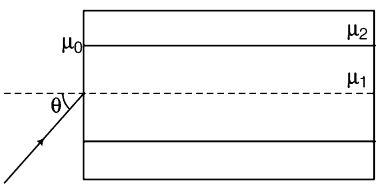
AMU

Engineering Entrance Exam

Solved Paper 2013

Physics

1. Light from a source located in a medium (refractive index $= \mu_0$) enters an optical fibre with core refractive index μ_1 and clad refractive index μ_2, as shown in the figure. The maximum value of incident angle θ which undergo total internal reflection in the fibre is



(a) $\theta = \cos^{-1}\left(\frac{\sqrt{\mu_1^2 - \mu_2^2}}{\mu_0}\right)$

(b) $\theta = \tan^{-1}\left(\frac{\mu_1 - \mu_2}{\mu_0}\right)$

(c) $\theta = \sin^{-1}\left(\frac{\sqrt{\mu_1^2 - \mu_2^2}}{\mu_0}\right)$

(d) None of the above

2. A cyclotron is operated at an oscillator frequency of 12 MHz and has a dee radius $R = 50$ cm. What is the magnitude of the magnetic field needed for a proton to be accelerated in the cyclotron?

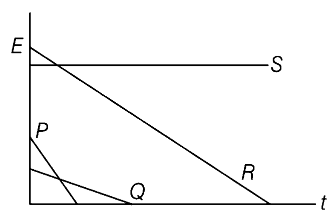
(a) 0.72 T (b) 0.65 T
(c) 0.39 T (d) 0.12 T

3. Which of the following expressions represents the relation between orbital magnetic moment and orbital angular momentum of an electron?

(a) $\mu_{orb} = -\frac{2m_e}{e}\mathbf{L}_{orb}$ (b) $\mu_{orb} = -2m_e\mathbf{L}_{orb}$

(c) $\mu_{orb} = -\frac{e}{2m_e}\mathbf{L}_{orb}$ (d) $\mu_{orb} = \frac{e}{2m_e}\mathbf{L}_{orb}$

4. In the following figure, the variation of electric field magnitude E *versus* time is shown for four uniform electric fields contained within identical circular regions. Arrange the fields according to the magnitudes of the magnetic fields they induce at the edge of the region, in decreasing order



(a) *P, R, Q, S* (b) *R, P, Q, S*
(c) *Q, R, P, S* (d) *S, P,Q, R*

5. In a double slit experiment, the distance between slits is 5.0 mm and the slits are 1.0 m from the screen. Two interference patterns can be seen on the screen : one due to light of wavelength 480 nm and the other due to light of wavelength 600 nm. What is the separation on the screen between the third order bright fringes of the two interference patterns?

(a) 0.02 mm (b) 0.05 mm
(c) 0.07 mm (d) 0.09 mm

6. A LED is constructed from a *p-n* junction based on a certain Ga–As–P semiconducting material whose energy gap is 1.9 eV. Identify the colour of the emitted light.

(a) Blue (b) Red
(c) Violet (d) Green

7. The binding energy per nucleon in a heavy nucleus is of the order of

(a) 8 MeV (b) 7 MeV
(c) 5 MeV (d) 2 MeV

8. In the following nuclear reaction $^{235}_{92}\text{U} + n \rightarrow X + Y + 2n$, which of the following pairs cannot represent X and Y?

(i) ^{141}Xe and ^{93}Sr (ii) ^{139}Cs and ^{95}Rb
(iii) ^{156}Nd and ^{79}Ge (iv) ^{141}Ba and ^{92}Kr
(a) (iii) and (iv) (b) (i) and (ii)
(c) (ii) dnd (iii) (d) (i) and (iv)

9. Which of the following fusion reactions will not result in the net release of energy?

(a) ^{6}Li + ^{6}Li (b) ^{4}He + ^{4}He
(c) ^{12}C + ^{12}C (d) ^{35}Cl + ^{35}Cl

10. For a damped harmonic oscillator of mass 250 g, the values of spring constant (k) and damping constant (b) are 85 N/m and 70 g/s, respectively. What is the period of motion?

(a) 2.5 s (b) 5.0 s
(c) 6.25 s (d) 7.2 s

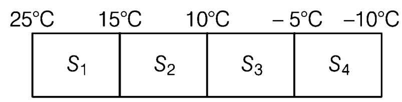
13. A string oscillates according to the equation, $y = (0.50\ \text{cm})\ \sin\left[\frac{\pi x}{3}\right] \cos\,[40\,\pi t]$. Find the distance between nodes

(a) 1.5 cm (b) 2.5 cm
(c) 3.0 cm (d) 3.5 cm

12. Two point sources, which are in phase and separated by distance $D = 1.5\,\lambda$, emit identical sound waves of wavelength λ. If a circle with a radius much greater than D, centered on the mid-point between the sources, what is the numebr of points around the circle at which the interference is fully constructive?

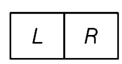
(a) 12 (b) 8
(c) 6 (d) 4

13. The figure shows the temperatures at four faces of a composite slab consisting of four materials S_1, S_2, S_3 and S_4 of identical thickness, through which the heat transfer is steady. Arrange the materials according to their thermal conductivities in decreasing order



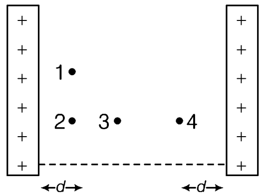
(a) S_2, S_4, S_1, S_3
(b) $S_2 = S_4, S_1, S_3$
(c) $S_1 = S_2, S_3, S_4$
(d) S_1, S_2, S_3, S_4

14. The figure shows two identical copper blocks of mass 0.5 kg. When they were not in contact, block L was at temperature 60°C and block R was at temperature 20°C. But, when the blocks bring in contact, they come to the equilibrium temperature 40°C. What is the net entropy change of the two block system during the irreversible process? (Specific heat of copper = 386 J/kg-K)



(a) 2.4 J/K
(b) 3.6 J/K
(c) 4.2 J/K
(d) 5.2 J/K

15. Two parallel large non-conducting sheets with identical (positive) uniform surface charge densities and a sphere A with a uniform (positive) volume charge density are arranged as shown in figure. Rank the points 1, 2, 3 and 4 according to the magnitudes of the net electric field in increasing order



(a) 1, 2, 3, 4
(b) 1, 2, 3 = 4
(c) 1 = 2, 3, 4
(d) 4, 3, 2, 1

16. Choose the electromagnetic radiation relevant to telecommunication.

(a) Ultraviolet (b) Infrared
(c) Visible (d) Microwave

17. A microscope is focussed at a point at the bottom of a beaker containing water. The microscope is then raised through 3 cm. To what height water must be added into the beaker to bring the point again in focus?

(a) 15 cm (b) 12 cm
(c) 10 cm (d) 8 cm

18. The electric field in a region is given by

$$\mathbf{E} = \frac{A}{x^3}\,\hat{\mathbf{i}} + B\,\hat{\mathbf{j}} + Cz^2\,\hat{\mathbf{k}}$$

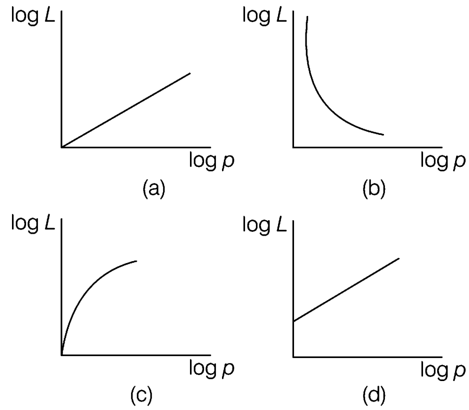
The SI units of A, B and C are, respectively

(a) $\frac{\text{N-m}^3}{\text{C}}$, V/m^2, $\frac{\text{N}}{\text{m}^2\text{-C}}$
(b) V-m^2, V/m, $\frac{\text{N}}{\text{m}^2\text{-C}}$
(c) V-m^2, V/m, $\frac{\text{N-C}}{\text{m}^2}$
(d) V/m, $\frac{\text{N-m}^3}{\text{C}}$, $\frac{\text{N-C}}{\text{m}}$

19. Find the component of vector $\mathbf{A} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ along the direction $(\mathbf{i} - \mathbf{j})$.

(a) $-\frac{1}{2}(\hat{\mathbf{i}} - \hat{\mathbf{j}})$
(b) $-\frac{1}{2}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$
(c) $\frac{1}{2}(\hat{\mathbf{i}} - \hat{\mathbf{j}})$
(d) $\frac{1}{2}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

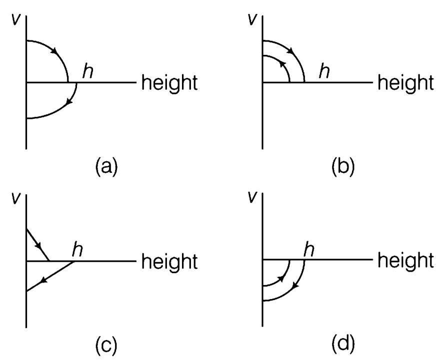
20. Angular momentum L is given by $L = p \cdot r$. The variation of $\log L$ and $\log p$ is shown by



21. By what velocity a ball be projected vertically upwards so that the distance covered in 5th second is twice of that covered in 6th second?

(a) 19.6 m/s (b) 58.8 m/s
(c) 49 m/s (d) 65 m/s

22. A ball is dropped vertically from a height h above the ground. After touching the ground, it bounces up to a height $h/2$. Neglecting the air resistance, its velocity v varies with the height above the ground as



23. A uniform rope of mass 0.1 kg and length 2.45 m hangs from a ceiling. The time taken by a transverse wave to travel the full length of the rope is

(a) 1.2 s (b) 1.0 s
(c) 2.2 s (d) 3.1 s

24. The magnitudes of gravitational field at distances r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are I_1 and I_2, respectively. Find the ratio (I_1 / I_2) if $r_1 > R$ and $r_2 < R$.

(a) $\frac{R^2}{r_1 r_2}$ (b) $\frac{R^3}{r_1 r_2^2}$

(c) $\frac{R^3}{r_1^2 r_2}$ (d) $\frac{R^4}{r_1^2 \cdot r_2^2}$

25. A toy car of mass 2.0 kg is moving towards negative Y-axis with a velocity of 0.5 m/s. It takes a turn towards X-axis with a velocity of 0.4 m/s. How much is the change in the linear momentum of the car due to the turn?

(a) $(0.6\hat{i} - 1.0\,\hat{j})$ kg.m/s

(b) $(0.8\,\hat{i} + 1.0\,\hat{j})$ kg.m/s

(c) $(0.8\,\hat{i} - 1.0\,\hat{j})$ kg.m/s

(d) $(0.8\,\hat{j} - 0.6\,\hat{j})$ kg.m/s

26. The potential energy of a particle of mass m varies as $U(x) = \begin{cases} E_0 & \text{for } 0 \le x \le 1 \\ 0 & \text{for } x > 1 \end{cases}$

The de-Broglie wavelength of the particle in the range $0 \le x \le 1$ is λ_1 and that in the range $x > 1$ is λ_2. If the total energy of the particle is $2E_0$, find $\frac{\lambda_1}{\lambda_2}$.

(a) $\sqrt{2}$ (b) $\sqrt{3}$

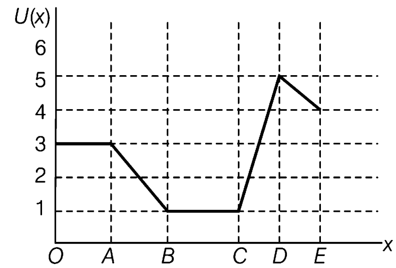
(c) $\sqrt{\frac{1}{2}}$ (d) $\sqrt{\frac{2}{3}}$

27. A car moves around a curved road of radius R_1 at constant speed v without sliding. If we double the car's speed, what is the least radius that would now keep the car from sliding?

(a) $2R_1$ (b) $4R_1$

(c) $6R_1$ (d) R_1

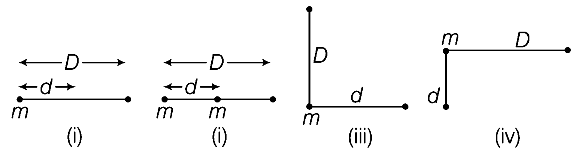
28. The figure shows the potential energy function $U(x)$ of a system in which a particle is in one-dimensional motion. Arrange regions AB, BC, CD and DE according to the magnitude of thefore on the particle in decreasing order



(a) *BC, DE, AB, CD* (b) *BC, AB, CD, DE*

(c) *AB, CD, BC, DE* (d) *CD, AB, DE, BC*

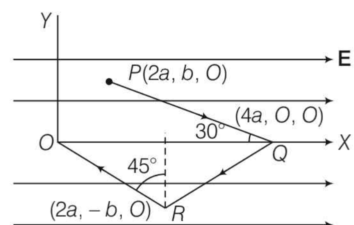
29. The figure shows four arrangements of three particles of equal masses. Arrange them according to the magnitude of the net gravitational force on the particle labelled m, in decreasing order



(a) (i), (iii) = (iv), (ii) (b) (i) = (iii), (ii), (iv)

(c) (i), (ii), (iii), (iv) (d) (iv), (iii), (ii), (i)

30. A point charge $+q$ moves from point P to origin O along the path $PQRO$ in a uniform electric field $\mathbf{E}$. Find the work done by the field



(a) $-2qEa$ (b) $2qEa$

(c) $8qEa$ (d) $-4qEa$

31. In a new system of units called star units, 1 kg* = 10 kg, 1 m* = 1 km and 1s* = 1 minute, what will be the value of 1J in the new system?

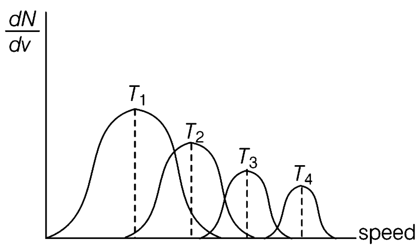
(a) 2.4×10^{-5} J* (b) 3.6×10^{-4} J*

(c) 4.2×10^{-3} J* (d) 4.2×10^{-2} J*

32. A wooden box has a $4\text{m} \times 4\text{m} \times 10$ cm metallic cover ($K = 1.26$ W/m – °C). At some instant, the temperature outside is 40°C and that inside is 26°C. Neglecting convection, the amount of heat flowing per second into the box through the cover is

(a) 1832 W
(b) 2212 W
(c) 2822 W
(d) 3122 W

33. The following figure shows the Maxwell's speed distribution plots at four different temperatures T_1, T_2, T_3 and T_4



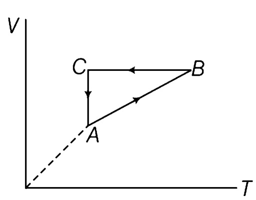
Which of the following gives the correct relation between temperatures?

(a) $T_4 > T_3 > T_2 > T_1$
(b) $T_4 < T_3 < T_2 < T_1$
(c) $T_1 = T_2 = T_3 = T_4$
(d) $T_1 > T_2, T_3 < T_4$

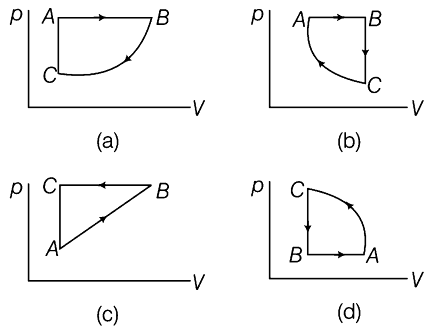
34. A thermodynamic state of a given sample of an ideal gas is completely described, if its

(a) pressure, volume and internal energy are known
(b) pressure, volume, temperature and internal energy are known
(c) pressure, volume and temperature are known
(d) pressure and volume are known

35. A cyclic process $ABCA$ on the V–T diagram (shown below) is performed with a constant mass of an ideal as



Which of the following figures corresponds to the same process on a p-V diagram?



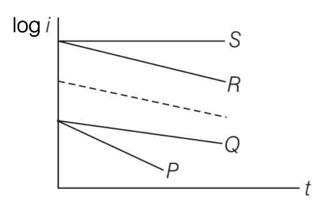
36. The ratio of the radii of the nuclei ${}_{13}\text{Al}^{27}$ and ${}_{52}\text{Te}^{125}$ is

(a) 13 : 52
(b) 27 : 125
(c) 3 : 5
(d) 14 : 73

37. How many NAND gates are needed to obtain OR gate?

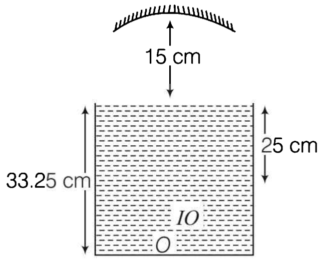
(a) 1
(b) 2
(c) 3
(d) 4

38. A capacitor is discharged through a resistance in R–C circuit. The variation of $\log_e i$ with time t is shown by a dotted line in the figure, where i is the discharging current. If the resistance in the circuit be doubled, the variation of $\log_e i$ with time t would be best represented by the line



(a) P
(b) Q
(c) R
(d) S

39. A container is filled with water ($\mu = 1.33$) upto a height of 33.25 cm. A concave mirror is held 15 cm above the water level and the image I of an object O placed at the bottom is formed 25 cm below the water level. The focal length of the mirror is roughly



(a) 10 cm (b) 15 cm (c) 20 cm (d) 25 cm

40. A current of 4.0 A is present in a wire of cross-sectional area 2.0 mm^2. Find the number of free electrons in each cubic metre of the wire, if the drift velocity is 2.1×10^{-4} m/s.

(a) $6.0 \times 10^{28}\ m^{-3}$ (b) $3.6 \times 10^{29}\ m^{-3}$
(c) $7.0 \times 10^{3}\ m^{-3}$ (d) $8.2 \times 10^{32}\ m^{-3}$

41. Doubly charged Mg^{24} ions are accelerated to kinetic energy 8 keV and are projected perpendicularly into a magnetic field of magnitude 1.2 T. Find the radius of the circle formed by the ions.

(a) 2.4 cm (b) 3.2 cm (c) 4.8 cm (d) 5.3 cm

42. When two radiations of wavelength λ_1 and λ_2 fall on a metallic surface, they produce photoelectrons with maximum energies k_1 and k_2, respectively. Which of the following relations is used to estimate the Planck constant?

(a) $h = \frac{k_1 - k_2}{c} \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}$ (b) $h = \frac{k_1 + k_2}{c} \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}$

(c) $h = \frac{k_1 - k_2}{c} \frac{\lambda_1\lambda_2}{\lambda_2 + \lambda_1}$ (d) $h = \frac{\sqrt{k_1^2 - k_2^2}}{c} \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}$

43. Which of the following phenomena establishes the wave nature of particles?

(a) Millikan oil drop experiment
(b) Davisson-Germer experiment
(c) Stern-Gerlach experiment
(d) Franck-Hertz experiment

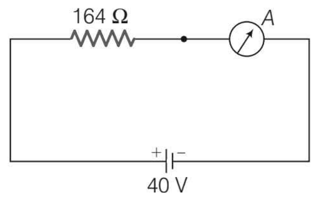
44. Find the de-Broglie wavelength for a 100 g bullet moving at 900 m/s.

(a) 3.7×10^{-35} m (b) 7.4×10^{-36} m
(c) 7.8×10^{-37} m (d) 8.2×10^{-39} m

45. The colours of the rings on a resistor are brown, yellow, green and gold as seen from the left to the right. The value of the resistance is

(a) (1.4 ± 0.07) MΩ (b) (2.4 ± 0.05) MΩ
(c) (3.4 ± 0.5) MΩ (d) (1.4 ± 0.05) MΩ

46. The ammeter, shown below, consists of a 360 Ω coil connected in parallel to a 40 Ω shunt. Find the reading of the ammeter



(a) 0.35 A (b) 0.4 A (c) 0.25 A (d) 0.2 A

47. You are given four semiconductors *P, Q, R* and *S* with respective band gaps 4 eV, 3 eV, 2 eV and 1 eV for use in a photodetector to detect $\lambda = 1400$ nm. Select the suitable semiconductor.

(a) *P* (b) *Q*
(c) *R* (d) *S*

48. When two waves of nearly equal frequenices ν_1 and ν_2 are produced simultaneously, the time interval between successive maxima is

(a) $\frac{1}{\nu_1 - \nu_2}$ (b) $k\frac{1}{\nu_1} - \frac{1}{\nu_2}$
(c) $\frac{1}{\nu_1} + \frac{1}{\nu_2}$ (d) $\frac{1}{\nu_1 + \nu_2}$

49. A particular emission line, detected in the light from a star, has a wavelength $\lambda_{det} = 1.1\lambda$, where λ is the proper wavelength of the line. What is the star's distance from us?

(a) 8.6×10^{8} ly (b) 1.6×10^{9} ly
(c) 3.2×10^{10} ly (d) 9.2×10^{11} ly

50. For sky wave propagation of 10 MHz signal, what should be the minimum electron density in ionosphere?

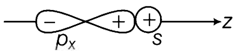
(a) $\sim 1.2 \times 10^{12}\ m^{-3}$ (b) $\sim 10^{6}\ m^{-3}$
(c) $\sim 2.3 \times 10^{14}\ m^{-3}$ (d) $\sim 10^{22}\ m^{-3}$

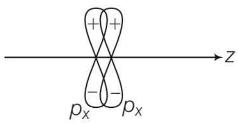
Chemistry

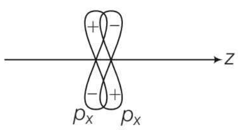
1. When a dilute solution of KNO_3 is mixed with a dilute solution of NaBr, the enthalpy change is expected to be

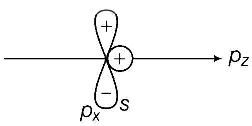
(a) Zero, 0 (b) > 0
(c) < 0 (d) All of these

2. Which of the following represents the zero overlap?

(a) 

(b) 

(c) 

(d) 

3. A compound alloy of copper and gold crystallises in a cubic lattice in which the copper atom occupy the centres of each of the cube faces and the gold atoms occupy the lattice point. The formula of compound is

(a) Au_3Cu
(b) $AuCu_3$
(c) Au_2Cu_3
(d) Au_3Cu_2

4. What volume of oxygen, at 18°C and 750 torr, can be obtained from 110 g of $KClO_3$?

(a) 32.6 L (b) 42.7 L
(c) 3.26 L (d) 4.27 L

5. CsBr crystallises in a body centred cubic unit lattice with an edge length of 4.287Å. How many molecules of CsBr will be present in the unit lattice?

(a) 1 (b) 2
(c) 3 (d) 4

6. Arrange the following solutions in the increasing order of their osmotic pressure

(i) 34.2 g/L sucrose ($M = 342$)
(ii) 60 g/L urea ($M = 60$)
(ii) 90 g/L glucose ($M = 180$)
(vi) 58.5 g/L NaCl ($M = 58.5$)

(a) Sucrose $<$ Urea $<$ Glucose $<$ NaCl
(b) Sucrose $<$ Glucose $<$ NaCl $<$ Urea
(c) Sucrose $<$ Glucose $<$ Urea $<$ NaCl
(d) NaCl $<$ Urea $<$ Glucose $<$ Sucrose

7. The magnitude of the charge on the electron is 4.8×10^{-10} esu. What is the magnitude of the charge on the proton on the nucleus of a helium atom?

(a) 4.8×10^{-10} esu (b) 9.6×10^{-10} esu
(c) 6.4×10^{-10} esu (d) 14.4×10^{-10} esu

8. In free radical polymerisation the extent of conversion increasing with

(a) increase in temperature
(b) increase in polymerisation time
(c) increase in monomer concentration
(d) All the above

9. 100 mL of 0.1 M H_2SO_4 is mixed with 100 mL of 0.1 NaOH. The normality of the solution obtained is

(a) 0.4 N (b) 0.05 N
(c) 0.04 N (d) 0.2 N

10. The thermodynamic equilibrium constant, K_c, for the reaction

$CuO(s) + H_2(g) \longrightarrow Cu(s) + H_2O(g)$ is given by

(a) $K_c = \frac{[Cu][H_2O]}{[CuO][H_2]}$ (b) $K_c = \frac{[Cu][H_2O]}{[H_2]}$

(c) $K_c = \frac{[H_2O]}{[CuO][H_2]}$ (d) $K_c = \frac{[H_2O]}{[H_2]}$

11. One mole of compound A, on heating with excess of conc. HI gave two moles of ethyl iodide. Compound A, is

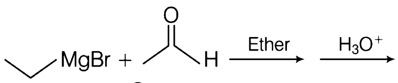
(a) ethoxy benzene (b) 1,2-dimethoxy ethane
(c) methoxy ethane (d) ethoxy ethane

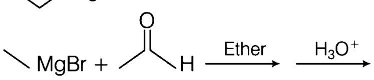
12. The appropriate reagent (*s*) for the following transformation is (are)

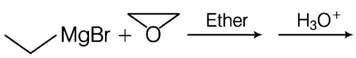
$$CH_3(CH_2)_9—\overset{O}{\overset{||}{C}}—OC_2H_5 \longrightarrow CH_3(CH_2)_9—CHO$$

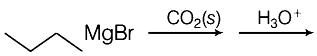
(a) $SnCl_2 + HCl$
(b) DIBAL – H
(c) CrO_2Cl_2
(d) CO,HCl, anhyd. $AlCl_3$

13. Which of the following reactions would provide the best synthesis of butan-2-ol?

(a) 

(b) 

(c) 

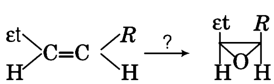
(c) 

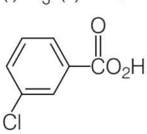
14. Pick out the electrophiles from following species

BF_3, NH_3, $Me_3C\oplus$, HCl

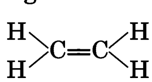
(a) BF_3 and NH_3
(b) $Me_3C\oplus$ and HCl
(c) BF_3 and $Me_3C\oplus$
(d) NH_3 and HCl

15. What reagent(*s*) is (are) needed to accomplish the following conversion?

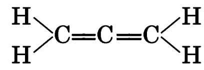


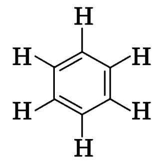
(a) (i) O_3 (ii) Zn/H^+
(b) $KMnO_4$ $\overline{O}H$, cold
(c) 
(d) H_3O^+

16. Which of the following compounds is strongest acid?


I

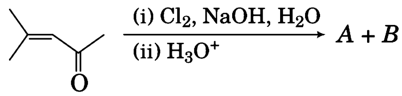
H—C≡C—H
II

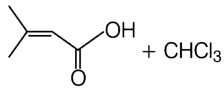

III

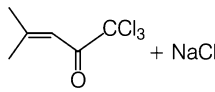

IV

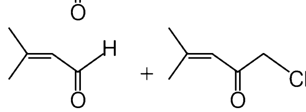
(a) I (b) II (c) III (d) IV

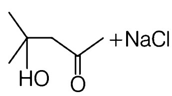
17. Major products *A* and *B*, of the following reaction are



(a) 

(b) 

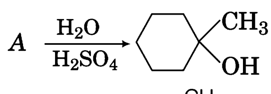
(c) 

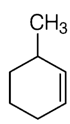
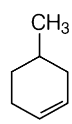
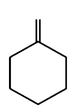
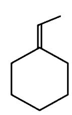
(d) 

18. The hydrolysis of which of the following takes the longest time?

(a) CH_3COCl
(b) $(CH_3CO)_2O$
(c) $CH_3COOC_2H_5$
(d) CH_3CONH_2

19. In the given reaction, *A*, may be

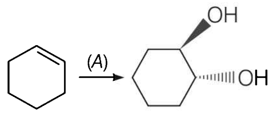


(a) 
(b) 
(c) 
(d) 

20. Wilkinson's catalyst is used for

(a) epoxidation
(b) hydrogenation
(c) polymerisation
(d) substitution

21. For the following conversion,



reagent /reagents (*A*) is /are

(a) OsO_4

(b) O_3

(c) I_2 and silver acetate under wet condition

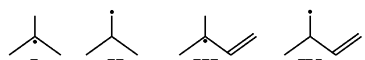
(d) peracid following by acid hydrolysis

22. An ether solution of benzoic acid (*A*) aniline (*B*) and toluene (*C*) is extracted with aqueous NaOH. The ether layer will contain what compound (s) after the extraction?

(a) *A* (b) *A* + *B*

(c) *B* + *C* (d) *A* + *C*

23. Which of the free radical is most stable?



I II III IV

(a) I (b) II (c) III (d) IV

24. The best reagent for converting 2-phenylpropanamide into 2-phenylpropanamine is

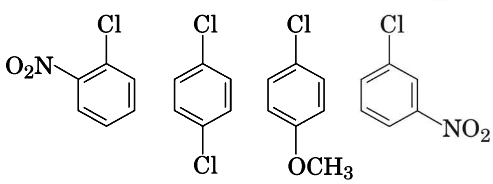
(a) Br_2 in aqueous NaOH

(b) excess of H_2

(c) iodine in the presence of red phosphorus

(d) $LiAlH_4$

25. Which of the following compounds reacts faster with sodium methoxide ($NaOCH_3$)?

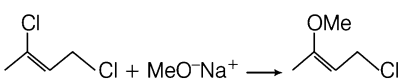


I II III IV

(a) I (b) II

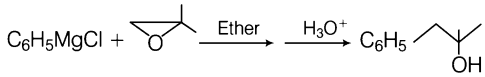
(c) III (d) IV

26. Which one of the following reactions is easily possible?

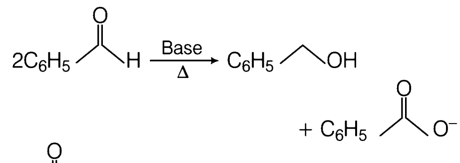
(a) 

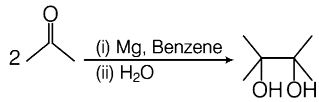
(b) + Cl + $MeO^-Na^+ \longrightarrow$ OMe

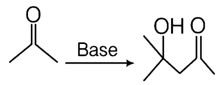
(c) $C_6H_5OH + HCl \longrightarrow C_6H_5Cl$

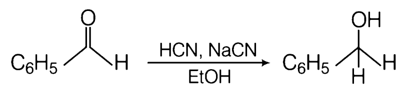
(d) 

27. Which one of the following is hydride transfer reaction?

(a) 

(b) 

(c) 

(d) 

28. Which of the following has maximum number of lone pairs associated with Xe?

(a) XeF_4 (b) XeF_6 (c) XeF_2 (d) XeO_3

29. The number of types of bonds between two carbon atoms in calcium carbide is

(a) one sigma, one pi

(b) two sigma, one pi

(c) two sigma, two pi

(d) one sigma two pi

30. The magnetic moment of transition metal ion is $\sqrt{15}$ BM. The number of unpaired electrons present in it are

(a) 4 (b) 1

(c) 2 (d) 3

31. The structure of H_2O_2 is

(a) planar (b) non-planar

(c) spherical (d) linear

32. Which one of the following decides the shapes of orbitals in a energy shell?

(a) Magnetic quantum number

(b) Principal quantum number

(c) Azimuthal quantum number

(d) Spin quantum number

33. The structure of paramagnetic nickel complex, $[NiCl_4]^{2-}$ is

(a) tetrahedral (b) square planar
(c) trigonal bipyramidal (d) distorted octahedral

34. The elements Re, Os and Ir belong to

(a) first transition series
(b) second transition series
(c) third transition series
(d) fourth transition series

35. Which of the following is not true?

(a) Halogens act as strong oxidising agents
(b) Fluorine oxidises water to oxygen and ozone
(c) Iodine has good solubility in water
(d) Solubility of iodine in water much increases by adding KI due to formation of I_3^- ion

36. Silica is attacked by

(a) conc. HNO_3 (b) conc. H_2SO_4
(c) aqua regia (d) hydrofluoric acid

37. $[Co(NH_3)_5Br)]\ SO_4$ and $[Co(NH_3)_5SO_4]\ Br$ are examples of which type of isomerism?

(a) Linkage (b) Geometrical
(c) Ionisation (d) Optical

38. Which of the following ions will exhibit colour in aqueous solution?

(a) Lu^{3+} ($Z = 71$) (b) Sc^{3+} ($Z = 21$)
(c) La^{3+} ($Z = 57$) (d) Ti^{3+} ($Z = 22$)

39. The correct order of C—O bond length among CO, CO_3^{2-}, CO_2 is

(a) $CO < CO_2 < CO_3^{2-}$ (b) $CO_2 < CO_3^{2-} < CO$
(c) $CO < CO_3^{2-} < CO_2$ (d) $CO_3^{2-} < CO_2 < CO$

40. In $Fe(CO)_5$, the Fe—C bond possesses

(a) ionic character (b) σ character only
(c) π character only
(d) Both σ and π character

41. The anion O^- is isoelectronic with

(a) N^{2-} (b) F^- (c) N^{3-} (d) Ne

42. Which of the following oxides is most acidic?

(a) BeO (b) MgO
(c) Al_2O_3 (d) Cl_2O_7

43. Analysis shows that a binary compound of X (atomic mass = 10) and Y (atomic mass = 20) contains 50% X. The formula of the compound is

(a) XY (b) X_2Y (c) XY_2 (d) X_2Y_3

44. The volume occupied by 16 g of oxygen gas at STP is

(a) 22.4 L (b) 44.8 L (c) 11.2 L (d) 5.6 L

45. If K_{sp} of $Ni(OH)_2$ is 2.0×10^{-15} M, the molar solubility of $Ni(OH)_2$ in 0.10 M NaOH is

(a) 2.0×10^{-15} M (b) 2.0×10^{-13} M
(c) 2.0×10^{-11} M (d) 2.0×10^{-9} M

46. Above the Boyle temperature, the compressibility factor of the real gases, Z is

(a) 1 (b) < 1 (c) > 1 (d) ≤ 1

47. The arsenic content of an agricultural insecticide was reported as 28% As_2O_5. What is the percentage of arsenic in this preparation?

(a) 16% (b) 18% (c) 15% (d) 20%

48. Calculate the maximum work that can be obtained from the cell

$$Zn\,|\,Zn^{2+}(1M)\,||\,Ag^{+}(1M)\,|\,Ag$$

where, $E^{\circ}_{Zn} = -0.76$ V and $E^{\circ}_{Ag} = 0.8$ V

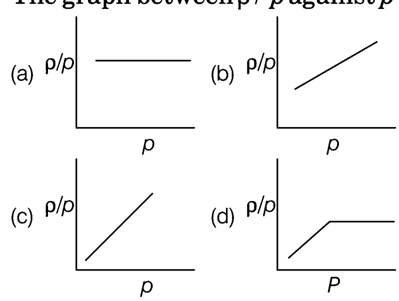
(a) 301.080 kJ (b) 201.830 kJ
(c) 112.830 kJ (d) 213.630 kJ

49. Which metal is protected from corrosion by a layer of its own oxide?

(a) Tl (b) Ag (c) Al (d) Au

50. The ideal gas equation, $pV = nRT$ can be written in terms of density, ρ, as $\rho/p = \frac{M}{RT}$.

The graph between ρ / p against p is given by



Mathematics

1. Max $z = x + y$

subject to $y \leq |x| - 1$

$y \geq -|x|$

$x, y \geq 0$

The Max z is equal to

(a) 0 (b) 1

(c) 2 (d) – 1

2. Max $z = 2x + 3y$

subject to $y \leq x - 1$

$y \geq x + 1$

$x, y \geq 0$

The Max z

(a) is 2 (b) is 3

(c) does not exist (d) is 10

3. For any two events A and B, which of the following result does not hold true in general

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(b) $P(A) = P(A \cap B) + P(A \cap \overline{B})$

(c) $P(B) = P(A \cap B) = P(\overline{A} \cap B)$

(d) $P(A \cup B) = P(A) + P(B)$

4. A fair die is rolled. The probability that the first time 1 occurs at the odd throw is

(a) $\frac{5}{11}$ (b) $\frac{6}{11}$

(c) $\frac{1}{6}$ (d) $\frac{31}{36}$

5. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then value of α for which $A^2 = B$ is

(a) $\alpha = 1$ (b) $\alpha = -1$

(c) $\alpha = 4$ (d) no real value of α

6. The number of positive roots of the equation $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$ is

(a) 1 (b) 2

(c) 3 (d) 0

7. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then A^3 is equal to

(a) $\begin{bmatrix} -\cos 3\theta & -\sin 3\theta \\ \sin 3\theta & \cos 3\theta \end{bmatrix}$ (b) $\begin{bmatrix} \cos 3\theta & -\sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$

(c) $\begin{bmatrix} \cos 3\theta & -\sin 3\theta \\ \sin 3\theta & \cos 3\theta \end{bmatrix}$ (d) $\begin{bmatrix} -\cos 3\theta & \sin 3\theta \\ -\sin 3\theta & -\cos 3\theta \end{bmatrix}$

8. The value of $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\gamma) \\ \sin\beta & \cos\beta & \sin(\beta+\gamma) \\ \sin\delta & \cos\delta & \sin(\gamma+\delta) \end{vmatrix}$ is

(a) $\sin\alpha \sin\beta \sin\delta$ (b) $\cos\alpha \cos\beta \cos\delta$

(c) 1 (d) 0

9. The numbers of real tangents that can be drawn from (1, 1) to the circle

$x^2 + y^2 - 6x - 4y + 4 = 0$, is

(a) 1 (b) 2

(c) 0 (d) 3

10. The line $x = my + c$ is normal to $x^2 = -4ay$, if c is equal to

(a) $-2am - m^3a$ (b) $-2am + am^3$

(c) $-2am - am^3$ (d) $2am + am^3$

11. If P is a point (x, y) and $P_1 = (3,0)$, $P_2 = (-3,0)$ and $16x^2 + 25y^2 = 400$, then $PP_1 + PP_2$ is equal to

(a) 8 (b) 10 (c) 5 (d) 4

12. Let $A = \{(x, y) : y = e^{-x}\}$ and

$B = \{(x, y) : y = -x\}$. Then,

(a) $A \cap B = \phi$ (b) $A \subset B$

(c) $B \subset A$ (d) $A \cap B = \{(0,1), (0,0)\}$

13. The set of zeros of $f(x) = 0$ is a non-empty set, when $f(x)$ is equal to

(a) $e^{-x} + x$ (b) $|x| + (x-2)^2$

(c) $x - \log_e x$ (d) $x + e^x$

14. Let $f(x) = (x+2)^2 - 2$, $x \geq -2$. Then, $f^{-1}(x)$ is equal to

(a) $-\sqrt{2+x} - 2$

(b) $\sqrt{2+x} + 2$

(c) $\sqrt{2+x} - 2$

(d) $-\sqrt{2+x} + 2$

15. Let z be the set of integers. Then, the relation $R = \{(a, b) : 1 + ab > 0\}$ on z is

(a) reflexive and transitive but not symmetric
(b) symmetric and transitve but not reflexive
(c) reflexive and symmetric but not transitive
(d) an equivalence relation

16. Let L denote the set of all straight lines in a plane. Let a relation R be defined by $l_1 R l_2$ if and only if the straight line l_1 is perpendicular to the straight line l_2. Then, R is

(a) symmetric (b) reflexive
(c) transitive (d) None of these

17. Which one of the following is not true?

(a) $|\sin x| \le 1$
(b) $-1 \le \cos x \le 1$
(c) $|\sec x| < 1$
(d) $\operatorname{cosec} x \ge 1$ or $\operatorname{cosec} x \le -1$

18. The general solution of $\sin 3x + \sin x - 3\sin 2x = \cos 3x + \cos x - 3\cos 2x$ is

(a) $\frac{n\pi}{2} + \frac{\pi}{8}$ for n integer (b) $\frac{n\pi}{2} - \frac{\pi}{8}$ for n integer
(c) $n\pi + \frac{\pi}{8}$ for n integer (d) $n\pi - \frac{\pi}{8}$ for n integer

19. The value of $\tan\left[\cos^{-1}\frac{3}{5} + \tan^{-1}\frac{2}{3}\right]$ is

(a) 18 (b) 2
(c) 9 (d) None of these

20. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then

(a) $x = 0, \ y = 1$ (b) $x = 1, \ y = 0$
(c) $x = 1, \ y = 1$ (d) None of these

21. The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$, is

(a) 4 (b) 2 (c) 8 (d) 16

22. If z_1, z_2 and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$, then $|z_1 + z_2 + z_3|$ is

(a) 3 (b) 1
(c) greater than 3 (d) less than 1

23. In a triangle the lengths of the largest and the smallest sides are 10 and 9 respectively. If the angles are in AP, then the length of the third side is

(a) $\sqrt{91}$ (b) 8 (c) $3\sqrt{3}$ (d) 5

24. Consider an infinite geometric series with first term a and common ratio r. If its sum is 4 and the second term is $\frac{3}{4}$, then

(a) $a = 2, \ r = \frac{1}{2}$ (b) $a = 2, \ r = \frac{3}{8}$
(c) $a = 1, \ r = \frac{3}{4}$ (d) None of these

25. If p, q are positive real numbers such that $pq = 1$, then the least value of $(1 + p)(1 + q)$ is

(a) 4 (b) 1
(c) 2 (d) None of these

26. If $49^n + 16n + p$ is divisible by 64 for all $n \in N$, then the least negative integral value of p is

(a) – 2 (b) – 3 (c) – 4 (d) – 1

27. If the 4th term in the expansion of $\left(ax + \frac{1}{x}\right)^n$ is $\frac{5}{2}$, for all $x \in R$, then the values of a and n are

(a) $\frac{1}{2}, 6$ (b) $6, \frac{1}{2}$
(c) 2,6 (d) None of these

28. Let n be a positive integer. If the coefficients of second, third and fourth terms in the expansion of $(1 + x)^n$ are in AP then n is equal to

(a) 2 (b) 6
(c) 7 (d) None of these

29. Total number of ways in which five '+' and three '–' signs can be arranged in a line such that no two '–' sign occur together is

(a) 10 (b) 20
(c) 15 (d) None of these

30. A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if atleast one black ball is to be included in the draw?

(a) 64 (b) 24 (c) 3 (d) 12

31. Let $P=(-1,0)$, $O=(0,0)$ and $Q=(3,3\sqrt{3})$ be three points. Then, the equation of the bisector of the $\angle POQ$ is

(a) $y=\sqrt{3}\,x$ (b) $\sqrt{3}\,y=x$
(c) $y=-\sqrt{3}\,x$ (d) $\sqrt{3}\,y=-x$

32. The lines $2x-3y=5$ and $3x-4y=7$ are diameters of a circle of area 154 sq units. Then, the equation of the circle is

(a) $x^2+y^2-2x+2y+47=0$
(b) $x^2+y^2+2x-2y-47=0$
(c) $x^2+y^2-2x+2y-47=0$
(d) $x^2+y^2-2x-2y-47=0$

33. The line $2x+y+k=0$ is a normal to the parabola $y^2=-8x$, if k is equal to

(a) –24 (b) 12 (c) 24 (d) –12

34. If t is a parameter, then $x=a(\sin t-\cos t)$, $y=b(\sin t+\cos t)$ represents

(a) circle (b) parabola
(c) ellipse (d) hyperbola

35. The eccentricity of the hyperbola n $9x^2-16y^2+72x-32y-16=0$ is

(a) 5/4 (b) 6/5
(c) 4/3 (d) 3/2

36. The value of λ such that $x-4=y-2=\frac{1}{2}(z-\lambda)$ lies in the plane $2x-4y+z=7$, is

(a) 7 (b) –7
(c) 4 (d) None of these

37. The $f(x)=\dfrac{\log(1+ax)-\log(1-bx)}{x}$ is not defined at $x=0$. The value which should be assigned to f at $x=0$, so that it is continuous at $x=0$, is

(a) $a-b$ (b) $a+b$
(c) $b-a$ (d) None of these

38. Let $f:R\to R$ be such that $f(1)=3$, and $f^1(1)=6$. Then, $\lim_{x\to 0}\left[\dfrac{f(1+x)}{f(1)}\right]^{1/x}$ is equal to

(a) 1 (b) $e^{1/2}$
(c) e^2 (d) e^3

39. If $f(x)$ is differentiable and strictly increasing function, then the value of $\lim_{x\to 0}\dfrac{f(x^2)-f(x)}{f(x)-f(0)}$ is equal to

(a) –1 (b) 0 (c) 1 (d) 2

40. The angle between the tangents drawn from the point (1, 4) to the parabola $y^2=4x$ is

(a) $\pi/2$ (b) $\pi/6$
(c) $\pi/4$ (d) $\pi/3$

41. The difference between the greatest and the least values of the function $f(x)=\sin 2x-x$ on $\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]$ is

(a) π
(b) $\sqrt{3}-\pi/3$
(c) $-\sqrt{3}+\pi/3$
(d) None of the above

42. $\displaystyle\int\frac{(x^2-1)}{x^3\sqrt{2x^4-2x^2+1}}\,dx$ is equal to

(a) $-\dfrac{1}{2}\sqrt{2-\dfrac{2}{x^2}+\dfrac{1}{x^4}}+C$
(b) $\dfrac{1}{2}\log(2x^2-2x^2+1)+C$
(c) $\dfrac{1}{2}\sqrt{2-\dfrac{2}{x^2}+\dfrac{1}{x^4}}+C$
(d) None of the above

43. $\displaystyle\int_{-\pi}^{\pi}\sin x[f(\cos x)]\,dx$ is equal to

(a) $2f(\pi)$ (b) $2f(2)$
(c) $2f(1)$ (d) None of these

44. The area bounded by the curves $y=x$ and $y=x^3$ is equal to

(a) 0 (b) $\dfrac{1}{4}$
(c) $\dfrac{1}{2}$ (d) 1

45. The area bounded by the curves $y = e^x, y = e^{-x}$, the ordinates $x = 0$ and $x = 1$ is given by

(a) $e + e^{-1} - 2$

(b) $e - e^{-1}$

(c) $e + e^{-1}$

(d) $e + e^{-1} + 2$

46. The differential equation of all non-vertical lines in a plane is

(a) $\frac{d^2y}{dx^2} = 0$ (b) $\frac{dy}{dx} = 0$

(c) $\frac{dx}{dy} = 0$ (d) None of these

47. A solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$ is

(a) $y = 2x$ (b) $y = -2x$

(c) $y = 2x - 4$ (d) $y = 2x + 4$

48. If $\mathbf{a}$ and $\mathbf{b}$ are unit vectors such that $\mathbf{a} + 3\mathbf{b}$ is perpendicular to $7\mathbf{a} - 5\mathbf{b}$, then the angle between $\mathbf{a}$ and $\mathbf{b}$ is

(a) $\frac{\pi}{3}$ (b) $\pi/6$

(c) $2\pi/3$ (d) None of these

49. The unit vector perpendicular to vectors $\mathbf{i} + \mathbf{j}$ and $\mathbf{i} - \mathbf{j}$ forming a right handed system is

(a) $\mathbf{k}$ (b) $2\mathbf{k}$

(c) $-\mathbf{k}$ (d) None of these

50. Let $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{ab} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{c} = \mathbf{i} - \mathbf{j} - \mathbf{k}$ be three vectors. A vector $\mathbf{v}$ in the plane of $\mathbf{a}$ and $\mathbf{b}$, whose projection on $\mathbf{c}$ is $\frac{1}{\sqrt{3}}$, is given by

(a) $3\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ (b) $3\mathbf{i} - \mathbf{j} + 3\mathbf{k}$

(c) $3\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ (d) $-3\mathbf{i} + \mathbf{j} + 3\mathbf{k}$

Answers

Physics

1. (c)	**2.** (a)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (b)	**7.** (a)	**8.** (a)	**9.** (d)	**10.** (*)
11. (c)	**12.** (d)	**13.** (b)	**14.** (a)	**15.** (b)	**16.** (d)	**17.** (*)	**18.** (a)	**19.** (*)	**20.** (d)
21. (d)	**22.** (a)	**23.** (b)	**24.** (c)	**25.** (b)	**26.** (a)	**27.** (b)	**28.** (a)	**29.** (a)	**30.** (a)
31. (b)	**32.** (c)	**33.** (a)	**34.** (c)	**35.** (b)	**36.** (c)	**37.** (b)	**38.** (b)	**39.** (c)	**40.** (a)
41. (*)	**42.** (a)	**43.** (b)	**44.** (b)	**45.** (d)	**46.** (*)	**47.** (d)	**48.** (a)	**49.** (b)	**50.** (a)

(*) None option is correct.

Chemistry

1. (b)	**2.** (d)	**3.** (b)	**4.** (a)	**5.** (a)	**6.** (c)	**7.** (a)	**8.** (a)	**9.** (b)	**10.** (d)
11. (d)	**12.** (b)	**13.** (a)	**14.** (c)	**15.** (c)	**16.** (b)	**17.** (a)	**18.** (a)	**19.** (c)	**20.** (b)
21. (d)	**22.** (c)	**23.** (c)	**24.** (d)	**25.** (a)	**26.** (d)	**27.** (a)	**28.** (c)	**29.** (d)	**30.** (d)
31. (b)	**32.** (c)	**33.** (a)	**34.** (c)	**35.** (b)	**36.** (d)	**37.** (c)	**38.** (d)	**39.** (a)	**40.** (d)
41. (a)	**42.** (d)	**43.** (b)	**44.** (c)	**45.** (b)	**46.** (c)	**47.** (b)	**48.** (a)	**49.** (*)	**50.** (a)

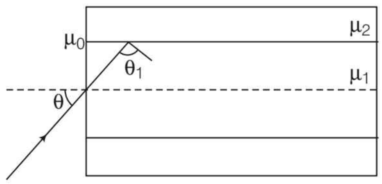
Mathematics

1. (b)	**2.** (c)	**3.** (d)	**4.** (b)	**5.** (d)	**6.** (b)	**7.** (c)	**8.** (d)	**9.** (c)	**10.** (d)
11. (b)	**12.** (a)	**13.** (b)	**14.** (c)	**15.** (d)	**16.** (a)	**17.** (c)	**18.** (a)	**19.** (a)	**20.** (d)
21. (a)	**22.** (b)	**23.** (a)	**24.** (c)	**25.** (a)	**26.** (d)	**27.** (a)	**28.** (c)	**29.** (b)	**30.** (a)
31. (c)	**32.** (c)	**33.** (c)	**34.** (c)	**35.** (a)	**36.** (a)	**37.** (b)	**38.** (c)	**39.** (a)	**40.** (d)
41. (b)	**42.** (c)	**43.** (d)	**44.** (b)	**45.** (a)	**46.** (a)	**47.** (c)	**48.** (a)	**49.** (c)	**50.** (b)

Hints & Solutions

Physics

1. From Snell's law



$$\mu_0 \sin\theta = \mu_1 \sin\theta_1$$

$$\sin\theta = \left(\frac{\mu_1}{\mu_0}\right)\sin\theta_1$$

If θ_c is critical angle, then $\sin\theta_c = \frac{\mu_2}{\mu_1}$

$$\sin\theta = \frac{\mu_1}{\mu_2},\ \sin\theta_1 = \left(\frac{\mu_1}{\mu_0}\right)\sin(90° - \theta_c)$$

$$= \frac{\mu_1}{\mu_2}\cos\theta_c = \left(\frac{\mu_1}{\mu_2}\right)\sqrt{1-\left(\frac{\mu_2}{\mu_1}\right)^2}$$

$$\sin\theta = \frac{\mu_1^2 - \mu_2^2}{\mu_0}$$

2. Given, $f = 12\text{ MHz} = 12 \times 10^{-6}$ Hz

$$R = 50\text{ cm} = 520 \times 10^{-2}\text{ m}$$

$$B = \frac{2\pi mf}{q}$$

$$= \frac{(12 \times 10^6) \times 2 \times \pi \times (1.67 \times 10^{-27})}{(1.6 \times 10^{-19})}$$

$$= 0.786 \approx 0.72\text{ T}$$

3. When an electron is revolving in orbit then orbital magnetic moment and orbital angular momentum are in opposite direction also

$$|\mu_{orb}| = \frac{eh}{4\pi m_e}$$

and $$|\mathbf{L}_{orb}| = \frac{h}{2\pi} \quad \text{(For } n = 1)$$

$\therefore$ $$\mu_{orb} = -\frac{e}{2m_e}\mathbf{L}_{orb}$$

4. Magnetic fields induced will be maximum if rate of charge of electric field is maximum.

$\Rightarrow$ In graph line having maximum slope will induce maximum magnetic field.

$\therefore$ Correct order is *P, R, Q, S*.

5. Position of bright fringes is given by $\lambda = n\lambda\frac{D}{d}$

For 3rd order bright fringes, $n = 3$

According to question,

$$(x)_{480} = \frac{3 \times (480 \times 10^{-9}) \times 1}{(5 \times 10^{-3})}$$

$$= 2.88 \times 10^{-4}\text{ m}$$

$$(x)_{600} = \frac{3 \times (600 \times 10^{-9}) \times 1}{(5 \times 10^{-3})}$$

$$= 3.6 \times 10^{-4}\text{ m}$$

$\therefore$ Separation between third order bright is

$$= (x)_{600} - (x)_{480}$$

$$= (3.6 \times 10^{-4}\text{ m}) - (2.88 \times 10^{-4}\text{ m})$$

$$= 7.2 \times 10^{-5}\text{ m}$$

$$= 0.072 \times 10^{-3}\text{ m}$$

$$= 0.07\text{ mm}$$

6. Wavelength corresponding to energy gap of 1.9 eV will be given by

$$\lambda = \frac{hc}{E} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.9 \times 1.6 \times 10^{-19}}$$

$$= 6.513 \times 10^{-7}\text{ m}$$

$$= 651.3 \times 10^{-9} \approx 651\text{ nm}$$

Which is wavelength of red light.

So, red colour will be emitted.

7. The binding energy per nucleon in a heavy nucleus is of order of 7.6 MeV for ${}_{92}U^{238}$.

So, order is ≈ 8 MeV.

8. $^{235}U + N \longrightarrow x + y + 2n$

$$236 + 1 \longrightarrow x + y + 2 \times 1$$

$$236 \longrightarrow 2 + x + y \Rightarrow x + y = 234$$

(i) ^{141}Xe and $^{93}S_r$, $x = 141$, $y = 93$

$$x + y = 141 + 93 = 234$$

(ii) ^{139}Cs and ^{95}Rb $\longrightarrow x = 139,\ y = 95$

$$x + y = 139 + 95 = 234$$

(iii) ^{156}Nd and ^{79}Ge $\longrightarrow x = 156, y = 79$

$$x + y = 156 + 29 = 185$$

(iv) ^{141}Ba and $^{92}Kr \longrightarrow x = 141,\ y = 92$

$$x + y = 141 + 92 = 233$$

Hence, option (a) is correct.

9. The reaction $^{35}Cl + {}^{35}Cl$ will not result in the net release of energy because heavy nucleus does not use in fusion reaction.

10. Given, mass of damped harmonic oscillator

$$m = 250\text{g} = \frac{250}{1000}\text{ kg}$$
$$= \frac{1}{4}\text{ kg}$$

Spring constant (k) = 85 Nm^{-1}

Damping constant (b) = 70 gs^{-1} = 0.07 kgs^{-1}

Frequency of damped oscillator

$$f = \frac{1}{2n}\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$
$$= \frac{1}{2 \times 3.14}\sqrt{\frac{85}{\frac{1}{4}} - 0.07 \times \frac{0.07}{4 \times \frac{1}{4} \times \frac{1}{4}}}$$
$$= \frac{1}{2 \times 3.14}\sqrt{340 - 0.0196}$$
$$= \frac{1}{2 \times 3.14} \times 18.43$$
$$= 2.93$$

Now $T = \frac{1}{f} = \frac{1}{2.93} = 0.34$ s

11. Given,

$$y = (0.50\text{ cm})\sin\left(\frac{\pi x}{3}\right)\cos[40\pi t]$$

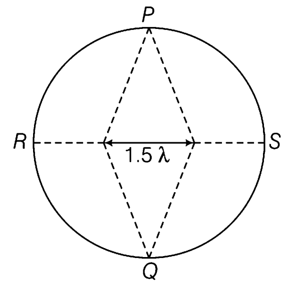
its compring with

$$y = 2a\sin\frac{22\pi x}{\lambda}\cos\frac{2\pi t}{\lambda}$$
$$\frac{2\pi x}{\lambda} = \frac{\pi x}{3}$$
$$\lambda = 6$$

Distance detween nodes

$$= \frac{\lambda}{2} = \frac{6}{2} = 30\text{ cm}$$

12. At point P and Q, both difference is equal to zero, so point P and Q are points of constructive interference now in points R and S path difference is equal to 1.5 λ and according to question path difference of 1.5 λ also corresponds to point of constructive difference.



So, P, Q, R and S are 4 points around the circle which are in constructive interference.

13. The steady state, rate of heat flow $\frac{\theta}{t}$ is constant.

$$\Rightarrow \quad \frac{\theta}{t} = \frac{kA(T_1 - T_2)}{L}$$
$$\Rightarrow \quad k = \frac{\theta L}{tA(T_1 - T_2)} \quad \Rightarrow \quad k \propto \frac{1}{(T_1 - T_2)}$$

(As all other factor are constant)

$(T_1 - T_2)$ is temperature difference

For S_1, temperature difference = 10°C

For S_2, temperature difference = 5°C

For S_3, temperature difference = 15°C

For S_4, temperature difference = 5°C

$\therefore$ Decreasing order of thermal conductivity is

$$S_2 = S_4, S_1, S_3$$

14. Change in entropy of the system

$$= \Delta S = \Delta S_L + \Delta S_R$$
$$= m_L S \ln\frac{T_{eq}}{(T_1)_L} + m_R S \ln\frac{T}{(T_1)_R}$$
$$= 0.5 \times 386\,[\ln T_{eq.} - \ln(T_1)L + \ln T_{eq.} - \ln(T_1)R]$$
$$= 193\,[2\ln T_{eq} - \ln(T_1)_L - \ln(T_2)_R]$$
$$= 193\,[2m\ 313 - \ln 333 - \ln 293]$$
$$= 193\,[2 \times 5.746 - 5.8 - 5.68]$$
$$= 193\,[11.492 - 5.8 - 5.68]$$
$$= 193(11.492 - 11.48)$$
$$= 2.316\text{ J/K} \approx 2.4\text{ J/K}$$

15. The net electric field due to both sheets will be equal and opposite in direction so net electric field will be zero.

$\therefore$ Points which are near sphere will have max value of electric field.

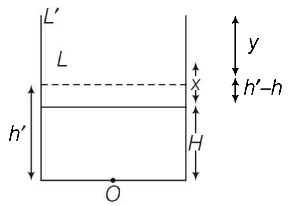
Point 3 and 4 are at equal distance so electric field at 3 and 4 will be same.

Increasing order of electric field is 1, 2, 3 = 4.

16. Microwave is used for telecommunication.

17. Suppose initial height of the water level was H.

Initial position of the observer was L and final position is L'.



(object)

Now for the image to be formed at the same place.

From adjacent figure

$$x + \frac{H}{\mu_w} = y + \frac{h'}{\mu_w}$$

(Object distance should be same)

$$\Rightarrow \quad y - x = \frac{H - h'}{\mu_w}$$

$$\Rightarrow \quad -3 = \frac{H - h'}{\mu_w}$$

$$\Rightarrow \quad h' - H = \mu_w (3) = \frac{4}{3} \times 3 = 4\,\text{cm}$$

Water level increased by 4 cm.

18. Here A, B and C must have same unit as of electric field.

So, $\quad \frac{A}{x^3} = \frac{\text{N}}{\text{C}}$

$$\Rightarrow \quad \frac{A}{\text{m}^3} \equiv \frac{\text{N}}{\text{C}}$$

$$\Rightarrow \quad A \equiv \frac{\text{N-m}^3}{\text{C}}$$

and $\quad By = \frac{V}{m}$

$$\Rightarrow \quad Bm \equiv \frac{V}{m}$$

$$\Rightarrow \quad B \equiv \frac{V}{m^2}$$

and $\quad Cz^2 \equiv \frac{\text{N}}{\text{C}}$

$$\Rightarrow \quad Cm^2 \equiv \frac{\text{N}}{\text{C}}$$

$$\Rightarrow \quad C \equiv \frac{\text{N}}{\text{C-m}^2}$$

19. Here, $\quad \mathbf{A} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$

Let $\quad \mathbf{B} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$

Now, component of vector A along vector

$$B = (\mathbf{A} \cdot \mathbf{B})\hat{\mathbf{B}}$$

$$= [(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}})] \frac{(\hat{\mathbf{i}} - \hat{\mathbf{j}})}{\sqrt{1^2 + 1^2}}$$

$$= (2 - 3) \frac{(\hat{\mathbf{i}} - \hat{\mathbf{j}})}{\sqrt{2}} = -\frac{(\hat{\mathbf{i}} - \hat{\mathbf{j}})}{\sqrt{2}}$$

$$= -\frac{1}{\sqrt{2}} (\hat{\mathbf{i}} - \hat{\mathbf{j}})$$

20. As, $L = p \cdot r$

So, $\quad \log L = \log p + \log r$

Comparing with, $y = mx + C$

$$y = \log L$$

$$x = \log p, \quad m = 1, \; c = \log r$$

If graph is drawn between $\log_e L$ and $\log_e p$, then it will be straight line which will not pass through the origin.

21. When a ball is projected vertically upward distance travelled in nth second is given by

$$S_n = 4 - \frac{g}{2}(2n - 1)$$

Now according to question

$$S_5 = 2S_6$$

$$\left[u - \frac{g}{2}(2 \times 5 - 1)\right] = 2\left[u - \frac{g}{2}(2 \times 6 - 1)\right]$$

$$u - \frac{g}{2}(10 - 1) = 2u - g\,(12 - 1)$$

$$\frac{-9g}{2} = u - 11g$$

$$u = 11g - \frac{9g}{2}$$

$$u = \frac{(22 - 9)\,g}{2}$$

$$u = 6.5\,g = 6.5 \times 10 \text{ m/s}$$

$$= 65 \text{ m/s}$$

22. Before heating the ground, the velocity v is given by

$$v^2 = 2gh$$

Further, $\quad v^2 = 2gh\left(\frac{h}{2}\right) = gh$

$$\therefore \quad \left(\frac{v}{v'}\right) = \sqrt{2} \text{ or } v = v'\sqrt{2}$$

As the direction is reversed and speed is decreased and hence, (a) represents these conditions correctly.

23. Velocity of transverse wave is given by

$$v = \sqrt{\frac{T}{m}}$$

T : Tension in string

m : Mass per unit length

$$\Rightarrow \quad v = \sqrt{\frac{0.1 \times 9.8}{(0.1/2.45)}}$$

$$= \sqrt{9.8 \times 2.45}$$

$$= 4.9 \text{ m/s}$$

Now, time taken by wave to travel full length of the rope

$$= \frac{2.45 \times 2}{4.9} = 1.0 \text{ s}$$

24. We consider the following two cases

(i) When $r_1 > R$

Gravitational field $\quad I_1 = \frac{5\,m}{r_1^2}$

$$\Rightarrow \quad I_1 = \frac{G\,\frac{4}{3}\pi R^3 \rho}{r_1^2} \quad \ldots\text{(i)}$$

(ii) When $r_2 > R$

Gravitational field $\quad I_2 = \frac{Gm_1}{r_1^2}$

where m_1 is the mass of the body within the sphere of radius r^2

$$m_1 = \frac{4}{3}\pi\rho r_2^3, \quad I_1 = \frac{G\frac{4}{3}\pi\rho r_2^3}{r_2^2}$$

Hence $\quad \frac{I_1}{I_2} = \frac{R^3}{r_1^2\, r_2^2}$

25. Given, mass of car, $m = 2$ kg

Initial velocity, $\mathbf{v}_i = -0.5\,\hat{\mathbf{j}}$ m/s

So, initial momentum $\mathbf{p}_i = 2 \times (-0.5\,\hat{\mathbf{j}})$

$$= -1.0\,\hat{\mathbf{j}} \text{ kg m/s}$$

Now, final velocity, $\mathbf{v}_f = 0.4\hat{\mathbf{i}}$ m/s

So, final momentum, $\mathbf{p}_f = 2 \times (0.4\,\hat{\mathbf{i}})$

$$= 0.8\,\hat{\mathbf{i}} \text{ kg m/s}$$

Now, change in momentum $= \mathbf{p}_f - \mathbf{p}_i$

$$= (0.8\,\hat{\mathbf{i}}) - (n - 10\,\hat{\mathbf{j}}) = 0.8\,\hat{\mathbf{i}} + \hat{\mathbf{j}}$$

$$= (0.8\,\hat{\mathbf{i}} + 1.0\,\hat{\mathbf{j}}) \text{ kg m/s}$$

26. We know that

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{h}{\sqrt{2mk}}$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{k_2}{k_1}} \quad (\because h \text{ and } m \text{ are constants})$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{2k}{k}} = \sqrt{2}$$

27. A car can move around a curve road, if

$$v = \sqrt{\mu\, R_1\, g} \quad \ldots\text{(i)}$$

$\because$ Speed is doubled and radius is changed

$$\Rightarrow \quad v' = \sqrt{\mu\, R_1' g}$$

$$\Rightarrow \quad 2v = \sqrt{\mu\, R_1' g} \quad \ldots\text{(ii)}$$

Dividing Eqs. (i) and (ii), we get

$$\frac{1}{2} = \sqrt{\frac{R_1}{R_1'}}$$

$$R_1' = 4R_1$$

28. Relation between force and potential energy is given by

$$F(x) = \frac{-dU(x)}{dx} = -\text{(slope of graph)}$$

From graph,

In case of (i), $\frac{dU}{dx}$ is maximum

$$\Rightarrow \quad -\frac{dU}{dx} \text{ is minimum}$$

In BC, $\quad \frac{dU}{dx} = 0 \;\Rightarrow\; F = 0$

If we compare $-\left(\frac{dU}{dx}\right)$ of DE and AB then magnitude of force of DE is greater then B.

So, (a) is correct.

29. Force in (i) case $= Gm^2\left(\frac{1}{d^2} + \frac{1}{D^2}\right)$

Force in (ii) case $= Gm^2\left(\frac{1}{d^2} - \frac{1}{D^2}\right)$

Force in (iii) case $= Gm^2\sqrt{\left(\frac{1}{d^2} + \frac{1}{D^2}\right)}$

Force in (iv) case $= Gm^2\sqrt{\left(\frac{1}{d^2} + \frac{1}{D^2}\right)}$

$\therefore$ Decreasing order is (i), (iii) = (iv), (ii)

30. As the electrostatic force are conservative, work done is independent of path. Effective displacement in going from P to D along any path $= \mathbf{PO}$

$= (0 - 2a)\,\hat{\mathbf{i}} + (0 - b)\,\hat{\mathbf{j}} + (0 - 0)\,\hat{\mathbf{k}} = -2a\,\hat{\mathbf{i}} - b\,\hat{\mathbf{j}}$

Force on charge, $\mathbf{F} = (qE)\,\hat{\mathbf{i}}$

Now, work done $= \mathbf{F} \cdot \mathbf{PO} = (qE\,\hat{\mathbf{i}}) \cdot (-2a\,\hat{\mathbf{i}} - b\,\hat{\mathbf{j}})$

$$= -2qE\,a$$

31. Given that

For star units $1\,kg^* = 10\,kg$

$1\,m^* = 1\,km$

and $1\,s^* = 1\,m$

The value of 1 J in star units

$1\ J = 1\ [ML^2T^{-2}] = 1[1 kg\ m^2 s^{-2}]$

$$= \left[1 \times \frac{1}{10} kg + \left(\frac{m^*}{1000}\right)^2 + \left(\frac{s^*}{60}\right)^{-1}\right] = 3.6 \times 154\ J^*$$

32. Thickness $= 10\,cm = 10 \times 10^{-2}$ m

$T_1 = 40°C$

$T_2 = 26°C$

$$\frac{\Delta Q}{\Delta t} = \frac{kA\,(T_1 - T_2)}{x} = \frac{1.26 \times 4 \times 4\,(40 - 26)}{10 \times 10^{-2}\ m}$$

$$= \frac{1.26 \times 16 \times 14}{10^{-1}}$$

$= 12.6 \times 16 \times 14$ W

$= 2822.4\ W \approx 2822\ W$

33. In Maxwell's speed distribution plot the total fraction of molecules when are lying between a particular velocity range is a constant at constant temperature.

So, here $T_4 > T_3 > T_2 > T_1$.

34. A thermodynamic state of a given sample of an ideal gas is completely described when its pressure, volume and temperature are known.

35. Following observation are made from given graph for part *AB*. In process *AB* prom given *V-T* curve

$$V \propto T$$

$\Rightarrow$ presure is constant and volume is increasing

($\because$ mass of gas is constant)

In process *BC*, $V =$ constant and T decreas

In process *CA*, $T =$ constant and V decreasing

$\therefore$ These process represent correctly of *p-V* curve by graph (ii)

T is constant and V is decreasing

So, $p \propto \dfrac{1}{V}$ (*P* will increase)

All there conditions are satisfied in graph (ii).

36. As $R \propto A^{1/3}$

$$\Rightarrow \frac{R_{Al}}{R_{Te}} = \frac{(27)^{1/3}}{(125)^{1/3}}$$

$$\frac{R_{Al}}{R_{Te}} = \frac{3}{5}$$

37. 2 NAND gates are needed to obtain OR gate.

38. In *R-C* circuit $i = \dfrac{E}{R} e^{-t/RC}$

$$\therefore \quad \log_e i = \frac{-t}{RC} + \log_e \frac{E}{R}$$

Value of R is double, the slope of curve will increase. At $t = 0$, for increased value of resistance , current will be minimum.

39. The object O will appear to be shifted upward by amount $\Delta t = 33.25 = 8.25$

$\therefore$ Position of object as seen first above another

$= 33.25 - 8.25 = 25\,cm$

According to question image is also formed at the same point

Now, $v = 15 + 25 = 40$ cm

$u = 15 + 25 = 40\,cm$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{40} + \frac{1}{40} = \frac{1}{f}$$

$f = 20\,cm$

40. As $I = neAv_d$

$4 = n\,(1.6 \times 10^{-19})\,(2\ mm^2)\,(2.1 \times 10^{-4})$

$$4 = n\,(1.6 \times 10^{-19})\left(\frac{2}{10^6}\ m^2\right)(2.1 \times 10^{-4})$$

$$n = \frac{4 \times 10^6}{(1.6 \times 10^{-19})\,(20)\,(2.1 \times 10^{-4})} = 5.95 \times 10^{28}\ m^{-3}$$

$\approx 6 \times 10^{28}\ m^{-3}$

41. Given that, mass of $mg^{24} = 24 \times 1.66 \times 10^{-27}$kg

$= 39.84 \times 10^{-27}$kg

$k = 8\,keV = 8 \times 1.6 \times 10^{-13} = 12.8 \times 10^{-13}$ J

$B = 1.2$ T

$q = 3.2 \times 10^{-19}$ C

$$r = \frac{\sqrt{2\,km}}{qB} = \frac{2 \times 8 \times 1.6 \times 10^{-13} \times 39.84 \times 10^{-27}}{32 \times 10^{-19} \times 1.2}$$

$$= \frac{31694 \times 10^{-20}}{3.2 \times 1.2 \times 10^{-19}} = 8.31 \times 10^{-1} = 0.831\ m$$

42. If W is work function of material

$$k_1 = \frac{hc}{\lambda_1} - W \quad \ldots(i)$$

and

$$k_2 = \frac{hc}{\lambda_2} - W \quad \ldots(ii)$$

Subtracting Eqs. (i) and (ii), we get

$$k_1 - k_2 = \frac{h_c}{\lambda} - \frac{h_c}{\lambda}$$

$$(k_1 - k_2) = \frac{h_c}{\lambda_1 \lambda_2}(\lambda_1 - \lambda_2)$$

$$\frac{k_1 - k_2}{c} \frac{\lambda_1 \lambda_1}{\lambda_2 - \lambda_1} = h$$

43. Davisson-Germer experiment establishes the wave nature of particles.

44. de-Broglie wavelength $\lambda = \frac{h}{mv} = \frac{6.6 \times 10^{-34}}{0.1 \times 900}$

$$= 7.3 \times 10^{-36} \text{ m}$$

45. Brown, yellow, green and gold

Here, $A \longrightarrow$ brown

$B \longrightarrow$ yellow

$C \longrightarrow$ green

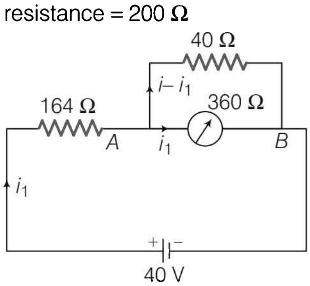
and $D \longrightarrow$ gold

From $R = AB \times C = DV$

$$= 14 \times 10^5 \pm \frac{5}{100} = (1.4 \pm 0.05) \text{ m}\Omega$$

46. As resistance 40 Ω and 360 Ω are in parallel, so their equivalent resistance is 360 Ω. Which is further connected in series with 164 Ω.

∴ Total resistance = 200 Ω



∴ current $i = \frac{40}{200} = 0.2$ A

Now, from A to B,

$$40(i - i_1) = i_1 \times 360$$

$\Rightarrow$ $40(0.2 - i_1) = 360\, i_1$

$\Rightarrow$ $8 - 40\, i_1 = 360\, i_1$

$$8 = 400\, i_1$$

$\Rightarrow$ $i_1 = \frac{8}{400} = 0.02$ A

47. $\lambda = 1400 \text{ nm} = 1400 \times 10^{-3}$ m corresponds to energy,

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1400 \times 10^{-9}}$$

$$= 1.41 \times 10^{-19} \text{ J}$$

$$= \frac{1.41 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV}$$

$$\approx 0.8 \text{ eV}$$

Which is closest to band gap of S so S will be suitable.

48. $y = a_1 \sin 2\pi\nu_1 t_1,\; y_2 = A \sin 2\pi\nu_2 t$

$$y = 2a \sin \frac{2(\nu_1 + \nu_2)\pi t}{2} \cos \frac{2(\nu^1 - \nu^2)}{2} \pi t$$

$$y = 2a \sin(\nu_1 + \nu_2)\pi t \cos(\nu_1 - \nu_2)\pi t$$

Comparing $y = 2a \sin \omega t$

$$A = \cos(\nu_1 - \nu_2)\pi t$$

For maxima $(\nu_1 - \nu_2)\pi t = k\pi$

$$t = \frac{k}{\nu_1 - \nu_2}$$

Now, $k = 0, 1, 2, 3 \ldots$

Then, $t = 0, \frac{1}{n_1 - n_2}, \frac{2}{n_1 - n_q}, \frac{3}{n_1 - n_2} \ldots$

∴ Time interval $= \frac{1}{\nu_1 - \nu_2}$.

49. $Z = \frac{v}{c} = \frac{\Delta\lambda}{\lambda} = \frac{1.1\lambda}{\lambda}$

$$c = 3 \times 10^8 \text{ m/s}$$

$$v = 3.3 \times 10^5 \text{ km/s}$$

From Hubble's law

Distance $d = \frac{3.3 \times 10^5}{73} = 0.045 \times 10^5$ Mpc

Also 1 Mpc = 3.26 ly

$$d = 1.6 \times 10^9 \text{ ly}$$

50. We know that,

$$f_c = 9(N_{max})^{1/2}$$

$$10 \times 10^6 = 9(N_{max})^{1/2}$$

$$\frac{10^7}{9} = 9(N_{max})^{1/2}$$

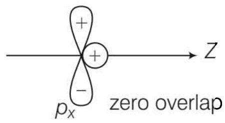
$$\frac{10^{14}}{81} = N_{max}$$

$$\frac{100}{81} = 10^{12}\, N_{max}$$

$$N_{max} = 1.2 \times 10^{12} \text{m}^{-3}$$

Chemistry

1. $\Delta H > 0$

2. 

When two atoms come close to each other, there is overlapping of atomic orbitals. This overlap may be positive, negative or zero depending upon the properties of overlapping of atomic orbitals.

3. Copper atoms occupy the centres of each of the cube faces, so number of copper atoms = $6 \times \frac{1}{2} = 3$ atoms Gold atoms occupy the lattice point so number of gold atoms

$$= 8 \times \frac{1}{8} = 1 \text{ atom}$$

Formula of the compound is $AuCu_3$

4. $2KClO_3 \longrightarrow 2KCl + 3O_2$

$2 \times 122.5 = 245$ g $\quad$ 3 mol O_2

245 g $KClO_3$ gives = 3 mol O_2

110 g $KClO_3$ will give = $\frac{3 \times 110}{245} = 1.346$ mole

$$pV = nRT$$

Given, $p = 750$ torr, $V = 7$

$R = 62.364 \text{ dm}^3 \text{ Torr K}^{-1} \text{ mol}^{-1}$

and $T = 273 + 18 = 291\text{K}$

$$V = \frac{1.346 \times 62.34 \times 291}{750}$$

$$= 32.569\text{L} \approx 32.6 \text{ L}$$

5. In CsBr unit cell structure Br^- ions are arranged in a simple cubic arrangement. In this arrangement the Br^- ions are present at all the corners of the cube. The Cs^+ ions occupy the cubic sites *i.e.,* Cs^+ ions are present at the body centre of the cube. Number of Br^- ions = $8 \times \frac{1}{8} = 1$ ions per unit cell.

There is also one Cs^+ ion in the body centre and its contribution per unit cell is 1.

Therefore, there is one Br^- ion for each Cs^+ ion.

There is one molecule of CsBr in the unit lattice.

6. Osmotic pressure is a colligative property, the property of the solution which depends only on the number of solute particles.

(i) 34.2 g/ L sucrose = 0.1 mole sucrose in 1 L solution

(ii) 60 g/L urea = 1 mole urea in 1 L solution.

(iii) 90 g/L glucose = 0.5 mole glucose in 1 L solution.

(vi) 58.5 g/L NaCl = 1 mol NaCl in 1L solution

Sucrose, urea and glucose do not dissociate into their ions while 1 mole NaCl gives 2 mole particles *i.e.,* Na^+ and Cl^- ions.

Hence, the correct increasing order of their osmotic pressure is sucrose < glucose < urea < NaCl. Solutions.

7. Charge on the proton is same as on the electron.

i.e., 4.8×10^{-10} esu.

or 1.6×10^{-19} C

8. In free radical polymerisation, the extent of conversion increasing with increase in temperature.

9. $N_1V_1 = 0.2 \times 100 = 20$

(0.1 M H_2SO_4 = 0.2 N H_2SO_4)acid

$N_2V_2 = 0.1 \times 100 = 10$ (base)

N_1V_1 acid > N_2V_2 base hence, resultant mixture is acidic.

Resultant normality

$$= \frac{N_1V_1 - N_2V_2}{V_1 + V_2} = \frac{20 - 10}{200}$$

$$= \frac{10}{200} = 0.05 \text{ N}$$

10. $CuO(s) + H_2(g) \rightleftharpoons Cu(s) + H_2O(g)$

$$K_c = \frac{[H_2O]}{[H_2]}$$

(As a general rule, the concentration of pure solids and pure liquids are not included when writing an equilibrium equation.)

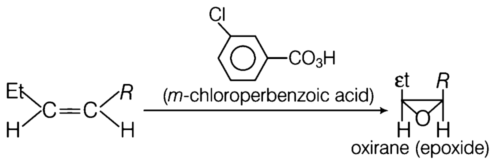
11. $\underset{\text{ethoxy ethane}}{C_2H_5—O—C_2H_5} + 2HI \text{ (excess)} \xrightarrow{375\text{ K}} \underset{\text{ethyl iodide}}{2C_2H_5I} + H_2O$

12. DIBAL-H (diisobutyl aluminium hydride) reduces esters into aldehydes.

13. $CH_3CH_2MgBr + CH_3CHO \xrightarrow[H_2O/H^+]{\text{Ether (solvent)}} CH_3\overset{OH}{\overset{|}{C}}HCH_2CH_3$

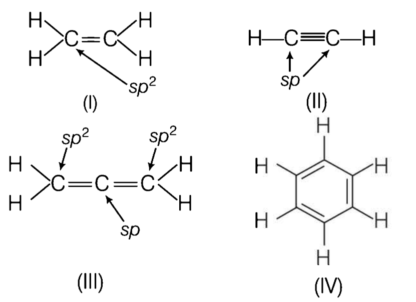
butan-2-ol

14. The neutral or positively charged species with deficiency of electrons and capable of accepting a pair of electrons (Lewis acid) are called electrophiles. *e.g.*, BF_3 and $Me_3C^{\oplus}$.

15. 

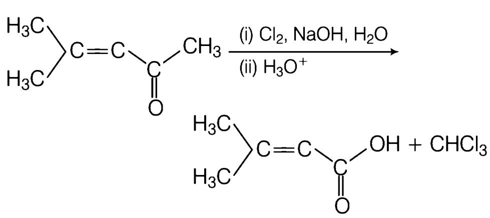
Alkenes give oxiranes or epoxides on reaction Oxirane (epoxide) with *m*-chloroperbenzoic acid.

16. H—C≡C—H is the strongest acid in the given species. Terminal alkynes have acidic H because C – H bond has ionic character due to resonance. Greater the *s*-character of carbon greater the electronegativity and hence greater the attraction for electrons.

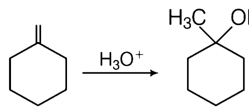


(IV) In benzene all carbon atoms are sp^2 hybridised.

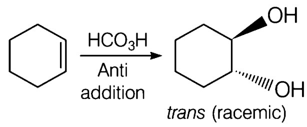
17. Hypohalite ($X\overline{O}$, X = Cl, Br, I) oxidises ketones with terminal (CH_3—C(=O)—) into acid. It is a case of haloform test.



18. CH_3COCl is hydrolysed to maximum extent.

19. 

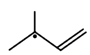
20. Rhodium complex $[(Ph_3P)_3RhCl]$, a Wilkinson catalyst is used for hydrogenation of alkenes.

21. 

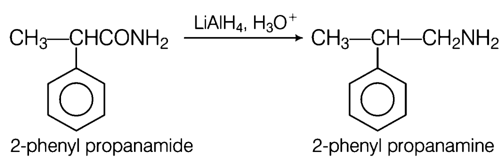
and enantiomer

Hydroxylation with peroxy acid is carried out by allowing alkenes to stand with the mixture of H_2O_2 and formic acid or benzoic acid for a few hours and and then heating the product with H_2O to hydrolyse the intermediate.

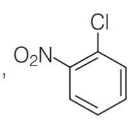
22. When an ether solution of benzoic acid (*A*) aniline (*B*) and toluene (*C*) is extracted with aqueous NaOH, then the benzoic acid (*A*) is dissolved in aqueous NaOH. Therefore, the ether layer will contain aniline (*B*) and toluene (*C*) .

23.  is more stable. The overall stability order among free radicals is

triphenyl methyl > benzyl > allyl > 3° > 2° > 1° > methyl > vinyl

24. 

25. Presence of deactivating group makes the nucleophilic aromatic sbstitution faster. Therefore,

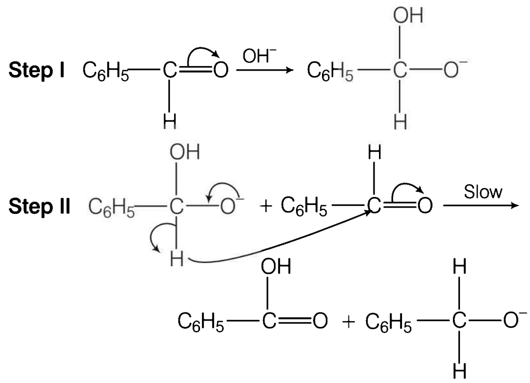
compound (I), 

faster.

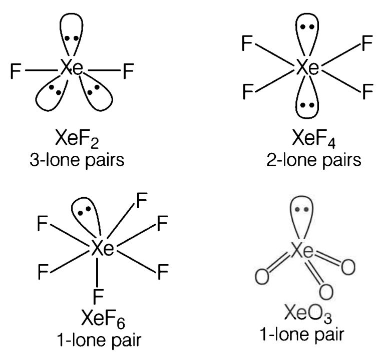
26. Because reaction of Grignard reagent with epoxide proceeds quickly. If epoxide is unsymmetrical, carbanion attacks the less hindered epoxide carbon atom.

27. $2\,C_6H_5CHO + \underset{50\%}{NaOH} \longrightarrow C_6H_5CH_2OH + C_6H_5COONa$

Cannizzaro reaction involving same hydride or different aldehydes is proton (H^+)-hydride (H^-) transfer reaction.



28.



29. Calcium carbide, CaC_2 contain C_2^{2-} ion.

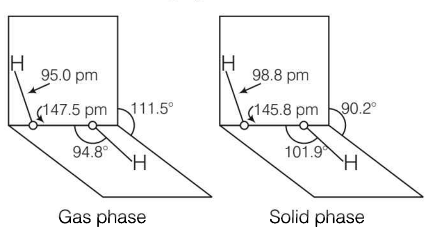
$(C \equiv C)^{2-}$ it contains 1 sigma and 2 π bonds.

30. $\mu = \sqrt{n(n+2)} = \sqrt{15}$ BM.

$$= \sqrt{3(3+2)}\ ;\ n = 3,$$

So, there are 3 unpaired electrons present in an transition metal ion.

31. The structure of H_2O_2 is a non-planar structure.



32. Shapes of orbitals in an energy shell is decided by azimuthal quantum number.

33. In $[Ni(Cl)_4]^{2-}$ complex ion, Ni is in +2 oxidation state. It has electronic configuration $3d^8$. It is paramagnetic and sp^3 hybridised so the four hybridised equivalent orbitals oriented tetrahedrally.

34. The elements Re, Os and Ir belong to third transition series.

35. Fluorine oxidises water to oxygen.

$2F_2(g) + 2H_2O(l) \rightarrow 4H^+(aq) + 4F^-(aq) + O_2(g)$

36. HF attacks on SiO_2 and gives white fumes of SiF_4.

$$SiO_2 + 4HF \rightarrow SiF_4 + 2H_2O$$

37. $[Co(NH_3)_5Br]SO_4$ and $[Co(NH_3)_5SO_4]$ Br are examples of ionisation isomerism. This form of isomerism arises when the counter ion in a complex salt is itself a potential ligand and can displace a ligand which can then become the counter ion.

38. $Lu^{3+} = 4f^{14}$ Colourless

$Sc^{3+} = 3d^0$ Colourless

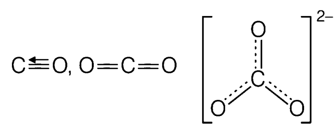
$La^{3+} = 4f^0$ Colourless

$Ti^{3+} = 3d^1$ Coloured (purple)

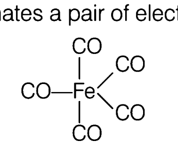
Transition metal ions and inner transition metal ions having d^1 to d^9 configuration and f^1 to f^{13} configuration are coloured due to d-d transition and f - f transition respectively. Ions having d° and d^{10} or f^0 and f^{14} configuration are colourless.

39. The correct order of C—O bond length among CO, CO_3^{2-}, CO_2 is

$CO < CO_2 < CO_3^{2-}$;



40. In $Fe(CO)_5$ the Fe – C bond posseses both σ and π – bonding. In this complex, carbon of carbon monoxide donates a pair of electrons to the metal.



41. The anion $\bar{O}$ $(8+1=9$ electrons) is isoelectronic with N^{2-} $(7+2=9$ electrons). Isoelectronic species have same number of electrons. $(N^{3-}=7+3=10e^-$, $F^-=9+1=10e^-$ and $Ne=10e^-)$.

42. BeO – amphoteric, MgO–Basic, Al_2O_3 – amphoteric ; Cl_2O_7 – most acidic

43. Percentage of X in X_2Y

$$=\frac{2\times10\times100}{(2\times10)+20}$$

$$=\frac{2\times10\times100}{20+20}=50\%$$

X_2Y compound contains 50% X therefore formula of the binary compound will be X_2Y.

44. 1 mole of a gas occupy = 22.4 L volume at STP therefore, 16 g O_2 gas = 0.5 mole of O_2 gas will occupy $=\frac{0.5\times22.4}{1}=11.2$ L at STP.

45. $K_{sp}=2.0\times10^{-15}$M

$$Ni(OH)_2 \rightleftharpoons \underset{s}{Ni^{2+}}+2\underset{2s}{O\bar{H}}$$

$$K_{sp}=[Ni^{2+}][OH^-]^2$$

$$=[s][0.10+2s]^2$$

(because total concentration of $[OH^-]$ ions = [0.10 + 2 s]

As K_{sp} is small $2s<<0.10$

Thus, $(0.10+2s)\approx0.10$

Hence, $2.0\times10^{-15}=s(0.10)^2$

$s=2.0\times10^{-13}\,M=[Ni^{2+}]$

46. The temperature at which a real gas obeys ideal gas law over an appreciable range of pressure is called **Boyle temperature or Boyle point**. Boyle point of a gas depends upon its nature. Above their Boyle point, real gases show positive deviation from ideality and Z values are greater than one. $Z>1$

47. There is 28% As_2O_5 in an insecticide. It means 28g As_2O_5 in 100 g insecticide.

Molar mass of $As_2O_5=(2\times75)+(5\times16)=230$ g

230 g As_2O_5 contains = 150 g As

28 g As_2O_5 contains

$$=\frac{150\times28}{230}$$

$$=18.26\text{ g As}$$

18.26 g As is present in 100 g insecticide so it is 18.26 % ≈ 18%

48. Electrical work done

$$=\Delta_rG°=-nFE°$$

$$E°=E°_{cathode}-E°_{anode}$$

$$=0.80-(-0.76)$$

$$=1.56\text{ V}$$

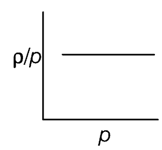
$$\Delta G°=-2\times96500\times1.56$$

$$=-301080\text{ J mol}^{-1}$$

$$=-301.080\text{ kJ mol}^{-1}$$

49. Aluminium is protected from corrosion by a layer of its own oxide Al_2O_3.

50. $pv=nRT=\frac{WRT}{m}$



$$m=\frac{WRT}{pv}$$

$$d=\frac{pm}{RT}$$

$$d\propto p$$

$\frac{d}{p}$ is constant.

Mathematics

1. Given, max $z = x + y$

subject to $y \le |x| - 1$

$y \ge 1 - |x|$

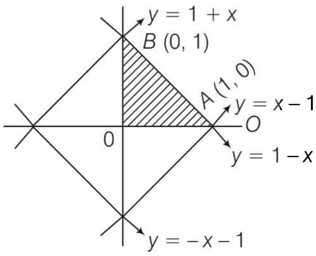
$x, y \ge 0$

Consider above inequalities as equations, we get

$$y = |x| - 1 = \begin{cases} -x - 1, & x < 0 \\ x - 1, & x > 0 \end{cases}$$

$$y = 1 - |x| = \begin{cases} 1 - x, & x > 0 \\ 1 + x, & x < 0 \end{cases}$$

On plotting these lines on the graph paper, we get



Here the feasible region is BOA and its corner points are $O(0, 0)$, $A(1, 0)$ and $B(0, 1)$

Now, by corner point method

Point	Max $z = x + y$
$O(0, 0)$	$z = 0$
$A(1, 0)$	$z = 1$
$B(0, 1)$	$z = 1$

Hence, the maximum value of z is 1.

2. Given, max $z = 2x + 3y$

subject to $y \le x - 1$

$y \ge x + 1$

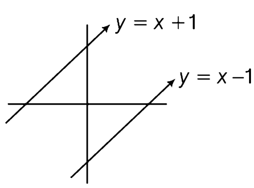
$x, y \ge 0$

Consider above inequalities as equations, we get

$y = x - 1$

$y = x + 1$

On plotting these lines on graph paper, we get



It is clear from the graph that both lines does not intersect each other. So, max z does not exist.

3. The result $P(A \cup B) = P(A) + P(B)$ hold only if A and B are mutually exclusive.

Since, if A and B are mutually exclusive, then $A \cap B = \phi$

$\therefore \quad P(A \cap B) = 0$

and then $P(A \cup B) = P(A) + P(B)$

4. Here, total number of outcomes = 6

Let $P(A)$ = probability of getting $1 = \frac{1}{6}$

$\therefore \quad P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{6} = \frac{5}{6}$

Since, first time 1 occurs at the odd throw.

So, the required probability $= P(A) + P(\bar{A})\,P(\bar{A})\,P(A) + P(\bar{A})\,P(\bar{A})\,P(\bar{A})\,P(\bar{A})\,P(A) + \ldots$

$$= \frac{1}{6} + \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + \ldots$$

$$= \frac{1}{6} + \frac{1}{6}\left(\frac{5}{6}\right)^2 + \frac{1}{6}\left(\frac{5}{6}\right)^4 + \ldots$$

$$= \frac{1/6}{1 - \left(\frac{5}{6}\right)^2} = \frac{6}{11}$$

5. Given, $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix}$$

Now, $\quad A^2 = B \quad$ (given)

$$\Rightarrow \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

On comparing, we get

$\alpha^2 = 1 \Rightarrow \alpha = \pm 1$

and $\quad \alpha + 1 = 5 \Rightarrow \alpha = 5 - 1 = 4$

Since, both equations gives different values of α. So, no real value of α exists.

6. Given, $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

$\Rightarrow x(x^2 - 12) - 3(2x - 14) + 7(12 - 7x) = 0$

$\Rightarrow x^3 - 12x - 6x + 42 + 84 - 49x = 0$

$\Rightarrow x^3 - 67x + 126 = 0$

$\Rightarrow (x - 2)(x^2 + 2x - 63) = 0$

$\Rightarrow (x - 2)(x^2 + 9x - 7x - 63) = 0$

$\Rightarrow\ (x-2)\{x(x+9)-7(x+9)\}=0$

$\Rightarrow\ (x-2)(x+9)(x-7)=0$

$\Rightarrow\ x=2,\ 7,\ -9$

Hence, number of positive roots is 2.

7. Given, $A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

$A^2=A\cdot A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

$=\begin{bmatrix}\cos^2\theta-\sin^2\theta & -\sin\theta\cos\theta-\cos\theta\sin\theta\\ \cos\theta\sin\theta+\sin\theta\cos\theta & -\sin^2\theta+\cos^2\theta\end{bmatrix}$

$=\begin{bmatrix}\cos 2\theta & -2\sin\theta\cos\theta\\ 2\sin\theta\cos\theta & \cos 2\theta\end{bmatrix}$

$=\begin{bmatrix}\cos 2\theta & -\sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{bmatrix}$

Similarly, $A^3=\begin{bmatrix}\cos 3\theta & -\sin 3\theta\\ \sin 3\theta & \cos 3\theta\end{bmatrix}$

8. Let $\Delta=\begin{vmatrix}\sin\alpha & \cos\alpha & \sin(\alpha+\gamma)\\ \sin\beta & \cos\beta & \sin(\beta+\gamma)\\ \sin\delta & \cos\delta & \sin(\gamma+\delta)\end{vmatrix}$

$\Delta=\begin{vmatrix}\sin\alpha & \cos\alpha & \sin\alpha\cos\gamma+\cos\alpha\sin\gamma\\ \sin\beta & \cos\beta & \sin\beta\cos\gamma+\cos\beta\sin\gamma\\ \sin\delta & \cos\delta & \sin\gamma\cos\delta+\cos\gamma\sin\delta\end{vmatrix}$

$[\because \sin(A+B)=\sin A\cos B+\cos A\sin B]$

On applying $C_3\to C_3-\cos\gamma\, C_1-\sin\gamma\, C_2$, we get

$$\Delta=\begin{vmatrix}\sin\alpha & \cos\alpha & 0\\ \sin\beta & \cos\beta & 0\\ \sin\delta & \cos\delta & 0\end{vmatrix}$$

$=0$ $[\because$ all elements in C_3 are zero$]$

9. Let $S_1\equiv x^2+y^2-6x-4y+4=0$

For the point (1, 1)

$S_1=1^2+1^2-6(1)-4(1)+4$

$=1+1-6-4+4$

$=-4<0$

$\Rightarrow\ S_1<0$

$\therefore$ Point (1, 1) lie inside the circle.

Hence, number of real tangents is zero.

10. We know that equation of normal to the parabola $x^2=-4ay$ in the slope form is

$$x=my+2am+am^3$$

and point of contact is $(2am,-am^2)$

So, line $x=my+c$ is normal to $x^2=-4ay$

if $c=2am+am^3$

11. Given equation of ellipse is $16x^2+25y^2=400$

$\Rightarrow\ \dfrac{x^2}{25}+\dfrac{y^2}{16}=1$

Its foci are $P_1(3,0)$, $P_2(-3,0)$

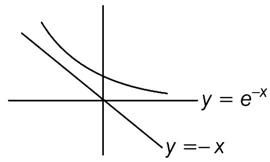
We know that, if $P(x,y)$ is any point on the ellipse, then sum of focal distances of any point on the ellipse is equal to the length of major axis.

$\therefore\ PP_1+PP_2=2a=2\sqrt{25}=10$

12. Given, $A=\{(x,y): y=e^{-x}\}$

and $B=\{(x,y): y=-x\}$

On putting the graph, we get



It is clear from the graph that there is no intersection point of both the graphs.

So, $A\cap B=\phi$

13. Take $f(x)=|x|+(x-2)^2$

When $x>0$, then $f(x)=x+(x-2)^2$

Put $f(x)=0$

$\Rightarrow\ x+x^2+4-4x=0$

$\Rightarrow\ x^2-3x+4=0$

$\Rightarrow\ x=\dfrac{3\pm\sqrt{9-4\times 4}}{2}=\dfrac{3\pm\sqrt{-7}}{2}$

No real zeros.

When $x<0$, then $f(x)=-x+(x-2)^2$

Put $f(x)=0$

$\Rightarrow\ -x+x^2+4-4x=0$

$\Rightarrow\ x^2-4x-x+4=0$

$\Rightarrow\ x(x-4)-1(x-4)=0$

$\Rightarrow\ (x-1)(x-4)=0\ \Rightarrow\ x=1,4$

Hence, set of zeros is non-empty when

$f(x)=|x|+(x-2)^2$

14. Given, $f(x)=(x+2)^2-2=y$ (let)

Then, $(x+2)^2=y+2$

$\Rightarrow\ x+2=\sqrt{y+2}$

$\Rightarrow\ x=\sqrt{y+2}-2$

$\Rightarrow\ f^{-1}(y)=\sqrt{y+2)}-2$

Hence, $f^{-1}(x)=\sqrt{x+2}-2$

15. Given, $R = \{(a, b): 1 + ab > 0\}$

For reflexive

$$a\,R\,a \Rightarrow 1 + a \cdot a > 0$$

$\therefore \quad (a, a) \in R$

$\therefore$ R is reflexive.

For symmetric $(a, b) \in R$

$\Rightarrow \quad 1 + a \cdot b > 0$

$\Rightarrow \quad 1 + b \cdot a > 0$

$\Rightarrow \quad (b, a) \in R$

$\therefore$ R is symmetric.

For transitive

$(a, b) \in R \Rightarrow 1 + a \cdot b > 0$

$(b, c) \in R \Rightarrow 1 + bc > 0$

$\Rightarrow \quad 1 + ac > 0 \qquad (\because a, b, c \in z)$

$\Rightarrow \quad (a, c) \in R$

$\therefore$ R is transitive.

Hence, R is an equivalence relation.

16. Given, $R = \{(l_1\ l_2): l_1$ is perpendicular to $l_2\}$

For reflexive : a line can never be perpendicular to itself.

So, R is not reflexive.

For symmetric : If l_1 is perpendicular to l_2, then l_2 is also perpendicular to l_1. So, R is symmetric.

17. Here, $|\sec x| < 1$ is not true

Since, $\sec x \geq 1$ or $\sec x \leq -1$

18. We have, $\sin 3x + \sin x - 3 \sin 2x$

$$= \cos 3x + \cos x - 3 \cos 2x$$

$$\Rightarrow 2 \sin\left(\frac{3x + x}{2}\right) \cos\left(\frac{3x - x}{2}\right) - 3 \sin 2x$$

$$= 2 \cos\left(\frac{3x + x}{2}\right) \cos\left(\frac{3x - x}{2}\right) - 3 \cos 2x$$

$$\Rightarrow 2 \sin 2x \cos x - 3 \sin 2x - 2 \cos 2x \cos x + 3 \cos 2x = 0$$

$$\Rightarrow \sin 2x (2 \cos x - 3) - \cos 2x (2 \cos x - 3) = 0$$

$$\Rightarrow (\sin 2x - \cos 2x)(2 \cos x - 3) = 0$$

$$\Rightarrow \sin 2x - \cos 2x = 0$$

$$\left(\because 2 \cos x - 3 \neq 0 \Rightarrow \cos x \neq \frac{3}{2}\right)$$

$$\Rightarrow \sin 2x = \cos 2x \quad \Rightarrow \quad 2x = 2n\pi \pm \left(\frac{\pi}{2} - 2x\right)$$

Taking +ve sign, we get

$$2x = 2n\pi + \frac{\pi}{2} - 2x$$

$\Rightarrow$ $x = \dfrac{n\pi}{2} + \dfrac{\pi}{8}$, where n is an integer.

19. $\tan\left[\cos^{-1} \dfrac{3}{5} + \tan^{-1} \dfrac{2}{5}\right]$

$$= \tan\left[\tan^{-1} \frac{\sqrt{1 - \left(\frac{3}{5}\right)^2}}{\frac{3}{5}} + \tan^{-1} \frac{2}{3}\right]$$

$$\left(\because \cos^{-1} x = \tan^{-1} \frac{\sqrt{1 - x^2}}{x}\right)$$

$$= \tan\left[\tan^{-1} \frac{\sqrt{25 - 9}}{5} \times \frac{5}{3} + \tan^{-1} \frac{2}{3}\right]$$

$$= \tan\left[\tan^{-1} \frac{4}{3} + \tan^{-1} \frac{2}{3}\right]$$

$$= \tan\left[\tan^{-1} \left\{\frac{\left(\frac{4}{3} + \frac{2}{3}\right)}{1 - \left(\frac{4}{3}\right)\left(\frac{2}{3}\right)}\right\}\right]^{-1}$$

$$= \tan\left[\tan^{-1} \left\{\frac{6}{3} \times \frac{9}{1}\right\}\right]$$

$$= \tan[\tan^{-1}(18)]$$

$$= 18$$

20. We have, $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$

$$\Rightarrow 6i\,(3i^2 + 3) + 3i\,(4i + 20) + 1\,(12` - 60i) = x + iy$$

$$\Rightarrow 6i\,(-3 + 3) + 12i^2 + 60i + 12 - 60i = x + iy$$

$$(\because i^2 = -1)$$

$$\Rightarrow 0 - 12 + 12 = x + iy$$

$$\Rightarrow 0 + i\,0 = x + iy$$

On comparing, we get

$$x = 0, \quad y = 0$$

21. $\left(\dfrac{1 + i}{1 - i}\right)^n = 1$

$$\Rightarrow \left(\frac{1 + i}{1 - i} \times \frac{1 + i}{1 + i}\right)^n = 1$$

$$\Rightarrow \left\{\frac{(1 + i)^2}{1 - i^2}\right\}^n = 1$$

$$\Rightarrow \left\{\frac{1 + i^2 + 2i}{1 + 1}\right\}^n = 1$$

$$\Rightarrow \left\{\frac{1 - 1 + 2i}{2}\right\}^n = 1$$

$\Rightarrow \quad (i)^n = 1 = i^4$

On comparing, we get

$n = 4$

22. Given, $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$

i.e., $\quad |z_k| = 1, \quad k = 1, 2, 3$

$\Rightarrow \quad |z_k|^2 = 1$

$\Rightarrow \quad z_k \bar{z}_k = 1$

$\Rightarrow \quad \bar{z}_k = \frac{1}{z_k}$

Now, $|z_1 + z_2 + z_3| = |\overline{z_1 + z_2 + z_3}| \quad [\because |z| = |\bar{z}|]$

$= |\bar{z}_1 + \bar{z}_2 + \bar{z}_3|$

$= \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$

$= 1$

23. Given, largest side = 10

Smallest side = 9

So, third side will be less than 10 and greater than 9. From options only $\sqrt{91}$ satisfies this condition.

Hence, option (a) is correct.

24. Let the infinite geometric series is

$a, ar, ar^2, ar^3 \ldots\ldots$

where, a = first term

and $\quad r$ = common ratio

Given, second term $= ar = \frac{3}{4}$

$\Rightarrow \quad r = \frac{3}{4a} \quad \ldots(i)$

and sum = 4

$\Rightarrow \quad \frac{a}{1-r} = 4$

$\Rightarrow \quad \frac{a}{1 - \frac{3}{4a}} = 4$

$\Rightarrow \quad \frac{4a^2}{4a - 3} = 4$

$\Rightarrow \quad a^2 = 4a - 3$

$\Rightarrow \quad a^2 - 4a + 3 = 0$

$\Rightarrow \quad (a-3)(a-1) = 0$

$\Rightarrow \quad a = 3, 1$

when $a = 3$, then from Eq. (i), we get

$r = \frac{3}{4 \times 3} = \frac{1}{4}$

when $a = 1$, then from Eq. (i),

$r = \frac{3}{4}$

25. Given, $pq = 1 \quad \Rightarrow \quad q = \frac{1}{p} \quad \ldots(i)$

Let $y = (1 + p)(1 + q) = 1 + p + q + pq$

$= 1 + p + \frac{1}{p} + 1 \quad$ [from Eq. (i)]

$\Rightarrow \quad y = 2 + p + \frac{1}{p}$

Now, we know that

AM $\geq$ GM

$\Rightarrow \quad \frac{p + \frac{1}{p}}{2} \geq p \cdot \frac{1}{p}$

$\Rightarrow \quad p + \frac{1}{p} \geq 2$

$\therefore$ Least value of $y = 2 + 2 = 4$

26. Let $f(x) = 49^n + 16n + p$

$= (1 + 48)^n + 16n + p$

$= 1 + {}^nC_1 (48) + {}^nC_2 (48)^2 + \ldots + {}^nC_n (48)^n + 16n + p$

$= (1 + p) + 48n + 16n + {}^nC_2 (48)^2 + {}^nC_3 (48)^3 + \ldots + {}^nC_n (48)^n$

$= (1 + p) + 64n + (48)^2 [{}^nC_2\, 6^2 + {}^nC_3\, 6^3 \cdot 8 + \ldots + {}^nC_n\, 6^n\, 8^{n-2}]$

Now, $f(x)$ will be divisible by 64 if

$1 + p = 0$

$\Rightarrow \quad p = -1$

27. In the expansion of $\left(ax + \frac{1}{x}\right)^n$, 4th term

$= {}^nC_3 (ax)^{n-3} \left(\frac{1}{x}\right)^3$

$= {}^nC_3\, a^{n-3}\, x^{n-6}$

But 4th term $= \frac{5}{2}$ (given)

$\therefore \quad {}^nC_3\, a^{n-3}\, x^{n-6} = \frac{5}{2}$

$\Rightarrow \quad n - 6 = 0$

$\Rightarrow \quad n = 6$

Now, $\quad {}^6C_3\, a^3 = \frac{5}{2}$

$\Rightarrow \quad a^3 = \frac{5}{2} \times \frac{1}{20} = \frac{1}{8}$

$\Rightarrow \quad a = \frac{1}{2}$

28. In the expansion of $(1+x)^n$,

Second term, $T_2 = nx$

Third term, $T_3 = \frac{n(n-1)}{2!}x^2$

and fourth term, $T_4 = \frac{n(n-1)(n-2)}{3!}x^3$

Given, coefficients of second, third and fourth terms are in AP

$$2\left(\frac{n(n-1)}{2!}\right) = n + \frac{n(n-1)(n-2)}{3!}$$

($\because$ a, b and c are in AP, then $2b = ac$)

$\Rightarrow$ $n(n-1) = n + \frac{(n^2-n)(n-2)}{6}$

$\Rightarrow$ $6n^2 - 6n = 6n + n^3 - 2n^2 - n^2 + 2n$

$\Rightarrow$ $n^3 - 9n^2 + 14n = 0$

$\Rightarrow$ $n(n^2 - 9n + 14) = 0$

$\Rightarrow$ $n(n^2 - 7n - 2n + 14) = 0$

$\Rightarrow$ $n(n-7)(n-2) = 0$

$\Rightarrow$ $n = 0, 2, 7$

Here, $n \neq 0$ and $n \neq 2$ since there are atleast four terms in the expansion of $(1+x)^n$.

Hence, $n = 7$

29. + + + + +

On fixing five positive signs, there are 6 places for negative signs.

$\therefore$ Required number of ways $= {}^6C_3 = 20$

30. Here, number of white balls = 2

Number of black balls = 3

Number of red balls = 4

Since, 3 balls are drawn

So, required number of ways

$= {}^6C_2 \times {}^3C_1 + {}^6C_1 \times {}^3C_2 + {}^6C_0 \times {}^3C_3$

$= \frac{6\times 5}{2} \times 3 + 6 \times 3 + 1$

$= 45 + 18 + 1$

$= 64$

31. Given, points are $P = (-1, 0)$, $O = (0, 0)$ and $Q = (3, 3\sqrt{3})$

Equation of line PO is

$$y - 0 = \frac{0-0}{0+1}(x-1)$$

$\Rightarrow$ $y = 0$

and equation of line OQ is

$$y - 0 = \frac{3\sqrt{3}-0}{3-0}(x-0)$$

$\Rightarrow$ $y = \sqrt{3}\,x$

$\Rightarrow$ $y - \sqrt{3}\,x = 0$

Now, equation of bisector of the $\angle POQ$ is

$$\frac{y}{\sqrt{1}} = \pm\left(\frac{y-\sqrt{3}\,x}{\sqrt{1+(-\sqrt{3})^2}}\right)$$

$$\frac{y}{\sqrt{1}} = \pm\left(\frac{y-\sqrt{3}x}{2}\right)$$

$\Rightarrow$ $y = \pm\left(\frac{y-\sqrt{3}x}{2}\right)$

$\Rightarrow$ $2y = \pm(y - \sqrt{3}\,x)$

Taking +ve sign, we get

$2y = y - \sqrt{3}\,x$

$\Rightarrow$ $y = -\sqrt{3}\,x$

Taking –ve sign, we get

$2y = -y - \sqrt{3}\,x$

$\Rightarrow$ $3y = -\sqrt{3}\,x$

Hence, the required equation bisector is

$y = -\sqrt{3}\,x$

32. Let radius of the required circle is r.

Given, area = 154

$\Rightarrow$ $\pi r^2 = 154$

$\Rightarrow$ $r^2 = \frac{154 \times 7}{22} = 49$

Also given, diameters are

$2x - 3y = 5$ ...(i)

$3x - 4y = 7$ ...(ii)

We know that intersection point of diameters gives the centre of the circle. So, on multiplying Eq. (i) by 3 and (ii) by 2 and then subtracting, we get

$-y = 1 \Rightarrow y = -1$

From Eq. (i), we get

$2x = 5 + 3(-1) = 2$

$\Rightarrow$ $x = 1$

$\therefore$ Centre of the circle is $(1, -1)$

Hence, equation of the required circle is

$(x-1)^2 + (y+1)^2 = 49$

$\Rightarrow$ $x^2 + 1 - 2x + y^2 + 1 + 2y = 49$

$\Rightarrow$ $x^2 + y^2 - 2x + 2y - 47 = 0$

33. We know that the equation of normal to the parabola $y^2 = -4ax$ is

$$y = mx + 2am + am^3 \quad \ldots\text{(i)}$$

Here, $4a = 8 \Rightarrow a = \frac{8}{4} = 2$

and equation of normal is

$$2x + y + k = 0$$

$$\Rightarrow y = -2x - k \quad \ldots\text{(ii)}$$

On comparing Eqs. (i) and (ii), we get

$$m = -2$$

and $-k = 2am + am^3$

$$\Rightarrow -k = 2(2)(-2) + 2(-2)^3$$

$$\Rightarrow -k = -8 - 16$$

$$\Rightarrow k = 24$$

34. Given, $x = a(\sin t - \cos t)$

$$\Rightarrow \frac{x}{a} = \sin t - \cos t \quad \ldots\text{(i)}$$

and $y = b(\sin t + \cos t)$

$$\Rightarrow \frac{y}{b} = \sin t + \cos t \quad \ldots\text{(ii)}$$

On squaring Eqs. (i) and (ii) and then adding, we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = (\sin t - \cos t)^2 + (\sin t + \cos t)^2$$

$$= \sin^2 t + \cos^2 t - 2\sin t \cos t + \sin^2 t + \cos^2 t + 2\sin t \cos t$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2 \Rightarrow \frac{x^2}{2a^2} + \frac{y^2}{2b^2} = 1$$

which represents the equation of an ellipse.

35. Given hyperbola is

$$9x^2 - 16y^2 + 72x - 32y - 16 = 0$$

$$\Rightarrow 9(x^2 - 8x) - 16(y^2 + 2y + 1) = 0$$

$$\Rightarrow 9(x^2 - 8x + 16 - 16) - 16(y + 1)^2 = 0$$

$$\Rightarrow 9(x - 4)^2 - 9 \times 16 - 16(y + 1)^2 = 0$$

$$\Rightarrow 9(x - 4)^2 - 16(y + 1)^2 = 144$$

$$\Rightarrow \frac{(x-4)^2}{\frac{144}{9}} - \frac{(y+1)^2}{144/16} = 1$$

$$\Rightarrow \frac{(x-4)^2}{16} - \frac{(y+1)^2}{9} = 1$$

Here, $a^2 = 16, \quad b^2 = 9$

$$\therefore e = \sqrt{\frac{b^2}{a^2} + 1} = \sqrt{\frac{9}{16} + 1}$$

$$= \sqrt{\frac{25}{16}} = \frac{5}{4}$$

36. Given equation of line is

$$x - 4 = y - 2 = \frac{1}{2}(z - \lambda) \quad \ldots\text{(i)}$$

$$\Rightarrow \frac{x-4}{1} = \frac{y-2}{1} = \frac{z-\lambda}{2} = r \quad \text{(say)}$$

$$\Rightarrow x = r + 4, \quad y = r + 2, \quad z = 2r + \lambda$$

and equation of plane is

$$2x - 4y + z = 7 \quad \ldots\text{(ii)}$$

Since, line (i) lie on the plane (ii), so the point $(r + 4, r + 2, 2r + \lambda)$ will satisfy the equation of plane

$$\therefore 2(r + 4) - 4(r + 2) + 2r + \lambda = 7$$

$$\Rightarrow 2r + 8 - 4r - 8 + 2r + \lambda = 7$$

$$\Rightarrow \lambda = 7$$

37. Given, $f(x) = \frac{\log(1 + ax) - \log(1 - bx)}{x}$

Now, $\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\log(1 + ax) - \log(1 - bx)}{x}$ $\left[\frac{0}{0} \text{ form}\right]$

$$= \lim_{x \to 0} \frac{\frac{1}{1+ax} \cdot a - \frac{1}{1-bx}(-b)}{1}$$

[by L-Hospital rule]

$$= \lim_{x \to 0} \frac{a}{1+ax} + \frac{b}{1-bx}$$

$$= \frac{a}{1+0} + \frac{b}{1-0}$$

$$= a + b$$

Hence, the value $a + b$ should be assigned to f at $x = 0$, so that it becomes continuous at $x = 0$.

38. Given, $f(1) = 3, \quad f'(1) = 6$

$$\lim_{x \to 0} \left[\frac{f(1+x)}{f(1)}\right]^{1/x} = e^{\lim_{x \to 0} \frac{1}{x}[\log f(1+x) - \log f(1)]}$$

$$= e^{\frac{\lim_{x \to 0} f'(1+x)/f(1+x)}{1}}$$

$$= e^{f'(1)/f(1)}$$

$$= e^{6/3} = e^2$$

39. $\lim_{x \to 0} \frac{f(x^2) - f(x)}{f(x) - f(0)}$

$$= \lim_{x \to 0} \frac{f'(x^2) \cdot 2x - f'(x)}{f'(x)}$$

$$= \frac{0 - f'(x)}{f'(x)} = -1$$

40. Given equation of parabola is $y^2 = 4x$

Here, $4a = 4 \Rightarrow a = 1$

Equation of tangent at (x, y) is

$$y = mx + \frac{a}{m}$$

Here, $(x, y) = (1, 4)$

$\therefore \quad 4 = m + \frac{1}{m}$

$\Rightarrow \quad m^2 - 4m + 1 = 0$

$\Rightarrow \quad m = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$

Let $m_1 = 2 + \sqrt{3}, \quad m_2 = 2 - \sqrt{3}$

Let angle between the tangents is θ,

Then, $\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$

$= \frac{2 + \sqrt{3} - 2 + \sqrt{3}}{1 + (2+\sqrt{3})(2-\sqrt{3})}$

$= \frac{2\sqrt{3}}{1 + 4 - 3}$

$\Rightarrow \quad \tan\theta = \sqrt{3} = \tan\frac{\pi}{3}$

$\therefore \quad \theta = \frac{\pi}{3}$

41. Given, $f(x) = \sin 2x - x$

$f'(x) = 2\cos 2x - 1$

$f''(x) = -4\sin 2x$

For maxima or minima, put $f'(x) = 0$

$\Rightarrow \quad 2\cos 2x - 1 = 0$

$\Rightarrow \quad \cos 2x = \frac{1}{2} = \cos\frac{\pi}{3}$

$\Rightarrow \quad \cos 2x = 2n\pi \pm \frac{\pi}{3}$

$\Rightarrow \quad x = n\pi \pm \frac{\pi}{6}$

For $n = 0, \quad x = \pm\frac{\pi}{6}$

Now, at $x = \frac{\pi}{6}$,

$f''(x) = -4 \times \frac{\sqrt{3}}{2} < 0 \qquad \therefore$ maxima

at $x = -\frac{\pi}{6}$,

$f''(x) = 4 \times \frac{\sqrt{3}}{2} > 0 \qquad \therefore$ minima

Now, $f\left(\frac{\pi}{6}\right) = \sin 2\left(\frac{\pi}{6}\right) - \frac{\pi}{6} = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$

$f'\left(-\frac{\pi}{6}\right) = \sin 2\left(-\frac{\pi}{6}\right) + \frac{\pi}{6} = -\frac{\sqrt{3}}{2} + \frac{\pi}{6}$

Thus, greatest value of $f(x) = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$

and least value of $f(x) = -\frac{\sqrt{3}}{2} + \frac{\pi}{6}$

Hence, required difference

$= \left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right) - \left(-\frac{\sqrt{3}}{2} + \frac{\pi}{6}\right)$

$= \frac{2\sqrt{3}}{2} - \frac{2\pi}{6}$

$= \sqrt{3} - \frac{\pi}{3}$

42. Let $I = \int \frac{(x^2 - 1)}{x^3\sqrt{2x^4 - 2x^2 + 1}}\,dx$

$= \int \frac{\left(\frac{1}{x^3} - \frac{1}{x^5}\right)}{\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}}}\,dx$

Put $2 - \frac{2}{x^2} + \frac{1}{x^4} = t$

$\Rightarrow \quad \left(\frac{4}{x^3} - \frac{4}{x^5}\right)dx = dt$

$\therefore \quad I = \frac{1}{4}\int\frac{dt}{\sqrt{t}} = \frac{1}{4}\frac{t^{1/2}}{1/2} + C$

$= \frac{1}{2}\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}} + C$

43. Let $I = \int_{-\pi}^{\pi} \sin x\,[f(\cos x)]\,dx \qquad \ldots(i)$

$= \int_{-\pi}^{\pi} \sin(-x)\,[f\cos(-x)]\,dx$

$\Rightarrow \quad I = -\int_{-\pi}^{\pi} \sin x\,[f(\cos x)]\,dx \qquad \ldots(ii)$

On adding Eqs. (i) and (ii), we get

$2I = 0$

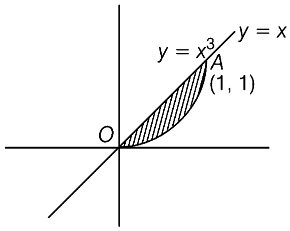
$\Rightarrow \quad I = 0$

44. Given curves are $y = x \qquad \ldots(i)$

$y = x^3 \qquad \ldots(ii)$

Intersection points of Eqs. (i) and (ii) are

$(0, 0), \ (1, 1)$ and $(-1, -1)$



Required area $= \int_0^1 [x - x^3]\, dx$

$$= \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$$

$$= \frac{1}{2} - \frac{1}{4} - 0 = \frac{1}{4}$$

45. Required area $= \int_0^1 (e^x - e^{-x})\, dx$

$$= [e^x + e^{-x}]_0^1$$

$$= e^1 + e^{-1} - e^0 - e^{-0}$$

$$= e + e^{-1} - 2$$

46. The general equation of non-vertical lines is

$$ax + by = 1$$

On differentiating w.r.t. x, we get

$$a + b\frac{dy}{dx} = 0$$

Again differentiating w.r.t. x, we get

$$\frac{bd^2y}{dx^2} = 0$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 0$$

47. Let $y = 2x - 4$ be a solution of the differential equation

$$\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) + y = 0$$

Now, we cross check this solution

LHS $\quad \left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) + y$

$$= \left\{\frac{d}{dx}(2x-4)\right\}^2 - x\left\{\frac{d}{dx}(2x-4)\right\} + (2x-4)$$

$$= (2)^2 - x(2) + 2x - 4$$

$$= 4 - 2x + 2x - 4$$

$= 0 =$ RHS

$\therefore$ $y = 2x - 4$ be a required solution.

48. Given, $|\mathbf{a}| = |\mathbf{b}| = 1$

and $\quad (\mathbf{a} + 3\mathbf{b}) \perp 7\mathbf{a} - 5\mathbf{b}$

$\therefore \quad (\mathbf{a} + 3\mathbf{b})\cdot(7\mathbf{a} - 5\mathbf{b}) = 0$

$\Rightarrow \quad 7|\mathbf{a}|^2 - 5\mathbf{a}\cdot\mathbf{b} + 21\mathbf{b}\cdot\mathbf{a} - 15|\mathbf{b}|^2 = 0$

$\Rightarrow \quad 7 - 5\mathbf{a}\cdot\mathbf{b} + 21\mathbf{a}\cdot\mathbf{b} - 15 = 0$

$[\because |\mathbf{a}|^2 = |\mathbf{b}|^2 = 1 \text{ and } \mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a}]$

$\Rightarrow \quad -8 + 16\mathbf{a}\cdot\mathbf{b} = 0$

$\Rightarrow \quad \mathbf{a}\cdot\mathbf{b} = \frac{8}{16} = \frac{1}{2}$

$\Rightarrow \quad |\mathbf{a}||\mathbf{b}|\cos\theta = \frac{1}{2}$

$\Rightarrow \quad \cos\theta = \frac{1}{2} = \cos\frac{\pi}{3} \quad [\because |\mathbf{a}| = |\mathbf{b}| = 1]$

$\therefore \quad \theta = \frac{\pi}{3}$

49. Let the required vector be $\mathbf{a}$. Then, $\mathbf{i} + \mathbf{j}$, $\mathbf{i} - \mathbf{j}$ and $\mathbf{a}$ form a right handed system

$\therefore \quad \mathbf{a} = (\mathbf{i} + \mathbf{j}) \times (\mathbf{i} - \mathbf{j})$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix}$$

$$= \mathbf{i}(0-0) - \mathbf{j}(0-0) + \mathbf{k}(-1-1)$$

$$= -2\mathbf{k}$$

Hence, the required unit vector is

$$\hat{\mathbf{a}} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{-2\mathbf{k}}{|-2\mathbf{k}|} = \frac{-2\mathbf{k}}{\sqrt{4}} = -\mathbf{k}$$

50. Since, $\mathbf{v}$ is the coplanar to $\mathbf{a}$ and $\mathbf{b}$

$\therefore \quad \mathbf{v} = \mathbf{a} + t\,\mathbf{b}^{-1}$

$$= (\mathbf{i} + \mathbf{j} + \mathbf{k}) + t(\mathbf{i} - \mathbf{j} + \mathbf{k})$$

$\Rightarrow \quad \mathbf{r} = (1+t)\mathbf{i} + (1-t)\mathbf{j} + (1+t)\mathbf{k} \quad \ldots$(i)

The projection of $\mathbf{r}$ on $\mathbf{c} = \frac{1}{\sqrt{3}}$ (given)

$\Rightarrow \quad \frac{\mathbf{v}\cdot\mathbf{c}}{|\mathbf{c}|} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \frac{|(1+t)1 - 1(1-t) - 1(1+t)|}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad 1 + t - 1 + t - 1 - t = 1$

$\Rightarrow \quad t = 2$

On putting the value of t in Eq. (i), we get

$$\mathbf{r} = 3\mathbf{i} + (-1)\mathbf{j} + (3)\mathbf{k}$$

$\Rightarrow \quad \mathbf{v} = 3\mathbf{i} - \mathbf{j} + 3\mathbf{k}$