y^2 = 100000$$

or $\quad u_y = 100\sqrt{10}\,\dfrac{m}{s} \quad \ldots\text{(i)}$

$$R = 20,000 \text{ m} = \frac{2u_x\, u_y}{g} = \frac{2 \times u_x}{10} \times 100\sqrt{10}$$

or $\quad u_x = 100\sqrt{10} \quad \ldots\text{(ii)}$

Now, $\quad u = \sqrt{u_x^2 + u_y^2} = 100\sqrt{20}$ m/s

25. We know that

$$\text{Angular velocity} = \frac{\text{Angular displacement } (\theta)}{\text{Time}}$$

As both cars complete their circle (2π) in equal time

$$\omega_1 = \frac{2\pi}{t} \quad \ldots\text{(i)}$$

$$\omega_2 = \frac{2\pi}{t} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{\omega_1}{\omega_2} = \frac{1}{1}$$

26. The acceleration of the (rope + block) system is given by $a = \dfrac{P}{m + M}$. Hence, the force on the block of mass M is equal to $\left(\dfrac{MP}{m + M}\right)$.

27. Given, Mass of bullet = 200 g

Speed of bullet = 5 m/s

Mass of gun = 1 kg

According to the question

$$\frac{200}{1000} \times 5 = 1 \times v$$

$$v = \frac{1000}{1000}$$

$$v = 1 \text{ m/s}$$

28. Given, $m = 80$ kg

$$a = 5 \text{ m/s}^2$$

$$g = 10 \text{ m/s}^2$$

We know that,

$$w = m(g + a) = 80(10 + 5)$$

$$= 80(15) = 1200 \text{ N}$$

29. Current, $i = \dfrac{E}{(R + V)} = \dfrac{2}{3 + R} = \dfrac{2}{(3 + R)}$

$$V = \frac{2}{(3 + R)} \times 3$$

$$\frac{V'}{l} = \frac{V}{L}$$

$$1 \times 10^{-3} = \frac{2 \times 3}{(3 + R) \times 100}$$

$$\frac{1}{10} = \frac{6}{(3 + R)}$$

$$3 + R = 60$$

$$R = 60 - 3$$

$$R = 57\,\Omega$$

30. As the body moves along the x-axis from $x_i = 3.0$ m to $x_f = 4.0$ m, the work done by the force is

$$W = \int_{x_i}^{x_f} f_x\, dx = \int_{x_i}^{x_f} -6x\, dx = -3(x_f^2 - x_i^2)$$

$$= -3[(4.0)^2 - (3.0)^2]11 = -21 \text{ J}$$

According to the work-kinetic energy theorem, this gives the change in the kinetic energy.

$$W = \Delta K = \frac{1}{2} m(v_f^2 - v_i^2)$$

where, v_i is the initial velocity (at x_i) and v_f is the final velocity (at x_f). The theorem yields.

$$v_f = \sqrt{\frac{2W}{m} + v_i^2} = \sqrt{\frac{2(-21)}{2.0} + (8.0)^2} = 6.6 \text{ m/s}$$

31. Given, power is $\frac{1}{4}nP = \frac{764}{4}\text{W} = 186.5\text{W}$

Since, efficiency of motor is 40%.

The power used in doing work is 40% of 186.5W, so

$$P = 186.5 \times \frac{40}{100}$$
$$= 74.6 \text{ W}$$

Angular velocity of the motor, ω = 600 rpm = 10 rps

$\Rightarrow \quad \omega = (600)(2\pi)60$ rad/s

$\Rightarrow \quad \omega = 20\pi$ rad/s

Let the torque be T.

So, $\quad P = T\omega$

$\Rightarrow \quad T = \frac{P}{\omega} = \frac{74.6}{20\pi}$

Now, work done in 1 rotation= $T(2\pi)$

$$W = \frac{74.6}{20\pi} \times 2\pi$$
$$W = 7.46 \text{ J}$$

32. Radius of earth = 6400 km

According to the question,

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = \frac{GMm}{(R+h)}$$
$$\frac{-GM}{R} + \frac{1}{2}v^2 = \frac{-GM}{R+h}$$
$$\frac{1}{2}v^2 = \frac{-GM}{(R+h)} + \frac{GM}{R}$$
$$\frac{1}{2}v^2 = GM\left(\frac{1}{R+h} - \frac{1}{R}\right)$$
$$\frac{1}{2} \times (10)^2 = 6.6 \times 10^{-11} \times 6.0 \times 10^{24} \left[\frac{1}{6400+h} - \frac{1}{6400}\right]$$
$$h = 2.5 \times 10^4 \text{ km}$$

33. To keep the centre of mass at the same position,velocity of centre of mass is zero, so

$$\frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0$$

(where, v_1 and v_2 are the velocities of particles 1 and 2 respectively)

$\Rightarrow \quad m_1 \frac{dr_1}{dt} + m_2 \frac{dr_2}{dt} = 0$

$$\left[\because v_1 = \frac{dr_1}{dt} \text{ and } v_2 = \frac{dr_2}{dt} \text{ and } m_1 m_2 \neq 0\right]$$

$\Rightarrow \quad m_1 dr_1 + m_2 dr_2 = 0$

(dr_1 and dr_2 represent the small changes in displacement, so that $dr_2 \to 0$ and $dr_2 \to 0$ of particles)

Let 2nd particle has been displaced by distance X, then

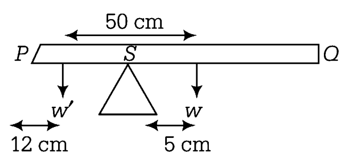
$$m_1(d) + m_2(X) = 0, \; X = -\frac{m_1 d}{m_2}$$

Negative sign shows that both the particle have to move in opposite directions.

So, $\frac{m_1}{m_2}d$ is the distance moved by 2nd particle to keep position of centre of mass unchanged.

34. Since, the acceleration of centre of mass in both the cases is same equal to g. So, the centre of mass of bodies B and C taken together does not shift compared to that of body A.

35. Let w and w' be the respective weight on the meter stick and the coin



The mass of meter stick is connected at its mid-point, i.e. at the 50 cm mark.

Mass of the meter stick = m

Mass of each coin, m = 5 g

When the coins are placed 12 cm away from the end P, the centre of mass gets shifted by 5 cm from R towards the end P. The centre of mass is located at a distance of 45 cm from point P.

The net torque will be conserved for rotational equilibrium about point R

$$10 + g(45-12) - m'g(50-45) = 0$$

$\therefore \quad m' = \frac{10 \times 33}{5} = 66$ g

Hence, the mass of meter stick is 66 g.

36. Angular momentum

$$L = I\omega \qquad \ldots\text{(i)}$$

Kinetic energy, $K = \frac{1}{2}I\omega^2 = \frac{1}{2}L\omega \qquad$ [from Eq. (i)]

$$L = \frac{2K}{\omega}$$

Now, the new angular momentum

$$L' = \frac{2\left(\frac{K}{2}\right)}{2\omega} \qquad \left[\because K' = \frac{K}{2} \text{ and } \omega' = 2\omega\right]$$

$\Rightarrow \quad L' = \frac{L}{4}$

37. The necessary centripetal force required for a planet to move round the sun

= gravitational force exerted on it

i.e. $$\frac{mv^2}{R} = \frac{GM_e m}{R^n}$$

$$v = \left(\frac{GM_e}{R^{n-1}}\right)^{1/2}$$

Now, $$T = \frac{2\pi R}{v}$$

$$= 2\pi R \times \left(\frac{R^{n-1}}{GM_e}\right)^{1/2}$$

$$\Rightarrow \quad 2\pi\left(\frac{R^2 \times R^{n-1}}{GM_e}\right)^{1/2}$$

$$T = 2\pi\left(\frac{R^{(n+1)/2}}{(GM_e)^{1/2}}\right)$$

$$\therefore \quad T \propto R^{(n+1)/2}$$

38. We know that

$$T^2 \propto R^3$$

or $$\left(\frac{T_2}{T_1}\right) = \left(\frac{R_2}{R_1}\right)^{3/2}$$

$$\frac{T_2}{T_1} = \left(\frac{6400}{36000}\right)^{3/2}$$

$$T_2 = \left(\frac{6400}{36000}\right)^{3/2} \times 24$$

$$T_2 = 2\text{ h}$$

39. Energy stored in the wire

$$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

and Young's modulus $= \dfrac{\text{Stress}}{\text{Strain}}$

where, strain $= \dfrac{S}{Y}$

$$\Rightarrow \quad \frac{\text{Energy stored in wire}}{2}$$

$$= \frac{1}{2} \times \text{stress} \times \text{strain}$$

$$= \frac{1}{2} \times S \times \frac{S}{Y} = \frac{S^2}{2Y}$$

40. Given, $\rho = 1.3 \times 10^3 \text{ kg/m}^3$

$$A = 4.00 \text{ cm}^2$$

$$= \frac{4.00}{100 \times 100} = 4.00 \times 10^{-4} \text{ m}^2$$

$$h_1 = 0.854 \text{ m}$$

$$h_2 = 1.560 \text{ m}$$

Now, work done by gravity = loss in PE

$$= \frac{A\rho_g}{4}(h_1 - h_2)^2$$

$$= \frac{4.00 \times 10^{-4} \times 1.3 \times 10^3 \times 10\,(1.560 - 0.854)}{4}$$

$$= 0.635 \text{ J}$$

41. Given, $\Delta l_1 = 1$ mm

We know that

$$\Delta l \propto \frac{F}{r^2}$$

or $$\frac{\Delta l_2}{\Delta l_1} = \frac{F_2}{F_1} \times \frac{r_1^2}{r_2^2}$$

$$\frac{\Delta l_2}{\Delta l_1} = 2 \times 2 \times 2 = 8$$

$$\Delta l_2 = 8\Delta l_1 = 8 \times 1 = 8 \text{ mm}$$

42. The energy density $\dfrac{u}{V}$ of an ideal gas is related to its pressure p as

$$\frac{u}{V} = \frac{3}{2}p$$

43. Thermal energy correspond to internal energy

Mass = 1 kg

Density = 4 kg/m^3

$$\Rightarrow \quad \text{Volume} = \frac{\text{Mass}}{\text{Density}} = \frac{1}{4}\text{ m}^3$$

$$\text{Pressure} = 8 \times 10^4 \text{ N-m}^2$$

∴ Internal energy (for diatomic gas) $f = 5$

$$= \frac{5}{2}p \times V = 5 \times 10^4 \text{ J}$$

$$\left[\because u = \frac{1}{2}fRT = \frac{1}{2}fpV \\ \text{where, } f \text{ is degree of freedom}\right]$$

44. Temperature inside the refrigerator,

$$T_1 = 9°\text{C} = 282 \text{ K}$$

Room temperature $T_2 = 36°$ C, = 309 K

$$\text{Coefficient of performance} = \frac{T_1}{T_2 - T_1}$$

$$= \frac{282}{309 - 289} = 10.44$$

45. According to Newton's law of cooling

$$\frac{d\theta}{dt} \propto \Delta\theta$$

But $$\frac{d\theta}{dt} \propto (\Delta\theta)^n$$

$$\therefore \quad n = 1$$

46. Energy radiated per second by a body which has surface area A at temperature T is given by Stefan's law,

$$E = \sigma A T^4$$

Therefore $$\frac{E_1}{E_2} = \left(\frac{r_1}{r_2}\right)^2 \left(\frac{T_1}{T_2}\right)^4$$

$$= \left(\frac{1}{4}\right)^2 \left(\frac{4000}{2000}\right)^4$$

$$\frac{E_1}{E_2} = \frac{16}{16} = \frac{1}{1}$$

47. Given that the velocity of sound in air = 300 m/s. If a source of sound is moving toward a stationary listener, the frequency heard by listener would be different from the actual frequency of the source, this apparent frequency is given by

$$f_{app} = \left(\frac{v_{\text{sound in air}}}{v_{\text{sound in air}} \pm v_{\text{source}}}\right),$$

where symbols have their usual meanings. In the denominator positive sign would be taken when source is recending away from the listener, while negative sign would be taken when source is approaching the listener, let v be the maximum value of source velocity for which the person is able to hear the sound, then

$$1000 = \left(\frac{300}{300 - v}\right) \times 9500$$

$\Rightarrow$ $$v = 15 \text{ m/s}$$

48. To determine the position velocity, etc at first we write the general representation of wave and then compare the given wave equation with general wave equation

Given $X = (2 \times 10^{-2}) \cos \pi t$

This gives $a = 2 \times 10^{-2} \text{ m} = 2 \text{ cm}$

At $t = 0$,

$$X = 2 \text{ cm}$$

i.e. the object is at positive extreme, so to acquire maximum speed (i.e. the reach mean position) it takes $\frac{1}{4}$th of the time period.

$\therefore$ Required time $= \frac{T}{4}$

where, $\omega = \frac{2\pi}{T} = \pi$

$\Rightarrow$ $T = 2 \text{ s}$

So, required time $= \frac{T}{4} = \frac{2}{4} = 0.5 \text{ s}$

49. We know that

$$V = \frac{1}{4\pi\varepsilon_0}, \frac{q}{r}$$

and $$q = 4\pi r^2 \sigma$$

$$V = \frac{1}{4\pi\varepsilon_0} \times \frac{4\pi r^2 \sigma}{r}$$

Now, potential at the centre

$$V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{4\pi r_1^2 \sigma}{r_1} + \frac{4\pi r_2^2 \sigma}{r_2}\right]$$

$$= \frac{\sigma}{\varepsilon_0}[r_1 + r_2]$$

50. Given, $\mathbf{E} = \frac{A}{x^3}\,\mathbf{i}$

We know that,

$$E = -\frac{dv}{dx}$$

$$\int dV = -\int -E\, dx$$

$$V = -\int \left(\frac{A}{x^3}\right) dx$$

$$V = \frac{1}{2} \cdot \frac{A}{x^2}$$

Chemistry

1. $CaSO_4 \longrightarrow Ca^{2+} + SO_4^{2-}$

$$K_{sp} = [Ca^{2+}][SO_4^{2-}]$$

$$K_{sp} = S^2$$

$$9.0 \times 10^{-6} = S^2$$

$$S = \sqrt{9.0 \times 10^{-6}} = 3.0 \times 10^{-3} \text{ mol/L}$$

Given, molecular mass = 136g mol^{-1}

Solubility of $CaSO_4 = 3.0 \times 10^{-3} \times 136 \text{ g/L}$

$$= 0.41 \text{g / L}$$

$\therefore$ To dissolve 0.41 of $CaSO_4$, water requires = 1 L

$\therefore$ To dissolve 1g of $CaSO_4$, water requires $= \frac{1}{0.41}$ L

$$= 2.439 \text{ L} = 2.44 \text{ L}$$

2. $$K_b = \frac{[NH_4^+][OH^-]}{[NH_4OH]} = \frac{[OH^-]^2}{[NH_4OH]}$$

$$[OH^-]^2 = K_b [NH_4OH]$$

$$= 1.8 \times 10^{-5} \times 0.1 \text{ M}$$

$$= 1.8 \times 10^{-6} \text{ M}$$

3. $N_2O_4 \rightleftharpoons 2NO_2$

	N_2O_4	$2NO_2$
Initial	1	0
At equil.	$1-x$	$2x$
	$=1-0.2$	2×0.2
	$=0.8$	$=0.4$

Mole of unreacted $N_2O_4 = 0.8$

$$n_1 = 1$$
$$n_2 = 1.2$$

From $pV = nRT$

$$\frac{p_1}{T_1 n_1} = \frac{p_2}{T_2 n_2}$$
$$\frac{1}{300\times1} = \frac{p_2}{600\times1.2}$$
$$\Rightarrow \quad p_2 = \frac{600\times1.2}{300} = 2.4 \text{ atm}$$

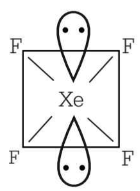
4. $H_V = 40.79 \text{ kJ mol}^{-1}$

Number of moles $(n) = \frac{80}{18}$

Heat, $q = nHv$

$$= \frac{80}{18}\times40.79$$
$$= 181.28 \text{ kJ}$$
$$= 1.81\times10^2 \text{ kJ}$$

5. XeF_4 has square planar structure.



it involves sp^3d^2 hybridisation of xenon, which adopts octahedral geometry in which two positions are occupied by lone pair of electrons.

6. Schrodinger wave equation for a particle in 1-D box is given as

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{h}(E-\infty)\psi = 0$$

where, ψ = wave function

E = total energy of a particle

7. 1M of HCl solution contains 6.022×10^{23} molecules of Cl^- ions.

$\therefore$ 0.001 M solution will contain

$$= 6.022\times10^{23}\times10^{-3} \text{ molecule}$$
$$= 6.002\times10^{20} \text{ molecule}$$

So, 6.022×10^{20} molecule are present in 1 L

or in 1000 mL, the number of molecules present

$$= 6.022\times10^{20}$$

$\therefore$ in 100 mL, the number of molecules present

$$= \frac{6.022\times10^{20}}{1000}\times100$$
$$= 6.022\times10^{19} \text{ molecules}$$

8. $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right] = 1.097\times10^7\left[\frac{1}{1}\right]$

$$\frac{1}{\lambda} = 1.097\times10^7$$
$$\lambda = \frac{1}{1.097\times10^7} = 0.9115\times10^{-7}$$
$$E = \frac{hc}{\lambda} = \frac{6.626\times10^{-34}\text{ Js}\times2.99\times10^8\text{ ms}^{-1}}{0.9115\times10^{-7}\text{ m}}$$
$$= \frac{6.626\times10^{-34}\text{ Js}\times2.99\times10^8\text{ ms}^{-1}}{0.9115\times10^{-7}\text{ m}}$$
$$= 21.7\times10^{-19}\text{ J} = 2.17\times10^{-18}\text{ J}$$

Options are not correct.

9. Energy of one photon is

$$E = h\nu$$
$$= 6.626\times10^{-34}\text{ Js}\times5\times10^{14}\text{ Hz}$$
$$= 3.313\times10^{-19}\text{ J}$$

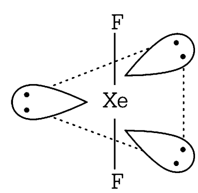
$\because$ One mole photons contains

$$= 6.022\times10^{23}\text{ photons.}$$

For 1 mole of photons, total energy

$$= 6.022\times10^{23}\times3.313\times10^{-19}\text{ J/mol}$$
$$= 19.9508\times10^4\text{ J mol}^{-1}$$
$$= 199.508\times10^3\text{ J mol}^{-1}$$
$$= 199.508\text{ kJ mol}^{-1}$$

10. XeF_2 has linear structure.



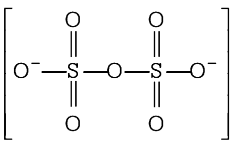
It involves sp^3d hybridisation on xenon which adopts trigonal bipyramidal geometry in which three positions are occupied by lone pairs. Because of the symmetrical arrangement of three lone pairs (each at an angle of 120°), the net repulsion on the Xe—F bond pairs is zero.

Thus, XeF_2 has linear geometry.

11. Mustard gas or sulphur mustard $(Cl—CH_2—CH_2)_2S$, is a chemical agent that causes burning of the skin, eyes and respiratory tract.

12. The constituent particles of amorphous solids have only short range order of arrangement, i.e. regular and periodical arrangement of particles is seen to a short distance only. The structures of amorphous solids are similar to that of liquids. Glass, rubber, plastics, etc., are some of the examples of amorphous solids. Amorphous solid are isotropic in nature, i.e. physical property of amorphous solid are same in all directions.

13. $S_2O_7^{2-}$ species having no S—S bond.



14. The cathode ray tube (CRT) is a vacuum tube containing one or more electron guns and a fluorescent screen used to view images. It has a means to accelerate and deflect the electron beam on the screen to create the images.

15. Volume of solution $= \frac{\text{Mass}}{\text{Density}}$

$$= \frac{100\text{g}}{1.25\text{g cm}^3} = 80 \text{ cm}^3 = 0.08 \text{ L}$$

Given, molarity of NaCl solution = 3 M

1000 cm^3 of 3M NaCl contain = 3 mol NaCl

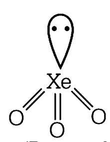
100 cm^3 of 3M NaCl will contain NaCl $= \frac{3}{1000} \times 100$

$= 0.3$ mol NaCl

Molality $= \frac{0.3 \text{ mol}}{0.08 \text{ L}} = 3.75 \text{ mol L}^{-1}$

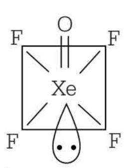
16. Molecular structure of XeO_3 and $XeOF_4$ respectively are

XeO_3



(Pyramidal)

$XeOF_4$



(Square pyramidal)

17. Conditions causing Frenkel defects are
(i) coordination number is low
(ii) anions are much larger in size than the cations

In pure alkali metals halides, these defects are not very common because the ions cannot get into interstitial positions due to their large sizes.

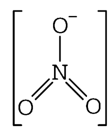
These defects can be found in silver halides, such as AgCl, AgBr, AgI and zinc sulphides, i.e. ZnS etc. Because of the small size of the Ag^+ and Zn^{2+} ions, these ions can go to the interstitial sites.

18. **Nitronium ion** (NO_2^+)

$$:\ddot{O}=\overset{+}{N}=\ddot{O}:$$

Hybridisation = *sp*
Geometry = linear

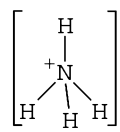
Nitrate (NO_3^-)



Hybridisation = sp^2

Geometry = trigonal planar

Ammonium ion (NH_4^+)



Hybridisation = sp^3

Geometry = tetrahedral

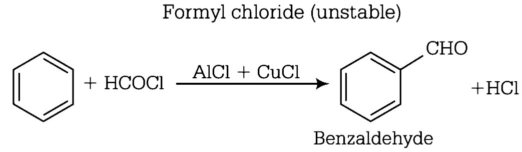
19. $NaCl < MgCl_2 < AlCl_3$ is the correct order of increasing polarising power of the cations. Because according to Fajan's rules for cations, high charge and small size of the cation have more polarising power that is ability to distort electron density on an cation and induce partial covalent character

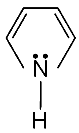
$$Na^+ < Mg^{2+} < Al^{3+}$$

20. N_2O is not an air pollutant. It is commonly known as laughing gas. It is a colourless, non-flammable gas possesing sweet odour and taste.

21. Gattermann-Koch reaction,

$CO + HCl \longrightarrow \underset{\text{Formyl chloride (unstable)}}{HCOCl}$



22.  is an aromatic compound

Because it follows all the conditions of aromaticity.

(i) It follows Huckel's rule $(4n+2)\ \pi = 6r$ electrons $(n = 0, 1, 2, ...)$

(ii) It is a planar, cyclic conjugated system.

(iii) π electrons are delocalised over the entire ring.

23. Possible alcohol isomers of $C_4H_{10}O$ is 4.

(i) $CH_3CH_2CH_2OH$

1-butanol (*n*-butyl alcohol)

(ii) $CH_3CH_2—CH(OH)—CH_3$

2-butanol (*sec*-butyl alcohol)

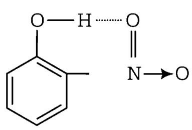
(iii) $(CH_3)_2\ CHCH_2OH$

2-methyl-1-propanol (iso-butyl alcohol)

(iv) $CH_3—C(CH_3)_2—OH$

2-methyl-2-propanol (*t*-butyl alcohol)

24. Intramolecular hydrogen bonding is found in *o*-nitrophenol.



The hydrogen bonding is between hydrogen of —OH group and oxygen of $—NO_2$ group which results decrease in its boiling point.

The two groups are close (—OH and $—NO_2$) enough to each other for intra molecular hydrogen bonding, make it less polar, steam volatile.

25. $CH_4(g) + 2O_2(g) \longrightarrow CO_2(g) + 2H_2O(l)$

Enthalpy change for a given reaction is equal to = (sum of standard enthalpies of formation of products) − (sum of standard enthalpies of formation of reactants).

Given,

enthalpy of $CO_2 = -393.5$ kJ / mol

enthalpy of $H_2O = -285.8$ kJ / mol

enthalpy of $CH_4 = -74.9$ kJ / mol

∴ Enthalpy of combustion of methane

$$= [-393.5 + 2 \times (-285.8)] - [-74.9 + 0]$$

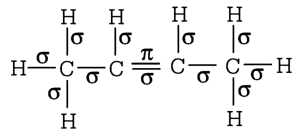
$$= -965.1 + 74.9$$

$$= -890.2 \text{ kJ/mol}$$

$$= \frac{890.2}{16} \text{ kJ/g}$$

$$= -55.6 \text{ kJ/g}$$

The standard enthalpy of formation is taken as zero, because it is most stable form at 298 K and 1 atm pressure.

26. 

$\Rightarrow$ 11 σ bonds + 1 π bond

27. Ethoxyethane is not soluble in water due to lack of H-bonding between oxygen atom in ethoxyethane molecule and hydrogen atom of water molecule.

28. Vitamin C is water soluble but vitamin D is not soluble in water. It is soluble in fat.

29. Higher carboxylic acid have odour.

30. Methanol (CH_3OH) is known as wood's spirit.

31. Valine is an essential amino acids.

Histidine, isoleucine, leucine, lysine, methionine phenylalanine, threonine, tryptophan are some essential amino acids.

Alanine, argine, asparagine, aspartic acids, cysteine, glutamic acid, glutamine, glycine, proline, serine are some non essential amino acids.

32. Copolymer have low degree of polymerisation.

33. The given reaction is known as carbylamine reaction.

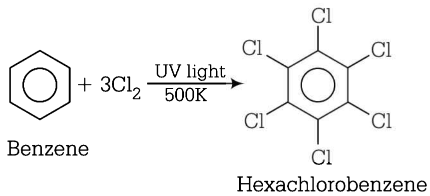
$$RNH_2 + CHCl_3 + 3KOH \xrightarrow{\Delta} R—NC + 3KCl + 3H_2O$$

It is a reaction for chemical test of primary amines.

In this reaction, the analyte is heated with alcohol, potassium hydroxide and chloroform. If primary amine is present, the isocyanide is formed.

34. General formula of Schiff's base is $RR'C = NR''$. Where R, R' represent hydrogen, an alkyl or an aryl and R'' is an alkyl or aryl. It is synthesised from an aliphatic or aromatic amine and a carbonyl compound followed by a dehydration.

35. The chemical reaction is an addition reaction.



36. $\ddot{C}Cl_2$ is an electrophile, because its octet is not complete to required two more electron to complete its octet.

Out of 4 valence electrons, 2 are shared with Cl-atoms and 2 are with C.

$\ddot{C}Cl_2 \rightarrow$ carbene present in two form singlet and triplet.

37. DIBAL-H is $AlH(i\text{-Bu})_2$ (diisobutylaluminium hydride).

38. Chemical adsorption is irreversible adsorption in nature.

39. 2.5 molal solution means 2.5 moles of solute are present in 1000 mL solution (in aqueous solution) moles of water

$$= \frac{1000}{18}$$
$$= 55.55 \text{ mol}$$

$\therefore$ Mole fraction of solute

$$= \frac{n_1}{n_1 + n_2}$$
$$= \frac{2.5}{2.5 + 55.5}$$
$$= \frac{2.5}{58.05}$$
$$= 0.043$$

40. $w_{\text{solute}} = 2.44$ g

$$W_{\text{solvent}} = 75 \text{ g}$$
$$T_b = 100.413^\circ \text{C}$$
$$\therefore \quad \Delta T_b = 100.413 - 100^\circ\text{C}$$
$$= 0.413^\circ\text{C}$$

Thus, $$M_A = \frac{K_b \times w_{\text{solute}} \times 1000}{\Delta T_b \times w_{\text{solvent}}}$$
$$= \frac{0.52 \times 2.44 \times 1000}{75 \times 0.413}$$
$$= 40.96 \text{ g mol}^{-1}$$

41. The cell reactions during charging of lead storage battery are

At anode $PbSO_4(s) + 2e^- \longrightarrow Pb(s) + SO_4^{2-}(aq)$

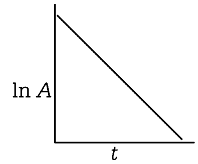
At cathode

$$PbSO_4(s) + 2H_2O \longrightarrow PbO_2(s) + SO_4^{2-}(aq) - 4H^+ + 2e^-$$

Thus, at cathode, formation of PbO_2 takes place.

42. For first order reaction,

$$[A] = [A]_0 e^{-kt} \text{ or } \ln[A] = \ln[A]_0 - kt$$



$\therefore$ ln A *versus* time plot is linear for first order reaction.

43. $\Delta E = -14.2$ kcal/mol

$$= -4.18 \times 14.2 \text{ kJ/mol} = -59.356 \text{ kJ/mol}$$

$\because \quad \Delta H = \Delta E + p\Delta V$

or $\quad \Delta H = \Delta E + \Delta_{ng} RT$

$$\Delta H = -59.356 \text{ kJ/mol} + 8.314 \text{ J/mol} \times 298 \text{ K}$$

$[\because \Delta_{ng} = 2 - 1 = 1]$

$$= -59.356 \text{ kJ/mol} + 2.48 \text{ kJ/mol}$$
$$= -56.87 \text{ kJ/mol}$$

44. $2NO_2 \underset{k_2}{\overset{k_1}{\rightleftharpoons}} N_2O_4$

The rate of disappearance of $NO_2 = \frac{-1}{2}\frac{d[NO_2]}{dt}$

$$\Rightarrow \quad \frac{-1}{2}\frac{d[NO_2]}{dt} = k_1[NO_2]^2 - k_2[N_2O_4]$$
$$\frac{-dNO_2}{dt} = 2k_1[NO_2]^2 - 2k_2[N_2O_4]$$

45. $M_{0.96}O_{1.00}$ may contains M^{2+}, M^{3+} ions. Let amount of M^{2+} ions in crystals = x,

then amount of M^{3+} ions will be $= 0.96 - x$.

And we know that negative and positive ions are equal.

$$\therefore \quad 2x + 3(0.96 - x) = 2$$
$$2x + 3(0.96 - x) - 2 = 0$$
$$2x + 2.88 - 3x - 2 = 0$$
$$0.88 - x = 0 \Rightarrow x = 0.88$$

Amount of M^{2+} ion = 0.88 ; and amount of

$$M^{3+} \text{ ion} = 0.96 - 0.88 = 0.08$$

% of $M^{2+} = \frac{0.88}{0.96} \times 100 = 91.67\%$

46. Given W_A, benzoic acid = 2 g

$$W_B\text{, Benzene} = 25 \text{ g}$$
$$K_f = 4.9 \text{ K kg mol}^{-1}$$
$$T_f = 1.62 \text{ K}$$
$$M_B = 241.98$$
$$2C_6H_5COOH \rightleftharpoons (C_6H_5COOH)_2$$

Let, x = degree of association of solute

$\therefore (1 - x)$ = moles of benzene left in unassociated form

$\frac{x}{2}$ = moles of benzoic acid in equilibrium

Total number of moles at equilibrium = $1 - x + \frac{x}{2}$

van,t Hoff factor, $i = 1 - \frac{x}{2}$

$$i = \frac{\text{Total number of moles of particles after association}}{\text{Number of moles of particle before association}}$$

$$i = \frac{122 \text{ g mol}}{241.98 \text{ g mol}}$$

or, $1 - \frac{x}{2} = \frac{122}{241.98}$

$\therefore \frac{x}{2} = 1 - \frac{122}{241.98}$

$\frac{x}{2} = 0.49$

$x = 2 \times 0.49$

$x = 0.9916$

$\therefore$ The degree of association of benzoic acid in benzene is 99.2%.

47. $CH_3COOH \rightleftharpoons CH_3COO^- + H^+$

Before dissolution	1	0	0
After dissolution	$(1-\alpha)$	α	α

$i = (1 - \alpha) + \alpha + \alpha$

$= 1 + \alpha$

Given that $i = 1.04$

$\therefore \quad 1.04 = 1 + \alpha$

$\alpha = 1.04 - 1$

$= 0.04$

48. 1 atm pressure of H_2 would be required to make emf of the hydrogen electrode zero in water at 25°C.

49. $\because v_{rms} = \sqrt{\frac{3RT}{M_m}}$

$\therefore \quad V \propto \sqrt{\frac{1}{M_m}}$

Where $\quad M_m$ = molar mass

$\because \quad \text{Density} = \frac{\text{Mass}}{\text{Volume}}$

$\therefore \quad$ Density $\propto$ Mass

or $\quad V \propto \frac{1}{\sqrt{\text{density}}}$

$\Rightarrow \quad V \propto \frac{1}{\sqrt{d}}$

50. $K_2Cr_2O_7 + 7H_2SO_4 + 6FeSO_4 \longrightarrow 3Fe_2(SO_4)_3 + Cr_2(SO_4)_3 + 7H_2O + K_2SO_4$

$= 294 \quad 6 \times 154 = 912$

$\because$ 912 g of $FeSO_4$ oxidises by = 294 g of $K_2Cr_2O_7$)

$\therefore$ 20 g of $FeSO_4$ oxidise by

$= \frac{294 \times 20}{6 \times 154}$ g $K_2Cr_2O_7$ = 6.45 g of $K_2Cr_2O_7$

Mathematics

1. Let A denotes the matrix, every element of which is unity. Then, all the 2-rowed minors of A obviously vanish. But A is a non-null matrix. Hence, rank of A is 1.

2. Given, $\sqrt{(x-2)^2+y^2}+\sqrt{(x+2)^2+y^2}=4$

$$\Rightarrow \quad \sqrt{(x-2)^2+y^2}=4-\sqrt{(x+2)^2+y^2}$$

On squaring both sides, we get

$$(x-2)^2+y^2=(4-\sqrt{(x+2)^2+y^2})^2$$

$$\Rightarrow \quad (x-2)^2+y^2=16+\{(x+2)^2+y^2\}-8\sqrt{(x+2)^2+y^2}$$

$$\Rightarrow \quad (x-2)^2=16+(x+2)^2-8\sqrt{(x+2)^2+y^2}$$

$$\Rightarrow \quad x^2+4-4x=16+x^2+4+4x-8\sqrt{(x+2)^2+y^2}$$

$$\Rightarrow \quad -8x=16-8\sqrt{(x+2)^2+y^2}$$

$$\Rightarrow \quad -x=2-\sqrt{(x+2)^2+y^2}$$

$$\Rightarrow \quad \sqrt{(x+2)^2+y^2}=x+2$$

Again, on squaring both sides, we get

$$(x+2)^2+y^2=(x+2)^2$$

$$\Rightarrow \quad y^2=0 \qquad \ldots\text{(i)}$$

Eq. (i) represents pair of straight lines.

3. $\dfrac{n!}{n^n}=\dfrac{1}{n}\cdot\dfrac{2}{n}\cdot\dfrac{3}{n}\cdot\ldots\cdot\dfrac{n}{n}$

Let $\quad A=\lim\limits_{n\to\infty}\dfrac{(n!)^{1/n}}{n}=\lim\limits_{n\to\infty}\left(\dfrac{n!}{n^n}\right)^{1/n}$

$$\Rightarrow \quad A=\lim_{n\to\infty}\left[\frac{1}{n}\cdot\frac{2}{n}\cdot\frac{3}{n}\cdot\ldots\cdot\frac{n}{n}\right]^{1/n}$$

$$\Rightarrow \quad \log A=\lim_{n\to\infty}\frac{1}{n}\left[\log\left(\frac{1}{n}\right)+\log\left(\frac{2}{n}\right)+\log\left(\frac{3}{n}\right)+\ldots+\log\left(\frac{n}{n}\right)\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\left(\frac{r}{n}\right)$$

$$=\int_0^1\log x\,dx=[x\log x-x]_0^1$$

$$=[1\log 1-1]-[\lim_{x\to 0}x\log x-0]=-1$$

$$\therefore \quad A=e^{-1}=\frac{1}{e}$$

4. Let $\quad I=\displaystyle\int_0^\infty\frac{dx}{(x^2+4)(x^2+9)}$

$$=\int_0^\infty\frac{x^2}{x^2(x^2+4)(x^2+9)}dx$$

$$=\int_0^\infty\frac{x^2}{(x^2+0)(x^2+4)(x^2+9)}dx$$

$$=\frac{\pi}{2(0+2)(2+3)(3+0)}$$

$$\left[\because \int_0^\infty\frac{x^2}{(x^2+a^2)(x^2+b^2)(x^2+c^2)}dx=\frac{\pi}{2(a+b)(b+c)(c+a)}\right]$$

$$=\frac{\pi}{60}$$

5. We have, $4^{\sin^2 x}+4^{\cos^2 x}\geq 2\sqrt{4^{\sin^2 x}\cdot 4^{\cos^2 x}}$

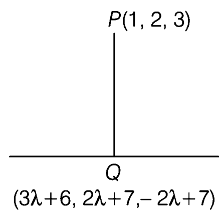
[$\because$ AM $\geq$ GM]

$$\Rightarrow \quad 4^{\sin^2 x}+4^{\cos^2 x}\geq 2\sqrt{4^{(\sin^2 x+\cos^2 x)}}$$

$$\Rightarrow \quad 4^{\sin^2 x}+4^{\cos^2 x}\geq 2\sqrt{4}$$

$$\Rightarrow \quad 4^{\sin^2 x}+4^{\cos^2 x}\geq 4$$

6. Let Q be the foot of the perpendicular drawn from the point $P(1, 2, 3)$ to the given line.



Let the coordinates of Q be

$$(3\lambda+6, 2\lambda+7, -2\lambda+7) \qquad \ldots\text{(i)}$$

$\therefore$ Direction ratios of PQ are proportional to

$$3\lambda+6-1, 2\lambda+7-2, -2\lambda+7-3$$

i.e. $\quad 3\lambda+5, 2\lambda+5, -2\lambda+4$

Direction ratios of the given line are proportional to $3, 2, -2$.

Since, PQ is perpendicular to the given line.

$\therefore 3(3\lambda+5)+2(2\lambda+5)+(-2)(-2\lambda+4)=0$

$$\Rightarrow \quad \lambda=-1$$

On putting $\lambda=-1$ in Eq. (i), we obtain the coordinates of Q as (3, 5, 9).

$\therefore \quad PQ=\sqrt{(3-1)^2+(5-2)^2+(9-3)^2}=7$ units

7. The smallest equivalence relation R_1 containing (1, 2) and (2, 1) is

$$R_1 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$$

Now, we are left with four ordered pairs namely (2, 3), (3, 2), (1, 3) and (3, 1). If we add any one say (2, 3) to R_1, then for symmetry, we must add (3, 2) and then for transitivity, we are forced to add (1, 3) and (3, 1). Thus, the only equivalence relation other than R_1 is the universal relation. Hence, the total number of equivalence relations containing (1, 2) and (2, 1) is 2.

8. Let $f(x) = y$

Then, $x^2 - x + 1 = y$

$\Rightarrow \quad x^2 - x + (1 - y) = 0$

$$\Rightarrow \quad x = \frac{1 \pm \sqrt{1 - 4(1 - y)}}{2}$$

$$\Rightarrow \quad x = \frac{1 \pm \sqrt{4y - 3}}{2}$$

$$\Rightarrow \quad x = \frac{1 + \sqrt{4y - 3}}{2} \qquad \left[\because x \geq \frac{1}{2}\right]$$

$$\Rightarrow \quad f^{-1}(y) = \frac{1 + \sqrt{4y - 3}}{2}$$

$$\Rightarrow \quad f^{-1}(x) = \frac{1 + \sqrt{4x - 3}}{2}$$

Now, $\quad f(x) = f^{-1}(x)$

$$\Rightarrow \quad x^2 - x + 1 = \frac{1 + \sqrt{4x - 3}}{2}$$

Clearly, it is satisfied by $x = 1$.

$\therefore \quad x = \{1\}$

9. We have, $z = x + iy$

$$\therefore \quad \left|\frac{z - 5i}{z + 5i}\right| = 1$$

$\Rightarrow \quad |z - 5i| = |z + 5i|$

$\Rightarrow \quad |x + iy - 5i| = |x + iy + 5i|$

$\Rightarrow \quad |x + (y - 5)i| = |x + (y + 5)i|$

$\Rightarrow \quad \sqrt{x^2 + (y - 5)^2} = \sqrt{x^2 + (y + 5)^2}$

$\Rightarrow \quad x^2 + (y - 5)^2 = x^2 + (y + 5)^2$

$\Rightarrow \quad (y - 5)^2 = (y + 5)^2$

$\Rightarrow \quad y^2 - 10y + 25 = y^2 + 10y + 25$

$\Rightarrow \quad 20y = 0$

$\Rightarrow \quad y = 0$

$\therefore$ z lies on X-axis.

10. Clearly, the curve which passing through (1, 1), is

$$y^3 + 2x - 3x^2 y = 0$$

Now, consider

$$y^3 + 2x - 3x^2 y = 0$$

On differentiating both sides w.r.t. x, we get

$$3y^2 \frac{dy}{dx} + 2 - 3\left(x^2 \frac{dy}{dx} + y \cdot 2x\right) = 0$$

$$\Rightarrow \quad (3y^2 - 3x^2)\frac{dy}{dx} + 2 - 6yx = 0$$

$$\Rightarrow \quad 3(y^2 - x^2)\frac{dy}{dx} = 6yx - 2$$

$$\Rightarrow \quad 3(y^2 - x^2)\, dy - 2(3yx - 1)\, dx = 0$$

which is not a given differential equation.

Hence, none of the given curves satisfy the given differential equation.

11. We have, $\lim\limits_{x \to a} \dfrac{a^x - x^a}{x^x - a^a} = -1$

$$\Rightarrow \quad \lim_{x \to a} \frac{a^x \log a - ax^{a-1}}{x^x (\log x + 1)} = -1$$

[using L'Hospital's rule]

$$\Rightarrow \quad \frac{a^a \log a - a \cdot a^{a-1}}{a^a(\log a + 1)} = -1$$

$$\Rightarrow \quad \frac{\log a - 1}{\log a + 1} = -1$$

$\Rightarrow \quad \log a - 1 = -\log a - 1$

$\Rightarrow \quad 2\log a = 0$

$\Rightarrow \quad \log a = 0$

$\Rightarrow \quad a = e^0$

$\Rightarrow \quad a = 1$

12. We have, $G(x) = hogof(x)$

$= h(g(f(x)))$

$= h(g(\sin x))$

$= h(\sin^2 x)$

$= \log \sin^2 x$

$= 2\log \sin x$

$$\Rightarrow \quad G'(x) = \frac{2}{\sin x} \cdot \cos x = 2\cot x$$

$\Rightarrow \quad G''(x) = -2\,\text{cosec}^2 x$

13. Let $f(x) = e^{\cos^2 x} \cdot \cos^3(2n + 1)\,x$

Then, $f(\pi - x) = e^{\cos^2 (\pi - x)} \cdot \cos^3[(2n + 1)\pi - (2n + 1)x]$

$= -\,e^{\cos^2 x} \cdot \cos^3(2n + 1)x$

$\Rightarrow \quad f(\pi - x) = -f(x)$

Hence, $\int_0^{\pi} e^{\cos^2 x} \cdot \cos^3(2n+1)x\, dx = 0$

$$\left[\because \int_0^{2a} f(x)\, dx = 0, \text{ if } f(2a-x) = -f(x)\right]$$

14. We have the following cases :

***Case* I** When $x < 2$ and $y < -1$, then we have

$$1 \le -(x-2) - (y+1) \le 2$$
$$\Rightarrow \quad 1 \le -x + 2 - y - 1 \le 2$$
$$\Rightarrow \quad 1 \le -x - y + 1 \le 2$$
$$\Rightarrow \quad 0 \le -x - y \le 1$$
$$\Rightarrow \quad -1 \le x + y \le 0 \qquad \ldots\text{(i)}$$

***Case* II** When $x < 2$ and $y \ge -1$, then we have

$$1 \le -(x-2) + y + 1 \le 2$$
$$\Rightarrow \quad 1 \le -x + 2 + y + 1 \le 2$$
$$\Rightarrow \quad 1 \le y - x + 3 \le 2$$
$$\Rightarrow \quad -2 \le y - x \le -1$$
$$\Rightarrow \quad 1 \le x - y \le 2 \qquad \ldots\text{(ii)}$$

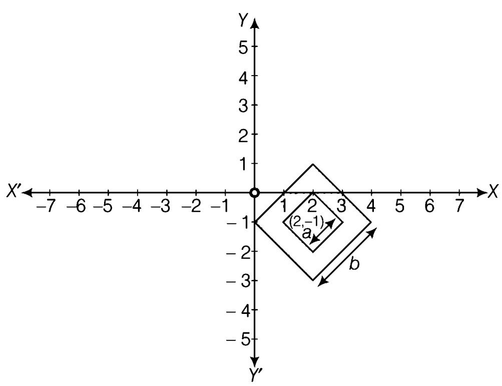
***Case* III** When $x \ge 2$ and $y < -1$, then we have

$$1 \le x - 2 - y - 1 \le 2$$
$$\Rightarrow \quad 1 \le x - y - 3 \le 2$$
$$\Rightarrow \quad 4 \le x - y \le 5 \qquad \ldots\text{(iii)}$$

***Case* IV** When $x \ge 2$ and $y \ge -1$, then we have

$$1 \le x - 2 + y + 1 \le 2$$
$$\Rightarrow \quad 1 \le x + y - 1 \le 2$$
$$\Rightarrow \quad 2 \le x + y \le 3 \qquad \ldots\text{(iv)}$$

Now, the required region is given by Case I, Case II, Case III and Case IV, which is as shown below :



Clearly, the required region is the region between two squares of sides $a = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$ units

and $\quad b = \sqrt{(2)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$ units

Hence, the required area

$$= (2\sqrt{2})^2 - (\sqrt{2})^2 = 6 \text{ sq units}$$

15. $\frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_2} + \ldots + \frac{15C_{15}}{C_{14}}$

$$= n + \frac{\frac{2n(n-1)}{2}}{n} + \frac{\frac{3n(n-1)(n-2)}{3\times 2}}{\frac{n(n-1)}{2}} + \ldots \text{upto 15 terms}$$

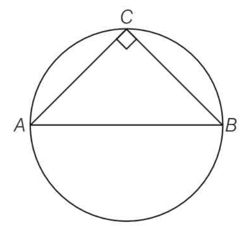
$$= n + (n-1) + (n-2) + \ldots + \text{ upto 15 terms}$$
$$= \frac{15(15+1)}{2} = 120$$

16. We have,

$$(hofog)(x) = h(fog(x)) = h(f(g(x))) = h(f(\sqrt{x^2+1}))$$
$$= h(x^2 + 1 - 1) = h(x^2)$$
$$= x^2 \qquad [\because x^2 \ge 0]$$

17. Clearly, $\angle C = 90°$, since angle in a semi-circle is a right angle. Now, area of triangle is maximum, when $AC = BC$.

i.e. Triangle is right angled isosceles.



18. We have, $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$

$$\Rightarrow \quad \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}(1 + abc) = 0$$

$$\Rightarrow \quad \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0 \text{ or } (1 + abc) = 0$$

But $\quad \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \ne 0$

as vectors $(1, a, a^2)$ $(1, b, b^2)$ and $(1, c, c^2)$ are non-coplanar.

$\therefore \quad 1 + abc = 0 \Rightarrow abc = -1$

19. The component of **a** along **b**

$$= \left(\frac{\mathbf{b}\cdot\mathbf{a}}{|\mathbf{b}|^2}\right)\mathbf{b} = \frac{(3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})\cdot(4\hat{\mathbf{i}} + 6\hat{\mathbf{j}})}{|5|^2}(3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$$

$$= \frac{18}{25}(3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$$

20. Given, $y = \sin x - \tan x$

$$\Rightarrow \quad \frac{dy}{dx} = \cos x - \sec^2 x$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\sin x - 2\sec^2 x \tan x < 0, \quad \forall x \in (0, \pi/2)$$

Hence, the tangent drawn to the curve will lie above the curve.

21. Consider, LHL $= \lim_{x \to 0^-} |x|^{[\cos x]} = \lim_{x \to 0} (-x)^0 = 1$

$[\because \text{as } x \to 0^- \Rightarrow 0 < \cos x < 1 \therefore [\cos x] = 0]$

and RHL $= \lim_{x \to 0^+} |x|^{[\cos x]}$

$= \lim_{x \to 0} x^0$

$[\because \text{as } x \to 0^+ \Rightarrow 0 < \cos x < 1 \therefore [\cos x] = 0]$

$= 1$

Hence, $\lim_{x \to 0} |x|^{[\cos x]} = 1$

22. Equation of all conics whose centre lies as origin can be taken as

$$ax^2 + 2hxy + by^2 + 2gx + 2fy = 0$$

On dividing the whole equation by f, we get

$$\frac{ax^2}{f} + \frac{2hxy}{f} + \frac{by^2}{f} + \frac{2gx}{f} + 2y = 0$$

Since, there are four arbitrary constants. So, its differential equation will be of order 4.

23. We have,

$$\mathbf{a} \times \mathbf{b} = \mathbf{c} \text{ and } \mathbf{c} \times \mathbf{a} = \mathbf{b}$$

$\Rightarrow \quad \mathbf{c} \perp \mathbf{a}, \mathbf{b}$ and $\mathbf{b} \perp \mathbf{c}, \mathbf{a}$

$\Rightarrow$ a is perpendicular to both b and c.

$\Rightarrow \quad \mathbf{a} \parallel \mathbf{b} \times \mathbf{c}$

24. We know that, $\cos^{-1} x$ and $\sin^{-1} x$ exist for $x \in [-1, 1]$.

Now, $\cos^{-1} x > \sin^{-1} x$

$$\Rightarrow \quad \cos^{-1} x > \frac{\pi}{2} - \cos^{-1} x$$

$$\Rightarrow \quad 2\cos^{-1} x > \frac{\pi}{2} \Rightarrow \cos^{-1} x > \frac{\pi}{4}$$

$$\Rightarrow \quad x \in \left[-1, \frac{1}{\sqrt{2}}\right)$$

25. If A is a diagonal matrix and $A^2 = A$

Then, $a_{ii}^2 = a_{ii}$ for $i = 1, 2, 3, 4 \Rightarrow a_{ii} = 0$

Hence, total number of 4×4 diagonal matrix

$= 2^4 = 16$

$\therefore$ Number of non-zero diagonal matrix $= 16 - 1 = 15$

26. For the first set, we have ${}^{52}C_{17}$ choices.

For the second set, we have ${}^{35}C_{17}$ choices. For the third set, we have ${}^{18}C_{17}$ choices and for the last set, we have ${}^{1}C_{1}$ choices.

Since, order of group is not important, so we have divide all arrangements by $\frac{1}{3!}$.

$\therefore$ Total number of ways of distribution

$$= {}^{52}C_{17} \times {}^{35}C_{17} \times {}^{18}C_{17} \times {}^{1}C_{1} \times \frac{1}{3!} = \frac{52!}{(17!)^3 3!}$$

27. Number of relations from A to B, if A has m elements and B has n elements $= 2^{m \times n}$

$\therefore$ Required relation on $A = 2^{n \times n} = 2^{n^2}$

28. Area $= \int_1^b f(x)\,dx = g(b) = (b-1)\sin(3b+4)$

On differentiating both sides with respect to b, we have

$f(b) - 0 = g'(b)$

$\Rightarrow \quad f(b) = \sin(3b+4) + (b-1)\cos(3b+4)\cdot 3$

$\Rightarrow \quad f(x) = \sin(3x+4) + 3(x-1)\cos(3x+4)$

29. Let the angle of ΔABC be $4x$, x and x.

Then, $4x + x + x = 180°$

$\Rightarrow \quad x = 30°$

So, the angles are $A = 120°$, $B = 30°$ and $C = 30°$

Now, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$

$$\Rightarrow \quad a = \frac{\sqrt{3}}{2}k, \; b = \frac{1}{2}k \text{ and } c = \frac{1}{2}k$$

$$\therefore \quad \text{Required ratio} = \frac{a}{a+b+c} = \frac{\frac{\sqrt{3}}{2}k}{\frac{\sqrt{3}}{2}k + \frac{1}{2}k + \frac{1}{2}k}$$

$$= \frac{\sqrt{3}}{2+\sqrt{3}}$$

$$= \sqrt{3} : 2 + \sqrt{3}$$

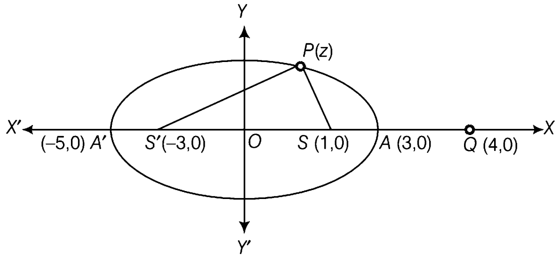
30. We have, $|z-1|+|z+3|=8$ ...(i)

$\Rightarrow |z-(1+0i)|+|z-(-3+0i)|=8$

Thus, if P, S and S' are three points in the argand plane representing complex numbers z, $1+0i$ and $-3+0i$, then from Eq. (i), we have

$$PS + PS' = 8$$

$\Rightarrow P$ lies on the ellipse whose two foci are at $S(1,0)$ and $S'(-3,0)$ and major axis $=8$



Now, $PQ=|z-4|$

Clearly, PQ is minimum or maximum according as P coincides with A (3, 0) and $A'(-5,0)$, respectively.

Thus, $PQ=|z-4|$ varies between $AQ=1$ and $A'Q=9$

Hence, $|z-4|\in[1,9]$

31. Since, a, b and c are the roots of equation $x^3-px^2+qx-r=0$.

$\therefore a+b+c=p, ab+bc+ca=q, abc=r$

Now, $\dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{c^2}=\dfrac{a^2b^2+b^2c^2+c^2a^2}{(abc)^2}$

$$=\frac{(ab)^2+(bc)^2+(ca)^2}{(abc)^2}$$

$$=\frac{(ab+bc+ca)^2-2(ab^2c+bc^2a+ca^2b)}{(abc)^2}$$

$$=\frac{(ab+bc+ca)^2-2abc(a+b+c)}{(abc)^2}$$

$$=\frac{q^2-2rp}{r^2}$$

$$=\frac{q^2-2pr}{r^2}$$

32. Let $\quad I=\int x^3 f(x^2)\,dx$

$$=\frac{1}{2}\int \underset{\text{I}}{x^2}\,\underset{\text{II}}{f(x^2)}\,d(x^2)$$

$$=\frac{1}{2}[x^2F(x^2)-\int F(x^2)\,d(x^2)]$$

$$=\frac{1}{2}[x^2F(x^2)-\int F(x^2)\,d(x^2)]$$

33. We have,

$$x^2-5x+6=0$$

$\Rightarrow \quad (x-2)(x-3)=0$

$\Rightarrow \quad x=2,3$

Clearly, these two equations represent two parallel planes parallel to YOZ plane.

34. We have, $f(x)=\log_x(\log x)=\dfrac{\log(\log x)}{\log x}$

Now, $\dfrac{d}{dx}(f(x))=f'(x)$

$$=\frac{\log x\cdot\dfrac{1}{\log x}\cdot\dfrac{1}{x}-\log(\log x)\cdot\dfrac{1}{x}}{(\log x)^2}$$

$$=\frac{\dfrac{1}{x}[1-\log(\log x)]}{(\log x)^2}$$

$$=\frac{1-\log(\log x)}{x(\log x)^2}$$

$\Rightarrow \dfrac{d}{dx}[f(x)]|_{\text{at } x=e}=\dfrac{1-\log(\log e)}{e(\log e)^2}$

$$=\frac{1-\log 1}{e(1)^2}=\frac{1}{e}$$

35. Since, $f(x)$ is continuous for all x.

$\therefore \quad \lim_{x\to 2} f(x)=f(2)$

$\Rightarrow \lim_{x\to 2}\dfrac{x^3+x^2-16x+20}{(x-2)^2}=b$

$\Rightarrow \quad \lim_{x\to 2}\dfrac{3x^2+2x-16}{2(x-2)}=b$

[using L'Hospital's rule]

$\Rightarrow \quad \lim_{x\to 2}\dfrac{6x+2}{2}=b$

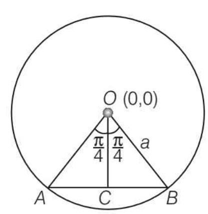
[using L'Hospital's rule]

$\Rightarrow \quad \dfrac{12+2}{2}=b$

$\Rightarrow \quad b=7$

36. The coordinates of the centre and radius of given circle are (1, 1) and a, respectively.

Let $C(x_1, y_1)$ be the mid-point of the chord AB.



In ΔCOB,

$$\sin\frac{\pi}{4} = \frac{BC}{OB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{BC}{a} \Rightarrow BC = \frac{a}{\sqrt{2}}$$

Using Pythagoras theorem,

$$OB^2 = OC^2 + CB^2$$

$$\Rightarrow \quad (a)^2 = x_1^2 + (y_1)^2 + \left(\frac{a}{\sqrt{2}}\right)^2$$

$$\Rightarrow \quad a^2 = x_1^2 + y_1^2 + \frac{a^2}{2}$$

$$\Rightarrow \quad x_1^2 + y_1^2 = \frac{a^2}{2}$$

Hence, locus of mid-point of chord is

$$x^2 + y^2 = \frac{a^2}{2}$$

37. We have, $y \log y \frac{dx}{dy} + x - \log y = 0$

$$\Rightarrow \quad y \log y \frac{dx}{dy} + x = \log y$$

$$\Rightarrow \quad \frac{dx}{dy} + \frac{x}{y \log y} = \frac{1}{y}$$

The above equation is a linear differential equation in x.

$$\therefore \quad \text{IF} = e^{\int \frac{1}{y\log y}dx} = e^{\log \log y} = \log y$$

38. When a fair die is rolled, then total outcomes = 6

Now, $P(A \cup B / C) = \frac{P[(A \cup B) \cap C]}{P(C)}$

$$= \frac{3/6}{4/6}$$

$[\because A \cup B = \{1, 2, 3, 5\}$ and $(A \cup B) \cap C = \{2, 3, 5\}]$

$$= \frac{3}{4}$$

39. We have, probability of getting atleast seven points

= probability of getting 7 points or 8 points.

Seven points in four matches can be obtained in the following four different ways:

2, 2, 2, 1; 2, 2, 1, 2; 2, 1, 2, 2; 1, 2, 2, 2

The probability of each of these ways

$$= (0.50)^3(0.05)$$

$$= 0.00625$$

$\therefore$ Probability of getting 7 points = 4(0.00625)

$$= 0.0250$$

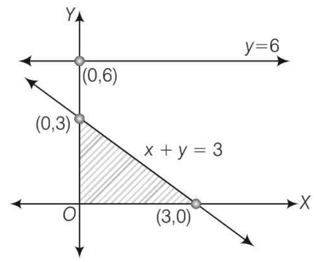
Eight points in four matches can be obtained only in one way, i.e. 2, 2, 2, 2.

$\therefore$ Probability of getting 8 points = $(0.50)^4 = 0.0625$

Thus, the required probability = 0.0250 + 0.0625

$$= 0.0875$$

40. The given region is bounded in first quadrant.



41. Since, $x^2 + px + 1$ is a factor of $ax^3 + bx + c$, so the remainder will be zero.

Now, $ax^3 + bx + c = (x^2 + px + 1)(ax - ap) + x(b - a + ap^2) + (c + ap)$

$$\Rightarrow \quad x(b - a + ap^2) + (c + ap) = 0$$

$$\Rightarrow \quad b - a + ap^2 = 0 \text{ and } c + ap = 0$$

On putting $p = -\frac{c}{a}$ in $b - a + ap^2 = 0$, we get

$$b - a + a\cdot\left(\frac{c^2}{a^2}\right) = 0$$

$$\Rightarrow \quad ab - a^2 + c^2 = 0$$

$$\Rightarrow \quad a^2 - c^2 = ab$$

42. We have, $\frac{\tan 3x - \tan 2x}{1 + \tan 3x \tan 2x} = 1$

$$\Rightarrow \quad \tan(3x - 2x) = 1$$

$$\Rightarrow \quad \tan x = 1$$

$$\Rightarrow \quad x = n\pi + \frac{\pi}{4}$$

But for this value of x, we have

$$\tan 2x = \tan\left(2n\pi + \frac{\pi}{2}\right) = \infty$$

which does not satisfy the given equation as it reduces to an indeterminate form.

$$\therefore \quad x = \phi$$

43. We have, $y = a\log x + bx^2 + x$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{a}{x} + 2bx + 1$$

For extremum, $\frac{dy}{dx} = 0$ at $x = -1$ and $x = 2$

$$\therefore \quad -a - 2b + 1 = 0 \Rightarrow a + 2b = 1 \quad \ldots(i)$$

and $\frac{a}{2} + 4b + 1 = 0 \Rightarrow a + 8b = -2$ ...(ii)

From Eqs. (i) and (ii), we have

$$a = 2 \text{ and } b = -\frac{1}{2}$$

44. Let the axes be rotated through an angle θ. Then,

$x = X\cos\theta - Y\sin\theta,\ y = X\sin\theta + Y\cos\theta$

$\therefore\quad ax^2 + 2hxy + by^2 + 2gx + 2fy + c$

$$= a(X\cos\theta - Y\sin\theta)^2 + 2h(X\cos\theta - Y\sin\theta)(X\sin\theta + Y\cos\theta) + b(X\sin\theta + Y\cos\theta)^2 + 2g(X\cos\theta - Y\sin\theta) + 2f(X\sin\theta + Y\cos\theta) + c$$

This will not contain the product XY, if coefficient of $XY = 0$

$$\Rightarrow -2a\cos\theta\sin\theta + 2h(\cos^2\theta - \sin^2\theta) + 2b\sin\theta\cos\theta = 0$$

$$\Rightarrow -a\sin 2\theta + 2h\cos 2\theta + b\sin 2\theta = 0$$

$$\Rightarrow (a-b)\sin 2\theta = 2h\cos 2\theta$$

$$\Rightarrow \tan 2\theta = \frac{2h}{a-b} \Rightarrow \theta = \frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right)$$

45. Given, $\cos A + \cos C = 4\sin^2 \frac{1}{2}B$

$$\Rightarrow 2\cos\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right) = 4\sin^2\frac{B}{2}$$

$$\Rightarrow 2\sin\frac{B}{2}\left[\cos\frac{A-C}{2} - 2\sin\frac{B}{2}\right] = 0$$

$$\Rightarrow \cos\left(\frac{A-C}{2}\right) - 2\cos\left(\frac{A+C}{2}\right) = 0$$

$$\Rightarrow -\cos\frac{A}{2}\cos\frac{C}{2} + 3\sin\frac{A}{2}\sin\frac{C}{2} = 0$$

$$\Rightarrow \tan\frac{A}{2}\tan\frac{C}{2} = \frac{1}{3}$$

$$\Rightarrow \sqrt{\frac{(s-b)(s-c)}{s(s-a)}\cdot\frac{(s-b)(s-a)}{s(s-c)}} = \frac{1}{3}$$

$$\Rightarrow \frac{s-b}{s} = \frac{1}{3}$$

$$\Rightarrow 3(s-b) = s$$

$$\Rightarrow 2s = 3b$$

$$\Rightarrow a + b + c = 3b$$

$$\Rightarrow a + c = 2b$$

Hence, a, b and c are in an AP.

46. General term of $(3 + 2x)^{74}$ is

$$T_{r+1} = {}^{74}C_r (3)^{74-r} 2^r x^r$$

Let two consecutive terms be T_{r+1}th and T_{r+2}th terms.

According to the question,

Coefficient of T_{r+1} = Coefficient of T_{r+2}

$$\Rightarrow {}^{74}C_r 3^{74-r} 2^r = {}^{74}C_{r+1} 3^{74-(r+1)} 2^{r+1}$$

$$\Rightarrow \frac{{}^{74}C_{r+1}}{{}^{74}C_r} = \frac{3}{2} \Rightarrow \frac{74-r}{r+1} = \frac{3}{2}$$

$$\Rightarrow 148 - 2r = 3r + 3$$

$$\Rightarrow r = 29$$

Hence, two consecutive terms are 30 and 31.

47. We have, $f(g(x)) = (\sin\sqrt{x})^2$

$$\Rightarrow g(x) = \sqrt{x} \text{ and } f(x) = (\sin^2 x)$$

48. Let e be the identity element for the binary operation *.

Then, $a * e = a$

$$\Rightarrow \frac{ae}{100} = a \Rightarrow e = 100$$

Let b be the inverse element for the operation. Then, $a * b = e$

$$\Rightarrow \frac{ab}{100} = 100 \qquad [\because e = 100] \quad \text{...(i)}$$

On putting $a = 0.1$ in Eq. (i), we get

$$\frac{0.1 \times b}{100} = 100 \Rightarrow b = 10^5$$

49. We have,

$$(a^2 + b^2 + c^2 + d^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \le 0$$

$$\Rightarrow (a^2p^2 - 2pab + b^2) + (b^2p^2 - 2pbc + c^2) + (c^2p^2 - 2pcd + d^2) \le 0$$

$$\Rightarrow (ap - b)^2 + (bp - c)^2 + (cp - d)^2 \le 0$$

$$\Rightarrow ap - b = 0,\ bp - c = 0,\ cp - d = 0$$

$$\Rightarrow p = \frac{b}{a},\ p = \frac{c}{b},\ p = \frac{d}{c}$$

$$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c} = p$$

$\therefore$ a, b, c and d are in GP.

50. Let P, Q and R be the given points. Then,

we have

$$\mathbf{PQ} = (2\mathbf{a} + 3\mathbf{b} - 4\mathbf{c}) - (\mathbf{a} - 2\mathbf{b} + 3\mathbf{c})$$
$$= \mathbf{a} + 5\mathbf{b} - 7\mathbf{c}$$
$$\mathbf{QR} = (-7\mathbf{b} + 10\mathbf{c}) - (2\mathbf{a} + 3\mathbf{b} - 4\mathbf{c})$$
$$= -2\mathbf{a} - 10\mathbf{b} + 14\mathbf{c}$$
$$= -2(\mathbf{a} + 5\mathbf{b} - 7\mathbf{c})$$
$$\Rightarrow \mathbf{QR} = -2\mathbf{PQ}$$

Hence, P, Q and R are collinear.