Solved Paper 2014

AMU

Engineering Entrance Exam

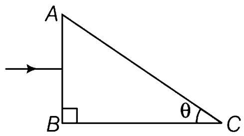
Physics

1. A parallel-plate capacitor with circular plates of radius R is being charged with a current i. Find the value $\oint \mathbf{B} \cdot d\,\mathbf{s}$ between the plates at radius $r = R/5$ from their centre
(a) $\mu_0 i/5$ (b) $\mu_0 i/25$
(c) $\mu_0 i$ (d) $5\mu_0 i$

2. A series RLC circuit, driven with $E_{rms} = 120$ V at frequency 50 Hz, contains an inductance with $X_L = 100\,\Omega$, a capacitance with $X_C = 110\,\Omega$ and an unknown resistance R. For what value of R, the power factor is 0.9?
(a) 20 Ω (b) 42 Ω
(c) 59 Ω (d) 110 Ω

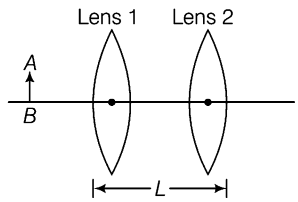
3. What inductance must be connected to a 17 pF capacitor in an oscillator capable of generating 550 nm electromagnetic waves?
(a) 2.3×10^{-25} H (b) 4.2×10^{-23} H
(c) 2.6×10^{-22} H (d) 5.0×10^{-21} H

4. In the figure, a ray of light is perpendicular to the face AB of a glass prism ($\mu = 1.52$). Find the value of θ so that the ray is totally reflected at face AC, if the prism is immersed in water.



(a) 45° (b) 30°
(c) 15° (d) None of these

5. Figure shows an object AB placed in front of two thin coaxial lenses 1 and 2 with focal lengths 24 cm and 9.0 cm, respectively. The object is 6.0 cm from the lens I and the lens separation is $L = 10$ cm. Where does the system of two lenses produce an image of the object AB?



(a) + 18 cm (b) – 18 cm
(c) + 24 cm (d) – 24 cm

6. A concave mirror has a radius of curvature of 35 cm. It is positioned so that the (upright) image of an object is 2.5 times, the size of the object. How far is the mirror from the object?
(a) 10.5 cm (b) 9.2 cm
(c) 8.7 cm (d) 6.7 cm

7. White light, with a uniform intensity, is perpendicularly incident on a water film of refractive index 1.33 and thickness 320 nm, that is suspended in air. At what wavelength is the light reflected by the film brightest to an observer?
(a) 459 nm
(b) 567 nm
(c) 623 nm
(d) 690 nm

8. Rank the following radiations according to their associated energies, greatest first.
(i) Yellow light from a sodium lamp
(ii) Gamma ray emitted by a radioactive nucleus
(iii) Radio wave emitted by the antenna
(iv) Microwave beam emitted by radar
(a) (ii), (i), (iv), (iii) (b) (i), (ii), (iii), (iv)
(c) (iii), (iv), (i), (ii) (d) (i), (ii), (iv), (iii)

9. Find the maximum wavelength of the light that will excite an electron in the valence band of diamond to the conduction band. The energy gap is 5.5 eV.
(a) 225 nm (b) 315 nm
(c) 352 nm (d) 412 nm

10. In Rutherford experiment, a 5.3 MeV alpha particle moves towards the gold nucleus ($Z = 79$). How close does the alpha particle get to the centre of the nucleus, before it comes momentarily to rest and reverses its motion? ($\varepsilon_0 = 8.8 \times 10^{-12}$ F/m)
(a) 3.4×10^{-15} m (b) 8.6×10^{-14} m
(c) 4.3×10^{-14} m (d) 1.6×10^{-14} m

11. Which of the following fusion reactions will not result in the net release of energy
(i) $^6\text{Li} + ^6\text{Li}$ (ii) $^4\text{He} + ^4\text{He}$
(iii) $^{12}\text{C} + ^{12}\text{C}$ (iv) $^{35}\text{Cl} + ^{35}\text{Cl}$
(a) (i) (b) (ii) (c) (iii) (d) (iv)

12. Which of the following values is the correct order of nuclear density?
(a) 5×10^5 kg/m^3 (b) 9×10^{10} kg/m^3
(c) 3×10^{21} kg/m^3 (d) 2×10^{17} kg/m^3

13. A man running at a speed 5 m/s is viewed in the side view mirror of radius of curvature $R = 2$ m of a stationary car. Calculate the speed of image when the man is at a distance of 9 m from the mirror
(a) 0.3 m/s (b) 0.2 m/s
(c) 0.1 m/s (d) 0.05 m/s

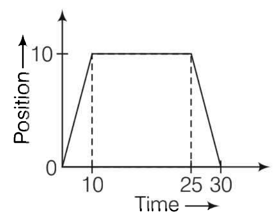
14. The image of an object in concave lens is formed at $f/2$, where f is the focal length of the lens. Find the distance of the object from the lens
(a) f (b) $2f$
(c) $f/2$ (d) infinity

15. The height of a man is measured by a metre scale having graduations m centimetre only and the height turns out to be 170 cm. The scientific method for reporting the measurement is
(a) 170×10^0 cm (b) 1.700 m
(c) 170 cm (d) 1.70×10^2 cm

16. In an experiment the values of two resistances are : $R_1 = (5.0 \pm 0.2)\,\Omega$ and $R_2 = (10.0 \pm 0.1)\,\Omega$. The total resistance when they are connected in parallel is
(a) 3.3 ± 7% (b) 3.3 ± 5%
(c) 3.3 ± 0.3% (d) 2.5 ± 7%

17. What are the dimensions of A/B in the relation $F = A\sqrt{x} + Bt^2$, where F is the force, x is distance and t is time?
(a) $[ML^2T^{-2}]$ (b) $[L^{-1/2}T^2]$
(c) $[L^{-1/2}T^{-1}]$ (d) $[LT^{-2}]$

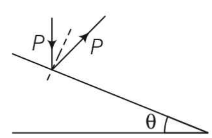
18. Figure shows the position-time graph of an object. Find the ratio of the velocities at 5th and 27th s



(a) 1/2 (b) −1/2 (c) 1 (d) −1

19. Two vectors **A** and **B** are such that **A** + **B** = **C** and |**A**| + |**B**| = |**C**|. Then the vectors **A** and **B** are
(a) parallel (b) perpendicular
(c) anti-parallel (d) null vectors

20. A particle P falling vertically from a height hits a plane inclined to the horizontal at an angle θ with speed v and rebounds elastically, as shown. The distance along the plane where it hits the second time is



(a) $\frac{4v^2 \sin^2\theta}{g}$ (b) $\frac{v^2 \sin\theta}{2g}$
(c) $\frac{4v^2 \sin\theta}{g}$ (d) $\frac{v^2 \sin\theta}{4g}$

21. An object takes n times as much time to slide down a 45° rough incline as it takes to slide down a perfectly smooth 45° incline. The coefficient to kinetic friction between the object and the incline is given by

(a) $\frac{1}{1-n^2}$ (b) $1-\frac{1}{n^2}$

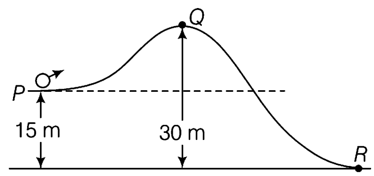
(c) $\sqrt{1-\frac{1}{n^2}}$ (d) $\sqrt{\frac{1}{1-n^2}}$

22. A particle of mass 2 mg moves with constant speed and is found to pass two points 5.0 m apart in a time interval of 5 ms. Find the kinetic energy of the particle

(a) 1.0 J (b) 2.0 J

(c) 3.0 J (d) 4.0 J

23. In the figure shown, a ball of mass 1.0 kg is rolled with a kinetic energy of 330 J. It reaches R through the path PQR. Find the speed of the ball at R, if its potential energy at P is zero



(a) 52 m/s (b) 45 m/s

(c) 40 m/s (d) 31 m/s

24. A force $\mathbf{F} = -(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$ acts on a particle moving in the x-y plane. Starting from the origin, the particle is taken along the positive X-axis to the point $(2a, 0)$ and then parallel to the Y-axis to the point $(2a, 2a)$. The total work done on the particle is

(a) $-4a^2$ (b) $-2a^2$

(c) $4a^2$ (d) $2a^2$

25. Two masses of 6 and 2 unit are at positions $(6\hat{\mathbf{i}} - 7\hat{\mathbf{j}})$ and $(2\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 8\hat{\mathbf{k}})$, respectively. The coordinates of the centre of mass (CM) are

(a) (2, – 5, 3) (b) (5, – 5, – 3)

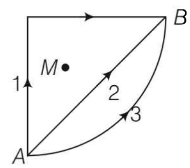
(c) (5, 4, – 2) (d) (5, – 4, – 4)

26. Find the linear velocity of a point on a rigid body rotating with angular velocity $\omega = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + \hat{\mathbf{k}}$; the radius vector of the point is $\mathbf{r} = 5\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$

(a) $-18\hat{\mathbf{i}} - 13\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ (b) $18\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

(c) $3\hat{\mathbf{i}} - 18\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ (d) $-3\hat{\mathbf{i}} - 18\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

27. If W_1, W_2 and W_3 are the work done in moving a particle from A to B along three different paths 1, 2, and 3 respectively (as shown) in the gravitational field of a point mass M, the relation between W_1, W_2 and W_3 is



(a) $W_1 > W_2 > W_3$ (b) $W_1 = W_2 = W_3$

(c) $W_1 < W_2 < W_3$ (d) $W_2 > W_1 > W_3$

28. The height of a mountain is H and the density of its rock is 3×10^3 kg/m^3. If the elastic limit of the rock is 3×10^8 N/m^2, find the height of the mountain

(a) 50 km (b) 30 km (c) 8 km (d) 10 km

29. A glass capillary of radius 0.4 mm is inclined at 60° with the vertical in water. Find the length of water in the capillary tube. (Surface tension of water = 7×10^{-2} N/m)

(a) 7.1 cm (b) 3.6 cm (c) 1.8 cm (d) 0.9 cm

30. The densities of a certain material at 10°C and 40°C are 2.5 g/cm^3 and 2.49 g/cm^3, respectively. The average value of the coefficient of linear expansion of the material in this temperature range is

(a) 4.0×10^{-7} /°C (b) 4.0×10^{-6} /°C

(c) 4.0×10^{-5} /°C (d) 2.0×10^{-3} /°C

31. Stars S_1 and S_2 emits maximum energy at wavelengths 5000 Å and 50 μm, respectively. The surface temperature of S_1 is 6000 K. Find the surface temperature of S_2

(a) 90 K (b) 80 K

(c) 70 K (d) 60 K

32. Which of the following statements is true for the specific heat of solids at constant volume (C_v)?

(a) It is independent of temperature

(b) It increases with rise in temperature and its value is different for different solids at high temperatures

(c) It increases with rise in temperature and its value become $3R$ for different solids at large temperatures

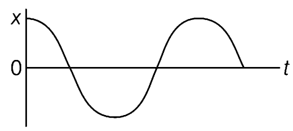
(d) Its value becomes zero for different solids at large temperatures

33. A particle of mass m is placed in a potential field $U(x) = U_0(1 - \cos\alpha x)$, where U_0 and α are positive constants. The time period of small oscillation would be

(a) $\frac{2\pi}{\alpha}\sqrt{\frac{m}{U_0}}$ (b) $\frac{2\pi}{U_0}\sqrt{\frac{m}{\alpha}}$

(c) $2\pi\sqrt{\frac{m}{\alpha U_0}}$ (d) $2\pi\frac{m}{U_0}$

34. The displacement-time graph of a particle executing SHM is shown below



Which of the following statement(s) is are true?

(i) The force is zero at $t = 3\,T/4$

(ii) The acceleration is maximum at $t = T$

(iii) The velocity is maximum at $t = T/4$

(iv) The potential energy is equal to kinetic energy at $t = T/2$

(a) (i) and (ii)
(b) (i), (ii) and (iii)
(c) (i) and (iv)
(d) All of the above

35. Fine the order of root mean square (rms.) velocity of molecules of gas, if the velocity of sound in the same gas is 330 m/s. ($\gamma = 1.41$)

(a) 481 m/s (b) 293 m/s
(c) 280 m/s (d) 260 m/s

36. Two progressive waves are represented by the following equations

$y_1 = 10\sin 2\pi(10t - 0.1x)$

$y_2 = 20\sin 2\pi(20t - 0.2x)$

Find the ratio of their intensities.

(a) 1/2 (b) 1/4
(c) 1/8 (d) 1/16

37. A resonance tube is resonated with a tuning fork of frequency 380 Hz. Two successive lengths of the resonated air-column are found to be 16 cm and 50 cm. Find the length of the third resonance

(a) 85 cm (b) 72 cm
(c) 69 cm (d) 92 cm

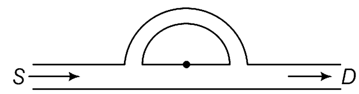
38. Some of the frequencies of tones produced by an organ pipe are

220, 440, 550, 660 Hz.

Find the effective length of the pipe. (Take the speed of sound in air = 330 m/s)

(a) 1.8 m (b) 1.7 m (c) 1.5 m (d) 1.3 m

39. As shown in figure, a sound wave of wavelength 2.28 m enters the tube at S. Find the smallest radius of the circular path to hear minimum sound at D.



(a) 1.6 m (b) 1.3 m
(c) 1.1 m (d) 1.0 m

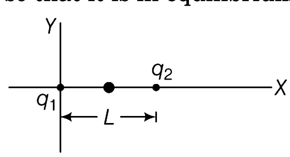
40. An air bubble of volume $20\,cm^3$ is at the bottom of a pond 40 m deep where the temperature is 4.0°C. What will be the volume of the bubble, if it rises to the surface, which is at a temperature of 20°C?

(a) $200\,cm^3$ (b) $100\,cm^3$
(c) $50\,cm^3$ (d) $20\,cm^3$

41. The volumes of containers A and B, connected by a tube and a closed valve are V and $4\,V$ respectively. Both the containers A and B have the same ideal gas at pressures (temperatures) 5.0×10^5 Pa (300 K) and 1.0×10^5 Pa (400 K), respectively. The valve is opened to allow the pressure to equalize, but the temperature of each container is kept constant at its initial value. Find the common pressure in the containers

(a) 2.5×10^5 Pa (b) 2.0×10^5 Pa
(c) 3.0×10^5 Pa (d) 1.5×10^5 Pa

42. Two particles of charges $q_1 = +8q$ and $q_2 = -2q$ are placed, as shown. At what point away from q_2 on the X-axis, can a proton be placed so that it is in equilibrium?



(a) $x = 2\,L$ (b) $x = 2.5\,L$
(c) $x = 3.0\,L$ (d) $x = 3.2\,L$

43. The potential energies associated with four orientation of an electric dipole in an electric field are : (i) $-5U_0$, (ii) $-7U_0$ (iii) $3U_0$ and (iv) $5U_0$ where U_0 is positive.

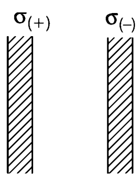
Rank the orientations according to the angle between the electric dipole moment $\boldsymbol{p}$ and the electric field $\mathbf{E}$, greatest first

(a) (i), (ii), (iii), (iv)
(b) (ii), (iii), (i), (iv)
(c) (iv), (iii), (i), (ii)
(d) (iv), (i), (iii), (ii)

44. A thin glass rod is bent into a semi-circle of radius R. A charge $+Q$ is uniformly distributed along the upper half and a charge $-Q$ is uniformly distributed along the lower half. The magnitude of the electric field at the centre of the semi-circle is

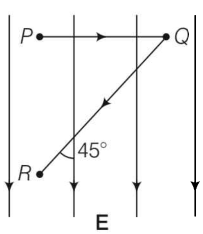
(a) $\frac{2Q}{\pi^2 \varepsilon_0 R^2}$
(b) $\frac{Q}{\pi^2 \varepsilon_0 R^2}$
(c) $\frac{Q}{3\pi^2 \varepsilon_0 R^2}$
(d) $\frac{2Q}{3\pi^2 \varepsilon_0 R^2}$

45. Figure shows the portions of two infinite parallel non-conducting sheets having the magnitude of the surface charge densities $\sigma_{(+)} = 6.8\ \mu C/m^2$ and $\sigma_{(-)} = 4.3\ \mu C/m^2$ for the positively and negatively charged sheets, respectively. Find the electric field $\mathbf{E}$ between the sheets.



(a) 6.3×10^5 N/C towards right
(b) 6.3×10^5 N/C towards left
(c) 1.41×10^5 N/C towards right
(d) 1.41×10^5 N/C towards left

46. Figure shows two points P and R, separated by a distance d, in a uniform electric field $\mathbf{E}$. Find the potential difference by moving the positive test charge q_0 from P to R along the path PQR.



(a) $\mathbf{E}d$
(b) $-\mathbf{E}d/\sqrt{2}$
(c) $\sqrt{2}\mathbf{E}d$
(d) $-\mathbf{E}d$

47. Two conductors are made of the same material and have the same length. Conductor A is a solid wire of diameter 1.0 mm and conductor B is a hollow tube of outside diameter 2.0 mm and inside diameter 1.0 mm. Find the resistance ratio R_A/R_B measured between their ends

(a) 3
(b) 2
(c) 1
(d) 0.5

48. The table shows four sets of values for the circuit elements, when the fully charged capacitor is discharged. Rank the sets according to the time required for the current to decrease to half its initial value, greatest first

	1	2	3	4
E(V)	12	12	10	10
R(Ω)	2	3	10	5
C (μF)	3	2	0.5	2

(a) 1, 2, 3, 4
(b) 3, 2 = 4, 1
(c) 4, 1 = 2, 3
(d) 1, 2 = 3, 4

49. A cyclotron is operated at an oscillator frequency of 24 MHz and has a dee radius of 60 cm. Find the magnitude of the magnetic field needed for deuterons to be accelerated in the cyclotron

(a) 6.4 T
(b) 3.2 T
(c) 1.6 T
(d) 0.9 T

50. A long solenoid with 10 turn/cm and a radius of 7.0 cm carries a current of 20.0 mA. A current of 6.0 A exists in a straight conductor located along the central axis of the solenoid. At what radial distance from the axis will the direction of the magnetic field be at 45° to the axial direction?

(a) 4.8 cm
(b) 8.1 cm
(c) 9.9 cm
(d) 10.6 cm

Chemistry

1. On complete combustion, 0.246 g of an organic compound gave 0.198 g of CO_2 and 0.1014 g of H_2O. The ratio of carbon and hydrogen atoms in the compound is
(a) 1 : 3 (b) 1 : 2
(c) 2 : 5 (d) 2 : 7

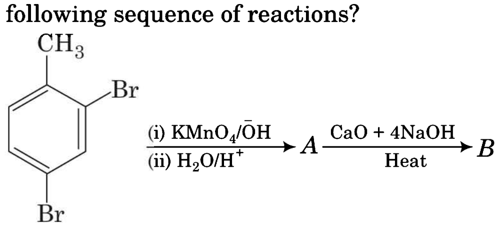
2. The correct order of basic strength in aqueous solution is
(a) $(CH_3)_2NH > CH_3NH_2 > (CH_3)_3N$
(b) $(CH_3)_3N > (CH_3)_2NH > CH_3NH_2$
(c) $CH_3NH_2 > (CH_3)_3N > (CH_3)_2NH$
(d) $(CH_3)_3N > CH_3NH_2 > (CH_3)_2NH$

3. $n\text{-}C_7H_{16} \xrightarrow[\text{10-20 atm}]{V_2O_5,500°C} A \xrightarrow{Cl_2/h\nu} B$

What is B in the above reaction?
(a) Benzyl chloride
(b) Benzal chloride
(c) Hexachlorobenzene
(d) Benzene hexachloride

4. What will be the product/s if benzal chloride is heated with a concentrated aqueous KOH solution?
(a) Benzaldehyde
(b) Benzoic acid
(c) Benzyl alcohol and sodium benzoate
(d) An aldol

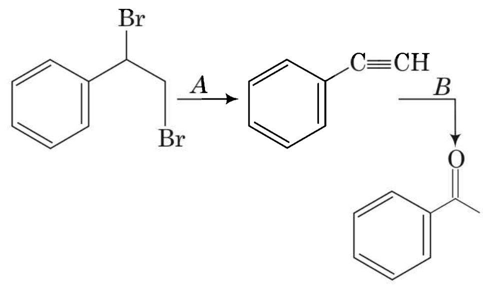
5. What will be the end product (B) in the following sequence of reactions?



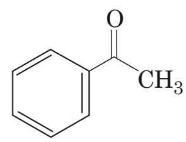
(a) 1, 2-dibromobenzene
(b) 1, 2-dibromobenzaldehyde
(c) 1, 3-dibromobenzene
(d) 1, 4-dibromobenzene

6. The monosaccharide constituents of lactose are
(a) α-D-glucose and β-D-fructose
(b) α-D-glucose only
(c) β-D-glucose only
(d) β-D-glucose and β-D-galactose

7. Identify the reagents in the following transformations



(a) alc. KOH and H_2O, $HgSO_4$, H_2SO_4
(b) alc. KOH and $KMnO_4$ / H^+
(c) $NaNH_2$ and H_2O, $HgSO_4$, H_2SO_4
(d) $NaNH_2$ and $KMnO_4$ / H^+

8. 

The above ketone will not be formed by
(a) reaction of benzene and acetyl chloride in the presence of $AlCl_3$
(b) reaction of acetonitrile with phenylmagnesium bromide in ether followed by hydrolysis
(c) treatment of acetyl chloride with debenzylcadminum
(d) Addition of water to phenylacetylene in the presence of mercuric sulphate and dilute sulphuric acid

9. What shall be the pH of a solution formed by mixing 10 mL of 0.1 M H_2SO_4 and 10 mL of $\frac{N}{10}$ KOH?
(a) 11.40 (b) 8.64
(c) 3.00 (d) 7.00

10. In the reaction at constant volume $C(s) + CO_2(g) \rightleftharpoons 2CO(g)$

Argon gas is added which does not take part in the reaction; choose the correct statement
(a) the equilibrium constant is unchanged
(b) the equilibrium shifts in the forward direction
(c) the equilibrium shifts in the backward direction
(d) the direction of equilibrium depends on the amount of argon added

11. The Balmer series in atomic hydrogen is observed in the following spectral region
(a) infrared (b) ultraviolet
(c) visible (d) far IR

12. The K_p value for the reaction, $H_2 + I_2 \rightleftharpoons 2HI$ at 460°C is 49. If the initial pressure of H_2 and I_2 is 0.5 atm, respectively, what will be the partial pressure of H_2 at equilibrium?
(a) 0.111 atm (b) 0.123 atm
(c) 0.113 atm (d) 0.222 atm

13. In the redox reaction,

$$\overset{+7}{M}nO_4 + 8H^+ + 5Br^- \longrightarrow Mn^{2+} + 4\overset{-1}{H_2}O + \frac{5}{2}Br_2$$

which one is the reducing agent?
(a) H^+ (b) MnO_4^- (c) Br^- (d) Mn^{2+}

14. The rate constant (k_1) of one of the reaction is found to be double that of the rate constant (k_2) of another reaction. The relationship between the corresponding activation energies of the two reactions E_{a1} and E_{a2} will be
(a) $E_{a1} < E_{a2}$ (b) $E_{a1} > E_{a2}$
(c) $E_{a1} = E_{a2}$ (d) $E_{a1} = 2E_{a2}$

15. The energy required to remove an electron from metal X is $E = 3.31 \times 10^{-20}$ J. Calculate the maximum wavelength of light that can photo eject an electron from metal X.
(a) 6.01×10^{-6} m (b) 3.10×10^{-3} m
(c) 5.01×10^{-6} m (d) None of these

16. A 5.82 g silver coin is dissolved in nitric acid. When sodium chloride is added to the solution, all the silver is precipitated as AgCl. The AgCl precipitate weighs 7.20 g. The percentage of silver in the coin is
(a) 60.3% (b) 80% (c) 93.1 % (d) 70%

17. For a reaction taking place in three steps, the rate constants are k_1, k_2 and k_3 and overall rate constant is $k = \frac{k_1 k_3}{k_2}$. If the energies of activation E_1, E_2 and E_3 are 60, 30 and 10 kJ mol^{-1}, respectively, then the overall energy of activation is
(a) 30 kJ mol^{-1} (b) 40 kJ mol^{-1}
(c) 60 kJ mol^{-1} (d) 100 kJ mol^{-1}

18. At 25°C, for the combustion of 1 mole of liquid benzene, the heat of reaction at constant pressure is given by

$$C_6H_6(l) + \frac{7}{2}O_2(g) \longrightarrow 6CO_2(g) + 3H_2O(l),$$

$$\Delta H = 780980 \text{ cal}$$

Calculate the heat of reaction at constant volume
(a) 780.086 kcal (b) –780.086 kcal
(c) –390.043 kcal (d) 390.043 kcal

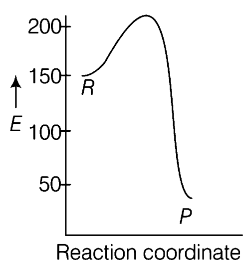
19. What will be solubility product of $Ca(OH)_2$ if its solubility is $\sqrt{3}$?
(a) 3 (b) $3\sqrt{3}$
(c) $12\sqrt{3}$ (d) 27

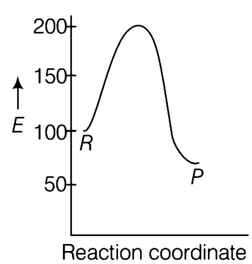
20. 25 mL of 3.0 M HCl are mixed with 75 mL of 4.0 M HCl. If the volumes are additive, the molarity of the final mixture will be
(a) 4.0 M (b) 3.75 M
(c) 4.25 M (d) 3.50 M

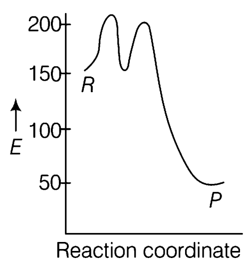
21. An exothermic chemical reaction proceeds in two stages

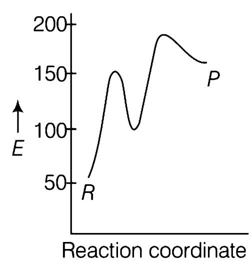
$$R \xrightarrow{\text{stage I}} \text{Intermediate} \xrightarrow{\text{stage II}} P$$

The activation energy of stage I is 50 kJ mol^{-1}. The enthalpy change of the reaction is –100 kJ mol^{-1}. Identify the energy level diagram for the reaction

(a)


(b)


(c)


(d)


22. Given the reduction potentials of Na^+, Mg^{2+}, Al^{3+} and Ag^+ as

$E^\circ_{Na^+/Na} = -2.71 V$; $E^\circ_{Mg^{2+}/Mg} = -2.37 V$

$E^\circ_{Al^{3+}/Al} = -1.66 V$; $E^\circ_{Ag^+/Ag} = 0.08 V$

The least stable oxide is

(a) Ag_2O (b) Al_2O_3
(c) MgO (d) Na_2O

23. The depression in freezing point of water observed for the same amount of acetic acid (I), trichloroacetic acid (II) and trichloroacetic acid (III) decreases in the order

(a) I> II> III (b) II > I > III
(c) III > I > II (d) III > II > I

24. The first ionisation potential of Na, Mg and Si are respectively 496,737 and 786 kJ mol^{-1}. The ionisation potential of Al will be closer to

(a) 760 kJ mol^{-1} (b) 575 kJ mol^{-1}
(c) 801 kJ mol^{-1} (d) 419 kJ mol^{-1}

25. Television picture tube is basically

(a) cathode ray tube
(b) anode ray tube
(c) hybrid of cathode and anode tube
(d) None of the above

26. The television picture on the screen results due to the phenomenon called

(a) phosphorescence
(b) fluoroscence
(c) chemofluoroscence
(d) fluorophosphorescence

27. The correct symbol of the species with number of electrons, protons and neutrons as 18, 16 and 16 respectively is

(a) $^{32}_{16}S$ (b) $^{32}_{18}S$
(c) $^{32}_{16}S^{2-}$ (d) $^{32}_{18}S^{2-}$

28. Which one of the following is least covalent in nature?

(a) NF_3 (b) $BiIF_3$ (c) PF_3 (d) SbF_3

29. Which one of the following acids is used as an oxidizer in rocket fuel ?

(a) $HClO_4$ (b) HNO_2
(c) H_3PO_4 (d) HNO_3

30. The paramagnetic species in the following is

(a) S_8 (b) S_6
(c) S_2 (d) S_2^{2-}

31. The spin only magnetic moment value (in BM unit) of $Cr(CO)_6$ is

(a) zero (b) 2.84
(c) 4.90 (d) 5.92

32. Which one of the following oxidation states is not possible in metal carbonyls

(a) + 1 (b) 0
(c) – 1 (d) + 2

33. The lanthanoid which exhibits +4 oxidation state

(a) Pm (b) Sm
(c) Ce (d) Gd

34. The production of dihydrogen gas *via* water-gas shift reaction

$$CO(g) + H_2O(g) \xrightarrow[\text{Catalyst}]{\Delta} CO_2(g) + H_2(g)$$

The CO_2 gas is removed by scrubbing with solution of

(a) sodium arsenite
(b) calcium oxide
(c) sodium phosphite
(d) aluminium oxide

35. How many hydrogen-bonded water molecule (s) are associated in $CuSO_4 \cdot 5H_2O$

(a) 5 (b) 1
(c) 4 (d) 3

36. The ratio of magnetic moment (spin only value) between $[FeF_6]^{3-}$ and $[Fe(CN)_6]^{3-}$ is approximately

(a) 4 (b) 2 (c) 5 (d) 3

37. Which is not true for describing the catalytic activity of transition metals?

(a) Their ability to adopt multiple oxidation states
(b) Their ability to form bonds between reactant molecule and atoms of the surface of catalysts
(c) Increasing the concentration of reactants at the catalyst surface
(d) Strengthening the bonds in the reacting molecules

38. The bond order between Ni—C bond in $Ni(CO)_4$ is

(a) one (b) two
(c) less than two (d) more than two

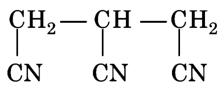
39. The interhalogen having dimeric structure is

(a) ClF_3 (b) BrF_3
(c) IF_3 (d) ICl_3

40. **Which one of the following transition element has the lowest value of enthalpy of atomisation?**
(a) Cr (b) Cu (c) Zn (d) Mn

41. **Which of the following stability order is correct**
(a) $O_2^{2-} > O_2^- > O_2 > O_2^+$ (b) $O_2^+ > O_2 > O_2^- > O_2^{2-}$
(c) $O_2^+ > O_2 < O_2^- > O_2^{2-}$ (d) $O_2 > O_2^+ > O_2^{2-} > O_2^-$

42. **The IUPAC name of the following compound is**



(a) 1, 2, 3-tricyanopropane
(b) propane-1, 2, 3-trinitrile
(c) 3-cyanopentane-1,5-dinitrile
(d) 1,3,5-pentanetrinitrile

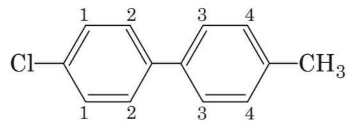
43. **Which is the least stable carbocation?**
I. $CH_3CH_2\overset{+}{C}H_2$ II. $CH_3\overset{+}{C}HCH_3$
III. $(CH_3)_4\overset{+}{C}$ VI. $(CH_3)_3C\overset{+}{C}H_2$
(a) I (b) II
(c) III (d) IV

44. **The order of compounds in their reactivity towards HCN is**
(a) acetaldehyde < ketone < methyl *tert*-butyl ketone < di-*tert*-butyl ketone
(b) di-*tert*-butyl ketone < methyl *tert*-butyl ketone < acetone < acetaldehyde
(c) di-*tert*-butyl ketone < acetone < methyl *tert*-butyl ketone < acetaldehyde
(d) Acetone < acetaldehyde < di-*tert*-butyl ketone < methyl *tert*-butyl ketone

45. **The product/s following reaction is (are)**
$C_2H_5Br + 2Na + CH_3Br \xrightarrow{\text{Dry ether}} ?$
(a) Ethane
(b) Propane
(c) Butane
(d) Ethane, propane and butane

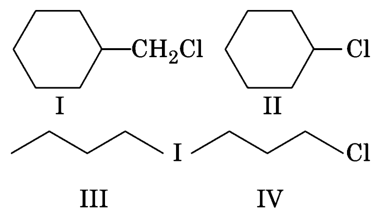
46. **Amongst the following, the most basic compound is**
(a) benzylamine (b) aniline
(c) acetanilide (d) *p*-nitroganiline

47. **Electrophilic substitution of compound *A* will be fastest at position**



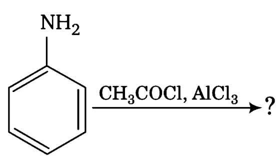
(a) 1 (b) 2 (c) 3 (d) 4

48. **Which of the following haloalkanes would undergo S_N2 reaction faster?**

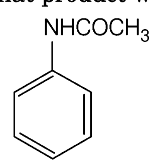


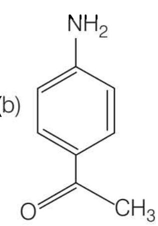
(a) I (b) II (c) III (d) IV

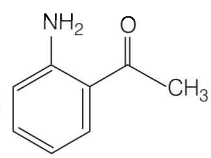
49. **Propene was oxidised by aqueous $KMnO_4$ to give a compound (*A*). Treatment of compound (*A*) with thionyl chloride gave**
(a) 1, 2-dichloropropane
(b) 1-chloropropanone
(c) 2-chloropropanoic acid
(d) 2-chloropropanal

50. 

What product will be obtained?

(a) 

(b) 

(c) 

(d) None of these

Mathematics

1. The distance between the lines $3x + 4y = 9$, $6x + 8y = 15$ is
(a) 3/10 (b) 7/10 (c) 3/2 (d) 2/3

2. If $-3 + ix^2y$ and $x^2 + y + 4i$ be conjugate complex numbers, then (x, y) is
(a) (1, – 4) (b) (–1, 4)
(c) (2, 1) (d) (–2, 1)

3. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with three vertices is equilateral equals
(a) $\frac{1}{2}$ (b) $\frac{1}{5}$ (c) $\frac{1}{10}$ (d) $\frac{1}{20}$

4. If $f(x) = \begin{cases} -x - \pi/2, & x \le -\pi/2 \\ -\cos x, & -\pi/2 < x \le 0 \\ x - 1, & 0 < x \le 1 \\ \ln x, & x > 1 \end{cases}$, then
(a) $f(x)$ is continuous at $x = -\frac{\pi}{2}$
(b) $f(x)$ is differentiable at $x = 0$
(c) $f(x)$ is not differentiable at $x = 1$
(d) $f(x)$ is not differentiable at $x = -\frac{3}{2}$

5. Which of the following is correct solution of
$$(1 + e^{x/y})\,dx + e^{x/y}\left[1 - \left(\frac{x}{y}\right)\right]dy = 0$$
(a) $x + ye^{x/y} = C$ (b) $y + xe^{x/y} = C$
(c) $x + y = Ce^{-x/y}$ (d) $y = x + Ce^{x/y}$

6. ${}^mC_{r+1} + \sum_{k=m}^{n} {}^kC_r =$
(a) ${}^nC_{r+1}$ (b) ${}^{n+1}C_{r+1}$
(c) ${}^nC_r$ (d) None of these

7. Let $f : \left[\frac{1}{2}, 1\right] \to R$ (the set of all real numbers) be a positive, non-constant and differentiable function such that $f'(x) < 2f(x)$ and $f\left(\frac{1}{2}\right) = 1$. Then, the value of $\int_{1/2}^{1} f(x)\,dx$ lies in the interval
(a) $(2e - 1, 2e)$ (b) $(e - 1, 2e - 1)$
(c) $\left(\frac{e-1}{2}, e - 1\right)$ (d) $\left(0, \frac{e-1}{2}\right)$

8. The existence of the unique solution of the system of equations
$$x + y + z = \beta$$
$$5x - y + \alpha z = 10$$
$$2x + 3y - z = 6$$
depends on
(a) α only (b) β only
(c) α and β both (d) neither β nor α

9. Let $f : [0, 1] \to R$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable, $f(0) = f(1) = 0$ and satisfies $f''(x) - 2f'(x) + f(x) \ge e^x$, $x \in [0, 1]$. If the function $e^{-x}f(x)$ assumes its minimum in the interval [0, 1] at $x = 1/4$, which of the following is true?
(a) $f'(x) < f(x)$, $\frac{1}{4} < x < \frac{3}{4}$
(b) $f'(x) > f(x)$, $0 < x < \frac{1}{4}$
(c) $f'(x) < f(x)$, $0 < x < \frac{1}{4}$
(d) $f'(x) < f(x)$, $\frac{3}{4} < x < 1$

10. If $\int_0^{\pi/3} \frac{\cos x}{3 + 4\sin x}\,dx = k \log\left(\frac{3 + 2\sqrt{3}}{3}\right)$, then k is
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

11. $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \ldots + n\frac{C_n}{C_{n-1}} =$
(a) $\frac{n(n-1)}{2}$ (b) $\frac{n(n+1)}{2}$
(c) $\frac{(n+1)(n+2)}{2}$ (d) None of these

12. If x, y and z are all different and $\begin{vmatrix} x & x^2 & 1 + x^3 \\ y & y^2 & 1 + y^3 \\ z & z^2 & 1 + z^3 \end{vmatrix} = 0$, then
(a) $xyz = -1$
(b) $xyz = 1$
(c) $xyz = -2$
(d) $xyz = 2$

13. Which of the following is a correct solution of $x \cos x \cdot \left(\frac{dy}{dx}\right) + y(x \sin x + \cos x) = 1$?

(a) $yx \sec y = C + \tan x$ (b) $yx \cos y = C + \tan x$
(c) $y \cdot x \sec x = C + \tan x$ (d) None of these

14. Let f be the function on $[-\pi, \pi]$ given by $f(0) = 9$ and $f(x) = \sin\left(\frac{9x}{2}\right) \Big/ \sin\left(\frac{x}{2}\right)$ for $x \neq 0$. The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$ is

(a) 0 (b) 4
(c) 8 (d) None of these

15. If $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + \ldots + a_{2n}x^{2n}$, then $a_0 + a_3 + a_6 + \ldots =$

(a) 3^{n+1} (b) 3^n
(c) 3^{n-1} (d) None of these

16. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then k is equal to

(a) 19 (b) $\frac{1}{19}$ (c) -19 (d) $-\frac{1}{19}$

17. Let $f(x)$ be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$. Then, $f(x)$ is

(a) $\frac{1}{3x} + \frac{2x^2}{3}$ (b) $-\frac{1}{3x} + \frac{4x^2}{3}$
(c) $-\frac{1}{x} + \frac{2}{x^2}$ (d) $\frac{1}{x}$

18. If α, β are the roots of the equation $ax^2 + bx + c = 0$ and $S_n = \alpha^n + \beta^n$, then $a\,S_{n+1} + b\,S_n + c\,S_{n-1}$ is equal to

(a) 0 (b) abc
(c) $a + b + c$ (d) None of these

19. Let V_r denote the sum of the first r terms of an arithmetic progression (AP) whose first term is r and the common difference is $(2r - 1)$. The sum $V_1 + V_2 + \ldots + V_n$ is

(a) $\frac{1}{12} n(n+1)(3n^2 - n + 1)$
(b) $\frac{1}{12} n(n+1)(3n^2 + n + 2)$
(c) $\frac{1}{2} n(2n^2 - n + 1)$
(d) $\frac{1}{3}(2n^3 - 2n + 3)$

20. If $x^p y^q = (x + y)^{p+q}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{y}{x}$ (b) $\frac{py}{qx}$
(c) $\frac{x}{y}$ (d) $\frac{qy}{px}$

21. Let $z = x + iy$ be a complex number where x and y are integers. Then, the area of the rectangle whose vertices are the roots of the equation $\bar{z}z^3 + z\bar{z}^3 = 350$ is

(a) 48 (b) 32 (c) 40 (d) 80

22. Let the straight line $x = b$ divide the area enclosed by $y = (1 - x)^2$, $y = 0$ and $x = 0$ into two parts $R_1 (0 \le x \le b)$ and $R_2 (b \le x \le 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then, b equals

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

23. If $\omega = \alpha - i\beta$, where $\beta \neq 0$ and $z \neq 1$, satisfies the condition that $\left(\frac{\omega - \bar{\omega}z}{1 - z}\right)$ is purely real, then the set of values of z is

(a) $\{z : |z| = 1\}$ (b) $\{z : z = \bar{z}\}$
(c) $\{z : z \neq 1\}$ (d) $\{z : |z| = 1, z \neq 1\}$

24. If the function $f(x) = \frac{a \sin x + 2\cos x}{\sin x + \cos x}$ is increasing for all values of x, then

(a) $a < 1$ (b) $a > 1$
(c) $a < 2$ (d) $a > 2$

25. If A is a square matrix of order $n \times n$, then adj (adj A) is equal to

(a) $|A|^n A$ (b) $|A|^{n-1} A$
(c) $|A|^{n-2} A$ (d) $|A|^{n-3} A$

26. Let, $\mathbf{a} = -\hat{\mathbf{i}} - \hat{\mathbf{k}}$, $\mathbf{b} = -\hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\mathbf{c} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ be three given vectors. If $\mathbf{r}$ is a vector such that $\mathbf{r} \times \mathbf{b} = \mathbf{c} \times \mathbf{b}$ and $\mathbf{r} \cdot \mathbf{a} = 0$, then the value of $\mathbf{r} \cdot \mathbf{b}$ is

(a) 3 (b) 6 (c) 9 (d) 12

27. Let n be a positive integer such that $\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \frac{\sqrt{n}}{2}$, then

(a) $6 \le n \le 8$ (b) $4 < n \le 8$
(c) $4 \le n < 8$ (d) $4 < n < 8$

28. Let $L = \lim_{x \to 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$, $a > 0$. If L is finite, then

(a) $a = 2$ (b) $a = 1$
(c) $a = \frac{1}{3}$ (d) None of these

29. If three-digit numbers A28, 3B9 and 62C, where A, B and C are integers between 0 and 9, are divisible by a fixed integer k, then the determinant $\begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$ is

(a) divisible by k (b) divisible by k^2
(c) divisible by $2k$ (d) None of these

30. The number of points of intersection of the two curves $y = 2 \sin x$ and $y = 5x^2 + 2x + 3$ is

(a) 0 (b) 1 (c) 2 (d) ∞

31. A straight line $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$ meets the plane $\mathbf{r} \cdot \hat{\mathbf{n}} = p$ in the point P whose position vector is

(a) $\mathbf{a} + \frac{\mathbf{a} \cdot \hat{\mathbf{n}}}{\mathbf{b} \cdot \hat{\mathbf{n}}} \mathbf{b}$ (b) $\mathbf{a} + \left(\frac{p - \mathbf{a} \cdot \hat{\mathbf{n}}}{\mathbf{b} \cdot \hat{\mathbf{n}}} \right) \mathbf{b}$
(c) $\mathbf{a} - \left(\frac{\mathbf{a} \cdot \hat{\mathbf{n}}}{\mathbf{b} \cdot \hat{\mathbf{n}}} \right) \mathbf{b}$ (d) None of these

32. The probability that a leap year selected at random contains 53 Sunday is

(a) $\frac{7}{366}$ (b) $\frac{28}{183}$ (c) $\frac{1}{7}$ (d) $\frac{2}{7}$

33. The principal value of $\sin^{-1} \left[\sin \left(\frac{2\pi}{3} \right) \right]$ is

(a) $\frac{-2\pi}{3}$ (b) $\frac{2\pi}{3}$
(c) $\frac{4\pi}{3}$ (d) None of these

34. Let R be the relation on the set R of all real numbers defined by aRb iff $|a - b| \le 1$, Then, R is

(a) reflexive (b) transitive
(c) anti-symmetric (d) None of these

35. Let $\mathbf{a} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{c} = \hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$. A vector in the plane of $\mathbf{a}$ and $\mathbf{b}$ whose projection on $\mathbf{c}$ is $\frac{1}{\sqrt{3}}$, is

(a) $4\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ (b) $3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$
(c) $2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ (d) $4\hat{\mathbf{i}} + \hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

36. If the squares of the tangents from a point P to the circles $x^2 + y^2 = a^2$, $x^2 + y^2 = b^2$ and $x^2 + y^2 = c^2$ are in AP, then

(a) a, b, c are in AP (b) a, b, c are in GP
(c) a^2, b^2, c^2 are in AP (d) a^2, b^2, c^2 are in GP

37. Set A and B have 3 and 6 elements respectively. What can be the minimum number of elements in $A \cup B$?

(a) 18 (b) 9 (c) 6 (d) 3

38. If a function $f : [2, \infty) \to A$ defined by $f(x) = x^2 - 4x + 5$ is a bijection, then A is equal to

(a) R (b) $[1, \infty)$
(c) $[2, \infty)$ (d) None of these

39. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius 2 having AB as its diameter, then the slope of the line joining A and B can be

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$
(c) 1 (d) None of these

40. For any two independent events E_1 and E_2, $P\{(E_1 \cup E_2) \cap (\overline{E}_1) \cap (\overline{E}_2)\}$ is

(a) $< \frac{1}{4}$ (b) $> \frac{1}{4}$
(c) $\ge \frac{1}{2}$ (d) None of these

41. Let, $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ be unit vectors such that $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$. Which one of the following is correct?

(a) $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{c} = \mathbf{c} \times \mathbf{a} = \mathbf{0}$
(b) $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{c} = \mathbf{c} \times \mathbf{a} \ne \mathbf{0}$
(c) $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{c} = \mathbf{a} \times \mathbf{c} = \mathbf{0}$
(d) $\mathbf{a} \times \mathbf{b}$, $\mathbf{b} \times \mathbf{c}$, $\mathbf{c} \times \mathbf{a}$ are mutually perpendicular

42. The locus of the orthocentre of the triangle formed by the lines
$(1 + a)x - ay + a(1 + a) = 0$,
$(1 + b)x - by + b(1 + b) = 0$ and $y = 0$, where $a \ne b$, is

(a) a hyperbola (b) a parabola
(c) an ellipse (d) a straight line

43. If $\mathbf{r} \cdot \mathbf{a} = \mathbf{r} \cdot \mathbf{b} = \mathbf{r} \cdot \mathbf{c} = 0$ for some non-zero vector $\mathbf{r}$, then the value of $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ is

(a) 2 (b) 3
(c) 0 (d) None of these

44. Which one of the following is correct solution of
$\left(\frac{dy}{dx}\right)\tan y = \sin(x+y) + \sin(x-y)$?
(a) $\sec x = C - 2\sec y$ (b) $\sec y = C + 2\cos y$
(c) $\sec y = C - 2\cos x$ (d) $\sec x = C - 2\cos y$

45. The line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is parallel to the plane
(a) $3x + 4y + 5z = 7$ (b) $2x + 3y + 4z = 0$
(c) $x + y - z = 2$ (d) $2x + y - 2z = 0$

46. The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is
(a) $2x - \sqrt{5}\,y - 20 = 0$ (b) $2x - \sqrt{5}\,y + 4 = 0$
(c) $3x - 4y + 8 = 0$ (d) $4x - 3y + 4 = 0$

47. Area bounded by the curve $x = 0$ and $x + 2|y| = 1$ is
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

48. Which of the following is a correct solution of $y^2 + \left(x - \frac{1}{y}\right)\frac{dy}{dx} = 0$?
(a) $y = C\,E^{1/x} + \frac{1}{y} + 1$ (b) $x = C\,E^{1/y} + \frac{1}{y} + 1$
(c) $x = C\,E^{1/x} + \frac{1}{y} + 1$ (d) $x = C\,E^{1/y} + \frac{1}{x} + 1$

49. For any real number x, let $[x]$ denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval $[-10, 10]$ by $f(x) = \begin{cases} x - [x] & \text{if } [x] \text{ is odd} \\ 1 + [x] - x & \text{if } [x] \text{ is even.} \end{cases}$

Then, the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x\, dx$ is
(a) 2 (b) 4
(c) 0 (d) None of these

50. Sum of the series
$(x+y)(x-y) + \frac{1}{2!}(x+y)(x-y)(x^2+y^2) + \frac{1}{3!}(x+y)(x-y)(x^4+y^4+x^2y^2) + \dots \infty$ is
(a) $e^x + e^y$ (b) $e^x - e^y$
(c) $e^{x^2} + e^{y^2}$ (d) $e^{x^2} - e^{y^2}$

Answers

Physics

1. (b)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (a)	**10.** (c)
11. (d)	**12.** (d)	**13.** (d)	**14.** (a)	**15.** (d)	**16.** (a)	**17.** (b)	**18.** (b)	**19.** (a)	**20.** (c)
21. (b)	**22.** (a)	**23.** (d)	**24.** (a)	**25.** (c)	**26.** (a)	**27.** (b)	**28.** (d)	**29.** (a)	**30.** (c)
31. (d)	**32.** (c)	**33.** (a)	**34.** (b)	**35.** (a)	**36.** (d)	**37.** (a)	**38.** (c)	**39.** (d)	**40.** (b)
41. (b)	**42.** (a)	**43.** (c)	**44.** (b)	**45.** (c)	**46.** (d)	**47.** (a)	**48.** (c)	**49.** (b)	**50.** (a)

Chemistry

1. (c)	**2.** (a)	**3.** (b)	**4.** (*)	**5.** (c)	**6.** (d)	**7.** (c)	**8.** (c)	**9.** (*)	**10.** (a)
11. (c)	**12.** (a)	**13.** (c)	**14.** (a)	**15.** (a)	**16.** (c)	**17.** (b)	**18.** (*)	**19.** (c)	**20.** (b)
21. (c)	**22.** (a)	**23.** (d)	**24.** (b)	**25.** (a)	**26.** (b)	**27.** (c)	**28.** (b)	**29.** (d)	**30.** (c)
31. (a)	**32.** (d)	**33.** (c)	**34.** (a)	**35.** (b)	**36.** (d)	**37.** (d)	**38.** (c)	**39.** (d)	**40.** (c)
41. (b)	**42.** (c)	**43.** (d)	**44.** (b)	**45.** (d)	**46.** (a)	**47.** (d)	**48.** (c)	**49.** (a)	**50.** (d)

(*) None option is correct.

Mathematics

1. (a)	**2.** (a)	**3.** (c)	**4.** (a)	**5.** (a)	**6.** (b)	**7.** (d)	**8.** (a)	**9.** (c)	**10.** (c)
11. (b)	**12.** (a)	**13.** (c)	**14.** (b)	**15.** (c)	**16.** (b)	**17.** (a)	**18.** (a)	**19.** (b)	**20.** (a)
21. (a)	**22.** (b)	**23.** (d)	**24.** (d)	**25.** (c)	**26.** (c)	**27.** (d)	**28.** (a)	**29.** (a)	**30.** (a)
31. (b)	**32.** (d)	**33.** (d)	**34.** (a)	**35.** (a)	**36.** (c)	**37.** (c)	**38.** (b)	**39.** (c)	**40.** (a)
41. (b)	**42.** (d)	**43.** (c)	**44.** (c)	**45.** (d)	**46.** (b)	**47.** (b)	**48.** (b)	**49.** (b)	**50.** (d)

Hints & Solutions

Physics

1. Surface integral of magnetic field between the circular plates of capacitor.

$$\oint \mathbf{B} \cdot d\mathbf{S} = |\mathbf{B}|\, 2\pi r = \mu_0\, \varepsilon_0 \frac{d}{dt} \int\int \mathbf{E} \cdot d\,\mathbf{A}$$

$$= \mu_0 \varepsilon_0 \frac{d|\mathbf{E}|}{dt} \pi r^2 = \mu_0 \varepsilon_0 \frac{d|\mathbf{E}|}{dt} \pi r^2$$

$$= \frac{\mu_0 i\ \pi r^2}{\pi R^2} = \frac{\mu_0 i\ r^2}{R^2}$$

Given, $r = \frac{R}{5}$

$\therefore \quad \oint \mathbf{B} \cdot d\,\mathbf{S} = \frac{\mu_0 i}{25}$

2. Given, $E_{\text{rms}} = 120\ \text{V}$

$f = 50\ \text{Hz}$

$X_L = 100\ \Omega$

$X_C = 110\ \Omega$

Power factor $= 0.9$

To find $R = ?$

As power factor in LCR circuit

$$\cos\phi = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$0.9 = \frac{R}{\sqrt{R^2 + (100 - 110)^2}}$$

$$0.9 = \frac{R}{\sqrt{R^2 + 100}}$$

$\Rightarrow \quad 0.81 = \frac{R^2}{R^2 + 100}$

$\Rightarrow \quad R \approx 20\ \Omega$

3. Given, $C = 17\ \text{pF}$

Wavelength of EM wave $\lambda = 550\ \text{nm}$

$= 550 \times 10^{-9}\ \text{m}$

For EM wave $v = c = 3 \times 10^8\ \text{m/s}$

$$f = \frac{v}{\lambda} = \frac{c}{\lambda} = \frac{3 \times 10^8}{550 \times 10^{-9}} = 5.4 \times 10^{14}\ \text{Hz}$$

Now, frequency of oscillation $f = \frac{1}{2\pi\sqrt{LC}}$

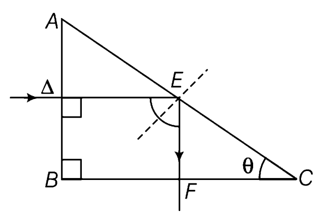
$$5.4 \times 10^{14} = \frac{1}{2 \times 3.14\sqrt{L \times 17 \times 10^{-12}}}$$

$$L = \frac{1}{2 \times 3.14 \times 5.4 \times 10^{14})^2 \times 17 \times 10^{-12}}$$

$= 5 \times 10^{-2}\ \text{H}$

4. For (TIR) Total Internal Reflection at face AC incident angle > critical angle

$i > C$



Now, $\sin C = \frac{1}{\mu}$

Here, $\sin C = \frac{\mu_w}{\mu_g} = \frac{1.33}{1.55} = 0.85 \approx \frac{\sqrt{3}}{2} \Rightarrow C = 60°$

If $\angle DEF = 60 \times 2 = 120°$

$\angle EDB = 90°$ and $\angle DBF = 90°$

$\therefore \quad \angle EFB = 60°$

$\Rightarrow \quad \angle EFC = 120°$

By using geometry, we find $\angle FEC = 30°$

$\therefore \quad \theta = 30°$

5. For first lens $f_1 = 24.0\ \text{cm}$

$u_1 = 6.0\ \text{cm}$

To find v_1

Using lens formula $\frac{1}{v} - \frac{1}{u_1} = \frac{1}{f_1}$

$$\frac{1}{v} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{24} - \frac{1}{6}$$

$$\frac{1}{v} = \frac{1-4}{24} = -\frac{3}{24}$$

$v = -8\ \text{cm}$

∴ For second lens

$$u_2 = -8 \text{ cm} + (-10 \text{ cm}) = -18 \text{ cm}$$

$$f_2 = 9 \text{ cm}$$

Again using lens formula

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$$\frac{1}{v} = \frac{1}{f_2} + \frac{1}{u_2} = \frac{1}{9} - \frac{1}{18}$$

$$= \frac{2-1}{18} = \frac{1}{18}$$

$\Rightarrow \quad v = 18 \text{ cm}$

∴ Final image will be formed at 18 cm to the right of second lens.

6. Here, focal length $f = \frac{R}{2} = \frac{35}{2} = 17.5$

As it is a concave mirror focal length will be negative $\quad f = -17.5$

Now $\quad m = -\frac{v}{u} = 2.5$

$\Rightarrow \quad v = -2.5\,u$

Using mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$-\frac{1}{17.5} = -\frac{1}{2.5\,u} + \frac{1}{u}$$

$\Rightarrow \quad u = -10.5 \text{ cm}$

7. The film appears bright due to interference of reflected rays.

The film will appear bright, when

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow 2 \times 1.33 \times 320 \text{ nm} \cos 0° = (2 \times 1 + 1)\frac{\lambda}{2}$$

$\Rightarrow \quad \lambda = 567 \text{ nm}$

8. Wave with higher frequency will have higher energy

Frequency in decreasing order is

Gamma rays > Yellow light > Microwave > Radio wave

2 > 1 > 4 > 3

9. Given, $E = 5.5 \text{ eV} = 5.5 \times 1.6 \times 10^{-19}$ J

To find $\lambda_{max} = ?$

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{\lambda}$$

$$5.5 \times 1.6 \times 10^{-19} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{\lambda}$$

$$\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{5.5 \times 1.6 \times 10^{-19}}$$

$$\lambda = 225 \times 10^{-9} \text{ m} = 225 \text{ nm}$$

10. The kinetic energy of α-particle is completely converted into potential energy

$$\frac{1}{2}mu^2 = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(Ze)}{r_0}$$

∴Distance of closest approach

$$r_0 = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{\frac{1}{2}mu^2}$$

$$= \frac{9 \times 10^9 \times 2 \times 79\,(1.6 \times 10^{-19})^2}{5.5 \times 1.6 \times 10^{-19} \times 10^6}$$

$$= \frac{9 \times 2 \times 79 \times 1.6 \times 10^9 \times 10^{-9}}{5.5 \times 10^6} = 4.3 \times 10^{-14} \text{ m}$$

11. The process involve fusion of very light nuclei chlorine is not so light to undergo fusion process.

12. Nuclear density of all atoms is same as it is independent of mass number. Its value is 2×10^{17} kg/m^3.

13. A convex mirror is used for this purpose.

According to mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Differentiate w.r.t. time t

$$\frac{d}{dt}\left[\frac{1}{v}\right] + \frac{d}{dt}\left[\frac{1}{u}\right] = \frac{d}{dt}\left[\frac{1}{f}\right]$$

$$-\frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$$

$$\frac{dv}{dt} = -\frac{v^2}{u^2}\frac{du}{dt}$$

Also to find v

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = 1 - \left(\frac{1}{-9}\right)$$

$$\frac{1}{v} = \frac{9+1}{9} = \frac{10}{9} \Rightarrow v = \frac{9}{10}$$

$$\therefore \quad \frac{dv}{dt} = -\left(\frac{9}{10}\right) \times \frac{1}{(-9)^2} \times (5) = 0.05 \text{ m/s}$$

14. Given, focal length (focal length is negative for concave lens)

$v = -\frac{f}{2}$ (image is always formed to the object side for concave lens]

To find, $u = ?$

Using lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\frac{1}{f} = -\frac{2}{f} - \frac{1}{u}$$

$$\frac{1}{u} = \frac{1}{f} - \frac{2}{f}$$

$$\frac{1}{u} = -\frac{1}{f} \Rightarrow u = -f$$

15. Height = 170 cm. It should be given in SI unit *i.e.*, in metre = 170 cm = 1.70 m.

(Also, decimal after one digit)

In scientific method it is written as 1.70×10^2 cm.

16. Given, $R_1 = (5.0 \pm 0.2)\ \Omega$, $R_2 = (10.0 \pm 0.1)\ \Omega$

To find R_P

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_p = \frac{R_1 R_2}{R_1 + R_2}$$

Ist to find the value of R_1R_2 with error

$$R_1R_2 = (5.0 \pm 0.2)\ (10.0 \pm 0.1)$$

As, $$\frac{\Delta R_1 R_2}{R_1 R_2} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2}$$

$$\Delta R_1 R_2 = 50\left(\frac{0.2}{5} + \frac{0.1}{10}\right)$$

$$= 50\ (0.04 + 0.01) = 50\ (0.05) \approx 2.5$$

$\therefore$ $R_1R_2 = 50 \pm 2.5$

Similarly, $R_1 + R_2 = (5.0 \pm 0.2)\ (10.0 \pm 0.1)$

$$= 15 \pm 0.3$$

Now, $$\frac{R_1R_2}{R_1 + R_2} = \frac{50 \pm 2.5}{15 \pm 0.3}$$

$$\Delta\left(\frac{R_1R_2}{R_1 + R_2}\right)\% = \left(\frac{2.5}{50} + \frac{0.3}{15}\right) \times 100$$

$\therefore$ $$\frac{R_1R_2}{R_1 + R_2} + \Delta\left(\frac{R_1R_2}{R_1 + R_2}\right)$$

$$= R_P + \Delta R_P = 3.3 \pm 7\%$$

17. Given, $F = A\sqrt{x} + Bt^2$

Substituting dimension of force, distance and time in given expression.

$$[\text{MLT}^{-2}] = A[\text{L}]^{1/2} + B[\text{T}]^2$$

Now, according to principle of homogeneity dimension of F will be same as dimension of $A\sqrt{x}$ and that of Bt^2.

$\therefore$ $$A[\text{L}]^{1/2} = [\text{MLT}^{-2}]$$

$\Rightarrow$ $$A = \frac{[\text{MLT}^{-2}]}{[\text{L}^{1/2}]}$$

$$= [\text{ML}^{+1/2}\ \text{T}^{-2}]$$

Similarly, $$B[\text{T}]^2 = [\text{MLT}^{-2}]$$

$$B = [\text{MLT}^{-4}]$$

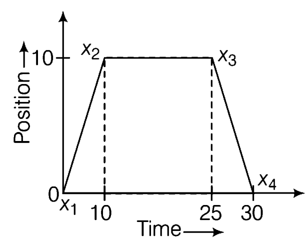
$\therefore$ Dimension of $$\frac{A}{B} = \frac{[\text{ML}^{1/2}\ \text{T}^{-2}]}{[\text{ML}\ \text{T}^{-4}]}$$

$$= [\text{L}^{-1/2}\ \text{T}^2]$$

18. Velocity at 5th second will be the velocity in interval (0 – 10) s

i.e., $$v_1 = \frac{x_2 - x_1}{t_2 - t_1}$$

$$= \frac{10 - 0}{10 - 0}$$

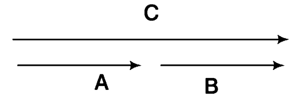


Similarly, velocity at 27th sec will be velocity in time interval (25 – 30)

$$v_2 = \frac{x_4 - x_3}{t_4 - t_3} = \frac{0 - 10}{30 - 25} = -2$$

$\therefore$ $$\frac{v_1}{v_2} = -\frac{1}{2}$$

19. The two conditions $\mathbf{A} + \mathbf{B} = \mathbf{C}$ and $|\mathbf{A}| + |\mathbf{B}| = |\mathbf{C}|$ may be satisfied when vectors are parallel.



20. We have two components for each initial velocity (v_0) and acceleration due to gravity (g). In this frame of reference the horizontal and vertical component of velocity and gravity are given by

$$v_x = v_0 \sin\theta,\ v_y = v_0 \cos\theta,$$
$$g_x = g\sin\theta,\ g_y = g\cos\theta$$

Now, $t_{\text{bounce}} = \dfrac{2v_y}{g_y} = \dfrac{2v_0\cos\theta}{g\cos\theta} = \dfrac{2v_0}{g}$

For horizontal distance $d = v_x t + \dfrac{1}{2} g_x t^2$

$$\therefore \quad d = v_0 \sin\theta \left(\frac{2v_0}{g}\right) + \frac{1}{2} g\sin\theta \left[\frac{2v_0}{g}\right]^2$$

$$= \frac{2v_0^2 \sin\theta}{g} + \frac{1}{2} g \frac{4v^2}{g^2}\sin\theta = \frac{4v_0^2 \sin\theta}{g}$$

21. For the motion on smooth inclined plane

$$S = \frac{1}{2} g\sin\theta\, t_1^2$$
$$S = \frac{1}{2} g\sin(45°)\, t_1^2$$
$$t_1^2 = \frac{2\sqrt{2}}{g} s \qquad \ldots\text{(i)}$$

For the motion on a rough inclined plane

$$s = \frac{1}{2} g\sin\theta\,(1-\mu_k)t_2^2$$
$$\frac{2\sqrt{2}\,s}{(1-\mu_k)} = t_2^2 \qquad \ldots\text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{t_2^2}{t_1^2} = n^2 = \frac{1}{1-\mu_k}$$
$$\Rightarrow \qquad \mu_k = 1 - \frac{1}{n^2}$$

22. Given, mass $= 2mg = 2\times 10^{-6}$ kg

Distance = 5.0 m

Time $t = 5$ ms $= 5\times 10^{-3}$ s

To find KE = ?

Now, velocity $= \dfrac{\text{Distance}}{\text{Time}} = \dfrac{5}{5\times 10^{-3}}$ m/s

$= 1000$ m/s

and KE $= \dfrac{1}{2} mv^2 = \dfrac{1}{2}\times 2\times 10^{-6}(10^3)^2$

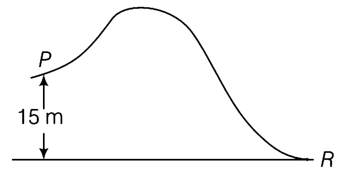
$= 1$ J

23. Given, mass of ball = 1.0 kg

KE = 330 J

To find, speed of ball at R

According to conservation of energy



$$\text{KE}_{\text{at}P} + \text{PE}_{\text{at}P} = \text{KE}_{\text{at}R} = \text{PE}_{\text{at}R}$$
$$\frac{1}{2} mv_1^2 + mgh = \frac{1}{2} mv_2^2 + 0$$
$$330 + 1\times 10\times 15 = \frac{1}{2}\times 1\times v^2$$
$$2(330+150) = v^2$$
$$960 = v^2 \Rightarrow v = 31 \text{ m/s}$$

24. Work done $= \mathbf{F}\cdot\mathbf{s}$

Work done is independent of path followed

$$F = [y\,\hat{\mathbf{i}} + x\,\hat{\mathbf{j}}]$$
$$\mathbf{s} = [2a, 2a] - [0, 0]$$
$$\mathbf{s} = (2a-0), (2a-0) = (2a, 2a)$$
$$\mathbf{F} = -[y\hat{\mathbf{i}} + x\hat{\mathbf{j}}) = 2a\hat{\mathbf{i}} + 2a\hat{\mathbf{j}}$$
$$\mathbf{F}\cdot\mathbf{s} = -[2a\hat{\mathbf{i}} + 2a\hat{\mathbf{j}}]\cdot[2a\hat{\mathbf{i}} + 2a\hat{\mathbf{j}}]$$
$$= \sqrt{(4a^2)^2 + (4a^2)^2}$$
$$= -4a^2 \text{ [}\mathbf{F} \text{ and } \mathbf{s} \text{ are in opposite directions]}$$

25. Given, masses $m_1 = 6$ unit

$m_2 = 2$ unit

Positions $6\hat{\mathbf{i}} - 7\hat{\mathbf{j}}$ and $2\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$

To find centre of mass

$$x_{\text{cor}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$
$$= \frac{6\times 6 + 2\times 2}{6+2} = \frac{36+4}{8} = 5\hat{\mathbf{i}}$$
$$y_{\text{cor}} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$
$$= \frac{6\times(-7) + 2\times(+5)}{6+2} = \frac{-42+10}{8} = 4\hat{\mathbf{j}}$$
$$Z_{\text{cor}} = \frac{m_1z_1 + m_2z_2}{m_1 + m_2}$$
$$= \frac{6\times(0) + 2\times(-8)}{2+6} = \frac{-16}{8} = -2\hat{\mathbf{k}}$$

$\therefore$ Centre of mass lies on $5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

26. Given, $\omega = 3\hat{i} - 4\hat{j} = \hat{k}$

$$r = 5\hat{i} - 6\hat{j} + 6\hat{k}$$

To find, $v = ?$

We know that $v = \omega \times r$

$$\therefore \quad v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix}$$

$$= \hat{i}\,(-24 + 6) - \hat{j}\,(18 - 5) + \hat{k}\,(-18 + 20)$$

$$= -18\hat{i} - 13\hat{j} + 2\hat{k}$$

27. Gravitational force is conservative work done by or against the force in moving a body depends only on the initial and final positions of the body and not on the nature of path followed by it

$$\therefore \quad W_1 = W_2 = W_3$$

28. Stress $= \dfrac{F}{A}$

$$= \frac{mg}{A} = \frac{Vdg}{A} = \frac{AHdg}{A} = Hdg$$

Elastic limit of the rock $= 3 \times 10^8$ N/m^2

$$3 \times 10^8 = Hdg$$

$$3 \times 10^8 = H \times 3 \times 10^4$$

$$\Rightarrow \quad H = 10 \text{ km}$$

29. If a capillary tube is dipped into a liquid and tilted at an angle α from vertical then the verical height of liquid column (h) remains same, whereas the length of the liquid column (l) in the capillary tube increases

$$h = l \cos\alpha$$

or $\quad l = \dfrac{h}{\cos\alpha}$

where $\quad h = \dfrac{2T\cos\theta}{\rho r g}$ $(\theta_{\text{water}} = 8° \approx 0°)$

$$\therefore \quad l = \frac{2p\cos\theta}{\rho\, rg\cos\alpha}$$

$$= \frac{2 \times 7 \times 10^{-2} \cos 0°}{10^3 \times 0.4 \times 10^{-3} \times 9.8 \cos 60°}$$

$$= 7.1 \text{ cm}$$

31. According to Wein's displacement law

$$\lambda t = \text{constant}$$

$$\therefore \quad \lambda_1 T_1 = \lambda_2 T_2$$

$$5000 \times 10^{-10} \times 6000 = 50 \times 10^{-6} \times T_2$$

$$\Rightarrow T_2 = \frac{5000 \times 10^{-10} \times 6000}{50 \times 10^{-6}} = 60 \text{ K}$$

32. Specific heat of solids at constant volume increases with rise in temperature and its value becomes $3R$ for different solids at larger temperature.

33. In a potential field.

$$F = -\nabla U$$

$$\therefore \quad F = -U_0 \alpha \sin\alpha x$$

For small value of (αx) $\sin\alpha x \approx \alpha x$

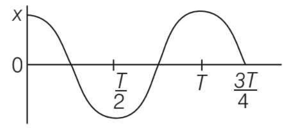
Now, $\quad F = -U_0 \alpha^2 x$

$$\Rightarrow \quad \frac{md^2x}{dt^2} + U_0\alpha^2 x = 0$$

$$\Rightarrow \quad \omega_0^2 = \frac{U_0\alpha^2}{m}$$

As, $\quad T = \dfrac{2\pi}{\omega_0} = \dfrac{2\pi}{\alpha}\sqrt{\dfrac{m}{U_0}}$

34. (i) The force will be zero at $t = \dfrac{3T}{4}$ (when the particle is at mean position)



(ii) The acceleration will be maximum at extreme position (for $t = T$)

(iii) Velocity will be maximum at mean position $\left(t = \dfrac{T}{4}\right)$ whereas at $\dfrac{T}{2}$ PE will be maximum KE will be zero.

35. $$\frac{v_{\text{rms}}}{v_s} = \frac{\sqrt{\dfrac{3RT}{M}}}{\sqrt{\dfrac{\gamma RT}{M}}} = \sqrt{\frac{3}{\gamma}} = \sqrt{2.2} = 1.48$$

$$= 330 \times 1.48 \approx 481 \text{ m/s}$$

36. Intensity of a progressive wave,

$$I = 2\pi^2 \rho f^2 A^2 v$$

where $\rho =$ density of medium

$f =$ frequency of wave

$v =$ wave velocity

$A =$ amplitude

For wave 1

$10 \sin 2\pi\,(10t - 0.1x)$

$$A \sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$$

$$A = 10$$

For frequency $\omega = \frac{2\pi}{T} = 20\pi$

$\Rightarrow \quad f = 10$

Wave velocity $= \lambda \times f = \frac{1}{(0.1)} \times (10)$

Similarly, for wave 2

$A_1 = 20$

Frequency $\omega_2 = \frac{2\pi}{T_2} = 40\pi \Rightarrow f = 20$

Wave velocity $v_2 = \frac{1}{0.2} \times (20)$

$$\therefore \quad \frac{2\pi^2 \rho A_1^2 f_1^2 v_1}{2\pi^2 \rho A_2^2 f_2^2 v_2} = \frac{(10)^2 (10)^2 (100)}{(20)^2 (20)^2 (100)}$$

$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{1}{16}$$

37. Difference between two successive lengths

$$l_1 - l_2 = \frac{\lambda}{2}$$

$$50 - 16 = \frac{\lambda}{2}$$

$$\Rightarrow \quad \lambda = 68 \text{ cm}$$

For third resonance

$$L = \frac{5\lambda}{4} = \frac{5 \times 68}{4} = 85 \text{ cm}$$

38. For destructive interference

Path difference $\pi r - 2r = \frac{1}{2}(2n-1)\lambda$

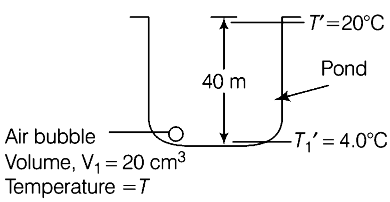
$$r(\pi - 2) = \frac{1}{2}(2n-1)\lambda$$

For minimum sound $n = 1$

$$r(3.14 - 2) = \frac{1}{2}(2 \times 1 - 1) \times 2.28$$

$$r = 1 \text{ m}$$

40.



$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

(Below water) (At surface)

Temperature of the bubble $= T_1 = T_2 = T$

$$\therefore \quad p_1 V_1 = p_2 V_2$$

(Below water) (At surface)

The rate of descent pressure under water is 1 atm/10 m. It means that, if bubble moves 10 m from the surface, it gets 1 atm additional pressure at the depth.

$$p = \frac{40 \text{ m}}{10 \text{ m/ atm}} = 4 \text{ atm}$$

Total p under water $= 4\,\text{atm} + 1\,\text{atm}$

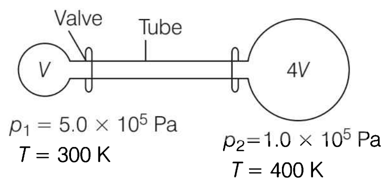
$= 5$ atmospheric pressure (atm)

$$\Rightarrow \quad p_1 V_1 = p_2 V_2$$

$$5 \times 20 = 1 \times V_2$$

$$V_2 = 100 \text{ cm}^3$$

41.



Since temperature are kept constant and $p_1 > p_2$. So to equalise pressure some of the p is reduced from p_1. Let it be p, so $p_A = p_1 - p$

$$\frac{p_A V_1}{T_1} = \frac{p_2 V_2}{T_2} \Rightarrow \frac{(p_1 - p) V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\frac{(5 \times 10^5 - p) V}{300} = \frac{p_2 \times 4V}{400}$$

$$5 \times 10^5 - p = 3p_2 = 3 \times 10^5$$

$$p = 5 \times 10^5 - 3 \times 10^5$$

$$= 2 \times 10^5 \text{ Pa}$$

42. At equilibrium

$$\frac{-2q}{4\pi \varepsilon_0 (x - L)^2} + \frac{8q}{4\pi\varepsilon_0 x^2} = 0$$

or $\quad x = 2L$

43. Potential energy for a dipole $U = -\mathbf{p} \cdot \mathbf{E}$

$$\therefore \quad U = -pE\cos\theta$$

$$U \propto -\cos\theta$$

For greater orientation value of U will be more positive.

$\therefore$ Orientation in decreasing order is as follow

(iv) > (iii) > (i) > (ii)

44. We see that the horizontal field component from the upper and lower quadrants cancel and downwards components and thus the field is pointed downwards. The calculation requires us to integrate the E_y components along the upper quadrants. This yield the integral

$$E_y = 2\int \frac{kdQ\sin\theta}{R^2}$$

where θ = angle of point on the arc from P runs from 0 to $\frac{\pi}{2}$

Here, $$dQ = \lambda R\, d\theta = \frac{2Q}{\pi d\theta}$$

and linear charge density $\lambda = \frac{Q}{\pi R/2}$

This calculation yields $E_y = \frac{4k\,Q}{\pi R^2} = \frac{Q}{\pi^2 \varepsilon_0 R^2}$

45. The electric field due to sheet of positive charge will be from left to right (along positive direction) and that due to sheet of negative charge will be from right to left (along negative direction).

$$\therefore \quad E = \frac{\sigma_{(+)}}{2\varepsilon_0} + \left[-\frac{\sigma_{(-)}}{2\varepsilon_0}\right]$$

$$E = \frac{1}{2\varepsilon_0}[\sigma_{(+)} - \sigma_{(-)}] = \frac{(6.8-4.3)\times 10^{-6}}{2\times 8.85\times 10^{-12}}$$

$= 1.41 \times 10^5$ N/C towards right

46. Potential difference

$$V_R - V_P = -\mathrm{E}d$$

Negative sign as $V_R < V_P$

47. Resistance of wire, $R_A = \frac{\rho l}{A} = \frac{\rho l}{\pi\left[\frac{D}{2}\right]^2} = \frac{\rho l}{\frac{\pi D^2}{4}}$

$$= \frac{\rho l}{\pi\frac{(1)^2}{4}} = \frac{4\rho l}{\pi}$$

Similarly, for hollow conductor B

$$R_B = \frac{\rho l}{A} = \frac{\rho l}{\pi\left[\frac{D_1^2}{4} - \frac{D_2^2}{4}\right]} = \frac{\rho l}{\pi\left[\frac{2^2}{4} - \frac{1^2}{4}\right]}$$

$$= \frac{4}{3}\frac{\rho l}{\pi}$$

$$\therefore \quad \frac{R_A}{R_B} = \frac{4\rho l}{\pi} \times \frac{3\pi}{4\rho l} = 3$$

48. The capacitance time constant CR-circuit is defined as the time in which charge decay from maximum to 0.368 of its maximum value

CR for $1 = 2 \times 3 = 6$

CR for $2 = 3 \times 2 = 6$

CR for $3 = 10 \times 0.5 = 5$

CR for $4 = 5 \times 2 = 10$

$\therefore$ Time required for the current to decrease to half will be in following order

$$t_1 > t_2 = t_3 > t_4$$

49. The cyclotron frequency

$$\nu = \frac{1}{T} = \frac{Bq}{2\pi m}$$

For deutron H_1^2 $\quad q = 1.6 \times 10^{-19}$ C

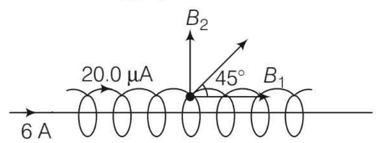
$$m = 2 \times 1.6 \times 10^{-27} \text{ kg}$$

$$\therefore 24 \times 10^6 = \frac{B \times 1.6 \times 10^{-19}}{2 \times 3.14 \times 2 \times 1.6 \times 10^{-27}}$$

$$\Rightarrow \quad B = \frac{2 \times 3.14 \times 2 \times 1.6 \times 10^{-27} \times 24 \times 10^6}{1.6 \times 10^{-19}}$$

$= 3.2$ T

50.



Given, for solenoid

$n = 10$ turn/cm

$I_1 = 20.0$ mA$= 20 \times 10^{-3}$ A

B_1 = magnetic field due to solenoid

B_2 = magnetic field due to wire

For wire $I_2 = 6$ A

Now, according to question if resultant should be at 45° then the two field B_1 and B_1 must be equal to each other

So, $\quad B_1 = B_2$

$$\Rightarrow \mu_0\, nI_1 = \frac{\mu_0 I_2}{2\pi r}$$

$$\Rightarrow r = \frac{I_2}{I_1 \times n 2\pi} = \frac{6}{20 \times 10^{-3} \times 10 \times 2 \times 3.14}$$

$$= \frac{6 \times 10^2}{20 \times 2 \times 3.14} = \frac{60}{4 \times 3.14}$$

$= 4.77$

$= 4.8$ cm (approx)

Chemistry

1. $\% \text{ of C} = \frac{12}{44} \times \frac{\text{weight of } CO_2 \times 100}{\text{weight of organic compound}}$

$$= \frac{12}{44} \times \frac{0.198 \times 100}{0.246} = 21.95\%$$

$\% \text{ of H} = \frac{2}{18} \times \frac{\text{weight of } H_2O \times 100}{\text{weight of organic compound}}$

$$= \frac{2}{18} \times \frac{0.1014 \times 100}{0.246} = 4.57$$

$\% \text{ of O} = (100 - 21.95 - 4.57) = 73.47$

$\text{Moles of C} = \frac{21.95}{12} = 1.83$

$\text{Atoms of C} = 1.83 \times N_A$

$\text{Moles of H} = \frac{4.57}{1} = 4.57$

$\text{Atoms of C} = 4.57 \times N_A$

$\text{Moles of O} = \frac{73.47}{16} = 4.59$

$\text{Atoms of O} = 4.59 \times N_A$

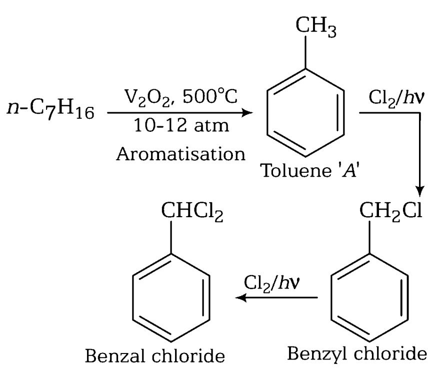
$\text{Simplest ratio} : C : H : O = 1.83 : 4.57 : 4.59$

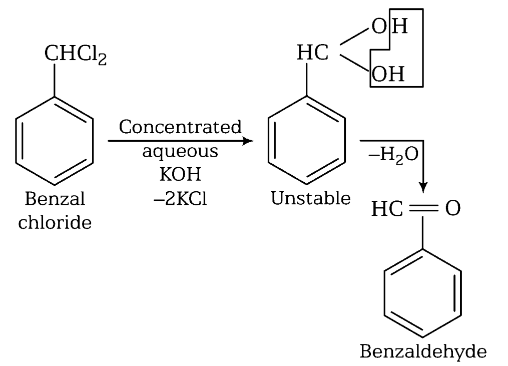
$$= 1 : 2.5 : 2.5 = 2 : 5 : 5$$

Thus, the ratio of atoms of carbon and hydrogen in the compound is 2 : 5.

2. In general, the basic strength of amines increases as the number of electron releasing groups (+ I showing group like —CH_3) increases due to more availability of nitrogen to donate its lone pair of electrons. But in aqueous solution, two other factors, *i.e.*, steric effect and solvation effect, also involves due to which tendency of tertiary (3°) amines to loose their lone pair of electrons *i.e.*, basicity reduces. Thus, the order of basicity, if the electron releasing group in methyl, in aqueous solution is

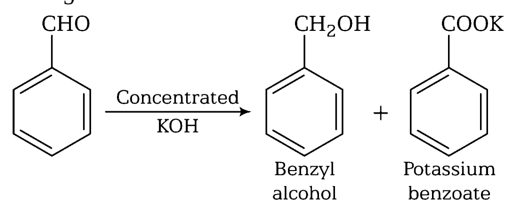
$(CH_3)_2NH > CH_3NH_2 > (CH_3)_3N$

3. 

4. 

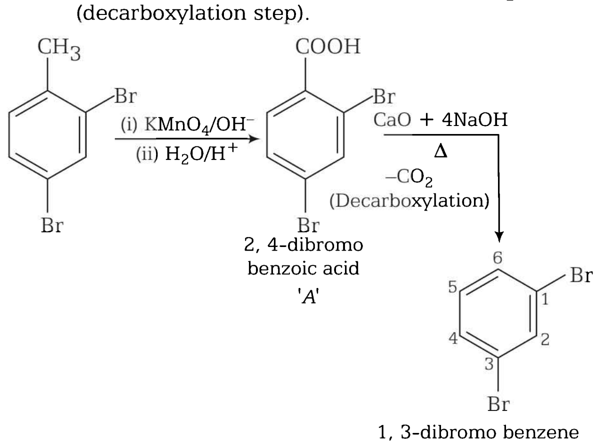
Thus, benzaldehyde is obtained when benzal chloride is heated with concentrated aqueous solution of KOH.

The obtained aldehyde because of the absence of α-hydrogen atom further reacts with concentrated aqueous KOH solution to give benzyl alcohol and potassium benzoate, *i.e.*, it undergoes cannizzaro reaction.

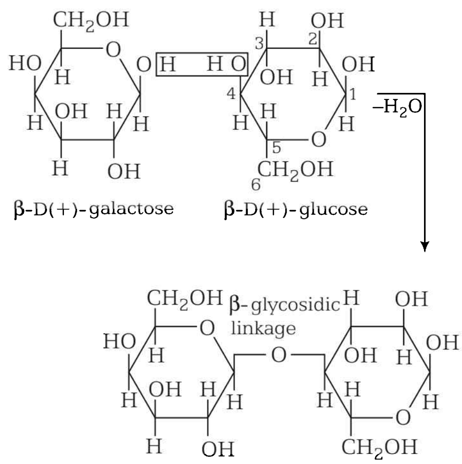


5. $KMnO_4$ (alkaline) converts the alkyl chain attached directly to benzene nucleus into —COOK group. Hydrolysis converts —COOK into —COOH group, *i.e.*, we get a carboxylic acid.

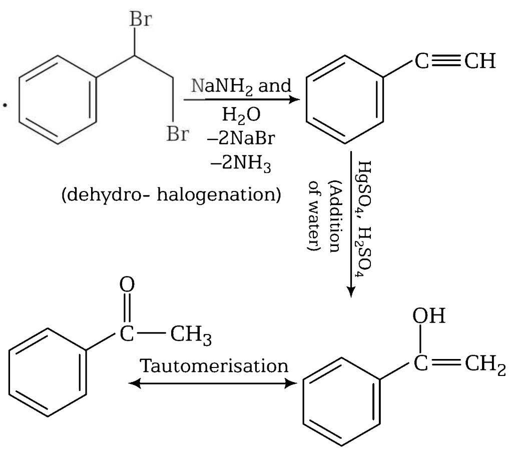
The carboxylic acid group when fused with hot sodalime (NaOH + CaO), it gets converted into —H with the removal of CO_2 gas, (decarboxylation step).



6. Lactose on hydrolysis with dilute acid, produces an equimolar mixture of β-D(+)-glucose and β-D(+)-galactose, thus, its monosaccharide constituents are β-D-glucose and β-D-galactose.



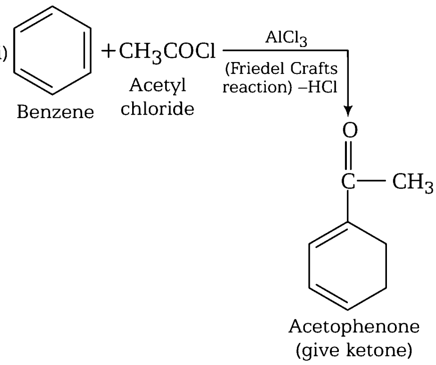
(+)-lactose

7. 

Thus, $A = NaNH_2$ and H_2O

$B = HgSO_4, H_2SO_4$

Note A cannot be alcohol KOH and H_2O as it stops the reaction at alkene level.

8. (i) 

(ii) $CH_3C{\equiv}N + C_6H_5MgBr \longrightarrow CH_3C(C_6H_5){=}NMgBr \xrightarrow{H_2O} CH_3-\overset{O}{\overset{||}{C}}-C_6H_5$

Acetophenone (given ketone)

(iii) $2CH_2COCl + (C_2H_5CH_2)_2Cd \xrightarrow[\text{ether}]{\text{Dry}}$

$2C_2H_5CH_2COCH_3 + CdCl_2$

Benzylmethyl keton

(iv) $C_6H_5-C{\equiv}CH \xrightarrow[H_2SO_4]{HgSO_4+} C_6H_5-C(OH){=}CH$

(Addition of water)

$\xrightarrow{\text{Tauto merisation}} C_6H_5-\underset{O}{\underset{||}{C}}-CH_3$

(Acetophenone given ketone)

Thus, acetophenone is not the product of reaction (c).

9. Milliequivalent of $H^+ = 10 \times 0.1 \times 2 = 0$

Milliequivalent of $OH^- = 10 \times \frac{1}{10} = 1$

Normality of solution $= \frac{2-1}{20} = \frac{1}{20} = 0.05$

Molarity of the resulting solution

$= 2 \times 0.05 = 0.1$

pH of the solution $= -\log [H^+] = -\log (0.1)$

$= -\log (10^{-1}) = 1$

10. Equilibrium constant depends only upon temperature, not on the concentration. Thus, it remains unchanged when an inert gas like argon is added.

Further, argon is added at the constant volume, *i.e.*, the relative molar concentration of the substance will not change. Hence, the equilibrium position of the reaction remains unaffected.

11. Balmer series in atomic hydrogen is observed in visible region.

Note Lyman series lies in UV region and Paschen lies in IR and Brackett and others lie in Far IR region.

12.
$$\begin{array}{lllll} H_2 & + & I_2 & \rightleftharpoons & 2HI \\ 0.5 \text{ atm} & & 0.5 \text{ atm} & & 0 \text{ initially} \\ (0.5-x) \text{ atm} & & (0.5-x) \text{ atm} & & 2x \text{ atm at equilibrium} \end{array}$$

$$K_p = \frac{(p_{HI})^2}{p_{H_2} \cdot p_{I_2}}$$

$$\Rightarrow \quad \frac{(2x)^2}{(0.5-x)(0.5-x)} = 49$$

or $$\frac{(2x)^2}{(0.5-x)^2} = 49 \text{ or } \frac{2x}{0.5-x} = 7$$

$$2x = 3.5 - 7x \Rightarrow 9x = 3.5$$

$$x = \frac{3.5}{9} = 0.3888$$

$$p_{H_2} = 0.5 - 0.3888 = 0.1111$$

13. Increase in oxidation number *i.e.*, oxidation

$$\overset{+7}{MnO_4^-} + 8H^+ + 5Br^- \longrightarrow Mn^{2+} + 4H_2O + \frac{5}{2}\overset{0}{Br_2}$$

Decrease in oxidation state, *i.e.*, reduction

Since, in this reaction Br^- reduces $KMnO_4^-$ into Mn^{2+} and itself gets oxidised into Br_2, so here it acts as the reducing agent.

Note In a reaction, the species which undergoes oxidation, generally acts as reducing agent.

14. From Arrhenius equation,

$K = Ae^{-E_a/RT}$, on taking log, we get

$$\log k = \log A - \frac{E_a}{2.303\ RT}$$

For reaction I, $\log k_1 = \log A - \frac{E_{a_1}}{2.303\ RT}$ ...(i)

For reaction II, $\log k_2 = \log A - \frac{E_{a_2}}{2.303\ RT}$ ...(ii)

Eq. (i) – Eq. (ii), we get

$$\log k_1 - \log k_2 = \log A - \frac{E_{a_1}}{2.303\ RT} - \left[\log A - \frac{E_{a_2}}{2.303\ RT}\right]$$

$$\log \frac{k_1}{k_2} = -\frac{E_{a_1}}{2.303\ RT} + \frac{E_{a_2}}{2.303\ RT}$$

$$\log 2 = \frac{1}{2.303\ RT}(-E_{a_1} + E_{a_2}) \text{ or}$$

$$E_{a_2} = E_{a_1} + 2.303\ RT \log 2$$

Thus, $E_{a_2} > E_{a_1}$

15. Energy, $E = \frac{hc}{\lambda}$

or $$\lambda = \frac{hc}{E} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{3.31 \times 10^{-20}}$$

$$= 6.00 \times 10^{-6} \text{ m}$$

16. Silver solution in HNO_3 + NaCl $\longrightarrow$ AgCl

108 + 35.5

Atomic mass of Ag = 108 g = 143.5

Weight of Ag = 5.82 g

∵ 143.5 g AgCl is obtained from Ag = 108 g

∴ 7 .20 g AgCl will be obtained from Ag

$$= \frac{108 \times 7.20}{143.5} = 5.418 \text{ g}$$

Weight of Ag taken = 5.82

$$\% \text{ purity} = \frac{5.418}{5.82} \times 100 = 93.09\%$$

17. Given, $k = \frac{k_1 k_3}{k_2}$

From Arrhenius equation, $k = Ae^{-E_a/RT}$

$$Ae^{-E_a/RT} = \frac{Ae^{-E_1/RT} \cdot Ae^{-E_3/RT}}{Ae^{-E_2/RT}}$$

$$\therefore \quad E_a = E_1 + E_3 - E_2 = 60 + 10 - 30$$

$$= 70 - 30 = 40 \text{ kJ mol}^{-1}$$

18. $\Delta H = \Delta U + \Delta n_g RT$

(where, ΔH = heat of reaction at constant pressure

ΔU = heat of reaction at constant volume).

For the given reaction,

$$\Delta n_g = 6 - \frac{7}{2} = \frac{5}{2} = 2.5$$

$$\therefore \Delta U = \Delta H - \Delta n_g RT$$
$$= 780980 - 2.5 \times 2 \times 298$$
$$= 780980 - 1490 = 779490 = 779.49 \text{ kcal}$$

19. In aqueous solution, $Ca(OH)_2$ is present as

$$Ca(OH)_2 \longrightarrow Ca^{2+} + 2OH^-$$

Let the solubility of $(Ca(OH)_2$ be s mol L^{-1}

s mol L^{-1} s mol L^{-1} 2s mol L^{-1}

$$K_{sp} = [Ca^{2+}][OH^-]^2 = s \cdot (2s)^2 = 4s^3$$

Given, solubility, $s = \sqrt{3}$

On substituting value of s, we get

$$K_{sp} = 4(\sqrt{3})^3 = 4 \times 3\sqrt{3} = 12 \cdot \sqrt{3}$$

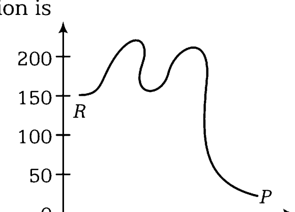
20. On mixing two solutions of same substance,

Molarity of the final mixture, $M = \dfrac{M_1V_1 + M_2V_2}{V_1 + V_2}$

$$= \frac{25 \times 3 + 75 \times 4}{25 + 75} = \frac{375}{100} = 3.75 \text{ M}$$

21. Since, the reaction is exothermic, so the energy of reactants (R) > energy of products (P)

Further, the reaction, involves intermediate step, so the energy profile diagram for the reaction is



22. As the value of reduction potential of the metal ion increases, the tendency of metal oxide to get reduce into metal increases.

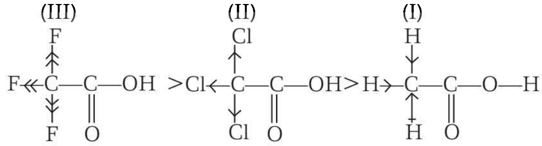
Since, reduction potential of only Ag is positive among the given, thus, Ag_2O readily gets reduced into Ag metal. In other words, it can be said that Ag_2O is the least stable oxide among the given.

23. Depression in freezing point is a colligative property *i.e.*, depends upon the number of ions present in the solution.

Further, more the number the electron, withdrawing groups ($-I$ showing groups like $-F > -Cl >$ etc.), more is the tendency of acid to get ionise, thus, more is the number of ions and hence, depression in freezing point.

Thus, the order of depression in freezing point in case of given carboxylic acids is

trifluoroacetic acid (III) > trichloroacetic acid (II) > acetic acid. (I)



24. Ionisation potential depends upon the effective nuclear charge which increases as we move along a period from left to right and hence, ionisation potential increases from left to right along a period but in case of elements having stable (completely or half filled) shells, the ionisation potential is more as compared to the element having non-stable configuration.

The electronic configuration of the given elements is

$_{11}Na = [Ne]\, 3s^1$

$_{12}Mg = [Ne]\, 3s^2$ [s-shell completely filled)

$_{13}Al = [Ne]\, 3s^2\, 3p^1$

$_{14}Si = [Ne]\, 3s^2 3p^2$

Thus, the order of ionisation potential will be

Na < Al < Mg < Si

496 < ? < 737 < 786

It means the IP of Al must lie in between 496 and 737. Thus, its IP should be 575 kJ mol^{-1}.

25. The picture tube which is used in television is actually the cathode ray tube, in which cathode rays are generated from cathode and move towards anode.

26. Television pictures are the result of fluorescene produced on the television screen which was coated with certain fluorescent or phosphorescent materials.

27. Number of protons = Atomic number = 16

Thus, the element is sulphur.

Further, mass number = number of protons + number of neutrons = 16 + 16 = 32

Since, number of electrons are greater than the number of protons, so it must be an anion (*i.e.*, obtained by accepting two electrons.

Hence, the correct symbol of the species must be $^{32}_{16}S^{2-}$.

28. In case of same anionic (non-metallic) species, the covalent character of molecule depends upon the size of central atom and decreases with increase in the size of central atom.

Here, the order of size of central atom is

$$N < P < Sb < Bi$$

Hence, BiF_3 is least covalent in nature.

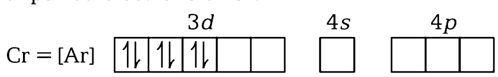
29. Rocket fuels generally contain two substance, one the fuel and other the oxidiser. Oxidiser is an oxygen rich substance and used to provide oxygen for burning process. Common oxidisers used for this purpose are nitric acid (HNO_3), hydrogen peroxide (H_2O_2), N_2O_4 etc.

30. Just like oxygen (O_2), S_2 is also a paramagnetic species because of the presence of two unpaired electrons in anti-bonding π-orbitals.

31. In $Cr(CO)_6$, Cr is present as Cr^0

Cr or Cr^0 = [Ar] $3d^5\ 4s^1$

CO being strong field ligand pair up the unpaired electrons of Cr.



In this condition,

number of unpaired electrons, $n = 0$

Magnetic moment, $\mu = \sqrt{n(n+2)}$

$= \sqrt{0(0+2)}$

$= 0$ BM

32. For the formation of π back bond (which is generally exist) between metal and the carbonyl carbon, the oxidation state of the metal must be lower than $+2$. Thus, $+2$ oxidation state is not possible in metal carbonyls.

33. The electronic configurations of the given elements are as follows

$${}_{61}Pm = [Xe]\ 4f^5,\ 5d^0,\ 6s^2$$

$${}_{62}Sm = [Xe]\ 4f^6,\ 5d^0,\ 6s^2$$

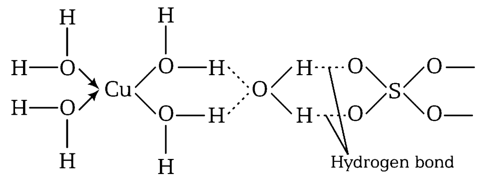
$${}_{58}Ce = [Xe]\ 4f^2,\ 5d^0 6s^2$$

$${}_{64}Gd = [Xe]\ 4f^7\ 5d^1\ 6s^2$$

From the electronic configurations, it is clear that only Ce can exhibit an oxidation state of $+4$ alongwith $+3$ because in that state it achieves the stable configuration of nobel gas xenon.

34. The given equation, *i.e.*, water gas shift reaction, is used to increase the production of hydrogen gas. Here, the obtained carbon dioxide (CO_2) is removed by scrubbing with a solution of sodium arsenite.

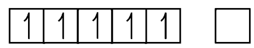
35. The structure of $CuSO_4 \cdot 5H_2O$ is as



Thus, only one water molecule is associated through hydrogen bonding in this molecule.

36. In $[FeF_6]^{3-}$ and $[Fe(CN)_6]^{30}$ both, Fe is present as Fe^{3+}.

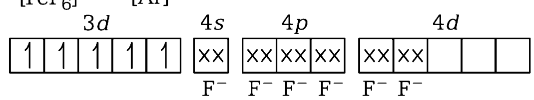
Fe^{3+} = [Ar] $3d^5$ $4s^0\ 4p^0\ 4d^0$



F^- being a weak field ligand, is not capable to pair up these unpaired electrons but CN^- does this.

Hence, is case of

$[FeF_6]^{3-}$ = [Ar]

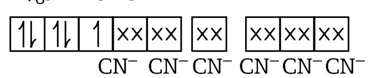


Number of unpaired electrons = 5

Magnetic moment, $\mu_1 = \sqrt{5\,(5+2)} = \sqrt{35}$

In case of

$[Fe\,(CN)_6]^{3-}$ = [Ar]



Number of unpaired electron = 1

Magnetic moment, $\mu_2 = \sqrt{1(1+2)} = \sqrt{3}$

Ratio of μ_1 and μ_2, $\frac{\mu_1}{\mu_2} = \frac{\sqrt{35}}{\sqrt{3}} = 3.41 \approx 3$

37. Catalytic activity of transition metals depends upon their ability to show variable (multiple) oxidation states and to form bonds with the reactant molecules, however not on the strength of the bonds formed. Further, increased concentration of reactants at the surface of the catalyst reduces the activity of catalyst.

38. CO form synergistic π back bonds with transition metals. It includes three components to give a partial triple bond. A σ bond is formed by the overlapping of non-bonding *sp*-hybridised electron pair of C with the blend of *d*, *s* and *p* orbitals of metal. Filled *d*-orbitals of the metal also overlaps with a pair of π anti-bonding orbitals of carbon of CO. To form such a bond, the oxidation state of metal atom must be lower than two.

39. Among the given interhalogen compounds, only ICl_3 exists as I_2Cl_6, *i.e.*, exists in dimeric form.

40. Electronic configuration of the given elements are as follows

$$_{24}Cr = [Ar]\ 3d^5 4s^1$$

$$_{29}Cu = [Ar]\ 3d^{10} 4s^1$$

$$_{30}Zn = [Ar]\ 3d^{10}\ 4s^2$$

$$_{25}Mn = [Ar]\ 3d^5\ 4s^2$$

Among these, only Zn has completely filled configuration. Thus, metallic bonding is weakest in case of Zn and hence, it has the lowest value of enthalpy of atomisation.

41. Higher the bond order of a molecule, more is the stability of molecular species.

Molecular orbital configuration and bond order of the given species are as follows

O_2^{2-} $(8 + 8 + 2 = 18) = \sigma 1s^2, \sigma^* 1s^2,$

$\sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2 \approx \pi 2p_y^2,$

$\pi^* 2p_x^2 \approx \pi^* 2p_y^2$

$$\text{Bond order} = \frac{N_b - N_a}{2} = \frac{10 - 8}{2} = 1$$

Similarly, O_2^- $(8 + 8 + 1 = 17)$

$$\text{Bond order} = \frac{N_b - N_a}{2} = \frac{10 - 7}{2} = 1.5$$

O_2 $(8 + 8 = 16)$

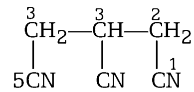
$$\text{Bond order} = \frac{N_b - N_a}{2} = \frac{16 - 6}{2} = 2$$

O_2^+ $(8 + 8 - 1 = 15)$

$$\text{Bond order} = \frac{N_b - N_a}{2} = \frac{10 - 5}{2} = 2.5$$

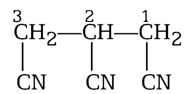
Thus, the order of bond order and stability of given species is $O_2^+ > O_2 > O_2^- > O_2^{2-}$.

42.



3-cyanopentane-1, 5-dinitrile

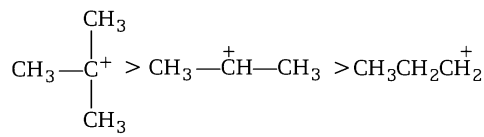
Note However, according to latest IUPAC recommendation, if three same functional groups are present, then all the three are not included in the main chain and the compound is name as.



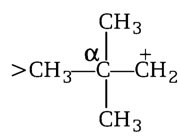
1,2,3-tri cyanopropane

43. Stability of carbocation depends upon the number of hyperconjugative structures. More the hyperconjugative structure (*i.e.*, more the number of hydrogen atoms attached directly to α-carbon atom), more is the stability of carbocation.

Thus, the order of stability of given carbocation is



III (9 α-H, 9, hyperconjugative structures) > II (6 α-H, 6, hyperconjugative structures) > I (2 α-H, 2, hyperconjugative structures)



(No α-H, so no hyperconjugative structures)

Thus, $(CH_3)_3C\overset{+}{C}H_2$ (IV) is least stable carbocation among the given.

44. As the number/ size of alkyl group attached to the carbonyl carbon increases, their reactivity towards. HCN (addition-elimination reaction) decreases due to steric hindrance.

Hence, the order of reactivity of different compounds towards HCN is

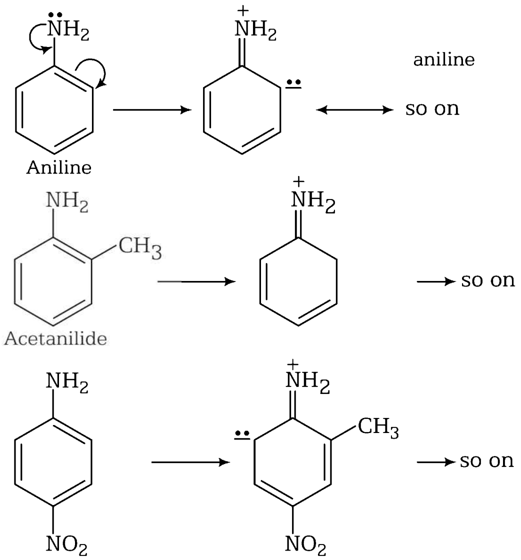
di *tert*-butyl ketone < methyl *tert*-butyl ketone < acetone < acetaldehyde.

or $[(CH_3)_3C]_2C{=}O < (CH_3)_3CCOCH_3 < CH_3COCH_3 < CH_3CHO$

45. When two different alkyl halides are heated with sodium metal in the presence of dry ether, a mixture of different alkanes (having double or more the number of carbon atoms as present in the parent halide) is obtained. This, reaction is called Wurtz reaction.

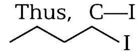
$$C_2H_5Br + 2Na + CH_3Br \xrightarrow{\text{Dry ether}} \underset{\text{ethane}}{CH_3—CH_3} + \underset{\text{Butane}}{C_2H_5—C_2H_5} + \underset{\text{Propane}}{C_2H_5—CH_3}$$

46. In benzylamine ($C_6H_5CH_2NH_2$) the lone pair of N does not involve in delocalisation and are available for donation but in other given compounds, the lone pair of N involves delocalisation and thus, are not available for donation as shown below

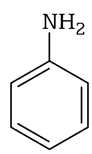


Thus, benzylamine is the most basic compound among the given compounds.

47. CH_3 is an activity group, *i.e.*, it activates the benzene nucleus towards electrophilic substitutuion reaction at *ortho-para* position. Cl, on the other hand, is a deactivating group towards electrophilic substitution. Since, the *para* position with respect to Cl is already occupied, thus, the substitution occurs fastest at *ortho* (4) position to CH_3.

48. Reactivity of haloalkanes in S_N1 as well as S_N2 reaction depends upon the strength of C—*X* bond. The strength of C—Cl bond is much greater as compared to C—I bond or the bond length of C—I bond is much greater as compared to C—Cl bond. Thus, C—I bond breaks readily and hence,  is the most reactive haloalkane among the given halide towards S_N2 as well as S_N1 reactions.

$$\underset{\text{Propene}}{CH_3CH=CH_2} \xrightarrow[\text{KMnO}_4]{\text{Aqueous}} \underset{(A)}{CH_3\underset{|}{\underset{OH}{CH}}—\underset{|}{\underset{OH}{CH_2}}} \xrightarrow{SOCl_2} \underset{\text{1,2-dichloropropane}}{CH_3—\underset{Cl}{CH}—\underset{Cl}{CH_2}}$$

50.  $\xrightarrow[\text{AlCl}_3]{CH_3COCl}$ No reaction

Note This reaction occurs if pyridine is present in place of $AlCl_3$.

Mathematics

1. Given, lines are $3x + 4y = 9$

and $6x + 8y = 15$ or $3x + 4y = \frac{15}{2}$

It is clear that given lines are parallel.

Here, $c_1 = 9$, $c_2 = \frac{15}{2}$, $a_1 = 3$ and $b_1 = 4$

∴ Distance between two lines

$$= \frac{|c_1 - c_2|}{\sqrt{a_1^2 + b_1^2}} = \frac{\left|9 - \frac{15}{2}\right|}{\sqrt{3^2 + 4^2}}$$

$$= \frac{|18 - 15|}{2\sqrt{9 + 16}} = \frac{3}{2\sqrt{25}} = \frac{3}{2 \times 5} = \frac{3}{10}$$

2. Let $z = -3 + ix^2y$

Now, $\bar{z} = -3 - ix^2y$

According to the question,

$$-3 - ix^2y = x^2 + y + 4i$$

On equating real and imaginary parts both sides, we get

$$-3 = x^2 + y \quad \text{and} \quad -x^2y = 4$$

$\therefore \quad -3 = x^2 - \frac{4}{x^2}$

$\Rightarrow \quad -3x^2 = x^4 - 4 \Rightarrow x^4 + 3x^2 - 4 = 0$

$\Rightarrow \quad x^4 + 4x^2 - x^2 - 4 = 0$

$\Rightarrow x^2(x^2 + 4) - 1(x^2 + 4) = 0$

$\Rightarrow \quad (x^2 - 1)(x^2 + 4) = 0$

$\Rightarrow \quad x^2 = 1 \text{ or } x^2 + 4 = 0$

$\Rightarrow \quad x = \pm 1 \qquad [\because x^2 \neq -4]$

$\Rightarrow \quad y = -4 \qquad [\because -x^2 y = 4]$

$\therefore \quad (x, y) = (1, -4), (-1, -4)$

3. In a regular hexagon, their are two equilateral triangles are possible.

$\therefore$ Required probability $= \frac{2}{{}^6C_3}$

$$= \frac{2}{\frac{6 \times 5 \times 4}{3 \times 2}} = \frac{2}{20} = \frac{1}{10}$$

4. Given $f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \leq -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2} < x \leq 0 \\ x - 1, & 0 < x \leq 1 \\ \ln x, & x > 1 \end{cases}$

Continuous at $x = -\frac{\pi}{2}$, LHL $= \lim_{x \to -\frac{\pi}{2}^-} f(x)$

$$= \lim_{h \to 0} -\left(-\frac{\pi}{2} - h\right) - \frac{\pi}{2} = \lim_{h \to 0} h = 0$$

RHL $= \lim_{x \to -\frac{\pi}{2}^+} f(x)$

$$= \lim_{h \to 0} -\cos\left(-\frac{\pi}{2} + h\right)$$

$$= \lim_{h \to 0} -\cos\left(\frac{\pi}{2} - h\right) = -\lim_{h \to 0} \sin h$$

$$= -\sin(0) = 0$$

and $f\left(-\frac{\pi}{2}\right) = -\left(-\frac{\pi}{2}\right) - \frac{\pi}{2} = 0$

Hence, $f(x)$ is continuous at $x = -\frac{\pi}{2}$.

5. Given, $(1 + e^{x/y}) \frac{dx}{dy} + e^{x/y}\left(1 - \frac{x}{y}\right) = 0$

Put $\quad x = vy$

$\Rightarrow \quad \frac{dx}{dy} = v + y\frac{dv}{dy}$

$\therefore (1 + e^{vy/y})\left(v + y\frac{dv}{dy}\right) + e^{vy/y}(1 - v) = 0$

$\Rightarrow (1 + e^v)\left(v + y\frac{dv}{dy}\right) + e^v(1 - v) = 0$

$\Rightarrow v + y\frac{dv}{dy} + ve^v + e^v y\frac{dv}{dy} + e^v - ve^v = 0$

$\Rightarrow y\frac{dv}{dy}(1 + e^v) + v + e^v = 0$

$\Rightarrow \quad y\frac{dv}{dy}(1 + e^v) = -(v + e^v)$

$\Rightarrow \quad \frac{(1 + e^v)\,dv}{(v + e^v)} = -\frac{1}{y}\,dy$

On integrating both sides, we get

$$\int \frac{1 + e^v}{v + e^v}\,dv = -\int \frac{1}{y}\,dy$$

Put $\quad v + e^v = t \Rightarrow (1 + e^v)\,dv = dt$

$$\int \frac{1}{t}\,dt = -\int \frac{1}{y}\,dy$$

$$\log t = -\log y + \log C$$

$\Rightarrow \quad \log(v + e^v) = \log\frac{C}{y}$

$\Rightarrow \log\left(\frac{x}{y} + e^{x/y}\right) = \log\frac{C}{y} \Rightarrow x + ye^{x/y} = C$

6. ${}^mC_{r+1} + \sum_{k=m}^{n} {}^kC_r$

$= ({}^mC_{r+1} + {}^mC_r) + {}^{m+1}C_r + {}^{m+2}C_r + \dots {}^{m+n}C_r$

$= ({}^{m+1}C_{r+1} + {}^{m+1}C_r) + {}^{m+2}C_r + \dots {}^{m+n}C_r$

$= {}^{m+2}C_{r+1} + {}^{m+2}C_r + \dots + {}^nC_r$

$\vdots \qquad \vdots \qquad \vdots$

$= {}^{n+1}C_{r+1}$

7. $f'(x) < 2f(x)$

$\Rightarrow \frac{f'(x)}{f(x)} < 2 \Rightarrow \int \frac{f'(x)\,dx}{f(x)} < \int 2\,dx$

$\Rightarrow \quad \log f(x) < 2x + C$

At $x = \frac{1}{2}$, $f\left(\frac{1}{2}\right) = 1$

$\Rightarrow \log(1) < 2 \times \frac{1}{2} + C \Rightarrow C > -1$

$\Rightarrow \log f(x) < 2x - 1 \Rightarrow f(x) < e^{2x-1}$

$\Rightarrow \int_{1/2}^{1} f(x)\,dx < \int_{1/2}^{1} e^{2x-1}\,dx$

$\Rightarrow \int_{1/2}^{1} f(x)\,dx < \left[\frac{e^{2x-1}}{2}\right]_{1/2}^{1}$

$\Rightarrow \int_{1/2}^{1} f(x)\,dx < \frac{1}{2}(e-1)$

$\therefore \int_{1/2}^{1} f(x)\,dx \in \left(0, \frac{e-1}{2}\right)$

8. Given, system of equation is $x + y + z = \beta$

$5x - y + \alpha z = 10$ and $2x + 3y - z = 6$

For unique solution $\begin{vmatrix} 1 & 1 & 1 \\ 5 & -1 & \alpha \\ 2 & 3 & -1 \end{vmatrix} \neq 0$

$\Rightarrow 1(1 - 3\alpha) - 1(-5 - 2\alpha) + 1(15 + 2) \neq 0$

$\Rightarrow 1 - 3\alpha + 5 + 2\alpha + 17 \neq 0$

$\Rightarrow -\alpha + 23 \neq 0 \Rightarrow \alpha \neq 23$

Hence, for the existence of the unique solution the system of equations depend on α only.

9. Hence, option (c) is true.

10. We have,

$$\int_0^{\pi/3} \frac{\cos x}{3 + 4\sin x}\,dx = k \log\left(\frac{3 + 2\sqrt{3}}{3}\right) \quad \ldots(i)$$

Let $I = \int_0^{\pi/3} \frac{\cos x}{3 + 4\sin x}\,dx$

Put $3 + 4\sin x = t$

$\Rightarrow 0 + 4\cos x\,dx = dt$

Upper limit, $x = \frac{\pi}{3}$, $t = 3 + 4\sin\frac{\pi}{3}$

$= 3 + 4 \times \frac{\sqrt{3}}{2} = 3 + 2\sqrt{3}$

and lower limit, $x = 0$,

$t = 3 + 4\sin 0 = 3$

$\therefore I = \int_3^{3+2\sqrt{3}} \frac{dt}{4t} = \frac{1}{4}[\log t]_3^{3+2\sqrt{3}}$

$= \frac{1}{4}[\log(3 + 2\sqrt{3}) - \log 3]$

$\Rightarrow I = \frac{1}{4}\log\left(\frac{3 + 2\sqrt{3}}{3}\right) \quad \ldots(ii)$

$\therefore$ From Eqs. (i) and (ii), we get

$k = \frac{1}{4}$

11. $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \ldots + n\frac{C_n}{C_{n-1}}$

$= \frac{{}^nC_1}{{}^nC_0} + 2\frac{{}^nC_2}{{}^nC_1} + 3\frac{{}^nC_3}{{}^nC_2} + \ldots + n\frac{{}^nC_n}{{}^nC_{n-1}}$

$= \frac{n}{1} + 2 \times \frac{\frac{n(n-1)}{2}}{n} + 3 \times \frac{\frac{n(n-1)(n-2)}{3 \times 2}}{\frac{n(n-1)}{2}} + \ldots + n \times \frac{1}{n}$

$= n + (n-1) + (n-2) + \ldots + 1$

$= \sum n = \frac{n(n+1)}{2}$

12. Given, $\begin{vmatrix} x & x^2 & 1 + x^3 \\ y & y^2 & 1 + y^3 \\ z & z^2 & 1 + z^3 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + xyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + xyz\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} = 0$

$\Rightarrow (1 + xyz)\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} = 0$

$\Rightarrow 1 + xyz = 0$

and $\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} \neq 0 \quad \therefore \quad xyz = -1$

13. Given, $x\cos x\left(\frac{dy}{dx}\right) + y(x\sin x + \cos x) = 1$

$\Rightarrow \frac{dy}{dx} + \frac{y(x\sin x + \cos x)}{x\cos x} = \frac{1}{x\cos x}$

On comparing with $\frac{dy}{dx} + Py = Q$, we get

$$P = \frac{x \sin x + \cos x}{x \cos x} = \tan x + \frac{1}{x}$$

$$\therefore \quad \text{IF} = e^{\int P\, dx}$$

$$= e^{\int \left(\tan x + \frac{1}{x}\right) dx} = e^{\log \sec x + \log x}$$

$$= e^{\log x \sec x} = x \sec x$$

$\therefore$ Solution is

$$y \times x \sec x = \int \frac{x \sec x}{x \cos x}\, dx = \int \sec^2 x\, dx$$

$$\Rightarrow \quad xy \sec x = \tan x + C$$

14. Given, $f(0) = 9$ and $f(x) = \dfrac{\sin\left(\frac{9x}{2}\right)}{\sin \frac{x}{2}}$

Now, $f(-x) = \dfrac{\sin\left(-\frac{9x}{2}\right)}{\sin\left(-\frac{x}{2}\right)} = \dfrac{-\sin\left(\frac{9x}{2}\right)}{-\sin\left(\frac{x}{2}\right)} = f(x)$

Hence, $f(x)$ is an even function.

$$\therefore I = \frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\, dx = \frac{2}{\pi} \times 2 \int_0^{\pi} f(x)\, dx$$

$$= \frac{4}{\pi}\int_0^{\pi} \frac{\sin \frac{9x}{2}}{\sin \frac{x}{2}}\, dx = \frac{4}{\pi}\int_0^{\pi} \frac{\sin\left(4x + \frac{x}{2}\right)}{\sin \frac{x}{2}}\, dx$$

$$= \frac{4}{\pi}\int_0^{\pi} \frac{\sin 4x \cos \frac{x}{2} + \sin \frac{x}{2} \cos 4x}{\sin \frac{x}{2}}\, dx$$

$$= \frac{4}{\pi}\left[\int_0^{\pi}\left(\sin 4x \cot \frac{x}{2}\right) dx + \int_0^{\pi} \cos 4x\, dx\right]$$

$$= \frac{4}{\pi}\left[\int_0^{\pi} \sin 4x \cot \frac{x}{2}\, dx\right] + 0$$

$$\left[\because \int_0^{\pi} \cos 4x\, dx = \left[\frac{\sin 4x}{4}\right]_0^{\pi} = \frac{1}{4}(\sin 4\pi - \sin) = \frac{1}{4}[0 - 0] = 0\right]$$

$$= -\frac{1}{4}[0 - 0] = 0$$

$$\therefore I = \frac{4}{\pi}\int_0^{\pi} \sin 4x \cot \frac{x}{2}\, dx \qquad \text{...(i)}$$

$$= \frac{4}{\pi}\int_0^{\pi} \sin(4\pi - 4x) \cot\left(\frac{\pi - x}{2}\right) dx$$

$$\Rightarrow I = \frac{4}{\pi}\int_0^{\pi} -\sin 4x \tan \frac{x}{2}\, dx \qquad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I = \frac{4}{\pi}\left[\int_0^{\pi} \sin 4x \left(\cot \frac{x}{2} - \tan \frac{x}{2}\right) dx\right]$$

$$= \frac{4}{\pi}\left[\int_0^{\pi} \sin 4x \frac{\left(\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}\right)}{\sin \frac{x}{2} \cos \frac{x}{2}}\, dx\right]$$

$$= \frac{4}{\pi}\left[\int_0^{\pi} \frac{\sin 4x \cos x}{\frac{1}{2}\sin x}\, dx\right]$$

$$= \frac{8}{\pi}\left[\int_0^{\pi} \frac{2 \sin 2x \cos 2x \cos x}{\sin x}\, dx\right]$$

$$= \frac{16}{\pi}\left[\int_0^{\pi} \frac{2 \sin x \cos x \cos 2x \cos x}{\sin x}\, dx\right]$$

$$= \frac{32}{\pi}\int_0^{\pi} \cos^2 x \cos 2x\, dx$$

$$= \frac{32}{\pi}\int_0^{\pi} \left(\frac{\cos 2x + 1}{2}\right) \cos 2x\, dx$$

$$= \frac{16}{\pi}\left[\int_0^{\pi} \cos^2 2x\, dx + \int_0^{\pi} \cos 2x\, dx\right]$$

$$= \frac{16}{\pi}\left[\int_0^{\pi} \frac{\cos 4x + 1}{2}\, dx + \int_0^{\pi} \cos 2x\, dx\right]$$

$$= \frac{16}{\pi}\left[\frac{1}{2}\left[\frac{\sin 4x}{4} + x\right]_0^{\pi} + \left[\frac{\sin 2x}{2}\right]_0^{\pi}\right]$$

$$= \frac{16}{\pi}\left[\frac{1}{2}[0 + \pi - 0 - 0] + \left[\frac{0 - 0}{2}\right]\right] = \frac{16}{\pi}\left[\frac{\pi}{2}\right]$$

$$2I \Rightarrow 8 \Rightarrow I = 4$$

15. Given, $(1 + x + x^2)^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_{2n} x^{2n}$

$$(1 + x + x^2)^n = (a_0 + a_3 x^6 + a_9 x^9 + \ldots) + x(a_1 + a_4 x^3 + a_7 x^6 + \ldots) + x^2(a_2 + a_5 x^3 + a_8 x^6 + \ldots)$$

Let $E_1 = a_0 + a_3 + a_9 + \ldots$

$E_2 = a_1 + a_4 + a_7 + \ldots$

and $E_3 = a_2 + a_5 + a_8 + \ldots$

Put $x = 1, \omega, \omega^2$ respectively, we get

$$(1+1+1)^n = E_1 + E_2 + E_3$$

$\Rightarrow \quad 3^n = E_1 + E_2 + E_3 \quad$ ...(i)

$$(1+\omega+\omega^2)^n = E_1 + \omega E_2 + \omega^2 E_3$$

$\Rightarrow \quad 0 = E_1 + \omega E_2 + \omega^2 E_3 \quad$ ...(ii)

and $(1+\omega^2+\omega^4)^n = E_1 + \omega^2 E_2 + \omega^4 E_3$

$\Rightarrow \quad 0 = E_1 + \omega^2 E_2 + \omega E_3 \quad$ ...(iii)

On adding Eqs. (i), (ii) and (iii), we get

$$3^n = 3E_1 + (1+\omega+\omega^2) E_2 + (1+\omega^2+\omega) E_3$$

$\Rightarrow 3^n = 3E_1 + 0 + 0 \Rightarrow E_1 = 3^{n-1}$

16. Given, $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$

$\therefore \quad A^{-1} = \frac{1}{-4-15}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$

$$= \frac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \frac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$$

[multiplying −1 each element of a matrix]

$$= \frac{1}{19} A$$

Given, $A^{-1} = kA \quad \therefore \quad k = \frac{1}{19}$

17. Given, $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$

Apply L'Hospital rule $\lim_{t \to x} \frac{2t\, f(x) - x^2 f'(t)}{1} = 1$

$\Rightarrow \quad 2x\, f(x) - x^2 f'(x) = 1$

$\Rightarrow \quad x^2 f'(x) - 2x\, f(x) = -1$

$\Rightarrow \quad f'(x) - \frac{2}{x} f(x) = -\frac{1}{x^2}$

It is a linear differential equation.

$\therefore \text{IF} = e^{\int -\frac{2}{x} dx} = e^{-2\log x} = e^{\log\left(\frac{1}{x^2}\right)} = \frac{1}{x^2}$

$\therefore$ Solution is

$$f(x) \times \frac{1}{x^2} = \int -\frac{1}{x^2} \times \frac{1}{x^2}\, dx + C$$

$$f(x) \times \frac{1}{x^2} = \frac{1}{3x^3} + C \Rightarrow f(x) = \frac{1}{3x} + Cx^2$$

At $\quad f(1) = 1 \Rightarrow 1 = \frac{1}{3} + C \Rightarrow C = \frac{2}{3}$

$\therefore \quad f(x) = \frac{1}{3x} + \frac{2}{3} x^2$

18. Given, α and β are the roots of equation

$$ax^2 + bx + c = 0.$$

$\therefore \quad \alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Now, $\quad S_{n+1} = \alpha^{n+1} + \beta^{n+1}$

$$= \alpha^{n+1} + \beta^{n+1} + \alpha^n\beta + \beta^n\alpha - \alpha^n\beta - \beta^n\alpha$$

$$= \alpha^n(\alpha+\beta) + \beta^n(\alpha+\beta) - \alpha\beta(\alpha^{n-1} + \beta^{n-1})$$

$$= (\alpha+\beta)(\alpha^n + \beta^n) - \alpha\beta(\alpha^{n-1} + \beta^{n-1})$$

$$= -\frac{b}{a} S_n - \frac{c}{a} S_{n-1}$$

$\Rightarrow S_{n+1} = \frac{-bS_n - cS_{n-1}}{a}$

$\therefore \quad aS_{n+1} + bS_n + cS_{n-1} = 0$

19. (b)

20. Given, $x^p y^q = (x+y)^{p+q}$

On differentiating both sides w.r.t. x, we get

$$x^p \frac{d}{dx} y^q + y^q \frac{d}{dx}(x^p) = \frac{d}{dx}(x+y)^{p+q}$$

$\Rightarrow \quad x^p q y^{q-1} \frac{dy}{dx} + y^q p x^{p-1}$

$$= (p+q)(x+y)^{p+q-1} \times \left(1 + \frac{dy}{dx}\right)$$

$\Rightarrow \quad \frac{qx^p y^q}{y} \frac{dy}{dx} + \frac{py^q x^p}{x}$

$$= (p+q)(x+y)^{p+q-1} \times \left(1 + \frac{dy}{dx}\right)$$

$\Rightarrow \quad x^p y^q \left(\frac{q}{y}\frac{dy}{dx} + \frac{p}{x}\right)$

$$= \frac{(p+q)(x+y)^{p+q}}{x+y} \times \left(1 + \frac{dy}{dx}\right)$$

$\Rightarrow \quad x^p y^q \left(\frac{q}{y}\frac{dy}{dx} + \frac{p}{x}\right) = \frac{(p+q)\, x^p y^q}{x+y}\left(1 + \frac{dy}{dx}\right)$

$\Rightarrow \frac{q}{y}\frac{dy}{dx} + \frac{p}{x} = \frac{p+q}{x+y}\left(1 + \frac{dy}{dx}\right)$

$\Rightarrow \frac{dy}{dx}\left(\frac{q}{y} - \frac{p+q}{x+y}\right) = \frac{p+q}{x+y} - \frac{p}{x}$

$\Rightarrow \frac{dy}{dx}\left(\frac{qx + qy - py - qy}{y(x+y)}\right) = \frac{px + qx - px - py}{x(x+y)}$

$\Rightarrow \frac{dy}{dx}\left(\frac{qx - py}{y}\right) = \frac{qx - py}{x} \quad \Rightarrow \frac{dy}{dx} = \frac{y}{x}$

21. Given, $z = x + iy$

Now, $\bar{z}z^3 + z\bar{z}^3 = 350$

$\Rightarrow (\bar{z}z)\, z^2 + (z\bar{z})\, \bar{z}^2 = 350$

$\Rightarrow |z|^2 (x+iy)^2 + |z|^2 (x-iy)^2 = 350$

$\Rightarrow (x^2+y^2)$

$[x^2 - y^2 + 2ixy + x^2 - y^2 - 2ixy] = 350$

$\Rightarrow 2(x^2+y^2)(x^2-y^2) = 350$

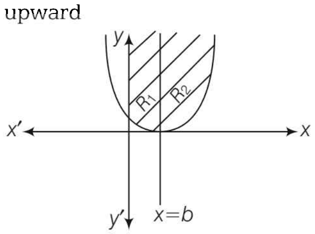
$\Rightarrow x^4 - y^4 = 175 \Rightarrow x = \pm 4, y = \pm 3$

$\therefore$ Vertices are $(-4, -3), (-4, 3), (4, -3)$ and $(4, 3)$.

$\therefore$ Length and breadth are 8 and 6.

$\therefore$ Area of the rectangle $= 8 \times 6 = 48$

22. Given, curve $y = (1-x)^2$ is a parabola, which open upward



$\therefore \quad R_1 = \int_0^b y\, dx$

$= \int_0^b (1-x)^2\, dx = \left[-\frac{(1-x)^3}{3}\right]_0^b$

$= \frac{-1}{3}[(1-b)^3 - 1]$ and $R_2 = \int_1^b y\, dx$

$= \int_1^b (1-x)^2\, dx = \left[-\frac{(1-x)^3}{3}\right]_1^b$

$= -\frac{1}{3}\left[-\frac{(1-b)^3}{3} - 0\right] = +\frac{1}{3}[(1-b)^3]$

But it is given, $R_1 - R_2 = \frac{1}{4}$

$\therefore -\frac{1}{3}[(1-b)^3 - 1] - \frac{1}{3}[(1-b)^3] = \frac{1}{4}$

$\Rightarrow -\frac{1}{3}[2(1-b)^3 - 1] = \frac{1}{4}$

$\Rightarrow 2(1-b)^3 = -\frac{3}{4} + 1$

$\Rightarrow (1-b)^3 = 1/8 \Rightarrow 1 - b = 1/2$

$\Rightarrow b = 1 - \frac{1}{2} = \frac{1}{2}$

23. Since, $\left(\frac{\omega - \bar{\omega} z}{1-z}\right)$ is purely real.

$\therefore \left(\frac{\omega - \bar{\omega} z}{1-z}\right) - \overline{\left(\frac{\omega - \bar{\omega} z}{1-z}\right)} = 0$

$\Rightarrow \frac{\omega - \bar{\omega} z}{1-z} - \frac{\bar{\omega} - \omega \bar{z}}{1-\bar{z}} = 0$

$\Rightarrow \frac{\omega - \bar{\omega} z - \bar{\omega} z + \bar{\omega} z\bar{z} - (\bar{\omega} - \bar{\omega} z - \omega\bar{z} + \bar{\omega} z\bar{z})}{(1-z)(1-\bar{z})} = 0$

$\Rightarrow \frac{\omega - \bar{\omega}}{(1-z)(1-\bar{z})} = 0 \Rightarrow \omega = \bar{\omega}, z \neq 1$ and $\bar{z} = 1$

Hence, option (d) is correct.

24. Given, $f(x) = \frac{a \sin x + 2\cos x}{\sin x + \cos x}$

On differentiating w.r.t. x, we get

$$f'(x) = \frac{\begin{bmatrix}(\sin x + \cos x)(a\cos x - 2\sin x) \\ -(a\sin x + 2\cos x)(\cos x - \sin x)\end{bmatrix}}{(\sin x + \cos x)^2}$$

For $f(x)$ to be increasing $f'(x) > 0$

$$\therefore \frac{\begin{bmatrix}(\sin x + \cos x)(a\cos x - 2\sin x) \\ -(a\sin x + 2\cos x)(\cos x - \sin x)\end{bmatrix}}{(\sin x + \cos x)^2} > 0$$

$\Rightarrow a\sin x\cos x - 2\sin^2 x + a\cos^2 x - 2\sin x\cos x$

$-(a\sin x\cos x - a\sin^2 x + 2\cos^2 x$

$-2\cos x\sin x) > 0$

$\Rightarrow a(\sin^2 x + \cos^2 x) - 2(\sin^2 x + \cos^2 x) > 0$

$\Rightarrow a - 2 > 0 \Rightarrow a > 2$

25. By the property of adjoint of a matrix

$\text{adj}(\text{adj } A) = |A|^{n-2} A$

26. Let $\mathbf{r} = x_1\hat{i} + x_2\hat{j} + x_3\hat{k}$

Now, $\mathbf{c} \times \mathbf{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 1 & 0 \end{vmatrix}$

$= \hat{i}(0-3) - \hat{j}(0+3) + \hat{k}(1+2) = -3\hat{i} - 3\hat{j} + 3\hat{k}$

and $\mathbf{r} \times \mathbf{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & x_2 & x_3 \\ -1 & 1 & 0 \end{vmatrix}$

$= \hat{i}(0 - x_3) - \hat{j}(0 + x_3) + \hat{k}(x_1 + x_2)$

$= -x_3\hat{i} - x_3\hat{j}(x_1 + x_2)\hat{k}$

Also, $\mathbf{r} \cdot \mathbf{a} = 0$

$\Rightarrow (x_1\hat{i} + x_2\hat{j} + x_3\hat{k}) \cdot (-\hat{i} - \hat{k}) = 0$

$\Rightarrow -x_1 - x_3 = 0$ ...(i)

But $\mathbf{r} \times \mathbf{b} = \mathbf{c} \times \mathbf{b}$

$\therefore \quad -x_3\hat{\mathbf{i}} - x_3\hat{\mathbf{j}} + \hat{\mathbf{k}}\,(x_1 + x_2) = -3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

On comparing both sides, we get

$$x_3 = 3 \quad \text{and} \quad x_1 + x_2 = 3 \qquad \ldots\text{(ii)}$$

On solving Eqs. (i) and (ii), we get

$$x_1 = -x_3 = -3 \;\text{ and }\; x_2 = 6$$

Now, $\mathbf{r} \cdot \mathbf{b} = (x_1\hat{\mathbf{i}} + x_2\hat{\mathbf{j}} + x_3\hat{\mathbf{k}}) \cdot (-\hat{\mathbf{i}} + \hat{\mathbf{j}})$

$$= -x_1 + x_2 = -(-3) + 6 = 9$$

27. $\sin\dfrac{\pi}{2n} + \cos\dfrac{\pi}{2n} = \dfrac{\sqrt{n}}{2}$

On squaring both sides, we get

$$\sin^2\frac{\pi}{2n} + \cos^2\frac{\pi}{2n} + 2\sin\frac{\pi}{2n}\cos\frac{\pi}{2n} = \frac{n}{4}$$

$$\Rightarrow \quad 1 + \sin\frac{2\pi}{2n} = \frac{n}{4} \;\Rightarrow\; 1 + \sin\frac{\pi}{n} = \frac{n}{4}$$

Consider $n = 6$,

$$1 + \frac{1}{2} = \frac{6}{4} \;\Rightarrow\; \frac{3}{2} = \frac{3}{2} \text{ satisfy}$$

$$\therefore \quad 4 < n < 8$$

28. Given, $L = \lim\limits_{x \to 0} \dfrac{a - \sqrt{a^2 - x^2} - \dfrac{x^2}{4}}{x^4} \quad \left[\text{form}\dfrac{0}{0}\right]$

$$= \lim_{x \to 0} \frac{0 - \dfrac{(0 - 2x)}{2\sqrt{a^2 - x^2}} - \dfrac{2x}{4}}{4x^3}$$

$$= \lim_{x \to 0} \frac{x\left(\dfrac{1}{\sqrt{a^2 - x^2}} - \dfrac{1}{2}\right)}{4x^3} = \lim_{x \to 0} \frac{\dfrac{1}{\sqrt{a^2 - x^2}} - \dfrac{1}{2}}{4x^2}$$

For limit to be exist, $a = 2$

29. Given, A28, 3B9 and 62C are divisible by k

$\therefore \quad \text{A28} = k$

$\Rightarrow 100\text{A} + 20 + 8 = k;\; 3\text{B}9 = k$

$\Rightarrow \quad 300 + 10\text{B} + 9 = k$ and $62\text{C} = k$

$\Rightarrow \quad 600 + 20 + \text{C} = k$

Let $\Delta = \begin{vmatrix} \text{A} & 3 & 6 \\ 8 & 9 & \text{C} \\ 2 & \text{B} & 2 \end{vmatrix}$

$$= \frac{1}{100} \times \frac{1}{10} \begin{vmatrix} 100\text{A} & 300 & 600 \\ 8 & 9 & \text{C} \\ 20 & 10\text{B} & 20 \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$

$$= \frac{1}{1000} \begin{vmatrix} 100\text{A} + 20 + 8 & 300 + 10\text{B} + 9 & 600 + 20 + \text{C} \\ 8 & 9 & \text{C} \\ 20 & 10\text{B} & 20 \end{vmatrix}$$

$$= \frac{1}{1000} \begin{vmatrix} k & k & k \\ 8 & 9 & \text{C} \\ 20 & 10\text{B} & 20 \end{vmatrix} = \frac{k}{100} \begin{vmatrix} 1 & 1 & 1 \\ 8 & 9 & \text{C} \\ 20 & 10\text{B} & 20 \end{vmatrix}$$

Hence, it is always divisible by k.

30. Given, $y = 2\sin x$ and $y = 5x^2 + 2x + 3$

$$= 5\left(x^2 + \frac{2}{5}x\right) + 3$$

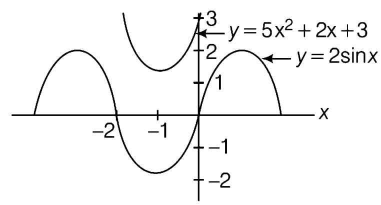
$$\Rightarrow y - 3 = 5\left(x^2 + \frac{2}{5}x + \frac{1}{25} - \frac{1}{25}\right)$$

$$\Rightarrow y - 3 = 5\left(x + \frac{1}{5}\right)^2 - \frac{1}{5}$$

$$\Rightarrow \quad y - 3 + \frac{1}{5} = 5\left(x + \frac{1}{5}\right)^2$$

$$\Rightarrow \quad 5\left(x + \frac{1}{5}\right)^2 = y - \frac{14}{5}$$

$$\Rightarrow \quad \left(x + \frac{1}{5}\right)^2 = \frac{1}{5}\left(y - \frac{14}{5}\right)$$



Here, we see that two curves do not intersect any point.

Hence, no solution exist.

31. The position p vector P of line $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ meets the plane $\mathbf{r} \cdot \hat{\mathbf{n}} = p$ is $\mathbf{a} + \left(\dfrac{p - \mathbf{a} \cdot \hat{\mathbf{n}}}{\mathbf{b} \cdot \hat{\mathbf{n}}}\right)\mathbf{b}$

32. A leap year has 366 days in which 52 weak and two days are extra.

i.e., (Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday)

So probability that a leap year contains 53 Sundays = 2/7

33. $\sin^{-1}\left(\sin\dfrac{2\pi}{3}\right) = \sin^{-1}\left[\sin\left(\pi - \dfrac{\pi}{3}\right)\right]$

$$= \sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$$

34. Given relation is $|a - b| \le 1$

Reflexive relation,

$aRa \Rightarrow |a - a| \le 1 = 0 \le 1$, which is true.

35. Given vectors are $\mathbf{a} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$,

$\mathbf{b} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{c} = \hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$

Any vector in the plane of **a** and **b** is

$$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda(\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$$
$$= (1+\lambda)\hat{\mathbf{i}} + (2-\lambda)\hat{\mathbf{j}} + \hat{\mathbf{k}}(1+\lambda)$$

The projection of **r** on **c** is $\frac{\mathbf{r}\cdot\mathbf{c}}{|\mathbf{c}|} = \frac{1}{\sqrt{3}}$

$$\therefore \quad \frac{[(1+\lambda)\hat{\mathbf{i}} + (2-\lambda)\hat{\mathbf{j}} + (1+\lambda)\hat{\mathbf{k}}]\cdot(\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{1+\lambda-2+\lambda-1-\lambda}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \lambda - 2 = 1 \Rightarrow \lambda = 3$$

$$\therefore \quad \mathbf{r} = (1+3)\hat{\mathbf{i}} + (2-3)\hat{\mathbf{j}} + (1+3)\hat{\mathbf{k}}$$
$$= 4\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

36. We know that the length of the tangent from a point $P(x_1, y_1)$ is $\sqrt{S_1}$, where S_1 is the equation of circle at point P.

Given equations of circle are

$$P \equiv x^2 + y^2 - a^2,\ Q \equiv x^2 + y^2 - b^2$$

and $\quad R \equiv x^2 + y^2 - c^2$

$\therefore$ Length of tangent at point $A(x_1, y_1)$ are

$$T_1 = \sqrt{P_1} = \sqrt{x_1^2 + y_1^2 - a^2}$$
$$T_2 = \sqrt{Q_1} = \sqrt{x_1^2 + y_1^2 - b^2}$$

and $\quad T_3 = \sqrt{R_1} = \sqrt{x_1^2 + y_1^2 - c^2}$

Since, it is given length of tangent are in AP

$$T_2^2 = \frac{T_1^2 + T_3}{2} \Rightarrow x_1^2 + y_1^2 - b^2$$
$$= \frac{(x_1^2 + y_1^2 - a^2) + (x_1^2 + y_1^2 - c^2)}{2}$$

$$\Rightarrow \quad x_1^2 + y_1^2 - b^2 = x_1^2 + y_1^2 - \frac{(a^2+c^2)}{2}$$

$$\Rightarrow \quad -b^2 = -\frac{(a^2+c^2)}{2} \Rightarrow b^2 = \frac{a^2+c^2}{2}$$

Hence, a^2, b^2 and c^2 are in AP.

37. Given, $n(A) = 3$ and $n(B) = 6$

$\therefore$ min $(A \cup B) = n(B) = 6$

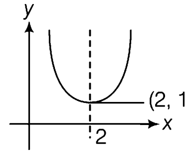
38. Given, function is $f(x) = x^2 - 4x + 5$

which is defined as $f : [2, \infty) \to A$

Let $\quad y = x^2 - 4x + 5$

$\Rightarrow \quad y = x^2 - 4x + 4 + 1$

$\Rightarrow \quad y - 1 = (x-2)^2$



From the curve, we see that

At $\quad x = 2,\ f(2) = 1,$

After that when we increase the value of x, the curve is increasing

$\therefore \quad A = [1, \infty)$

39. Given, equation of parabola is $y^2 = 4x$.

Let two point on the parabola be.

$A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$

Here, $\quad a = 1$

$\therefore \quad A(t_1^2, 2t_1)$ and $B(t_2^2, 2t_2)$

Since, A and B are the end points of a diameter.

$\therefore$ Mid-point of AB is $\left(\frac{t_1^2 + t_2^2}{2}, t_1 + t_2\right)$

Also, given radius of circle r.

$\therefore$ Equation of circle is

$$\left(x - \frac{t_1^2 + t_2^2}{2}\right) + [y - (t_1 + t_2)]^2 = (2)^2$$

$$\Rightarrow x^2 + \left(\frac{t_1^2 + t_2^2}{2}\right)^2 - 2x\left(\frac{t_1^2 + t_2^2}{2}\right)$$
$$+ y^2 + (t_1 + t_2)^2 - 2y(t_1 + t_2) = 4$$

$$\Rightarrow x^2 + y^2 - 2x\frac{(t_1^2 + t_2^2)}{2} - 2y(t_1 + t_2)$$
$$+ \left(\frac{t_1^2 + t_2^2}{2}\right)^2 + (t_1 + t_2)^2 - 4 = 0$$

Here, $\quad g = \frac{t_1^2 + t_2^2}{2}$

and $\quad c = (t_1^2 + t_2^2)^2 + (t_1 + t_2)^2 - 4$

Since, circle touch x-axis

$\therefore \quad g^2 = c$

$$\Rightarrow \quad \left(\frac{t_1^2 + t_2^2}{2}\right)^2 = \left(\frac{t_1^2 + t_2^2}{2}\right)^2 + (t_1 + t_2)^2 - 4$$

$$\Rightarrow \quad (t_1 + t_2)^2 = 4 \Rightarrow t_1 + t_2 = 2$$

Slope of $AB = \frac{2(t_2 - t_1)}{(t_2^2 - t_1^2)}$

$$= \frac{2(t_2 - t_1)}{(t_2 - t_1)(t_2 + t_1)} = \frac{2}{(t_2 + t_1)} = \frac{2}{2} = 1$$

40. For two events, given probability is less than 1/4.

41. Given, $\mathbf{a}+\mathbf{b}+\mathbf{c}=0$

Taking cross product of both sides, we get

$(\mathbf{a}+\mathbf{b}+\mathbf{c})\times\mathbf{a}=0\times\mathbf{a}$

$\Rightarrow \mathbf{a}\times\mathbf{a}+\mathbf{b}\times\mathbf{a}+\mathbf{c}\times\mathbf{a}=0$

$\Rightarrow 0+\mathbf{b}\times\mathbf{a}+\mathbf{c}\times\mathbf{a}=0 \Rightarrow -\mathbf{a}\times\mathbf{b}+\mathbf{c}\times\mathbf{a}=0$

$\Rightarrow \mathbf{a}\times\mathbf{b}=\mathbf{c}\times\mathbf{a}$

Similarly, $\mathbf{b}\times\mathbf{c}=\mathbf{c}\times\mathbf{a}$

$\therefore\ \mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{c}=\mathbf{c}\times\mathbf{a}\neq 0$

42. Given lines are $(1+a)\,x-ay+a(1+a)=0$,

$(1+b)\,x-by+b(1+b)=0$ and $y=0$

The point of intersection of lines are

$(-a,0)$, $(-b,0)$ and $[ab,(1+a)(1+b)]$

Equation of perpendicular line CD is

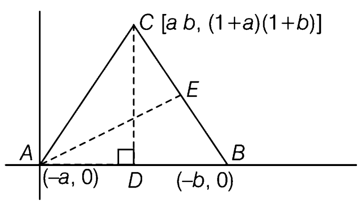
$$x=ab \qquad \text{...(i)}$$

and equation of perpendicular line AE is

$$y-0=\frac{-(ab+b)}{(1+a)(1+b)}(x+a)$$

$$\left[\because -\frac{1}{m}=\frac{-(ab+b)}{(1+a)(1+b)}\right]$$

$$y=\frac{-a}{(1+a)}(x+a) \qquad \text{...(ii)}$$



$\therefore$ The point of intersection of lines AE and CD is

$(ab,-ab)$, which is the coordinate of orthocentre.

Let, locus of orthocentre is (x,y).

$\therefore\quad x=ab$ and $y=-ab$

$\Rightarrow\quad x=-y \Rightarrow x+y=0$

which represent a equation of straight line

43. Given, $\mathbf{r}\cdot\mathbf{a}=0\Rightarrow\mathbf{r}\perp\mathbf{a}$

$\mathbf{r}\cdot\mathbf{b}=0\Rightarrow\mathbf{r}\perp\mathbf{b}$ and $\mathbf{r}\cdot\mathbf{c}=0\Rightarrow\mathbf{r}\perp\mathbf{c}$

$\Rightarrow$ **a**, **b** and **c** are coplanar.

$\therefore\ [\mathbf{a}\ \mathbf{b}\ \mathbf{c}]=0$

44. Given, differential equation is

$$\left(\frac{dy}{dx}\right)\tan y=\sin(x+y)+\sin(x-y)$$

$$\Rightarrow\left(\frac{dy}{dx}\right)\tan y=2\sin\left(\frac{x+y+x-y}{2}\right)\cos\left(\frac{x+y-x+y}{2}\right)$$

$$\Rightarrow\left(\frac{dy}{dx}\right)\tan y=2\sin(x)\cos(y)$$

$$\Rightarrow\quad\frac{\tan y}{\cos y}\,dy=2\sin x\,dx$$

$$\Rightarrow\quad\frac{\sin y}{\cos^2 y}\,dy=2\sin x\,dx$$

On integrating both sides, we get

$$\int\frac{\sin y}{\cos^2 y}\,dy=2\int\sin x\,dx$$

$$\Rightarrow\quad\frac{-(\cos y)^{-2+1}}{(-2+1)}=-2\cos x+C$$

$$\Rightarrow\quad\frac{1}{\cos y}=-2\cos x+C\ \Rightarrow \sec y=C-2\cos x$$

45. We know that, any line is parallel to the plane, then normal to the plane is perpendicular to the line.

Given, line is $\dfrac{x-2}{3}=\dfrac{y-3}{4}=\dfrac{z-4}{5}$

So, Dr's of a line are (3, 4, 5).

Let us consider the equation of plane is

$$2x+y-2z=0$$

Hence, Dr's of a plane are $(2,1,-2)$

Now, $3\times2+4\times1+5\times(-2)=6+4-10$

$=10-10=0$

Hence, option (d) is correct.

46. Equation of tangent to the hyperbola

$\dfrac{x^2}{9}-\dfrac{y^2}{4}=1$ is

$$y=mx+\sqrt{9m^2-4} \qquad \text{...(i)}$$

Since, Eq. (i) tagent to the circle

$\therefore$ Perpendicular distance from centre (0, 4, 0) to the circle is equal to the radius of the circle

$$\therefore\ \frac{|4m+0+\sqrt{9m^2-4}|}{\sqrt{1^2+m^2}}=4$$

$$\Rightarrow\quad(4m+\sqrt{9m^2-4})=4\sqrt{1+m^2}$$

$$\Rightarrow 16m^2+9m^2-4+8m\sqrt{9m^2-4}=16(1+m^2)$$

$$\Rightarrow 9m^2-20=-8m\sqrt{9m^2-4}$$

$\Rightarrow 81m^4 + 400 - 360m^2 = 64m^2(9m^2 - 4)$

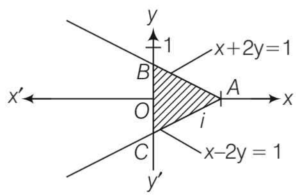
$\Rightarrow 495m^4 + 104m^2 - 400 = 0 \Rightarrow m = 2/\sqrt{5}$

$\therefore$ From Eq. (i), $y = \dfrac{2x}{\sqrt{3}} + \sqrt{9 \times \dfrac{4}{5} - 4}$

$\sqrt{5}y = 2xT + \sqrt{36 - 20} \Rightarrow 2x - \sqrt{5}y + 4 = 0$

Option (b) is correct.

47. Given, curve is $x = 0$ and $x + 2|y| = 1$



Now, $x + 2|y| = 0$

When $y > 0$, $x + 2y = 1$; When $y < 0$, $x - 2y = 1$

$\therefore$ Area of bounded region = ABC

$$= 2AOB = 2\int_0^1 \left(\frac{1-x}{2}\right) dx = \left[x - \frac{x^2}{2}\right]_0^1$$

$$= \left[1 - \frac{1}{2} - (0 - 0)\right] = \frac{1}{2}$$

48. Given, differential equation is

$$y^2 + \left(x - \frac{1}{y}\right)\frac{dy}{dx} = 0 \text{ or } y^2\frac{dx}{dy} + x = \frac{1}{y}$$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$$

It is a linear equation of the form of

$\dfrac{dx}{dy} + Px = Q$ Here, $P = \dfrac{1}{y^2}$ and $Q = \dfrac{1}{y^3}$

$$\therefore \quad \text{IF} = e^{\int \frac{1}{y^2} dy} = e^{-y^{-1}} = e^{-\frac{1}{y}}$$

$\therefore$ Solution is $x\,e^{-\frac{1}{y}} = \int e^{-\frac{1}{y}} \times \dfrac{1}{y^3}\,d$

Put $\dfrac{1}{y} = t \Rightarrow -\dfrac{1}{y^2}dy = dt = -\int e^{-t}\,t\,dt$

$$= -\left[-t\,e^{-t} - \int -1\,e^{-t}\,dt\right]$$

$$= -[-t\,e^{-t} - e^{-t}] = e^{-t}(t + 1) + C$$

$$\Rightarrow \quad x\,e^{-\frac{1}{y}} = e^{-\frac{1}{y}}\left(\frac{1}{y} + 1\right) + C$$

$$\therefore \quad x = \frac{1}{y} + 1 + C\,e^{1/y}$$

49. Let $I = \dfrac{\pi^2}{10}\left[\int_{-10}^{10} f(x)\cos \pi x\,dx\right]$

$$= \frac{\pi^2}{10}\left[\int_{-10}^{-9} f(x)\cos\pi x\,dx + \int_{-9}^{-8} f(x)\cos\pi x dx + \ldots + \int_9^{10} f(x)\cos\pi x\,dx\right]$$

Let $I_1 = \int_{-10}^{-9} f(x)\cos\pi x\,dx$

For $-10 < x < -9$, $f(x)$ is an even function.

Put $[x] = -10$

$$\therefore I_1 = \int_{-10}^{-9}(1 - 10 - x)\cos\pi x\,dx$$

$$= -\int_{-10}^{-9}\underset{\text{I}}{(9 + x)}\,\underset{\text{II}}{\cos\pi x}\,dx$$

$$= -\left[(9 + x)\frac{\sin\pi x}{\pi} - \int\frac{1 \times \sin\pi x}{\pi}dx\right]_{-10}^{-9}$$

$$= -\left[(9 + x)\frac{\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right]_{-10}^{-9}$$

$$= -\left[0 + \frac{\cos(-9\pi)}{\pi^2} - \left(0 + \frac{\cos(-10\pi)}{\pi^2}\right)\right]$$

$$= -\left[-\frac{1}{\pi^2} - \frac{1}{\pi^2}\right] = \frac{2}{\pi^2}$$

Similarly, $I_2 = \int_{-9}^{-8}\{x - (-9)\}\cos\pi x\,dx = \dfrac{2}{\pi^2}$

$$\therefore I = \frac{\pi^2}{10}\left[\left(\frac{2}{\pi^2}\right) \times 20\right] = 4$$

[$\because$ in $\int_{-10}^{10}$ there are such 20 intervals in all intervals we get equal values *i.e.*, $2/\pi^2$]

50. $(x + y)(x - y) + \dfrac{1}{2!}(x + y)(x - y)(x^2 + y^2)$

$$+ \frac{1}{3!}(x + y)(x - y)(x^4 + y^4 + x^2y^2) + \ldots$$

$$= (x^2 - y^2) + \frac{1}{2!}(x^4 - y^4) + \frac{1}{3!}(x^2 - y^2)(x^4 + y^4 + x^2y^2)$$

$$= (x^2 - y^2) + \frac{1}{2!}(x^4 - y^4) + \frac{1}{3!}(x^6 - y^6) + \ldots$$

$$= \left(x^2 + \frac{1}{2!}x^4 + \frac{x^6}{3!} + \ldots\right) - \left(y^2 + \frac{y^4}{2!} + \frac{y^6}{3!} + \ldots\right)$$

$$= \left(1 + \frac{x^2}{1!} + \frac{(x^2)^2}{2!} + \frac{(x^2)^3}{3!} + \ldots\right)$$

$$- \left(1 + \frac{y^2}{1!} + \frac{(y^2)^2}{2!} + \frac{(y^2)^3}{3!} + \ldots\right) = e^{x^2} - e^{y^2}$$