AMU

Engineering Entrance Exam

Solved Paper 2012

Physics

1. A heating coil is used to heat water in a container from 15°C to 50°C in 20 min. Two such coils are then joined in series to heat the same amount of water for the same temperature difference from the same constant voltage source. The time taken now is

(a) 5 min (b) 10 min
(c) 20 min (d) 40 min

2. Vector **A** has a magnitude of 5 units, lies in the *xy*- plane and points in a direction 120° from the direction of increasing *x*. Vector **B** has a magnitude of 9 units and points along the *z*-axis. The magnitude of cross product $\mathbf{A} \times \mathbf{B}$ is

(a) 30 (b) 35
(c) 40 (d) 45

3. What is approximately the centripetal acceleration (in units of acceleration due to gravity on earth $g = 10\ \text{m/s}^2$) of an air craft flying at a speed of 400 m/s through a circular arc of radius 0.6 km?

(a) 26.7 (b) 16.9
(c) 13.5 (d) 30.2

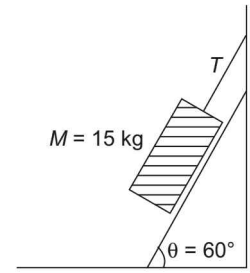
4. A parallel plate capacitor is charged to potential difference of 50 V. It is then discharged through a resistance for 2 s and its potential drops by 10 V. Calculate the fraction of energy stored in the capacitance.

(a) 0.14 (b) 0.25
(c) 0.50 (d) 0.64

5. A particle of mass 200 g is making SHM under the influence of a spring of force constant $k = 90$ N/m and a damping constant $b = 40$ g/s Calculate the time elapsed for the amplitude to drop to half its initial value
[Given ln (1/2) = 0.693]

(a) 7s (b) 9s
(c) 4s (d)11s

6. A block of mass 15 kg is held by a string on an inclined plane (angle 60°). The tension T in the string is ($g = 10\ \text{m/s}^2$)



(a) 55 N (b) 60 N
(c) 75 N (d) 90 N

7. A cosmonaut is circling the earth in a satellite at 7 km/s at a height of 630 km above the surface of earth. Calculate the centripetal force acting on the cosmonaut if his mass is 80 kg (take $R_E = 6.37 \times 10^6\text{m}$)

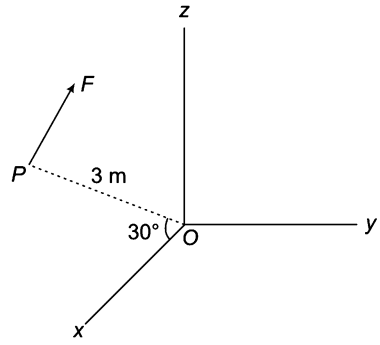
(a) zero (b) 560 N
(c) 600 N (d) 650 N

8. Two identical sinusoidal waves each of amplitude 5 mm with a phase difference of $\pi/2$ are travelling in the same direction in a string. The amplitude of the resultant wave (in mm) is

(a) zero (b) $5\sqrt{2}$
(c) $5/\sqrt{2}$ (d) 2.5

9. The rotor's velocity of a helicopter engine changes from 330 rpm to 110 rpm in two min. How long does the rotor blades take to stop?
(a) 3 min (b) 4 min
(c) 5 min (d) 6 min

10. A force $F = 2.0$ N acts on a particle P in the xz-plane. The force F is parallel to x-axis. The particle P (as shown in the figure) is at a distance 3 m and the line joining P with the origin makes angle 30° with the x-axis. The magnitude of torque on P with respect to origin O (in N-m) is



(a) 2 N-m (b) 3 N-m
(c) 4 N-m (d) 5 N-m

11. An aircraft has a mass 4×10^5 kg with total wing area 50 m^2 flying at a speed of 720 km/h. The density of air at it height is 1.2 kg/m^3. Estimate the fractional increase in air speed on the upper surface of its wings relative to lower surface.
(a) 0.04 (b) 0.08
(c) 0.17 (d) 0.32

12. A boat goes across a river with a velocity 12km/h. The magnitude of its resultant speed in flowing water is 13 km/h. The velocity of water flow in the river is
(a) 5 km/h (b) 7 km/h
(c) 9 km/h (d) 1 km/h

13. A steel rod has a radius $R = 9.5$ mm and length $L = 81$ cm. A force $F = 6.2 \times 10^4$ N stretches it along its length. What is the stress in the rod?
(a) 0.95×10^8 N/m^2
(b) 1.1×10^8 N/m^2
(c) 2.2×10^8 N/m^2
(d) 3.2×10^8 N/m^2

14. Calculate the acceleration due to gravity on the surface of a pulsar of mass $M = 1.98 \times 10^{30}$ kg and radius $R = 12$ km rotating with time period $T = 0.041$ s. ($G = 6.67 \times 10^{11}$ MKS)
(a) 9.2×10^{11} m/s^2 (b) 8.15×10^{11} m/s^2
(c) 7.32×10^{11} m/s^2 (d) 6.98×10^{11} m/s^2

15. A cord is wound over the rim of a flywheel of mass 20 kg and radius 25 cm. A mass 2.5 kg attached to the cord is allowed to fall under gravity. Calculate the angular acceleration of the flywheel.
(a) 25 rad/s^2 (b) 20 rad/s^2
(c)10 rad/s^2 (d) 5 rad/s^2

16. A shell of mass 200 g is fired by a gun of mass 100 kg. If the muzzle speed of the shell is 80 m/s. Calculate the recoil speed of the gun.
(a) 16 cm/s (b) 8 m/s
(c) 4 cm/s (d) 16 m/s

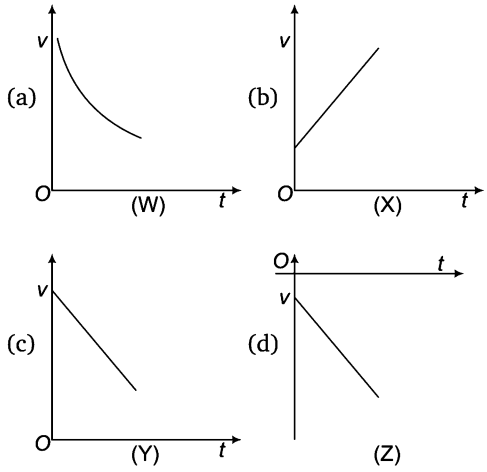
17. A body of mass 0.4 kg starting at origin at $t = 0$ with a speed of 10 m/s in the positive x-axis direction is subjected to a constant force $F = 8$ N towards negative x-axis. Calculate the position of the particle after 25 s.
(a) –6000 m (b) –8000 m
(c) +4000 m (d) +7000 m

18. Hot food cools from 94° C to 86°C in 2 min when the room temperature is 20° C. How long would the food take to cool from 71° C to 69°C?
(a) 12 s (b) 25 s
(c) 16 s (d) 42 s

19. A police car moving on a highway at 30 km/h fires a bullet on a speeding car of thieves at 150 km/h. If the muzzle speed of bullet is 150 m/s, calculate the speed with which the bullet strikes the car of the thieves.
(a) 95 m/s (b) 105 m/s
(c) 116 m/s (d) 192 m/s

20. Two trains A and B each of length 400 m are moving on two parallel tracks in the same direction (with A ahead of B) with same speed 72 km/h. The driver of B decides to overtake A and accelerates by 1 m/s^2. If after 50 s, B just brushes past A, calculate the original distance between A and B
(a) 750 m (b) 1000 m
(c) 1250 m (d) 2250 m

21. Given below are four curves describing variation of velocity with time of a particle. Which one of these describe the motion of a particle initially in positive direction with constant negative acceleration?



22. If mass is measure in units of α kg, length in β m and time in γ s then calorie would be

(a) $4.2\alpha\,\beta^2\gamma^{-2}$

(b) $4.2\alpha^{-1}\,\beta^2\gamma^2$

(c) $4.2\alpha^{-1}\,\beta^{-2}\gamma^2$

(d) $4.2\alpha^{-2}\,\beta^{-1}\gamma^{-2}$

23. The distance travelled s by an accelerated particle of mass M is given by the following relation (in MKS units) $s = 6t + 3t^2$

(a) 6 (b) 12

(c) 18 (d) 24

24. A new unit of length is so chosen that the speed of light in vacuum is unity. Calculate the distance (in this new unit) between the sun and the earth if light takes 8 min and 20 s to reach earth from sun.

(a) 300 (b) 400

(c) 500 (d) 600

25. Calculate the neutron separation energy from the following data:

$m(^{40}_{20}\text{Ca}) = 39.962591$ u,

$m(^{41}_{20}\text{Ca}) = 40.962278$ u,

$m_n = 1.00865$, 1u = 931.5 MeV/c^2

(a) 7.57 MeV (b) 8.36 MeV

(c) 9.12 MeV (d) 9.56 MeV

26. The following fusion reaction takes place

$$^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + n + 3.27 \text{ MeV}$$

If 2 kg of deuterium is subjected to above reaction, the energy released is used to light a 100 W lamp. How long will the lamp glow?

(a) 2×10^6 year (b) 3×10^5 year

(c) 5×10^4 year (d) 7×10^3 year

27. Calculate the highest frequency of the emitted photon in the Paschen series of spectral lines of the hydrogen atom

(a) 3.7×10^{14}Hz (b) 9.1×10^{15}Hz

(c) 10.23×10^{14}Hz (d) 29.7×10^{15}Hz

28. A block of mass 500 g is connected to a spring of spring constant $k = 312.4$ N/m on a frictionless table. The spring is held firmly at the other end. The block is pulled a distance of 5 cm and then released to make SHM. Calculate the time period of its oscillations.

(a) 2.0 s (b) 1.75 s

(c) 0.5 s (d) 0.25 s

29. The image of an electric bulb fixed in a wall is to be obtained on the wall opposite to it at a distance of 3 m. The maximum possible focal length of the convex lens is

(a) 3.25 m (b) 1.55 m

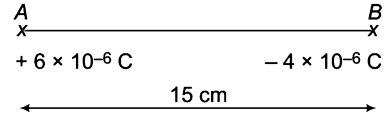
(c) 0.75 m (d) 0.25 m

30. A monochromatic source of light emits photons of frequency 6×10^{14} Hz. The power emitted by the source is 8×10^{-3}W. Calculate the number of photons emitted per second (Take $h = 6.63\times10^{34}$J - s)

(a) 6×10^{14} (b) 4×10^{15}

(c) 2×10^{16} (d) 1×10^{17}

31. Two charges +6 μC and − 4 μC are placed apart as shown. At what distances from A to its right, the electrostatic potential is zero
(distances in cm)



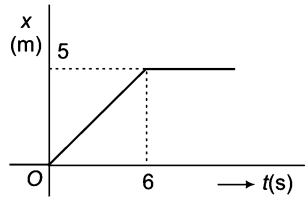
(a) 4, 9, and 60

(b) 9, 45, infinity

(c) 20, 45, infinity

(d) 9, 15, 45

32. The position-time graph of a particle of mass 4 kg is shown in the figure. Calculate the impulse (in MKS units) at time $t = 0$ and $t = 6$ s respectively



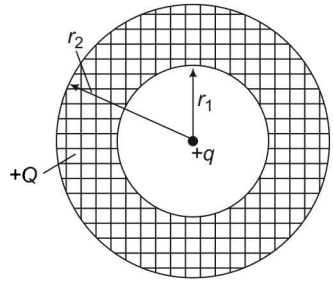
(a) +6.31 and – 6.31
(b) +3.33 and –3.33
(c) +5.25 and – 5.25
(d) +3.25 and –3.25

33. Pulse rate of a normal person is 75 per minute. The time period of heart is
(a) 0.8 s (b) 0.75 s
(c) 1.25 s (d) 1.75 s

34. Calculate the linear momentum of a 3 MeV photon
(a) 0.01 eV-s/m (b) 0.02 eV-s/m
(c) 0.03 eV-s/m (d) 0.04 eV-s/m

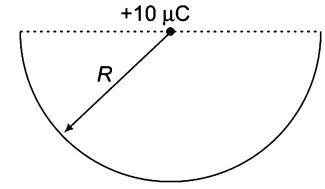
35. The far point of a myopic person is 80 cm in front of his eyes. The power of the lens required to see very distant objects is
(a) – 1.25 D (b) +0.85 D
(c) + 1.50 D (d) – 0.75 D

36. A thick metallic spherical shell of inner radius r_1 and outer radius r_2 has a charge $+Q$. A charge $+q$ is placed at the centre of the shell. The charge per unit area on the outer surface is



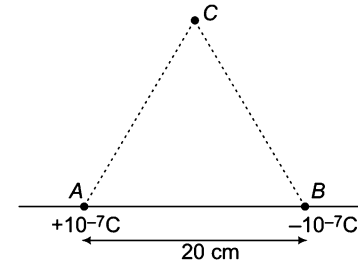
(a) $\frac{(Q-q)}{4\pi(r_2^2 - r_1^2)}$ (b) $\frac{(Q-q)}{4\pi\, r_2^2}$
(c) $\frac{(Q+q)}{4\,\pi\, r_2^2}$ (d) $\frac{(Q+q)}{4\,\pi\,(r_2^2 + r_2^2)}$

37. A charge 10 μC is placed at the centre of a hemisphere of radius $R = 10$ cm as shown.The electric flux through the hemisphere (in MKS units) is



(a) 20×10^5 (b) 10×10^5
(c) 6×10^5 (d) 2×10^5

38. Two point charges $+10^{-7}$ C and -10^{-7} C are placed at A and B, 20 cm apart as shown in the figure. Calculate the electric field at C, 20 cm apart form both A and B.



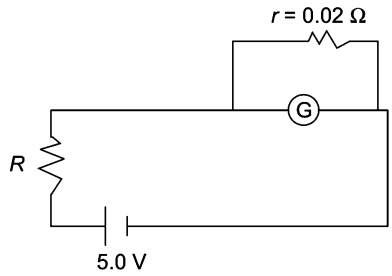
(a) 1.5×10^{-5} N/C (b) 2.2×10^4 N/C
(c) 3.5×10^6 N/C (d) 3.0×10^5 N/C

39. A particle is thrown vertically up with an initial velocity 9 m/s form the surfaces of earth (take $g = 20$ m/ s^2). The time taken by the particle to reach a height of 4 m from the surface second time (in second) is
(a) 1.3 (b) 1.2
(c) 1.1 (d) 1.0

40. An electric circuit requires a total capacitance of 2 μF across a potential of 1000 V. Large number of 1 μF capacitances are available each of which would breakdown if the potential is more than 350 V. How many capacitances are required to make the circuit?
(a) 24 (b) 20
(c) 18 (d) 12

41. A battery of emf 8V with internal resistance 0.5 is being charged by a 120 V DC supply using a series resistance of 15.5 Ω. The terminal voltage of the battery is
(a) 20.5 V (b) 15.5V
(c) 11.5 V (d) 2.5 V

42. In the circuit shown, the galvanometer G of resistance 60 Ω is shunted by a resistance $r = 0.02\,\Omega$. The current through R is nearly 1A. The value of resistance R is (in ohm) is nearly



(a) 1.00 Ω (b) 5.00 Ω
(c) 11.00 Ω (d) 60.00 Ω

43. A circular coil of radius $R = 10$ cm having 500 turns and total resistance 2 Ω is placed initially perpendicular to the earth's magnetic field of 3×10^{-5} T. The coil is rotated about its vertical diameter by an angle 2π in 0.5 s. The induced current in the coil is

(a) 0.5 mA (b) 1.0 mA
(c) 1.5 mA (d) 3.0 mA

44. An electromagnetic wave is propagating along x-axis. At $x = 1$ m and $t = 10$ s, its electric vector $|\mathbf{E}| = 6$ V/m , then the magnitude of its magnetic vector is

(a) 2×10^{-8} (b) 3×10^{-7}
(c) 6×10^{-8} (d) 5×10^{-7}

45. A charged capacitor $C = 30\,\mu$F is connected to an inductor $L = 27$ mH. The angular frequency of their oscillations is

(a) 9.0×10^{3} (b) 3.0×10^{3}
(c) 1.1×10^{3} (d) 0.3×10^{3}

46. A 4.5 cm object is placed perpendicular to the axis of a convex mirror of focal length 15 cm at a distance of 12 cm. The size of the image is

(a) 6.0 cm (b) 4.5 cm
(c) 3.0 cm (d) 2.5 cm

47. The energy of a photon of wavelength 390 nm is nearly

(a) 6.6 eV (b) 3.2 eV
(c) 5.5 eV (d) 1.2 eV

48. The angle between the vectors: $\mathbf{a} = 3\mathbf{j} - 4\mathbf{j}$ and $\mathbf{b} = -2\mathbf{i} + 3\mathbf{k}$ is

(a) $\cos^{-1}(-1/3)$ (b) $\cos^{-1}(-1/4)$
(c) $\cos^{-1}(-1/2)$ (d) $\cos^{-1}(-1/6)$

49. Alpha particles of kinetic energy 7.7 MeV are being scattered by the nucleus of gold which has 79 electrons. The distance of closest approach of the alpha particles is

(Take $\frac{1}{4\pi\varepsilon_o} = 9 \times 10^{9}$ MKS)

(a) 4×10^{-14} m
(b) 30×10^{-15} m
(c) 10×10^{-15} m
(d) 7.9×10^{-14} m

50. In a Bipolar Junction Transistor (BJT), the current gain β is defined is

(a) the ratio of change in collector current to the change in emitter current for a constant collector voltage in the common base configuration
(b) the ratio of change in emitter current to the change in base current for constant emitter voltage in common emitter configuration
(c) the ratio of change in collector current to the change in base current for constant collector voltage in collector emitter configuration
(d) the ratio of change in base current to the change in collector current for constant collector voltage in collector emitter configuration

Chemistry

1. Which of the following orbitals has zero probability of finding the electron in the *xy*-plane?
(a) p_x (b) p_z (c) d_{yz} (d) $d_{x^2-y^2}$

2. 1M solution each of $Cu(NO_3)_2$, $AgNO_3$, $Hg_2(NO_3)_2$ and $Mg(NO_3)_2$ is electrolysed using Pt-electrodes. The values of standard electrode potentials in volts are

$Ag^+/Ag = +0.80$ V, $Cu^{2+}/Cu = +0.34$ V,
$Hg_2^{2+}/Hg = +0.79$ V, $Mg^{2+}/Mg = -2.37$ V

The sequence of deposition of metals on the cathode will be
(a) Mg, Ag, Cu (b) Mg, Cu, Ag
(c) Ag, Hg, Cu (d) Cu, Hg, Ag

3. Which of the following acids is stronger than benzoic acid ($K_a = 6.3 \times 10^{-5}$)?
(a) $AK_a = 1.67 \times 10^{-8}$ (b) B ($pK_a = 6.0$)
(c) C ($pK_a = 4.0$) (d) $D(K_a = 1.0 \times 10^{-5})$

4. The number of moles of $Ca(OH)_2$ that must be dissolved to make 250 mL solution in water of pH = 10.65 is
(a) 5.6×10^{-5} (b) 6.5×10^{-5}
(c) 4.5×10^{-5} (d) 5.4×10^{5}

5. The change of energy on vaporizing 1.00 kg of liquid water at 0°C and 1 atm is
(a) 2367 kJ kg^{-1} (b) –2367 kJ kg^{-1}
(c) –2367 kJ mol^{-1} (d) –2367 kJ g^{-1}

6. Cl—O bond order in perchlorate ion is
(a) 1.33 (b) 1.50 (c) 1.75 (d) 1.90

7. The relative basic character of the following is
(a) $ClO^- < ClO_2^- < ClO_3^- < ClO_4^-$
(b) $ClO_4^- < ClO_3^- < ClO_2^- < ClO^-$
(c) $ClO_3^- < ClO_4^- < ClO_2^- < ClO^-$
(d) $ClO_2^- < ClO^- < ClO_3^- < ClO_4^-$

8. The density of a 3M sodium thiosulphate ($Na_2S_2O_3$) solution is 1.25 g/mL. Calculate the per cent by weight of sodium thiosulphate
(a) 12. 64% (b) 37.92%
(c) 0.87% (d) 63.21%

9. In $[NiCl_4]^{2-}$, the type of hybri dization for Ni is
(a) sp^3d^2 (b) dsp^3 (c) sp^3 (d) dsp^2

10. The hydrogen bonds are encountered in HF, H_2O, NH_3 and HF_2^-. The relative order of energies of hydrogen bonds is
(a) $HF > H_2O > NH_3 > HF_2^-$
(b) $H_2O > HF_2^- > HF > NH_3$
(c) $HF > HF_2^- > H_2O > NH_3$
(d) $HF_2^- > HF > H_2O > NH_3$

11. Which of the following is sparingly soluble in water?
(a) 2,6-dihydrobenzoic acid
(b) *p*-nitrophenol
(c) *o*-nitrophenol
(d) Ethanoic acid

12. A solid has a structure in which , atoms of W,O and Na are located respectively at the corners, centre of edges and at the centre of the cubic lattice. The compound is
(a) $NaWO_2$ (b) $NaWO_3$
(c) Na_2WO_3 (d) $NaWO_4$

13. Two solution of HCl, *A* and *B*, have concentrations of 0.5N and 0.1M respectively. The volume of solutions *A* and *B* required to make 2 L of 0.2 N HCl are
(a) 0.5 L of *A* + 1.5 L of *B*
(b) 1.5 L of *A* + 0.5 L of *B*
(c) 1.0 L of *A* + 1.0 L of *B*
(d) 0.57 L of *A* + 1.25 L of *B*

14. Which of the following is antipyertic and analgesic?
(a) Sulphaguanidine (b) Paracetamol
(c) Penicilline (d) Phenol

15. The initial rate, $-\frac{d[A]}{dt}$ at $t = 0$ was found to be 2.6×10^2 mol L^{-1} s^{-1} for the reaction

$$A + 2B \longrightarrow \text{Products}$$

The initial rate, $-\frac{d[B]}{dt}$, at $t \approx 0$ is
(a) 0.10 mol L^{-1} s^{-1}
(b) 2.6×10^{-2} mol L^{-1} s^{-1}
(c) 5.2×10^{-2} mol $L^{-1}s^{-1}$
(d) 6.5×10^{-3} mol L^{-1} s^{-1}

16. For the second order reaction,

$$A + B \longrightarrow \text{Products}$$

When a moles of A reacts with b moles of B, the rate equation is given by

$$k_2 t = \frac{1}{(a-b)} \ln \frac{b(a-x)}{a(b-x)}$$

When $a >> b$, the rate expression becomes that of

(a) first order
(b) zero order
(c) unchanged, second order
(d) third order

17. Maximum value of paramagnetism is shown by

(a) $[Fe(CN)_6]^{3-}$ (b) $[Cr(CN)_6]^{3-}$
(c) $[Co(CN)_6]^{3-}$ (d) $[Sc(CN)_6]^{3-}$

18. From the following data at 25°C,

$Cr^{3+}(aq) + e^- \longrightarrow Cr^{2+}(aq)$, $E° = -0.424$ V

$Cr^{2+}(aq) + 2e^- \longrightarrow Cr(s)$, $E° = -0.900$ V.

Find $E°$ at 25°C for the reaction,

$$Cr^{3+} + 3e^- \longrightarrow Cr(s)$$

(a) –0.741 V (b) –1.324 V
(c) –0.476V (d) + 0.741V

19. A sample of liquid in a thermally insulated container is stirred for 1h by a mechanical attachment to a motor in the surroundings, which of the following thermodynamic quantity for the system is zero?

(a) Work (W)
(b) Change in internal energy (ΔE)
(c) Change in enthalpy (ΔH)
(d) None of the above

20. The highest osmotic pressure corresponds to the following solution

(a) M/10 urea (b) M/10 glucose
(c) M/10 HCl (d) M/10 $BaCl_2$

21. The following is a conjugated diene

(a) $CH_3—CH=C=CH—CH_3$
(b) $CH_2=CH—CH_2—CH=CH_2$
(c) $CH_2=CH—CH_2—CH_2—CH=CH_2$
(d) $CH_2=\underset{\displaystyle CH_3}{\overset{|}{C}}—CH=CH_2$

22. For the reaction given below,

$$5Br^-(aq) + BrO_3^-(aq) + 6H^+ \longrightarrow 3Br_2(l) + 3H_2O(l)$$

The rate of formation of Br_2 is related to the rate of consumption of Br^- by the following relation

(a) $\frac{d[Br_2]}{dt} = -\frac{5d[Br^-]}{3dt}$
(b) $\frac{d[Br_2]}{dt} = -\frac{d[Br^-]}{dt}$
(c) $\frac{d[Br_2]}{dt} = \frac{5}{3}\frac{d[Br^-]}{dt}$
(d) $\frac{d[Br_2]}{dt} = -\frac{3}{5}\frac{d[Br^-]}{dt}$

23. For a diprotic acid, which of the following is true for first and second ionization constants? (K_{a_1} and K_{a_2}).

(a) $Ka_1 = Ka_2$ (b) $Ka_1 > Ka_2$
(c) $Ka_2 > Ka_1$ (d) $Ka_2 \geq Ka_1$

24. The strongest base of the following is

(a) NaOH (b) KOH
(c) LiOH (d) CsOH

25. If the solubility of $PbBr_2$ is S g-mol per litre, its solubility product, considering it to be 80% ionized, is

(a) $2.048\ s^2$ (b) $20.48\ s^3$
(c) $2.048\ s^3$ (d) $2.048\ s^4$

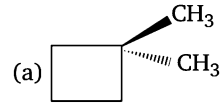
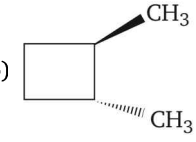
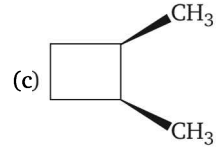
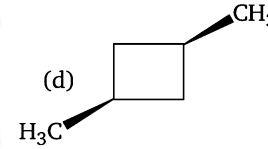
26. Solid PCl_5 exists as

(a) PCl_4^+ (b) PCl_5
(c) PCl_4^+ and PCl_6^- (d) PCl_6^-

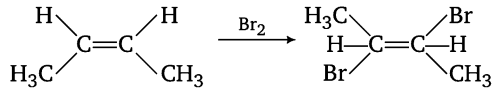
27. During charging of lead storage battery, the reaction occurring at the cathode is

(a) $Pb^{2+} + 2e^- \longrightarrow Pb$
(b) $Pb \longrightarrow Pb^{2+} + 2e^-$
(c) $PbSO_4 + 2H_2O \longrightarrow 2PbO_2 + 4H^+ + SO_4^{2-} + 2e^-$
(d) $Pb^{2+} + SO_4^{2-} \longrightarrow PbSO_4$

28. Which of the following is a chiral molecule?

(a) 
(b) 
(c) 
(d) 

29. Products of the reaction



are
(a) *meso*-compounds
(b) racemic mixtures
(c) mixtures of racemic and *meso*-compounds
(d) None of the above

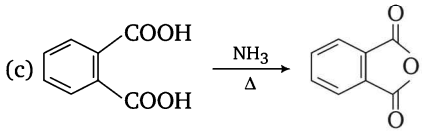
30. Ibuprofen contains
(a) only *S*-enantiomer
(b) only *R*-enantomer
(c) racemic mixture of both *R* and *S* enantiomer
(d) both *R*-and *S*-enantiomer are active pain killers

31. Which one of the following is an example of Hell-Volhard-Zelinsky reaction?

(a) $RCOOH \xrightarrow[(ii)\ H_3O^+]{(i)\ B_2H_6} RCH_2OH$

(b) $R_2CHCOOH \xrightarrow[(ii)\ H_2O]{(i)\ B_2/Red\ P} R_2\underset{\displaystyle Br}{\underset{|}{C}}COOH$

(c)



(d) $RCOOH \xrightarrow{PCl_5} RCOCl$

32. Ziesel method is used to estimate
(a) alcoholic group (b) amino group
(c) methoxy group (d) halo group

33. For an isolated system, $\Delta U = 0$, then
(a) $\Delta S = 0$
(b) $\Delta S < 0$
(c) $\Delta S > 0$
(d) The value of ΔS cannot be predicted

34. Hydrolysis of proteins give
(a) α-amino acids only
(b) β-amino acids only
(c) γ-amino acids only
(d) mixture of all *i.e.* α, β and γ-amino acids

35. Which one of the following is not a condensation polymer?
(a) Nylon 6,6 (b) Nylon 6
(c) Dacron (d) Buna-S

36. The correct order of equilibrium constants for the reactions is

$$H_2CO + H_2O \overset{K_1}{\rightleftharpoons} H_2C(OH)_2$$

$$CH_3CH_2CHO + H_2O \overset{K_2}{\rightleftharpoons} CH_3CH_2CH(OH)_2$$

$$CH_3COCH_3 + H_2O \overset{K_3}{\rightleftharpoons} CH_3C(OH)_2CH_3$$

(a) $K_1 > K_2 > K_3$ (b) $K_1 < K_2 < K_3$
(c) $K_1 > K_3 > K_2$ (d) $K_1 < K_3 < K_2$

37. The enolic form of acetone contains
(a) 9 sigma bonds, 2 pi bonds and 1 lone pair
(b) 9 sigma bonds, 1 pi bond and 2 lone pairs
(c) 8 sigma bonds, 2 pi bonds and 2 lone pairs
(d) 10 sigma bonds, 1 pi bond and 1 lone pair

38. What happens when aluminium and zinc salts react with an excess of NaOH?
(a) White precipitate is formed
(b) White precipitate of both Zn and Al first formed redissolve in excess of NaOH
(c) White precipitate of Al redissolves but that of Zn does not
(d) White precipitate of Zn redissolves and that of Al does not

39. Four compounds, toluene (I), *o*-dichlorobenzene (II), *m*-dichlorobenzene (III) and *p*-dichlorobenzene (IV) are arranged in the order of increasing dipole moment. The correct order is
(a) IV < I < III < II
(b) I < II < III < IV
(c) II < IV < III < I
(d) IV < III < II < I

40. The correct order of increasing boiling points of the following aqueous solutions

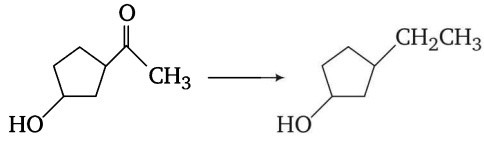
0. 0001 M NaCl (I), 0. 0001 M Urea (II), 0.001 M $MgCl_2$ (III), 0.01 M NaCl (IV) is

(a) I<II<III<IV (b) IV<III<II<I
(c) II<I<III<IV (d) III<II<IV<I

41. The correct order of acidic strength is
(a) $K_2O > CaO > MgO$
(b) $CO_2 > N_2O_5 > SO_3$
(c) $Na_2O > MgO > Al_2O_3$
(d) $Cl_2O_7 > SO_2 > P_4O_{10}$

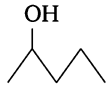
42. The reddish brown gas produced by heating KCl with $K_2Cr_2O_7$ (solid) and conc. H_2SO_4 is
(a) Cl_2 (b) $CrO_{-2}Cl_2$
(c) CrO_3 (d) H_2CrO_4

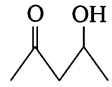
43. The appropriate reagent for the following transformation

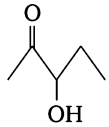


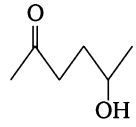
(a) NH_2NH_2, OH^-
(b) $NaBH_4$
(c) H_2 / Ni
(d) $AlCl_3$

44. Which one of the following will most readily be dehydrated in acidic solutions?

(a) 

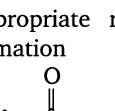
(b) 

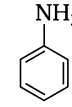
(c) 

(d) 

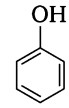
45. The reactivity order of alkyl halides towards nucelophiles is
(a) *R*F>*R*Br>*R*I>*R*Cl
(b) *R*I>*R*Cl>*R* Br>*R*F
(c) *R*F>*R*Cl>*R* Br>*R*I
(d) *R*I>*R*Br>*R*Cl>*R*F

46. Which of the following compound reacts slower than benzene in electrophilic substitution?

(a) 

(b) 

(c)

(d) 

47. The IUPAC name of

$CH_3CHOHCH_2CH(CH_3)CHO$ is
(a) 2-hydroxy-4-methyl pentanal
(b) 4-hydroxy-2-methyl pentanal
(c) 2-hydroxy-4-menthyl pentanal
(d) 2-methylpent-4-ol–1-al

48. Negatively charged colloidal soultion of clay in water will need for precipitation the minimum amount of
(a) aluminium sulphate (b) potassium sulphate
(c) sodium hydroxide (d) hydrochloric acid

49. The polarizing power of the following anions, N^{3-}, $O^{2-}$and F^-, follow the order
(a) $N^{3-} > F^- > O^{2-}$ (b) $O^{2-} > N^{3-} > F^-$
(c) $O^{2-} > F^- > N^{3-}$ (d) $N^{3-} > O^{2-} > F^-$

50. A ten fold increase in the $[H^+]$ of a solution
(a) increases its pH by one unit
(b) decreases its pH by one unit
(c) increases its pH by 10 unit
(d) decreases its pH by 10 unt

Mathematics

1. If $|z-\bar{z}|+|z+\bar{z}|=2$, then z lies on
(a) a circle (b) a square
(c) an ellipse (d) a line

2. If the product of the roots of the equation $x^2 - 2\sqrt{2}\,kx + 2e^{2\log k} - 1 = 0$ is 31, then the roots of the equation are real for k is equal to
(a) – 4 (b) 1
(c) 4 (d) 0

3. The temperature of a body lies between 10°C and 85°C. The corresponding temperature in °F will lie between
(a) 50 and 180 (b) 50 and 185
(c) 15 and 185 (d) None of these

4. The total number of ways is which 30 books can be distributed among 5 students is
(a) ${}^{30}C_5$ (b) ${}^{34}C_5$
(c) ${}^{30}C_4$ (d) ${}^{34}C_4$

5. If ${}^{12}P_r = {}^{11}P_6 + 6^{11}P_5$, then r is equal to
(a) 7 (b) 5
(c) 6 (d) 4

6. If the $(3r)$th and $(r+2)$th terms in the binomial expansion of $(1+x)^{2n}$ are equal, then
(a) $n = r$
(b) $n = r + 1$
(c) $n = 2r$
(d) $n = 2r - 1$

7. The total number of terms in the expansion of $(1+x)^{2n} - (1-x)^{2n}$ after simplification is
(a) $n + 1$ (b) $n - 1$
(c) n (d) $4n$

8. If $\log_{10} 2, \log_{10}(2^x - 1)$ and $\log_{10}(2^x + 3)$ are in AP, then x is equal to

(a) $\log_2 5$ (b) $\log_2(-1)$

(c) $\log_2(1/5)$ (d) $\log_5 2$

9. If the sum of first n natural numbers is 1/5 times the sum of their squares, then n is equal to

(a) 7 (b) 8

(c) 6 (d) 5

10. The equation of the plane through the points (2, 2, 1) and (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z - 1 = 0$ is

(a) $3x + 4y + 5z + 9 = 0$

(b) $3x + 4y - 5z + 9 = 0$

(c) $3x - 4y + 5z + 9 = 0$

(d) $3x + 4y - 5z - 9 = 0$

11. Let A and B be any two events. Which one of the following statements is always true?

(a) $P(A'/B) = P(A/B)$

(b) $P(A'/B) = P(B'/A)$

(c) $P(A'/B) = 1 - P(A/B)$

(d) $P(A'/B) = 1 - P(A/B')$

12. The inverse of a symmetric matrix is

(a) skew-symmetric (b) symmetric

(c) diagonal matrix (d) None of these

13. If

$$f(x) = \begin{vmatrix} x+1 & x & 1 \\ x(x+1) & x(x-1) & 2x \\ x(x+1)(x-1) & x(x-1)(x-2) & 3x(x-1) \end{vmatrix},$$

then $f(1000)$ is equal to

(a) 1 (b) 1000

(c) –1000 (d) 0

14. If $\begin{vmatrix} (x+a) & b & c \\ a & (x+b) & c \\ a & b & (x+c) \end{vmatrix} = 0$, then x is equal to

(a) $0, -(a+b+c)$ (b) 0

(c) $-(a+b+c)$ (d) $(a+b+c)$

15. If $f(x) = \log_x \{\ln(x)\}$, then $f'(e)$ is equal to

(a) $1/e$ (b) e

(c) $-e$ (d) e^2

16. The fixed point P on the curve $y = x^2 - 4x + 5$ such that the tangent at P is perpendicular to the line $x + 2y = 7$ is given by

(a) (3, 2) (b) (2, 1)

(c) (3, – 2) (d) (2, – 3)

17. Which of the following function is decreasing on $(0, \pi/2)$?

(a) $\sin 2x$ (b) $\cos 3x$

(c) $\tan x$ (d) $\cos 2x$

18. The least value of the function $f(x) = ax + b/x,\ a > 0, b > 0, x > 0$ is

(a) $\sqrt{ab}$ (b) $2\sqrt{\frac{a}{b}}$

(c) $2\sqrt{\frac{b}{a}}$ (d) $2\sqrt{ab}$

19. $\int \frac{2^x}{\sqrt{1-4^x}}\,dx$ is equal to

(a) $(\log 2)\sin^{-1} 2^x + C$

(b) $\frac{1}{2}\sin^{-1} 2^x + C$

(c) $\frac{1}{\log 2}\sin^{-1} 2^x + C$

(d) $2\log 2 \sin^{-1} 2^x + C$

20. If $\int_0^x f(t)dt = x + \int_x^1 tf(t)\,dt$, then $f(x)$ is equal to

(a) $\frac{1}{1-x}$ (b) $\frac{1}{x-1}$

(c) $\frac{1}{1+x}$ (d) $\frac{1}{x}$

21. The area of the region lying between the curve $y = x^2$ and the line $y = x + 2$ in the first quadrant is

(a) 10/3 (b) 10/6 (c) 10/2 (d) 9/2

22. A particle moves in a straight line with a velocity given by $\frac{dx}{dt} = x + 1$, where x is the distance traveled in time t. The time taken by the particle to traverse a distance of 999 m is

(a) $\log_{1000} e$ (b) $\log_e 100$

(c) $3\log_e 10$ (d) $4\log_e 10$

23. The number of vectors of unit length perpendicular to the vectors $\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{i} + \mathbf{j} + \mathbf{k}$ is

(a) 2 (b) 1

(c) 3 (d) infinite

24. If $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are three non-zero vectors such that $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, then

(a) $\mathbf{a} \perp \mathbf{b}$ and $\mathbf{c}$

(b) either $\mathbf{a} \perp (\mathbf{b} - \mathbf{c})$ or $\mathbf{b} = \mathbf{c}$

(c) $\mathbf{a} \perp (\mathbf{b} - \mathbf{c})$

(d) $\mathbf{b} = \mathbf{c}$

25. The equation of the plane containing the line

$$\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$$

is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$, where

(a) $ax_1+by_1+cz_1=1$ (b) $a/l=b/m=c/n$

(c) $lx_1+my_1+nz_1=0$ (d) $al+bm+cn=0$

26. Which of the following sets are not convex?

(a) $\{(x, y): 8x^2+6y^2\le 24\}$

(b) $\{(x, y): 6\le x^2+y^2\le 36\}$

(c) $\{(x, y): y\ge 3, y\le 30\}$

(d) $\{(x, y): x^2\le y\}$

27. Consider the Linear Programming Problem

Maximum $Z=4x+y$

Subject to $x+y\le 50$

$x+y\ge 100$

$x, y\ge 0$

The maximum value of Z is

(a) 0 (b) 50

(c) 100 (d) does not exist

28. Suppose A, B and C are three events of a sample space. Which of the following is not true?

(a) $P(A\cap B\cap C)=P(A)\cdot P(B/A)\cdot P(C/(A\cap B)\}$

(b) $P(A\cap B\cap C)=P(B)\cdot P(A/B)\cdot P\{C/(A\cap B)\}$

(c) $P(A\cap B\cap C)=P(A)\cdot P(B/A)\cdot P\{(A\cap B)/C\}$

(d) $P(A\cap B\cap C)=P(C)\cdot P(B/C)\cdot P\{A/(B\cap C)\}$

29. If $z_r=\cos\frac{\pi}{2^r}+i\sin\frac{\pi}{2^r}$, then $z_0\, z_1\, z_2\ldots$ is equal to

(a) 1 (b) -1 (c) i (d) $-i$

30. The area of the circle centered at $(-92, 103)$ and passing through $(-95, 99)$ is

(a) 3π (b) 4π

(c) 25π (d) 5π

31. The number of normals that can be drawn from a point inside an ellipse is

(a) 3 (b) 4 (c) 2 (d) 1

32. If $f(x)=\begin{cases} ax^2-b, & -1<x<1 \\ \frac{1}{|x|}, & |x|\ge 1 \end{cases}$ is differentiable at $x=1$, then

(a) $a=-\frac{1}{2}, b=-\frac{3}{2}$ (b) $a=-\frac{1}{2}, b=\frac{3}{2}$

(c) $a=\frac{1}{2}, b=-\frac{3}{2}$ (d) $a=\frac{1}{2}, b=\frac{3}{2}$

33. If for a matrix A, $A^2+I=0$, where I is the identity matrix of order 2, then A is equal to

(a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}i & 0\\ 0 & i\end{bmatrix}$

(c) $\begin{bmatrix}i & 0\\ 0 & 1\end{bmatrix}$ (d) $\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$

34. Let $f(x)$ be a function such that $f(x+y)=f(x)+f(y)$ and $f(x)=\sin x\, g(x)$ for $x, y\in R$. If $g(x)$ is a continuous function such that $g(0)=C$, then $f'(x)$ is equal to

(a) $C\sin x$ (b) C

(c) $C\cos x$ (d) $\cos x\, g(x)$

35. The value of α for which the equation $x^3-3x-\alpha=0$ has two distinct roots in $[-1, 2]$ is

(a) 1 (b) -1

(c) 2 (d) None of these

36. Let X be a binomial random variable and $X=\{0, 1, 2, \ldots, n\}$. For $r=0, 1, 2, \ldots, n$, the following holds

(a) $P(X=r)={}^nP_r p^r q^{n-r}$

(b) $P(X=r)={}^nC_r\, p^r q^{n-r}$

(c) $P(X=r)={}^nC_r\, p^r$

(d) $P(X=r)={}^nC_r\, p^{n-r}$

37. The degree of the differential equation satisfying $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ is

(a) 1 (b) 3

(c) 2 (d) None of these

38. The area enclosed by $2|x|+3|y|\le 6$ is

(a) 12 (b) 6 (c) 18 (d) 24

39. One of the diameters of the circle $x^2+y^2-12x+4y+6=0$ is given by

(a) $x-3y=0$ (b) $3x+y=0$

(c) $3x-y=0$ (d) $x+3y=0$

40. The curve represented by

$x=a(\sin h\,\theta+\cosh\theta)$

$y=b(-\sin h\,\theta+\cosh\theta)$ is

(a) a hyperbola (b) a parabola

(c) an ellipse (d) a circle

41. The parametric coordinates of any point on the parabola $y^2=4ax$ are given by

(a) $(-at^2, 2at)$ (b) $(a\sin^2 t, -2a\sin t)$

(c) $(-at^2, -2at)$ (d) $(a\sin t, -2a\sin t)$

42. If the two pair of lines

$x^2 - 2mxy - y^2 = 0$ and $x^2 - 2nxy - y^2 = 0$

are such that one of them represents the bisector of the angles between the other, then

(a) $mn = 1$ (b) $m + n = mn$
(c) $mn = -1$ (d) $m - n = mn$

43. Let R be a relation on the set N, defined by $\{(x, y) : 2x - y = 10\}$, then R is

(a) reflexive (b) symmetric
(c) transitive (d) None of these

44. If $aN = \{ax : x \in N\}$ and $bN \cap cN = dN$, where b and c are relative prime, then

(a) $b = cd$ (b) $c = bd$
(c) $d = bc$ (d) None of these

45. If $A = \{4^n - 3n - 1 : n \in N\}$ and $B = \{9(n-1) : n \in N\}$, then

(a) $B \subset A$ (b) $A \cup B = N$
(c) $A \subset B$ (d) None of these

46. If $f(x) = |\sin x|$ has an inverse, then its domain is

(a) $[-\pi/2, \pi]$ (b) $[0, \pi/2]$
(c) $[0, 2\pi]$ (d) $[-\pi, \pi]$

47. If $c^2 = a^2 + b^2$, $a + b + c = 2s$, then $4s(s-a)(s-b)(s-c)$ equals

(a) a^2b^2 (b) a^2c^2
(c) b^2c^2 (d) s^4

48. If $\tan^{-1}(a/x) + \tan^{-1}(b/x) = \pi/2$, then x is equal to

(a) $\sqrt{\frac{b}{a}}$ (b) $\sqrt{\frac{a}{b}}$
(c) $-\sqrt{ab}$ (d) $\sqrt{ab}$

49. For a natural number n, which one is the correct statement?

(a) $1^3 + 2^3 + \ldots + n^3 = (1 + 2 + \ldots + n)^2$
(b) $1^3 + 2^3 + \ldots + n^3 > (1 + 2 + \ldots + n)^2$
(c) $1^3 + 2^3 + \ldots + n^3 < (1 + 2 + \ldots + n)^2$
(d) $1^3 + 2^3 + \ldots + n^3 \neq (1 + 2 + \ldots + n)^2$

50. $\left(\frac{1+i}{1-i}\right)^n$ is real

(a) for every real number n
(b) for every odd integer n
(c) for every rational n
(d) for every even positive integer n

Answers

Physics

1. (d)	**2.** (d)	**3.** (a)	**4.** (d)	**5.** (a)	**6.** (d)	**7.** (b)	**8.** (b)	**9.** (a)	**10.** (b)
11. (c)	**12.** (a)	**13.** (c)	**14.** (a)	**15.** (c)	**16.** (a)	**17.** (a)	**18.** (d)	**19.** (c)	**20.** (c)
21. (c)	**22.** (c)	**23.** (c)	**24.** (c)	**25.** (b)	**26.** (c)	**27.** (a)	**28.** (d)	**29.** (c)	**30.** (c)
31. (b)	**32.** (b)	**33.** (a)	**34.** (a)	**35.** (a)	**36.** (c)	**37.** (b)	**38.** (b)	**39.** (d)	**40.** (c)
41. (c)	**42.** (b)	**43.** (d)	**44.** (a)	**45.** (c)	**46.** (d)	**47.** (b)	**48.** (a)	**49.** (b)	**50.** (b)

Chemistry

1. (b)	**2.** (c)	**3.** (c)	**4.** (c)	**5.** (a)	**6.** (c)	**7.** (b)	**8.** (b)	**9.** (c)	**10.** (d)
11. (c)	**12.** (b)	**13.** (a)	**14.** (b)	**15.** (c)	**16.** (a)	**17.** (b)	**18.** (b)	**19.** (d)	**20.** (d)
21. (d)	**22.** (d)	**23.** (b)	**24.** (d)	**25.** (c)	**26.** (c)	**27.** (c)	**28.** (b)	**29.** (b)	**30.** (c)
31. (b)	**32.** (c)	**33.** (c)	**34.** (a)	**35.** (d)	**36.** (a)	**37.** (b)	**38.** (b)	**39.** (a)	**40.** (c)
41. (d)	**42.** (b)	**43.** (a)	**44.** (b)	**45.** (d)	**46.** (c)	**47.** (b)	**48.** (a)	**49.** (d)	**50.** (b)

Mathematics

1. (b)	**2.** (c)	**3.** (b)	**4.** (a)	**5.** (c)	**6.** (c)	**7.** (c)	**8.** (a)	**9.** (a)	**10.** (d)
11. (c)	**12.** (d)	**13.** (d)	**14.** (a)	**15.** (a)	**16.** (a)	**17.** (d)	**18.** (d)	**19.** (c)	**20.** (c)
21. (a)	**22.** (c)	**23.** (a)	**24.** (c)	**25.** (d)	**26.** (d)	**27.** (d)	**28.** (c)	**29.** (b)	**30.** (c)
31. (b)	**32.** (a)	**33.** (b)	**34.** (b)	**35.** (c)	**36.** (b)	**37.** (a)	**38.** (a)	**39.** (d)	**40.** (a)
41. (b)	**42.** (c)	**43.** (a)	**44.** (c)	**45.** (c)	**46.** (b)	**47.** (a)	**48.** (d)	**49.** (a)	**50.** (d)

Hints & Solutions

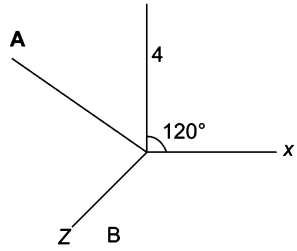
Physics

1. When two coils are connected in series, then time taken

$$t_p = t_1 + t_2$$
$$t_p = 20 + 20$$
$$= 40 \text{ min}$$

2. $|\mathbf{A}| = 5, |\mathbf{B}| = 9$

Vector **B** makes angle 90° from **A**



$$\mathbf{A} \times \mathbf{B} = |\mathbf{A}|\ |\mathbf{B}| \sin\theta$$
$$= 5 \times 9 \sin 90°$$
$$= 45$$

3. Centripetal acceleration

$$a = \frac{v^2}{r}$$
$$a = \frac{400 \times 400}{0.6}$$
$$= 267 \text{ m/s}^2$$

Now, $\frac{a}{g} = \frac{267}{10}$

$$= 26.7$$

4. Initial energy $E_1 = \frac{1}{2} C V^2$

$$= \frac{1}{2} C (50)^2$$

When capacitor is discharged, then its potential drops by 10 V. Therefore

Potential $V' = 50 - 10 = 40$ V

and energy $E_2 = \frac{1}{2} C V'^2 = \frac{1}{2} C (40)^2$

$\therefore$ The fraction of energy stored in the capacitor

$$= \frac{E_2}{E_1}$$
$$= \frac{\frac{1}{2} \times C \times (40)^2}{\frac{1}{2} \times C \times (50)^2}$$
$$= 0.64$$

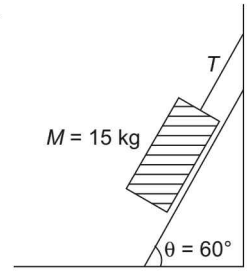
5. The amplitude decreases continuously with time

$$x = x_m e^{-(b/2m)t}$$
$$\frac{1}{2} x_m = x_m e^{-(40/2 \times 200)t}$$
$$\frac{1}{2} = e^{-t/10}$$

or $e^{(t/10)} = 2$

$$t/10 = \log_e 2$$
$$t = 10 \times 0.693$$
$$t = 7.00 \text{ s (approximately)}$$

6.



$$T = mg \sin 60°$$
$$= 15 \times 10 \times \frac{\sqrt{3}}{2}$$
$$T = 129 \text{ N}$$

7. The orbital velocity of cosmonaut

$$v_o = 7 \text{ km/s} = 7000 \text{ m/s}$$

Radius $r = R_E + h$

$$= (6.37 \times 10^6 + 0.63 \times 10^6)$$
$$= 7 \times 10^6 \text{ m}$$

Centripetal force

$$F = \frac{mv^2}{r}$$

$$= \frac{80 \times (7000)^2}{7 \times 10^6}$$

$$= 560 \text{ N}$$

8. Resultant amplitude

$$R = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos\phi}$$

where ϕ is the phase difference

$$R = \sqrt{(5)^2 + (5)^2 + 2 \times 5 \times 5 \times \cos\frac{\pi}{2}}$$

$$R = 5\sqrt{2}$$

9. Acceleration of helicopter blades

$$\alpha = \frac{\omega_2 - \omega_1}{t}$$

$$\alpha = \frac{330 - 110}{2} = 110 \text{ rev/min}^2$$

Final angular velocity of blades will be zero
i. e., $\omega = 0$

From equation of rotational motion

$$\omega = \omega - \alpha t$$

$$0 = 330 - 110 \times t$$

$$t = 3 \text{ min}$$

10. Torque $\tau = F \cdot r \sin\theta$

$$= 2 \times 3 \times \sin 30°$$

$$= 6 \times \frac{1}{2}$$

$$\tau = 3 \text{ N-m}$$

11. If v_1, v_2 are the speed of air on the lower and upper surface of the wings of aeroplane, and p_1, p_2 are the pressure there then using Bernoulli's theorem, we have

$$p_1 + \frac{1}{2}\rho d v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$p_1 - p_2 = \frac{1}{2}(v_2^2 - v_1^2)$$

$$\Delta p = \rho\left(\frac{v_2 + v_1}{2}\right)(v_2 - v_1)$$

Here, $v_{av} = \frac{v_2 + v_1}{2}$

$$= 720 \text{ km/h}$$

$$= 200 \text{ m/s}$$

$$\frac{v_2 - v_1}{v_{av}} = \frac{\Delta p}{\rho v_{av}^2}$$

$$\frac{v_2 - v_1}{v_{av}} = \frac{\frac{mg}{A}}{p v_{av}^2}$$

$$= \frac{4 \times 10^5 \times 10}{500 \times 1.2 \times (200)^2}$$

$$= 0.166 = 0.17$$

12. Resultant speed $\mathbf{v}_{br} = \mathbf{v}_b + \mathbf{v}_r$

$$\mathbf{v}_{br} = \sqrt{\mathbf{v}_b^2 + \mathbf{v}_r^2}$$

$$(13)^2 = (12)^2 + \mathbf{v}_r^2$$

$$\mathbf{v}_r^2 = 169 - 144$$

Velocity of water in the river

$$\mathbf{v}_r = \sqrt{25} = 5 \text{ km/h}$$

13. Stress $= \frac{F}{A}$

$$= \frac{6.2 \times 10^4}{\pi r^2}$$

$$= \frac{6.2 \times 10^4}{3.1 \times (9.5)^2 \times (10^{-3})^2}$$

$$= 2.2 \times 10^8 \text{ N/m}^2$$

14. The acceleration due to gravity

$$g = \frac{GM}{R^2}$$

$$= \frac{6.67 \times 10^{-11} \times 1.98 \times 10^{30}}{12000 \times 12000}$$

$$= 9.2 \times 10^{11} \text{ m/s}^2$$

15. Given $M = 20$ kg

$$R = 0.25 \text{ cm}$$

$$R = \frac{1}{4} \text{ m}$$

$$F = mg$$

$$= 2.5 \times 10 = 25 \text{ N}$$

Torque $= F \times R$

$$= 25 \times \frac{1}{4}$$

$$= \frac{25}{4}$$

Moment of inertia $= \frac{1}{2}MR^2$

$$= \frac{1}{2} \times 20 \times \left(\frac{1}{4}\right)^2$$

$$= \frac{5}{8}$$

We have $\tau = I\alpha$

$\therefore$ Angular acceleration of flywheel

$$\alpha = \frac{\tau}{I}$$

$$= \frac{25/4}{5/8}$$

$$\alpha = 10 \text{ rad/s}^2$$

16. From conservation of linear momentum

$$M_2 v_2 = m_1 v_1$$

$$100\, v_2 = \frac{200}{1000} \times 80$$

$$v_2 = \frac{200 \times 80}{1000 \times 100} = 0.16 \text{ m/s}$$

$$= 16 \text{ cm/s}$$

17. Constant force = 8 N

Mass of body = 0.4 kg

We know that

$$F = ma$$

$$8 = 0.4 \times a$$

$$a = \frac{8}{0.4} = -20 \text{ m/s}^2$$

$$s = ut + \frac{1}{2} at^2$$

$$= 10 \times 25 + \frac{1}{2} \times (-20) \times (25)^2$$

$$= 250 - 10 \times 625$$

$$= -6000 \text{ m}$$

18. From Newton's law of cooling

$$\frac{\theta_1 - \theta_2}{t} = k\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$$

$$= \frac{94 - 86}{2} = k\left[\frac{94 + 86}{2} - 20\right]$$

$$4 = k\,[70]$$

$$k = \frac{4}{70}$$

Again

$$\frac{71 - 69}{t} = \frac{4}{70}\left[\frac{71 + 69}{2} - 20\right]$$

$$\frac{2}{t} = \frac{4}{70}\,[50]$$

$$t = \frac{2 \times 70}{4 \times 50}$$

$$t = 0.7 \text{ min}$$

$$t = 0.7 \times 60$$

$$= 42 \text{ s}$$

19. Muzzle speed of bullet = 150 m/s = 540 km/h

Since the bullet is sharing the velocity of the car, its effective velocity = 30 + 540 = 570 km/h.

The speed of the bullet w.r.t. the thieves car moving in the same direction

$$= 570 - 150 = 420 \text{ km/h}$$

$$= \frac{420 \times 5}{18}$$

$$= 116 \text{ m/s}$$

20. Distance covered by train B

$$s = ut + \frac{1}{2} at^2$$

$$= 20 \times 50 + \frac{1}{2} \times 1 \times (50)^2$$

$$s_1 = 2250 \text{ m}$$

Distance covered by train A

$$s_2 = ut$$

$$= 20 \times 50$$

$$s_2 = 1000$$

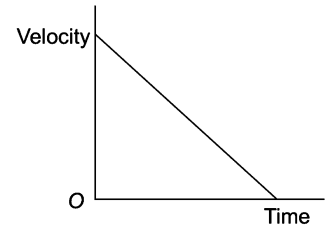
The distance between A and B

$$s = s_1 - s_2$$

$$= 2250 - 1000$$

$$= 1250 \text{ m}$$

21. When an object is moving with constant negative acceleration, having positive initial velocity, then velocity time graph is a straight line shown below



22. The SI system, the unit of heat is J or kg $m^2 s^{-2}$,

The magnitude of calorie in SI unit is 4.2 kg m^2 s^2

We have

$(n_1v_1) = (n_2u_2)$

$4.2\,\text{kgm}^2\text{s}^{-2} = \alpha\beta^2\gamma^2\text{kgm}^2\text{s}^{-2}$

$n_2 = 4.2\,\alpha^{-1}\,\beta^{-2}\gamma^2$

23. Distance travelled by particle $s = 6t + 3t^2$

$\frac{ds}{dt} = 6 + 3 \times 2t$

$v = 6 + 6t$

The velocity of the particle after 2 s

$v = 6 + 6 \times 2$

$= 6 + 12$

$= 18$ m/s

24. Speed of light = 1 unit

Time = 8 min+ 20 s

= 500 s

Distance = Speed × Time

$= 1 \times 500$

$= 500$

25. Mass defect = $m\,(^{40}\text{Ca}_{20}) + m_n - m\,(^{41}\text{Ca}_{20})$

$= 39.962591 + 1.00865 - 40.962278$

$= 8.963 \times 10^{-3}$

$= 8.963 \times 10^{-3} \times 931.5$

$= 8.3490$ MeV

26. Let t be the time.

According to the Avogadro number concept

Number of atoms in 2g of deuterium

$= 6.02 \times 10^{23}$

Number of atoms in 2 kg of deuterium

$= \frac{6.023 \times 10^{23} \times 2 \times 10^3}{2}$

$= 6.023 \times 10^{26}$ Nuclei

From given equation energy released during fusion of two deuterium = 3.27 MeV

∴Energy released by one deuterium

$= \frac{3.27}{2} = 1.635$ MeV

Energy released in 6.023×10^{26} deuterium atoms

$= 1.635 \times 6.023 \times 10^{26}$

$= 9.846 \times 10^{26}$ MeV

$= 9.848 \times 10^{26} \times 1.6 \times 10^{-13}$

$= 15.75 \times 10^{13}$ s

Time $= \frac{15.75 \times 1 \times 10^{13}}{100}$

$= 15.75 \times 10^{11}$ s

$= \frac{15.75 \times 10^{11}}{60 \times 24 \times 60 \times 365}$

$= 4.97 \times 10^4$

$= 5 \times 10^4$ years

27. $\frac{1}{\lambda} = R\left[\frac{1}{(n_1)^2} - \frac{1}{(n_2)^2}\right]$

For Paschen series

$\frac{1}{\lambda} = R\left[\frac{1}{(3)^2} - \frac{1}{(\infty)^2}\right]$

$= R\left[\frac{1}{9} - 0\right] = R\,\frac{1}{9}$

$\frac{1}{\lambda} = \frac{R}{9}$

$\lambda = \frac{9}{R}$

$\lambda = \frac{9}{1.09 \times 10^7} = 8.25 \times 10^{-7}$

Frequency $n = \frac{c}{\lambda} = \frac{3 \times 10^8}{8.25 \times 10^{-7}}$

$n = 3.7 \times 10^{14}$ Hz

28. Time period, $T = 2\pi\sqrt{\frac{m}{k}}$

$= 2 \times 3.14\sqrt{\frac{500/1000}{312.5}}$

$= 0.25$ s

29. For fixed distance s between object and screen, the lens equation does not give a real solution for u or u if f is greater than s/4.

Therefore focal length $= \frac{s}{4}$

$= \frac{3}{4} = 0.75$ m

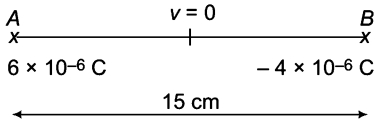
30. The number of photons emitted per second from a source of monochromatic radiation of wavelength λ and power P is given by

$$n = \frac{P}{h\nu}$$

$$= \frac{8 \times 10^{-3}}{6.63 \times 10^{-34} \times 6 \times 10^{14}}$$

$$= 2 \times 10^{16}$$

31.



$$V_1 + V_2 = 0$$

$$\frac{Q_1}{4\pi\varepsilon_0 r_1} + \frac{Q_2}{4\pi\varepsilon_0 r_2} = 0$$

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{6 \times 10^{-6}}{x} + \frac{-4 \times 10^{-6}}{(15-x)}\right] = 0$$

$$9 \times 10^9 \left[\frac{6 \times 10^{-6}}{x} + \frac{(-4 \times 10^{-6})}{(15-x)}\right] = 0$$

$$\frac{6 \times 10^{-6}}{x} = \frac{4 \times 10^{-6}}{(15-x)}$$

$$6(15-x) = 4x$$

$$90 - 6x = 4x$$

$$10x = 90$$

$$x = 9 \text{ cm}$$

Again at 45 cm

Potential $V = \dfrac{6 \times 10^{-6}}{45} + \dfrac{(-4 \times 10^{-6})}{30}$

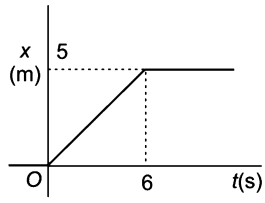
$$= 0.13 \times 6^{-6} - 0.13 \times 10^{-6}$$

$$= 0$$

At infinity

Potential $V = \dfrac{6 \times 10^{-6}}{\infty} + \dfrac{4 \times 10^{-6}}{\infty} = 0$

32. The position-time graph gives the velocity



According to this graph, velocity at $t = 6$ s

$$v = \frac{5}{6} \text{ m/s}$$

Impulse = Change in momentum

$$= m_1 v_2 - m_2 v_1$$

$$= 4 \times 5/6 - 4 \times 0 = 20/6 = 3.33$$

So option (b) is correct.

33. Time period of heart, $T = \dfrac{1}{\text{Frequency}}$

$$T = \frac{1}{75/60} = 0.8 \text{ s}$$

34. Momentum $p = \dfrac{E}{c}$

$$= \frac{3 \text{ MeV}}{3 \times 10^8 \text{ m/s}} = \frac{3 \times 10^6 \text{ eV}}{3 \times 10^8 \text{ m/s}}$$

$$= 0.01 \text{ eV-s/m}$$

35. The far point of a myopic person is 80 cm

i. e., $v = -80$ cm

$u = \infty$

From lens formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

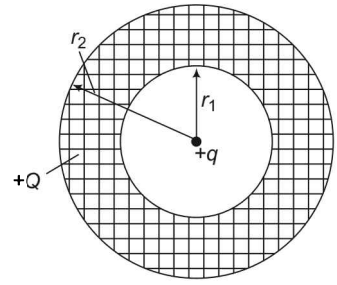
$$\frac{1}{f} = \frac{1}{-80} - \frac{1}{\infty}$$

$$\frac{1}{f} = -\frac{1}{80} \text{ cm}$$

Power, $P = \dfrac{1}{f} = -\dfrac{100}{80}$ D

$$= -1.25 \text{ D}$$

36.



Charge on the outer surface $= Q + q$

$\therefore$ The charge per unit area $= \dfrac{Q+q}{4\pi r_2^2}$

37. Given charge $q = 10\,\mu$C

Radius $R = 10$ cm

Electric flux $\phi = \dfrac{q}{\varepsilon_0}$

$$= \frac{10 \times 10^{-6}}{8.85 \times 10^{-12}}$$

$$= 11 \times 10^{-5}$$

$$\simeq 10 \times 10^{-5}$$

38. The magnitude of each electric field vector at point C due to charge q_1 and q_2

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

$$E = \frac{9 \times 10^9 \times 10^{-7}}{(20 \times 10^{-2})^2}$$

$$= 2.2 \times 10^4 \text{ N/C}$$

39. From equation of motion

$$h = ut - \frac{1}{2} gt^2$$

$$4 = 9t - \frac{1}{2} \times 10 \times t^2$$

$$4 = 9t - 5t^2$$

$$5t^2 - 9t + 4 = 0$$

$$5t^2 - (5 + 4)t + 4 = 0$$

$$5t^2 - 5t - 4t + 4 = 0$$

$$5t\ (t - 1) - 4\ (t - 1) = 0$$

$$(5t - 4)\ (t - 1)$$

We take only $(t - 1) = 0$

$\therefore \quad t = 1$ s

40. Suppose C' = Capacitance of the each capacitor

$= 1\ \mu F$

C = Required capacitance of the combination $= 2\ \mu F$

Maximum potential difference across each capacitor = 400 V

Potential difference $= 1000$ V = potential difference across each row n capacitor in a row will stand a voltage $= 350 \times n$

$$350 \times n = 1000$$

$$n = \frac{1000}{350} = 3$$

Now,

$$\frac{1}{C'} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = 3$$

$$C' = \frac{1}{3}\ \mu F$$

Total capacitance of m such row in parallel

$$C = m \times C'$$

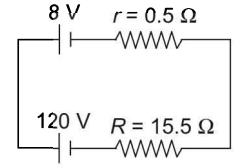
$$m = \frac{C}{C'} = 2/(1/3) = 2 \times 3 = 6$$

$$M = 6 \times 3 = 18$$

41. Emf of the battery $e = 8$V, emf of DC supply $V = 120$ V

Since, the battery is bring changes, so effective emf in the circuit

$$E = V - e = 120 - 8 = 112 \text{ V}$$



Current in the circuit $I = \dfrac{\text{Effective emf}}{\text{Total resistance}}$

$$= \frac{E}{r + R}$$

$$= \frac{112}{0.5 + 15.5}$$

$$= \frac{112}{16} = 7 \text{ A}$$

The battery of 8 V is being charged by 120 V, so the terminal potential across battery of 8 V will be greater than its emf.

Terminal potential difference

$$V = E + Ir$$

$$= 8 + 7(.5)$$

$$= 11.5 \text{ V}$$

42. Equivalent resistance of the combination

$$= \frac{GS}{G + S} = \frac{60 \times 0.02}{60 + 0.02}$$

$$= 0.0199$$

Total resistance $= R + 0.0199$

$$V = iR'$$

$$5 = 1 \times R + 0.0199$$

$$R = 5 - 0.0199 = 4.98 \approx 5.00\ \Omega$$

43. Induced current $i = \dfrac{NBA\omega}{R}$

$$i = \frac{NBA\theta}{Rt} \qquad \left(\because \omega = \frac{\theta}{t}\right)$$

$$i = \frac{500 \times 3 \times 10^{-5} \times \pi \times (10)^2 \times 2\pi \times 10^{-4}}{2 \times 0.5}$$

$$= 2.9 \times 20^{-3}$$

$$= 3 \text{ mA}$$

44. The magnitude of its magnetic vector

$$B = \frac{E}{c}$$

$$= \frac{6}{3 \times 10^8} = 2 \times 10^{-8}$$

45. The angular frequency of their oscillations

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{27 \times 10^{-3} \times 30 \times 10^{-6}}}$$

$$= 1.1 \times 10^3$$

46. Here, $u = -12\,\text{cm}$, $f = 15\,\text{cm}$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{15} + \frac{1}{12}$$

$$\frac{1}{v} = \frac{5+4}{60}$$

$$v = \frac{60}{9}\,\text{cm}$$

Now,

$$\frac{I}{O} = \frac{u}{v}$$

$$\frac{4.5}{O} = \frac{12}{60/9}$$

$$\text{Length of object} = \frac{4.5 \times 5}{9} = 2.5\,\text{cm}$$

47. Energy of photon, $E = \frac{hc}{\lambda}$

$$E = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{390 \times 10^{-9}}$$

$$= 5.0 \times 10^{-19}\ \text{J}$$

$$E = \frac{5.0 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.2\,\text{eV}$$

48. Here, $\mathbf{a} = 3\mathbf{i} - 4\mathbf{j}$

$$a = \sqrt{(3)^2 + (4)^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5$$

$$\mathbf{b} = -2\mathbf{i} + 3\mathbf{k}$$

$$= \sqrt{(-2)^2 + (3)^2} = \sqrt{4+9} = \sqrt{13}$$

The angle between the vector

$$\theta = \cos^{-1}\left(\frac{\mathbf{A} \cdot \mathbf{B}}{\mathbf{AB}}\right)$$

$$= \cos^{-1}\left[\frac{(3\mathbf{i} - 4\mathbf{j})(-2\mathbf{i} + 3\mathbf{k})}{\sqrt{25} \times \sqrt{13}}\right]$$

$$= \cos^{-1}\left[-\frac{6}{18}\right] = \cos^{-1}\left(-\frac{1}{3}\right)$$

49. We know that

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{(Ze)(2e)}{r_0} = K$$

$$9 \times 10^9 \times \frac{79 \times (1.6 \times 10^{-19})^2}{r_0} \times 2$$

$$= 7.7 \times 10^6 \times 1.6 \times 10^{-19}$$

$$r_0 = \frac{9 \times 10^9 \times 79 \times (1.6 \times 10^{-19})^2 \times 2}{7.7 \times 10^6 \times 1.6 \times 10^{-19}}$$

$$= 2.9 \times 10^{-14}$$

$$r_0 = 3.0 \times 10^{-14}$$

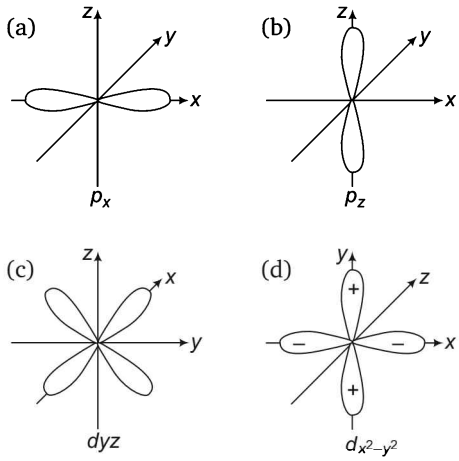
$$= 30 \times 10^{-15}\ \text{m}$$

50. In a Bipolar Junction Transistor (BJT), the current gain β is defined as the ratio of change in collector current to change in base current for constant collector voltage in collector-emiitter configuration

i. e., $\beta = \frac{\Delta I_C}{\Delta I_B}$

Chemistry

1. Shape of given orbitals may be drawn as



It is clear from the above shapes, p_z orbitals has zero probability of finding the electron in the xy-plane.

2. According to electrochemical series, metal having more standard electrode potential will deposit rapidly. Therefore, sequence of deposition of metals on the cathode will be

$$\underset{(+\,0.80V)}{Ag} > \underset{(+\,0.79V)}{Hg} > \underset{(+\,0.34V)}{Cu}$$

3. $\because pK_a = -\log K_a$

$\therefore pK_a$ for benzoic acid $= -\log(6.3\times10^{-5})$

$= -\log 6.3 - \log 10^{-5}$

$= -0.799 + 5$

$= 4.201$

(a) pK_a for $A = -\log(1.67\times10^{-8})$

$= -\log 1.67 - \log 10^{-8}$

$= -0.222 + 8 = 7.778$

(b) pK_a for $B = 6.0$

(c) pK_a for $C = 4.0$

(d) pK_a for $D = -\log 1.0\times10^{-5}$

$= -\log 1.0 - \log 10^{-5}$

$= 0 + 5 = 5$

For a stronger acids, pK_a must be less. Therefore, acid C will be stronger than benzoic acid.

4. $\because$ pH of solution = 10.65

$\therefore$ pOH of solution $= 14 - 1.0765 = 3.35$

$\therefore \quad [OH^-] = 10^{-pOH}$

$= 10^{-3.35}$

$\log[OH^-] = \log 10^{-3.35}$

$= -3.35$

$[OH^-] = \text{antilog}(-3.35)$

$= \text{antilog}(4.65)$

$= 0.0004467$

$\because Ca(OH)_2$ is a diacidic base

$\therefore \quad N = \dfrac{0.0004467}{z} = 0.00022335$

From $\dfrac{w}{E} = \dfrac{NV}{1000}$

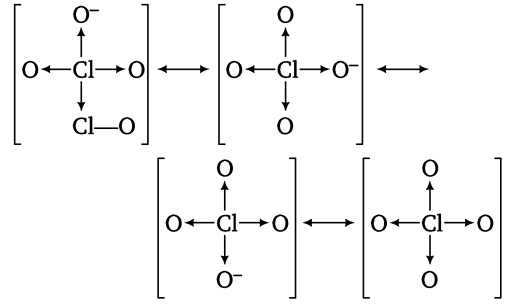
No. of moles $= \dfrac{0.00022335\times250}{1000}$

$= 0.0000558$

$= 5.58\times10^{-5}$

5. The change of energy on vaporizing 1.00 kg of any liquid at 0°C and 1 atm is called its latent heat of vaporization. For water its value is 2367 kJ kg^{-1}.

6. Perchlorate ion (ClO_4^-) exists in fiollowing racemic forms



Thus, Cl—O bond order

$$= \frac{\text{Total no. of bonds between two atoms in all resonating forms}}{\text{No. of resonating structures}}$$

$$= \frac{2\times3+1}{4} = \frac{7}{4}$$

$= 1.75.$

7. Generally with decrease in number of oxygen atoms, basic character decreases.

Therefore, the order of basic character is as follows :

$$ClO_4^- > ClO_3^- > ClO_2^- > ClO^-$$

8. Molecular weight of

$$Na_2S_2O_3 = 2\times 23 + 2\times 32 + 3\times 16$$
$$= 158$$

From molarity

$$\% \text{ by weight of solute} \times$$
$$= \frac{\text{density of solution} \times 10}{M}$$
$$3 = \frac{\% \text{ by weight of } Na_2S_2O_3 \times 1.25 \times 10}{158}$$
$$\therefore \% \text{ by weight of } Na_2S_2O_3 = \frac{3\times 158}{1.25\times 10}$$
$$= 37.92\%$$

9. For $[NiCl_4]^{2-}$

$H = \frac{1}{2}$ (number of valency electrons in central atom + number of monovalent atoms surrounding central atoms + charge on ion with sign)

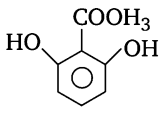
$$= \frac{1}{2}(6 + 4 - 2)$$
$$= 4$$

$\therefore$ Hybridisation for Ni in $[NiCl_4]^{2-}$ is sp^3.

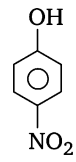
10. When a hydrogen bond is sandwiched between two highly electronegative elements (F, O, N), it shows a unique property to form a bridge between them which is termed as hydrogen bond. Its strength depands upon the electronegativity of associated atom. Thus, the order of hydrogen bond must be

$$HF_2^- > HF > H_2O > NH_3$$

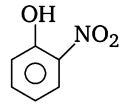
11.



(a)



(b)



(c)

CH_3COOH

(d)

In *o*-nitrophenol, due to intermolecular H-bonding, solubility decreases. Therefore, it is less soluble.

12. Number of W atoms present at each corner

$$= 8 \times \frac{1}{8} = 1$$

Number of O atoms present at the centre of edges $= 12 \times \frac{1}{4} = 3$

Number of Na atoms present at the centre of cube = 1

$\therefore$ Formula of cube = WO_3Na or $NaWO_3$

13. $N_A = 0.5$, $V_A = V$ L (let)

$N_B = M_B = 0.1$, $V_B = (2 - V)$ L

From normality equation

$$NV = N_AV_A + N_BV_B$$
$$0.2\times 2 = 0.5\times V + 0.1\times(2 - V)$$
$$0.4V = 0.5V + 0.2 - 0.1V$$
$$0.4V = 0.2$$
$$V = \frac{0.2}{0.4} = 0.5$$

$\therefore$ $V_A = 0.5$ L

and, $V_B = (2 - 0.5)$

$= 1.5$ L

14. Antipyretic drugs reduce body temperature in fever and analgesic drugs provide relief from pain. Paracetamol and aspirin act as antipyretic as well as analgesic.

15. $A + 2B \rightarrow$ Products

From rate law

$$\frac{-d[A]}{dt} = -\frac{1}{2}\cdot\frac{d[B]}{dt}$$

$\therefore$ $\frac{d[B]}{dt} = 2\cdot\frac{d[A]}{dt}$

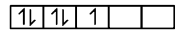
$$= 2\times 2.6\times 10^{-2}$$
$$= 5.2\times 10^{-2} \text{ mol } L^{-1}s^{-1}$$

16. $A + B \rightarrow$ Products

When $a >> b$, *i. e.*, reactant 'A' is present in large excess, rate of reaction does not depend upon its concentration. Hence, it will be of first order.

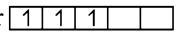
17. Paramagnetism is shown due to unpaired electrons. Since, CN^- is a strong field ligand, it causes pairing of electrons.

(a) In $[Fe(CN)_6]^{3-}$, $Fe^{3+} = Ar[3d^5]$

$= Ar$ 

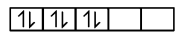
1 unpaired electron

(b) In $[Cr(CN)_6]^{3-}$, $Cr^{3+} = Ar[3d^3]$

$= Ar$ 

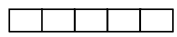
3 unpaired electrons.

(c) In $[Co(CN)_6]^{3-}$, $Co^{3+} = Ar[3d^6]$

$= Ar$ 

No unpaired electron.

(d) In $[Sc(CN)_6]^3$, $Sc^{3+} = Ar[3d]^\circ$

$= Ar$ 

No unpaired electron.

Hence, $[Cr(CN)_6]^{3-}$ will show maximum paramagnetism.

18. (i) $Cr^{3+} + e^- \rightarrow Cr^{2+}$, $E_1^\circ = -0.424$ V

(ii) $Cr^{2+} + 2e^- \rightarrow Cr^-$, $E_2^\circ = -9.00$V

(iii) $Cr^{3+} + 3e^- \rightarrow Cr$, $E^\circ = ?$

Since, reaction (i) and (ii) are performed at cathode. Therefore, for reaction (iii)

$$E^\circ = E_1^\circ + E_2^\circ$$
$$= (-0.424) + (-0.900)$$
$$= -1.324 \text{ V}$$

19. For the system, work done is equal to change in energy. So, there will be none of W, ΔE and ΔH will be zero in the system.

20. Osmotic pressure, $\pi = icRT$

For urea and glucose, $i = 1$

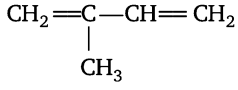
For HCl and $BaCl_2$, $i > 1$

Due to high ionisation of $BaCl_2$ than HCl, $i_{BaCl_2} > i_{HCl}$

$\therefore$ Order of osmotic pressure will be

BaCl > HCl > Urea = glucose

21. Conjugate diene have alternative single and double bonds. *e. g.*,



22. $5Br^- + BrO_3^- + 6H^+ \rightarrow 3Br_2 + 3H_2O$

$$\therefore \quad \frac{1}{3}\cdot\frac{d[Br_2]}{dt} = -\frac{1}{5}\cdot\frac{d[Br^-]}{dt}$$
$$= -\frac{d[BrO_3^-]}{dt}$$
$$\therefore \quad \frac{d[Br_2]}{dt} = -\frac{3}{5}\cdot\frac{d[Br^-]}{dt}$$

23. For a diprotic acid, further ionization is difficult, therefore

$$K_{a_1} > K_{a_2}$$

24. Since, basicity increases from top to bottom in a group, therefore, order of basicity will be

$$LiOH < NaOH < KOH < CsOH$$

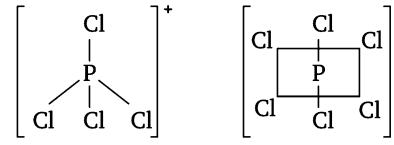
25.

$PbBr_2 \rightleftharpoons$	Pb^{2+}	+	$2Br^-$
Initially	1s	0	0
After 80% ionization	$(1 - 0.80) = 0.20s$	$0.80\ S$	$2 \times 0.80 = 1.60s$

$\therefore$ Solubility product, $K_{sp} = [Pb^{2+}][Br^-]^2$

$$= (0.80s)(1.60s)^2$$
$$2.048\ s^3$$

26. X-ray studies have been shown solid PCl_5 is a salt containing tetrahedral cation, $[PCl_4]^+$ and octahedral anion, $[PCl_6]^-$.



Terachlorophosphonium Hexachlorophosphate

In vapour phase, PCl_5 exists having a trigonal bipyramid shape.

27. During charging of lead storage battery, following reactions take place.

At anode

$$PbSO_4(s) + 2e^- \longrightarrow Pb(s) + SO_4^{2-}(aq)$$

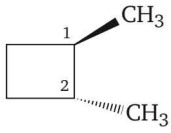
At cathode

$$PbSO_4(s) + 2H_2O \longrightarrow PbO_2(s) + SO_4^{2-}(aq) + 4H^+ + 2e^-$$

Total cell reaction is

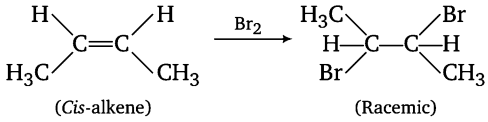
$$2PbSO_4(s) + 2H_2O \longrightarrow Pb(s) + PbO_2(s) + 2SO_4^{2-}(aq) + 4H^+(aq)$$

28.



Here C_1 and C_2 are asymmetrical, due to which it is a chiral molecule.

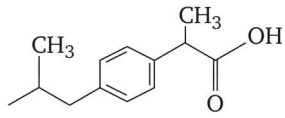
29.



(*Cis*-alkene) (Racemic)

Since, reactant is *cis*-alkene, therefore, product will be racemic. If the reactant is *trans* then the product will be *meso*-compound.

30. Ibuprofen is a racemic mixture of both *R* and *S* enantiomers.



(*RS*)-2-(4-(2-methylpropyl) phenyl) propanoic acid.

It is an analgesic drug.

31. When aliphatic carboxylic acids react with bromine in presence of red phosphorus give α-halogenated acids. This reaction is known as Hell-Volhard-Zelinsky reaction.

$$R_2CHCOOH \xrightarrow[(ii)\ H_2O]{(i)\ Br_2/Red\ P} R_2\underset{\displaystyle Br}{\underset{|}{C}}—COOH$$

32. Zeisel's method is used to estimate the methoxy group in an organic compound. In this method, organic compound having methoxy group is treated with HI and the alkyl halide thus formed is further treated with $AgNO_3$ to precipitate AgI.

33. For an isolated system,

$$T\Delta S = \Delta UT\ \ p\Delta U$$

If $\Delta U = 0$

$$T\Delta S = p\Delta V$$

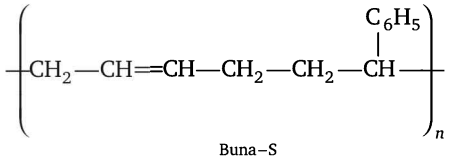
$$\Delta S = \frac{p\Delta V}{T}$$

$$> 0$$

34. Proteins are macromolecular polymers of α-amino acids bonded by peptide linkage. They on hydrolysis, yield α-amino acids further.

35. Buna-S is an additive polymer of 75% butadiene and 25% styrene. It is also known as artificial/synthetic rubber.

$$CH_2{=}CH—CH{=}CH_2 + CH_2{=}\underset{\displaystyle C_6H_6}{\underset{|}{C}}H \xrightarrow{\text{Polymerisation}}$$



Buna–S

36. $H_2CO + H_2O \overset{K_1}{\rightleftharpoons} H_2(COH)_2$

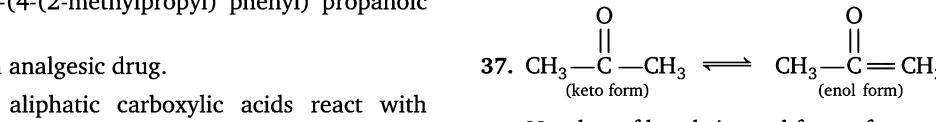
$CH_3CH_2CHO + H_2O \overset{K_2}{\rightleftharpoons} CH_3CH_2CH(OH)_2$

$CH_3COCH_3 + H_2O \overset{K_3}{\rightleftharpoons} CH_3C(OH)_2CH_3$

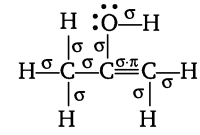
Due to steric effect, the order of equilibrium constants will be

$$K_1 > K_2 > K_3$$

37.



Number of bonds in enol form of acetone are



9σ, 1π, 2 lone pair of *e*

38. When NaOH reacts with salts of Al and Zn, it forms white precipitate which dissolve in excess of NaOH.

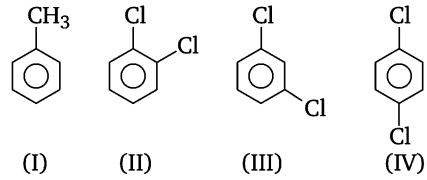
(i) $AlCl_3 + 3NaOH \longrightarrow \underset{\text{white ppt}}{Al(OH)_3}\downarrow + NaCl$

$Al(OH)_3 + NaOH \longrightarrow \underset{\substack{\text{sodium meta}\\ \text{aluminate (soluble)}}}{NaAlO_2} + 2H_2O$

(ii) $ZnSO_4 + 2NaOH \longrightarrow \underset{\text{white ppt}}{Zn(OH)_2}\downarrow + Na_2SO_4$

$Zn(OH)_2 + 2NaOH \longrightarrow \underset{\substack{\text{sodium zincate}\\ \text{(soluble)}}}{Na_2ZnO_2} + 2H_2O$

39.



(I) (II) (III) (IV)

The order of dipole moment will be

IV < I < III < II

40. ∵ Order of ionisation is

$$\underset{\text{(IV)}}{0.01\text{ M NaCl}} > \underset{\text{(III)}}{0.001\text{ M } MgCl_2} > \underset{\text{(I)}}{0.0001\text{ M NaCl}} > \underset{\text{(II)}}{0.0001\text{ M urea}}$$

∴ Order of boiling point will be same.

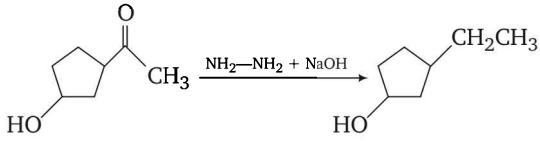
41. Since acidity of oxide increases on moving left to right in a period and decreases on moving from top to bottom, the correct order of acidic strength is

$$Cl_2O_7 > SO_2 > P_4O_{10}$$

42. When KCl is heated with acidic $K_2Cr_2O_7$, reddish brown gas of chromyl chloride evaporated.

$$4KCl + K_2Cr_2O_7 + 3H_2SO_4 \xrightarrow{\Delta} \underset{\text{chromyl chloride}}{2CrO_2Cl_2}\uparrow + 2Na_2SO_4 + K_2SO_4 + 3H_2O$$

43. In Wolff-Kishner reduction, carbonyl groups reduce into hydrocarbon in the presence of hydrazine and a strong base.



44.

Due to presence of tautomeric effect, α – H is acidic with respect to —OH group. So, it will dehydrated readily in acidic solutions.

45. The order of electronegativity increases from top to bottom in a group. Thus, its order for halogens is as follows

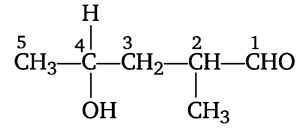
F > Cl > Br > I

Since, nucleophiles are also electron rich species, thus they repel halide ions. Hence, reactivity order of alkyl halides towards nucleophiles will be

*R*I > *R*Br > *R*Cl > *R*F

46. Due to presence of electron withdrawing group, nitrobenzene reacts slower than benzene in electrophilic reactions.

47.



IUPAC name is-4-hydroxy-2-methyl pentanal

48. (a) $Al_2(SO_4)_3 \longrightarrow 2Al^{3+} + 3SO_4^{2-}$

(b) $K_2SO_4 \longrightarrow 2K^+ + SO_4^{2-}$

(c) $NaOH \longrightarrow Na^+ + OH^-$

(d) $HCl \longrightarrow H^+ + Cl^-$

Since, aluminum sulphate has maximum positively charged ions, thus it will be coagulate maximum. Therefore, for precipitation, its minimum amount will be required.

49. Since, polarising power increases with the size of anion, thus the order will be

$$N^{3-} > O^{2-} > F^-$$

50. $pH_1 = -\log[H_1^+]$

$pH_2 = -\log[10H_1^+]$

$= -\log[H^+] - \log 10$

$= -\log[H^+] - 1$

$= pH_1 - 1$

Mathematics

1. Let $z = x + iy$

Given, $|z - \bar{z}| + |z + \bar{z}| = 2$

$\therefore |(x + iy) - (x - iy)| + |(x + iy) + (x - iy)| = 2$

$\Rightarrow \quad |2iy| + |2x| = 2$

$\Rightarrow \quad \pm y \pm x = 1$

Hence, it represents a square.

2. Given, $x^2 - 2\sqrt{2}kx + 2e^{2\log k} - 1 = 0$

$\therefore$ Product of roots $= 2e^{2\log k} - 1$

$= 31$ (given)

$\Rightarrow \quad 2e^{2\log k} = 32$

$\Rightarrow \quad e^{\log k^2} = 16$

$\Rightarrow \quad k^2 = 16$

$\Rightarrow \quad k = \pm 4$

But $k = -4$, $\log k$ is not defined.

Hence, required value of k is 4.

3. $\because {}^\circ C = \frac{5}{9}({}^\circ F - 32)$

(i) $\therefore \frac{10 \times 9}{5} = {}^\circ F - 32$

$\Rightarrow \quad \frac{90}{5} = {}^\circ F - 32$

$\Rightarrow \quad 18 = {}^\circ F - 32$

$\Rightarrow \quad F = 50$

(ii) $\frac{85 \times 9}{5} = {}^\circ F - 32$

$\Rightarrow \quad 17 \times 9 = {}^\circ F - 32$

$\Rightarrow \quad 153 + 32 = F$

$\Rightarrow \quad F = 185$

4. The total number of ways in which 30 books can be distributed among 5 students is ${}^{30}C_5$.

5. Given, ${}^{12}P_r = {}^{11}P_6 + 6 \cdot {}^{11}P_5$

$\Rightarrow \quad {}^{12}P_r = \frac{11!}{5!} + \frac{6 \times 11!}{6!}$

$\Rightarrow \quad \frac{12!}{(12-r)!} = \frac{11!}{5!} + \frac{11!}{5!}$

$\Rightarrow \quad 6 \times 5! = (12 - r)!$

$\Rightarrow \quad (12 - r)! = 6!$

$\Rightarrow \quad r = 6$

6. Given expansion is $(1 + x)^{2n}$.

According to the given condition,

${}^{2n}C_{3r-1} = {}^{2n}C_{r+1}$

$\Rightarrow \quad (2n - 3r + 1) = (r + 1)$

$\Rightarrow \quad 2n = 4r$

$\Rightarrow \quad n = 2r$

7. $(1 + x)^{2n} - (1 - x)^{2n}$

$= ({}^{2n}C_0 + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \ldots + {}^{2n}C_{2n} x^{2n})$

$- ({}^{2n}C_0 - {}^{2n}C_1 x + {}^{2n}C_2 x^2 - {}^{2n}C_3 x^3$

$+ \ldots + (-1)^n \, {}^{2n}C_{2n} x^{2n})$

$= 2\{{}^{2n}C_1 x + {}^{2n}C_3 x^3 + \ldots + {}^{2n}C_{2n-1}(x)^{2n-1}\}$

Hence, total number of terms is n.

8. Since, $\log_{10} 2$, $\log_{10}(2^x - 1)$ and $\log_{10}(2^x + 3)$ are in AP.

$\therefore \quad \log_{10}(2^x - 1) = \frac{\log_{10} 2 + \log_{10}(2^x + 3)}{2}$

$\Rightarrow \quad 2\log_{10}(2^x - 1) = \log_{10} 2(2^x + 3)$

$\Rightarrow \quad (2^x - 1)^2 = 2(2^x + 3)$

$\Rightarrow \quad 2^{2x} + 1 - 2 \cdot 2^x = 2 \cdot 2^x + 6$

$\Rightarrow \quad 2^{2x} - 4 \cdot 2^x - 5 = 0$

$\Rightarrow \quad 2^{2x} - 5 \cdot 2^x + 2^x - 5 = 0$

$\Rightarrow \quad 2^x(2^x - 5) + 1(2^x - 5) = 0$

$\Rightarrow \quad (2^x + 1)(2^x - 5) = 0$

$\Rightarrow \quad 2^x - 5 = 0, \; 2^x \neq -1$

$\Rightarrow \quad 2^x = 5$

$\Rightarrow \quad x = \log_2 5$

9. According to the given condition,

$1 + 2 + \ldots + n = \frac{1}{5}(1^2 + 2^2 + \ldots + n^2)$

$\Rightarrow \quad \frac{n(n+1)}{2} = \frac{1}{5}\left[\frac{n(n+1)(2n+1)}{6}\right]$

$\Rightarrow \quad 15 = 2n + 1$

$\Rightarrow \quad 2n = 14$

$\Rightarrow \quad n = 7$

10. Equation of plane passing through (2, 2, 1) is

$a(x - 2) + b(y - 2) + c(z - 1) = 0 \quad \ldots(i)$

Also, it passes through (9, 3, 6).

$\therefore \quad 7a + b + 5c = 0 \quad \ldots(ii)$

Also, plane (i) is perpendicular to the given plane

$2x + 6y + 6z - 1 = 0$

$\therefore \quad 2a + 6b + 6c = 0 \quad \ldots(iii)$

$(\because a_1a_2 + b_1b_2 + c_1c_2 = 0)$

From Eqs. (ii) and (iii), we get

$$\frac{a}{6-30} = \frac{b}{10-42} = \frac{c}{42-2}$$

$$\Rightarrow \quad \frac{a}{-24} = \frac{b}{-32} = \frac{c}{40}$$

$$\Rightarrow \quad \frac{a}{6} = \frac{b}{8} = \frac{c}{-10}$$

$\therefore$ From Eq. (i), we get

$$6(x-2) + 8(y-2) - 10(z-1) = 0$$

$$\Rightarrow \quad 6x + 8y - 10z - 18 = 0$$

$$\Rightarrow \quad 3x + 4y - 5z - 9 = 0$$

11. We know that,

$$P\left(\frac{A'}{B}\right) + P\left(\frac{A}{B}\right) = 1$$

$$\Rightarrow \quad P\left(\frac{A'}{B}\right) = 1 - P\left(\frac{A}{B}\right)$$

12. Let $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 3 & -5 \\ -1 & 5 & 1 \end{bmatrix}$ be asked symmetric matrix

$\therefore \quad |A| = 1(3+25) + 2(2-5) + 1(10+3)$

$= 28 - 6 + 13 = 35$

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} 28 & 7 & 7 \\ 3 & 2 & 7 \\ 13 & -3 & 7 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{35}\begin{bmatrix} 28 & 7 & 7 \\ 3 & 2 & 7 \\ 13 & -3 & 7 \end{bmatrix}$$

Hence, inverse of a skew symmetric matrix is not skew symmetric matrix.

13. Given,

$$f(x) = \begin{vmatrix} x+1 & x & 1 \\ x(x+1) & x(x-1) & 2x \\ x(x+1)(x-1) & x(x-1)(x-2) & 3x(x-1) \end{vmatrix}$$

Taking x common from R_2 and $x(x-1)$ common from R_3,

$$= x \times x(x-1)\begin{vmatrix} x+1 & x & 1 \\ x+1 & x-1 & 2 \\ x+1 & x-2 & 3 \end{vmatrix}$$

$$= x^2(x-1)(x+1)\begin{vmatrix} 1 & x & 1 \\ 1 & x-1 & 2 \\ 1 & x-2 & 3 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$,

$$= x^2(x-1)(x+1)\begin{vmatrix} 1 & x & 1 \\ 0 & -1 & 1 \\ 0 & -2 & 2 \end{vmatrix}$$

$= x^2(x^2-1)[1(-2+2)] = 0$

$$\Rightarrow \quad f(x) = 0$$

$$\therefore \quad f(1000) = 0$$

14. Let $\Delta = \begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$ and taking common from C_1,

$$= (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$,

$$= (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix}$$

$= (x+a+b+c)(x^2) = 0 \quad$ (given)

$$\Rightarrow \quad x = 0, -(a+b+c)$$

15. Given, $f(x) = \log_x \{\log(x)\}$

$$= \frac{\log \log x}{\log x} \qquad \left(\because \log_a b = \frac{\log b}{\log a}\right)$$

On differentiating w.r.t. 'x', we get

$$f'(x) = \frac{\log x\left[\frac{1}{\log x} \times \frac{1}{x} - \log\log x \times \frac{1}{x}\right]}{(\log x)^2}$$

$$= \frac{\left[\frac{1}{x \log x} - \frac{\log \log x}{x}\right]}{\log x}$$

At $x = e$,

$$f'(e) = \frac{\frac{1}{e \log e} - \frac{\log \log e}{e}}{\log e}$$

$$= \frac{\frac{1}{e} - 0}{1} = \frac{1}{e} \quad (\because \log \log e = \log 1 = 0)$$

16. Let fixed point be $P(x_1, y_1)$ on a given curve

$$y = x^2 - 4x + 5$$

$\therefore \quad y_1 = x_1^2 - 4x_1 + 5$

On differentiating given curve, we get

$$\frac{dy}{dx} = 2x - 4$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 2x_1 - 4$$

But it is given $\Rightarrow \frac{dy}{dx}(x_1, y_1)$

= perpendicular slope of given line

$\therefore \quad 2x_1 - 4 = 2$

$\Rightarrow \quad 2x_1 = 6$

$\Rightarrow \quad x_1 = 3$

$\therefore \quad y_1 = (3)^2 - 4(3) + 5$

$= 9 - 12 + 5$

$\Rightarrow \quad y_1 = 2$

$\therefore$ Required point is (3, 2).

17. We know that, cos x is decreasing in $(0, \pi)$.

$\therefore \cos 2x$ is decreasing in $\left(0, \frac{\pi}{2}\right)$.

18. Given, $f(x) = ax + \frac{b}{x}$

On differentiating w.r.t. 'x', we get

$$f'(x) = a - \frac{b}{x^2}$$

Put $f'(x) = 0$,

$$\Rightarrow \quad a - \frac{b}{x^2} = 0$$

$$\Rightarrow \quad x = \pm\sqrt{\frac{b}{a}}$$

$$\therefore \quad f''(x) = \frac{2b}{x^3}$$

At $x = \sqrt{\frac{b}{a}}$,

$$f''(x) = \frac{2b}{\left(\sqrt{\frac{b}{a}}\right)^3} > 0, \text{ minima}$$

$\therefore$ Minimum value of $f(x)$ is

$$f\left(\sqrt{\frac{b}{a}}\right) = a\sqrt{\frac{b}{a}} + \frac{b}{\sqrt{\frac{b}{a}}}$$

$$= \sqrt{ab} + \sqrt{ab} = 2\sqrt{ab}$$

19. Let $I = \int \frac{2^x}{\sqrt{1 - 4^x}}\, dx$

$$= \int \frac{2^x}{\sqrt{1 - (2^x)^2}}\, dx$$

Put $2^x = t$

$$\Rightarrow \quad 2^x \log 2\, dx = dt$$

$$\therefore \quad I = \int \frac{\frac{1}{\log 2}}{\sqrt{1 - t^2}}\, dt$$

$$= \frac{1}{\log 2} \cdot \sin^{-1} t + C$$

$$= \frac{1}{\log 2} \sin^{-1} 2^x + C$$

20. Given $\int_0^x f(t)\, dt = x + \int_x^1 t\, f(t) dt$

Apply Leibnitz's rule

$$f(x) = 1 - [x f(x)]$$

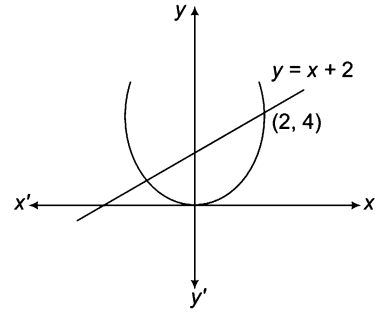
$$\Rightarrow \quad f(x)(1 + x) = 1$$

$$\Rightarrow \quad f(x) = \frac{1}{1 + x}$$

21. Intersection point of line

$y = x + 2$ and curve $y = x^2$ is $x^2 = x + 2$.

$$\Rightarrow \quad x^2 - x - 2 = 0$$



$\Rightarrow \quad (x-2)(x+1)=0$

$\Rightarrow \quad x=-1, 2$

$\Rightarrow \quad y=1, 4$

$\therefore$ Area of shaded region $= \int_0^2 (y_2 - y_1)\, dx$

$$= \int_0^2 [(x+2)-(x^2)]\, dx$$

$$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_0^2$$

$$= \left[2 + 4 - \frac{8}{3} - 0\right] = \frac{10}{3}$$

22. $\frac{dx}{dt} = x + 1$

$\Rightarrow \quad \frac{dx}{x+1} = dt$

On integrating both sides, we get

$$\log(x+1) = t + C$$

Initially $x = 0, t = 0$

$\Rightarrow \quad C = 0$

At $x = 999$ m,

$\therefore \quad \log_e (999 + 1) = t$

$\Rightarrow \quad t = \log_e 1000 = 3 \log_e 10$

23. Let $\mathbf{a} = \mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}$

$$\therefore \quad \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \mathbf{i}(1+1) - \mathbf{j}(1+1) + \mathbf{k}(1-1)$$

$$= 2\mathbf{i} - 2\mathbf{j}$$

$\therefore$ Unit vector $= \pm \frac{\mathbf{a} \times \mathbf{b}}{|\mathbf{a} \times \mathbf{b}|}$

$$= \pm \frac{2\mathbf{i} - 2\mathbf{j}}{\sqrt{4+4}}$$

$$= \pm \frac{\mathbf{i} - \mathbf{j}}{\sqrt{2}}$$

Hence, number of vector perpendicular to the unit vector is 2.

24. $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$

$\Rightarrow \quad \mathbf{a} \cdot (\mathbf{b} - \mathbf{c}) = 0$

$\Rightarrow \quad \mathbf{a} \perp (\mathbf{b} - \mathbf{c}) = 0$

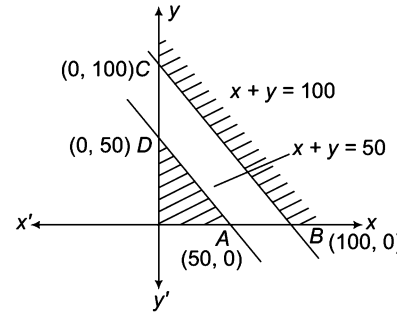
25. The equation of plane containing the given line is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$.

$\therefore$ Normal to the plane is perpendicular to the line.

$\therefore \quad al + bm + cn = 0$

26. Hence option (d) is correct.

27. Given, $Z = 4x + y$



and $x + y \le 50, x + y \ge 100$

$x, y \ge 0$

Hence, it is clear from the graph that it is not bounded, region. so maximum value cannot be determined.

28. Hence, option (c) is not true.

29. $z_r = \cos\frac{\pi}{2^r} + i \sin\frac{\pi}{2^r}$

$\therefore \quad z_0 z_1 z_2 \ldots =$

$$\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\left(\cos\frac{\pi}{2^2} + i\sin\frac{\pi}{2^2}\right)\ldots$$

$$= \cos\left(\frac{\pi}{2} + \frac{\pi}{2^2} + \ldots\right) + i \sin\left(\frac{\pi}{2} + \frac{\pi}{2^2} + \ldots\right)$$

$$= \cos\left(\frac{\pi/2}{1 - \frac{1}{2}}\right) + i \sin\left(\frac{\pi/2}{1 - \frac{1}{2}}\right)$$

$$= \cos(\pi) + i \sin(\pi)$$

$$= -1$$

30. $\therefore$ Radius, $r = \sqrt{(-95+92)^2 + (99-103)^2}$

$$= \sqrt{(-3)^2 + (-4)^2}$$

$$= \sqrt{9+16} = 5$$

$\therefore$ Area of circle $= \pi r^2$

$$= \pi(5)^2 = 25\pi$$

31. There are four normals can be drawn from a point inside an ellipse.

32. Given, $f(x)=\begin{cases} ax^2-b, & -1<x<1 \\ \dfrac{1}{|x|}, & |x|\geq 1 \end{cases}$

$$=\begin{cases} ax^2-b, & -1<x<1 \\ \dfrac{1}{x}, & x\geq 1 \end{cases}$$

Since, it is differentiable at $x=1$.

$\therefore$ LHD = RHD

$$\frac{d}{dx}(ax^2-b)=\frac{d}{dx}\left(\frac{1}{x}\right)$$

$$\Rightarrow \quad 2ax=-\frac{1}{x^2}$$

At $x=1$,

$$\Rightarrow \quad 2a=-1$$

$$\Rightarrow \quad a=-\frac{1}{2}$$

Also, it is continuous at $x=1$.

$\therefore$ LHL = RHL

$$a(1)-b=1$$

$$\Rightarrow \quad b=a-1=-\frac{1}{2}-1$$

$$\Rightarrow \quad b=-\frac{3}{2}$$

33. Let $A=\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$

$$\therefore \quad A^2=\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$

$$=\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\therefore A^2+I=\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}+\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}=O, \text{ which is true.}$$

34. $$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(x)+f(h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(h)}{h}$$

$$=\lim_{h\to 0}\frac{\sin g(h)}{h}$$

$$=g(0)\times\lim_{h\to 0}\frac{\sin h}{h}$$

$$f'(x)=C\times 1$$

$$\Rightarrow \quad f'(x)=C$$

35. Given, $x^3-3x-\alpha=0$

Let $\alpha=2$

$$\therefore \quad x^3-3x-2=0$$

$$\Rightarrow \quad (x+1)(x^2-x-2)=0$$

$$\Rightarrow \quad (x+1)(x+1)(x-2)=0$$

$$\Rightarrow \quad (x+1)^2(x-2)=0$$

$$\Rightarrow \quad x=-1, 2\in[-1, 2]$$

Hence, our assumption is true.

36. For a random variable X,

$$P(X=r)={}^nC_r\, p^r q^{n-r}$$

37. Given, $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ ...(i)

On differentiating w.r.t. x, we get

$$\frac{(-2x)}{2\sqrt{1-x^2}}+\frac{(-2y)}{2\sqrt{1-y^2}}\cdot\frac{dy}{dx}=a\left(1-\frac{dy}{dx}\right)$$

$$\Rightarrow \quad \frac{-x}{\sqrt{1-x^2}}-\frac{y}{\sqrt{1-y^2}}\cdot\frac{dy}{dx}=a\left(1-\frac{dy}{dx}\right)$$

$$\Rightarrow \quad a=\frac{\dfrac{x}{\sqrt{1-x^2}}+\dfrac{y}{\sqrt{1-y^2}}\cdot\dfrac{dy}{dx}}{\left(\dfrac{dy}{dx}-1\right)}$$

$\therefore$ From Eq. (i), we get

$$\sqrt{1-x^2}+\sqrt{1-y^2}$$

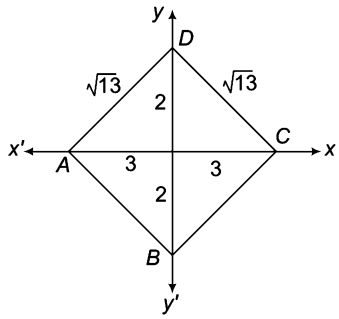
$$=\frac{\dfrac{x}{\sqrt{1-x^2}}+\dfrac{y}{\sqrt{1-y^2}}\cdot\dfrac{dy}{dx}}{\left(\dfrac{dy}{dx}-1\right)}(x-y)$$

$$\Rightarrow \quad \left(\frac{dy}{dx}-1\right)(\sqrt{1-x^2}+\sqrt{1-y^2})$$

$$=\frac{x}{\sqrt{1-x^2}}+\frac{y}{\sqrt{1-y^2}}\cdot\frac{dy}{dx}$$

Hence, degree is 1.

38. Given equation can be rewritten as



$$\frac{|x|}{3}+\frac{|y|}{2}\le 1$$

$$\Rightarrow \quad \pm\frac{x}{3}+\frac{(\pm y)}{2}\le 1$$

Hence, given equation form a rhombus.

$\therefore$ Area of rhombus $=\frac{1}{2}\times d_1 d_2$

$$=\frac{1}{2}\times 6\times 4$$

$$=12$$

39. $\therefore$ Centre of circle is $(6, -2)$.

We know that, every diameter of the circle passes through the centre.

By taking option (d),

$$x+3y=0$$

At point $(6, -2)$,

$$6-6=0$$

Hence, option (d) is correct.

40. Given equation can be rewritten as

$$\frac{x}{a}=\sinh\theta+\cosh\theta$$

and $\frac{y}{b}=-\sinh\theta+\cosh\theta$

$$\Rightarrow \quad \frac{x}{a}\times\frac{y}{b}=\cosh^2\theta-\sinh^2\theta=1$$

$$\Rightarrow \quad xy=ab$$

Hence, it represents a hyperbola.

41. Given curve is $y^2=4ax$.

Taking option (b),

At point $(a\sin^2 t, -2a\sin t)$,

$$(-2a\sin t)^2=4a(a\sin^2 t)$$

$$\Rightarrow \quad 4a^2\sin^2 t=4a^2\sin^2 t$$

Hence, option (b) is correct.

42. Bisector of the angles between the lines $x^2-2mxy-y^2=0$ is

$$\frac{x^2-y^2}{xy}=\frac{1-(-1)}{-m}$$

$$\Rightarrow \quad mx^2+2xy-my^2=0$$

But it represents by $x^2-2nxy-y^2=0$

$$\therefore \quad \frac{m}{1}=\frac{2}{-2n}$$

$$\Rightarrow \quad mn=-1$$

43. Given, $\{(x, y): 2x-y=10\}$

Reflexive, $xRx \Leftrightarrow 2x-x=10$

$$\Rightarrow \quad x=10$$

$$\Rightarrow \quad y=10$$

$\therefore$ Point $(10, 10)\in N$.

Hence, R is reflexive.

44. Given, $aN=\{ax: x\in N\}$

$$\therefore \quad bN\cap cN=bcN$$

($\because$ b and c are relatively prime)

$$\therefore \quad bc=d$$

45. Given, $A=\{4^n-3n-1: n\in N\}$

and $B=\{9(n-1): n\in N\}$

$$\therefore \quad A=(1+3)^n-3n-1$$

$$={}^nC_0 1^n+{}^nC_1 1^{n-1}\cdot 3+{}^nC_2\,1^{n-2}\cdot 3^2+\ldots$$

$$+{}^nC_n 3^n-3n-1$$

$$={}^nC_2 3^2+{}^nC_3\cdot 3^3+\ldots+{}^nC_n\cdot 3^n$$

$$=9\{{}^nC_2+{}^nC_3(3)+\ldots+{}^nC_n 3^{n-2}\}$$

$\therefore 4^n-3n-1$ is a multiple of 9 for $n\ge 2$.

For $n=1$,

$$4^n-3n-1=4-3-1=0$$

For $n=2$,

$$4^n-3n-1=16-6-1=9$$

$\therefore 4^n-3n-1$ is a multiple of 9 for all $n\in N$.

$$\therefore \quad A\subseteq B$$

46. Given, $f(x) = |\sin x|$

$\therefore$ The domain of an inverse of $f(x)$ is $\left[0, \frac{\pi}{2}\right]$.

47. $\because c^2 = a^2 + b^2$

$\therefore \Delta ABC$ is an right angled triangle at C.

$\therefore 4s(s-a)(s-b)(s-c) = 4\Delta^2$

$= 4\left(\frac{1}{2} \times \text{base} \times \text{height}\right)^2$

$= a^2b^2$

48. Given, $\tan^{-1}\left(\frac{a}{x}\right) + \tan^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}$

$\Rightarrow \quad \tan^{-1}\left(\frac{\frac{a}{x} + \frac{b}{x}}{1 - \frac{a}{x} \times \frac{b}{x}}\right) = \frac{\pi}{2}$

$\Rightarrow \quad \frac{(a+b)x}{x^2 - ab} = \tan\frac{\pi}{2} = \frac{1}{0}$

$\Rightarrow \quad x^2 = ab$

$\Rightarrow \quad x = \pm\sqrt{ab}$

$\Rightarrow \quad x = \sqrt{ab}$

($\because x = -\sqrt{ab}$ does not satisfy the given equation)

49. $1^3 + 2^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

and $(1 + 2 + \ldots + n)^2 = \left[\frac{n(n+1)}{2}\right]^2$

50. $\left(\frac{1+i}{1-i}\right)^n = \left[\frac{(1+i)^2}{1^2 + 1^2}\right]^n$

$= \left[\frac{1 - 1 + 2i}{2}\right]^n$

$= [i]^n$

For $(i)^n$ is real, n should be even integer.