AMU

Engineering Entrance Exam

Solved Paper 2011

Physics

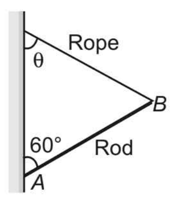
1. A string that is stretched between fixed supports separated by 75.0 cm has resonant frequencies of 420 Hz and 315 Hz, with no intermediate resonant frequencies. What is the lowest resonant frequency?

(a) 250 Hz (b) 317 Hz
(c) 180 Hz (d) 105 Hz

2. Suppose 4.0 mol of an ideal gas undergoes a reversible isothermal expansion from volume V_1 to volume $V_2 = 2.0 V_1$ at temperature $T = 400$ K. Find the entropy change of the gas. (Take $\ln 2 = 0.693$)

(a) 23.0 J/K (b) 42.0 J/K
(c) 51.6 J/K (d) 56.9 J/K

3. A uniform rod AB of mass m is hinged to a wall at its lower end, while its upper end is held by a rope attached to the wall. For what value of θ, the tension in the rope is equal to $\frac{mg}{2}$?



(a) 30° (b) 60°
(c) 45° (d) None of these

4. Three sinusoidal waves of the same frequency travel along a string in the positive x-direction. Their amplitudes are y, $\frac{y}{2}$ and $\frac{y}{3}$ and their phase constants are 0, $\frac{\pi}{2}$ and π respectively. What is the amplitude of the resultant wave?

(a) $0.63 y$ (b) $0.72 y$
(c) $0.83 y$ (d) $0.52 y$

5. A sample of gas expands from an initial pressure and volume of 10 Pa and 1.0 m^3 to a final volume of 2.0 m^3. During the expansion, the pressure and volume are related by the equation $p = aV^2$, where $a = 10$ N/m^8. Find the work done by the gas during the expansion.

(a) 23 J (b) 18 J
(c) 9 J (d) 43 J

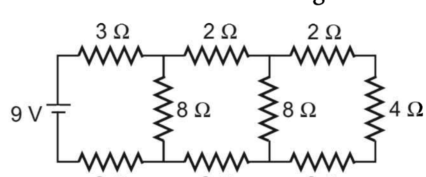
6. Two point +ve charges q each are placed at $(-a, 0)$ and $(a, 0)$. A third +ve charge q_0 is placed at $(0, y)$. Find the value of y for which the force at q_0 is maximum.

(a) $\frac{a}{\sqrt{3}}$ (b) $\frac{a}{\sqrt{2}}$
(c) a (d) $2a$

7. A particle of mass 40 mg and carrying a charge 5×10^{-9} C is moving towards a fixed charge of magnitude 10^{-8} C. When it is at a distance of 10 cm from the fixed charge, it has a velocity of 50 cm/s. At what distance from the fixed charge will the particle come momentarily to rest?

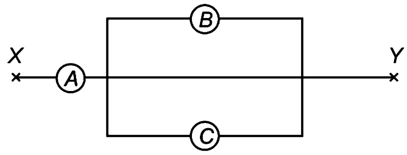
(a) 1.3×10^{-3} m (b) 1.9×10^{-3} m
(c) 3.9×10^{-2} m (d) 4.7×10^{-2} m

8. In the circuit shown in the figure



The current through 3 Ω resistance is
(a) 0.5 A (b) 0.7 A
(c) 1.0 A (d) 1.2 A

9. A, B and C are voltmeters of resistance $R, 1.5R$ and $3R$ respectively. When some potential difference is applied between X and Y, the voltmeter readings are V_A, V_B and V_C respectively. Then



(a) $V_A = V_B = V_C$ (b) $V_A \neq V_B = V_C$
(c) $V_A = V_B \neq V_C$ (d) $V_A \neq V_B \neq V_C$

10. A galvanometer has a resistance of 30 Ω and a current of 2.0 mA gives full scale deflection. How will you convert this galvanometer into a voltmeter of 0.2 V range?
(a) 700 Ω resistance should be connected parallel to the galvanometer
(b) 70 Ω resistance should be connected to the galvanometer
(c) 700 Ω resistance should be connected in series with the galvanometer
(d) 70 Ω resistance should be used in series with the galvanometer

11. A beam of 450 nm light is incident on a metal having work function 2 eV and placed in a magnetic field B. If the most energetic electrons emitted are bent into circular arc of radius 0.2 m, find B.
(a) 2.36×10^{-4} T (b) 1.46×10^{-5} T
(c) 6.9×10^{-5} T (d) 9.2×10^{-6} T

12. The de-Broglie wave length is given by
(a) $p = \frac{2\pi\hbar}{\lambda}$ (b) $p = \frac{\hbar}{2\lambda}$
(c) $p = \frac{2\pi}{\hbar\lambda}$ (d) $p = \frac{2\pi}{\lambda}$

13. Which of the following truth tables corresponds to NAND gate.

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(i)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(ii)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(iii)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	1

(iv)

(a) (iv) (b) (iii)
(c) (ii) (d) (i)

14. The range of nuclear force is of the order of
(a) 2×10^{-10} m (b) 1.5×10^{-20} m
(c) 1.2×10^{-4} m (d) 1.4×10^{-15} m

15. What is the momentum of a photon having frequency 1.5×10^{13} Hz?
(a) 3.3×10^{-29} kg-m/s
(b) 3.3×10^{-34} kg-m/s
(c) 6.6×10^{-34} kg-m/s
(d) 6.6×10^{-32} kg-m/s

16. The two headlights of an approaching car are 1.4 m apart. At what maximum distance will the eye resolve them. Assume that the pupil diameter is 5.0 mm and the wavelength of light is 550 nm.
(a) 5 km (b) 10 km
(c) 8 km (d) 5.3 km

17. Find the wavelength of light that may excite an electron in the valence band of diamond to the conduction band. The energy gap is 5.50 eV.
(a) 226 nm (b) 312 nm
(c) 432 nm (d) 550 nm

18. A copper wire of length 50.0 cm and total resistance of 1.1×10^{-2} Ω is formed into a circular loop and placed perpendicular to a uniform magnetic field that is increasing at the constant rate of 10.0 mT/s. At what rate is thermal energy generated in the loop?
(a) 1.32×10^{-8} W (b) 2.36×10^{-4} W
(c) 3.68×10^{-6} W (d) 4.23×10^{-5} W

19. An electron is moving at a speed of 100 m/s along the x-axis through uniform electric and magnetic fields. The magnetic field is directed along the z-axis and has magnitude 5.0 T. In unit vector notation, what is the electric field?
(a) $100\mathbf{j}\,\frac{\text{V}}{\text{m}}$ (b) $-100\mathbf{k}\,\frac{\text{V}}{\text{m}}$
(c) $-100\mathbf{k}\,\frac{\text{V}}{\text{m}}$ (d) $500\mathbf{j}\,\frac{\text{V}}{\text{m}}$

20. The half-life of $_{92}U^{238}$ undergoing α-decay is 1.5×10^{17} s. What is the activity of 238 g sample of $_{92}U^{238}$?

(a) $2.8 \times 10^6\ s^{-1}$ (b) $3.9 \times 10^7\ s^{-1}$
(c) $4.3 \times 10^8\ s^{-1}$ (d) $5.6 \times 10^9\ s^{-1}$

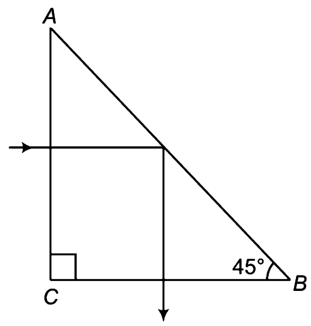
21. An intrinsic semiconductor has a resistivity of 0.50-Ω m at room temperature. Find the intrinsic carrier concentration, if the mobilities of electrons and holes are $0.39\ m^2/V\text{-}s$, and $0.11\ m^2/V\text{-}s$ respectively
(a) $1.2 \times 10^{18}/m^3$ (b) $2.5 \times 10^{19}/m^3$
(c) $1.9 \times 10^{20}/m^3$ (d) $3.1 \times 10^{21}/m^3$

22. The wavelength of spectral line coming from a distant star shifts from 600 nm to 600.1 nm. The velocity of the star relative to earth is
(a) 50 km/s (b) 100 km/s
(c) 25 km/s (d) 200 km/s

23. A bulb is placed at a depth of $2\sqrt{7}$ m in water $\left(\mu_w = \frac{4}{3}\right)$ and a floating opaque disc is placed over the bulb so that the bulb is not visible from the surface. What is the minimum diameter of the disc?
(a) 8 m (b) 12 m
(c) 15 m (d) 20 m

24. What is the refractive index of material of a plano-convex lens, if the radius of curvature of the convex surface is 10 cm and focal length of the lens is 30 cm?
(a) $\frac{6}{5}$ (b) $\frac{7}{4}$
(c) $\frac{2}{3}$ (d) $\frac{4}{3}$

25. A ray of light incident normally on one of the faces of a right angle prism is found to be totally reflected as shown in figure. What is the minimum value of the refractive index of the material of the prism?



(a) $\sqrt{2}$ (b) $\sqrt{\frac{3}{2}}$
(c) $\frac{4}{3}$ (d) None of these

26. In a two slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the plane of slits. If the screen is moved by 5×10^{-2} m towards the slits, the change in fringe width is 3×10^{-5} m. If the distance between slits is 10^{-3} m, the wavelength of light will be
(a) 3000 Å (b) 4000 Å
(c) 6000 Å (d) 7000 Å

27. For base station to mobile communication, the required frequency bond is
(a) 540-1600 kHz
(b) 200-325 MHz
(c) 5.9-6.42 GHz
(d) 840-935 MHz

28. A Carnot refrigerator extracts 35.0 kJ as heat during each cycle, operating with a coefficient of performance of 4.60. Find the energy per cycle transferred as heat to the surroundings
(a) 42.6 kJ (b) 53.2 kJ
(c) 63.9 kJ (d) 72.5 kJ

29. A carrier wave of peak voltage 10 V is used to transmit a message signal. What should be the peak voltage of the modulating signal in order to have a modulation index of 80%?
(a) 8 V (b) 10 V
(c) 12 V (d) 14 V

30. In the following equation, x, t and F represent respectively, displacement, time and force

$$F = a + bt + \frac{1}{c + dx} + A \sin(\omega t + \phi)$$

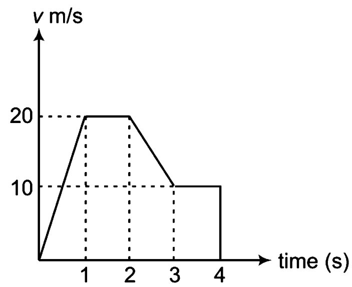
The dimensional formula for $A \cdot d$ is
(a) $[T^{-1}]$ (b) $[L^{-1}]$
(c) $[M^{-1}]$ (d) $[TL^{-1}]$

31. The angle between the vectors $\mathbf{A} = \mathbf{i} + \mathbf{j}$ and $\mathbf{B} = \mathbf{i} + \mathbf{j} + c\mathbf{k}$ is 30°. Find the unknown c.
(a) zero (b) ± 1
(c) $\pm\sqrt{\frac{2}{3}}$ (d) $\pm\frac{1}{2}$

32. Resultant of two vectors **A** and **B** is of magnitude P. If **B** is reversed, then resultant is of magnitude Q. What is the value of $P^2 + Q^2$?

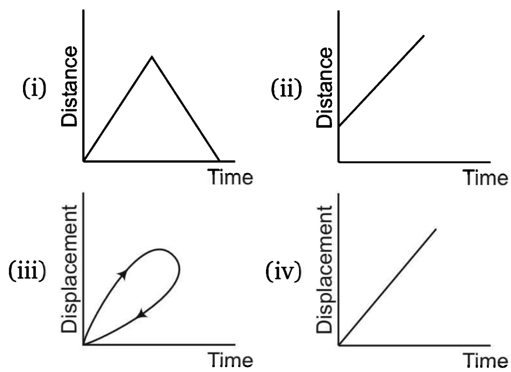
(a) $2(A^2 + B^2)$ (b) $2(A^2 - B^2)$
(c) $(A^2 - B^2)$ (d) $(A^2 + B^2)$

33. From the adjoining graph, the distance traversed by the particle is 4 s is



(a) 60 m (b) 25 m
(c) 55 m (d) 30 m

34. Which of the following graphs is/are not possible.



(a) (i) and (iii)
(b) (i) only
(c) (ii) and (iii)
(d) (iii) only

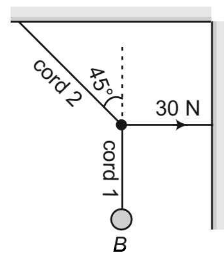
35. A body travelling along a straight line traverse one-third the distance with a velocity of 5 m/s. The remaining part of the distance was covered with velocity 3 m/s for half the time and with velocity 2 m/s for the other half of the time. The average velocity of the body over the whole time of motion will be

(a) 3 m/s (b) 5 m/s
(c) 2 m/s (d) 2.5 m/s

36. A projectile is thrown with an initial velocity of $\mathbf{v} = (p\mathbf{i} + q\mathbf{j})$ m/s. If the range of the projectile is double the maximum height reached by it, then

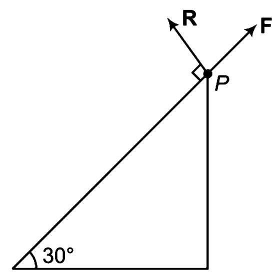
(a) $p = 2q$ (b) $q = 4p$
(c) $q = 2p$ (d) $q = p$

37. In the figure shown, the tension in the horizontal cord is 30 N. Find the weight of the body B.



(a) 40 N (b) 30 N
(c) 20 N (d) 10 N

38. In the following figure, an object of mass 1.2 kg is at rest at point P. If R and F are the reaction and the frictional force, respectively, then



(a) $R = 6\,\text{N}; F = 6\sqrt{3}\,\text{N}$
(b) $R = 3\,\text{N}; F = 3\sqrt{3}\,\text{N}$
(c) $R = 6\,\text{N}; F = 3\,\text{N}$
(d) $R = 6\sqrt{3}\,\text{N}; F = 6\,\text{N}$

39. A body of mass 1.0 kg strikes elastically with another body at rest and continues to move in the same direction with one-fourth of its initial velocity. The mass of the other body is

(a) 0.6 kg (b) 2.4 kg
(c) 3.0 kg (d) 4.0 kg

40. Moment of inertia does not depend on

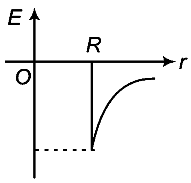
(a) mass distribution of body
(b) torque
(c) shape of the body
(d) the position of axis of rotation

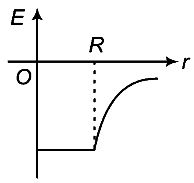
41. Three thin uniform rods each of mass M and length L are placed along the three axes of a Cartesian coordinate system. The moment of inertia of the system about z-axis is

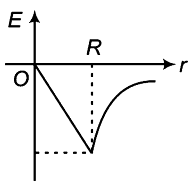
(a) $ML^{2/3}$ (b) $2ML^{2/3}$

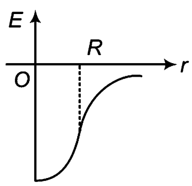
(c) $ML^{2/6}$ (d) ML^2

42. Which of the following graphs represents the gravitational field intensity due to solid sphere of radius R?

(i)


(ii)


(iii)


(iv)


(a) (i) only (b) (ii) only

(c) (iii) only (d) (iv) only

43. If a graph is plotted between T^2 and r^3 for a planet, then its slope will be

(a) $\frac{4\pi^2}{GM}$ (b) $\frac{GM}{4\pi^3}$

(c) $4\pi GM$ (d) GM

44. Three particles of equal mass m are situated at the vertices of an equilateral triangle of side l. What should be the velocity of each particle, so that they move on a circular path without changing l?

(a) $\sqrt{\frac{Gm}{2l}}$ (b) $\sqrt{\frac{Gm}{l}}$

(c) $\sqrt{\frac{2Gm}{l}}$ (d) $\sqrt{\frac{3}{2}\frac{Gm}{l}}$

45. A projectile is fired vertically upward from the surface of earth with a velocity of kv_e, where v_e is the escape velocity and $k < 1$. Neglecting air resistance, the maximum height to which it will rise, measured from the centre of the earth, is

(R = radius of earth)

(a) $\frac{R}{1-k^2}$ (b) $\frac{R}{k^2}$

(c) $\frac{1-k^2}{R}$ (d) $\frac{k^2}{R}$

46. The velocities of a particle in SHM at positions x_1 and x_2 are v_1 and v_2 respectively. Its time period will be

(a) $2\pi\sqrt{(v_1^2 - v_2^2)/(x_2^2 - x_1^2)}$

(b) $2\pi\sqrt{(x_1^2 + x_2^2)/(v_2^2 - v_1^2)}$

(c) $2\pi\sqrt{(x_2^2 - x_1^2)/(v_1^2 - v_2^2)}$

(d) $2\pi\sqrt{(x_2^2 + x_1^2)/(v_1^2 + v_2^2)}$

47. When a closed pipe is suddenly opened, the second overtone of closed pipe and first overtone of open pipe differ by 100 Hz. The fundamental frequency of the closed pipe will be

(a) 200 Hz (b) 150 Hz

(c) 100 Hz (d) 50 Hz

48. The phenomenon of beats can take place

(a) for longitudinal waves only

(b) for transverse waves only

(c) for sound waves only

(d) for both longitudinal and transverse waves

49. A solid sphere of mass 1.0 kg and diameter 0.3 m is suspended from a wire. If the twisting couple per unit twist for the wire is 6×10^{-3} N-m/rad, then the time period of small oscillations will be

(a) 0.7 s (b) 7.7 s

(c) 77 s (d) 777 s

50. A train approaching a railway crossing at a speed of 120 km/h sounds a whistle at frequency 640 Hz when it is 300 m away from the crossing. The speed of sound in air is 340 m/s. What will be the frequency heard by a person standing on a road perpendicular to the track through the crossing at a distance of 400 m from the crossing?

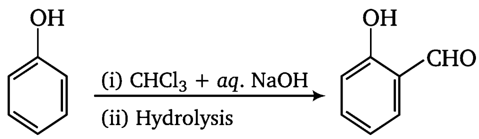
(a) 680 Hz

(b) 640 Hz

(c) 720 Hz

(d) 358 Hz

Chemistry

1. 

The above transformation proceeds through
(a) electrophilic addition
(b) electrophilic substitution
(c) activated nucleophilic substitution
(d) benzyne intermediate

2. In the diazotization of aryl amine, the use of nitrous acid is that
(a) it suppresses hydrolysis of phenol
(b) it is a source of electrophilic nitrosonium ion
(c) it neutralizes the base liberated
(d) All of the above

3. When MnO_2 is fused with KOH, a coloured compound is formed, the product and its colour are
(a) $KMnO_4$, purple
(b) K_2MnO_4, dark green
(c) Mn_2O_3, brown
(d) Mn_3O_4, black

4. The decay of ${}_{92}U^{238}$ nucleus by an α-particle emission produces a thorium nucleus
(a) ${}_{90}Th^{237}$ (b) ${}_{92}Th^{234}$
(c) ${}_{90}Th^{236}$ (d) ${}_{90}Th^{234}$

5. Considering the elements B, C, N, F and Si, the correct order of their non-metallic character is
(a) B > C > Si > N > F
(b) Si > C > B > N > F
(c) F > N > C > B > Si
(d) F > N > C > Si > B

6. Complete the following nuclear reaction by choosing the correct option
$${}_{95}Am^{241} + {}_{2}He^{4} \longrightarrow \ldots\ldots\ldots + 2{}_{0}n^{1}$$
(a) ${}_{97}Bk^{241}$ (b) ${}_{97}Bk^{243}$
(c) ${}_{97}Am^{243}$ (d) ${}_{96}Cm^{242}$

7. P_4O_{10} dissolves in water to give
(a) phosphorous acid
(b) orthophosphoric acid
(c) hypophosphorus acid
(d) pyrophosphoric acid

8. Which among the following expressions is not correct?
(a) $\mu^{\infty} = \gamma_{+}\lambda_{+}^{\infty} + \gamma_{-}\lambda_{-}^{\infty}$
(b) $\lambda^{\infty} = \dfrac{1}{n^{+}}\lambda_{+}^{\infty} + \dfrac{1}{n^{-}}\lambda_{-}^{\infty}$
(c) $\lambda_{\text{cation}}^{\infty} = \mu_{\text{cation}}^{\infty} \times \text{faraday}$
(d) $\lambda_{\text{anion}}^{\infty} = \mu_{\text{cation}}^{\infty} \times \text{faraday}$

9. The correct expression for Arrhenius equation showing the effect of temperature on the rate constant is $(T_2 > T_1)$
(a) $\log_{10}\dfrac{k_2}{k_1} = \dfrac{E_a}{2.303R}\left[\dfrac{T_1T_2}{T_2 - T_1}\right]$
(b) $\log_{10}\dfrac{k_2}{k_1} = \dfrac{R}{2.303E_a}\left[\dfrac{T_2 - T_1}{T_1T_2}\right]$
(c) $\log_{10}\dfrac{k_2}{k_1} = \dfrac{E_a}{R}\left[\dfrac{T_2 - T_1}{T_1T_2}\right]$
(d) $\log_{10}\dfrac{k_2}{k_1} = \dfrac{E_a}{2.303R}\left[\dfrac{T_2 - T_1}{T_2T_1}\right]$

10. Which of the following relation is correct?
(i) $\dfrac{x}{m}$ = constant (at high pressure)
(ii) $\dfrac{x}{m}$ = constant $\times p^{1/n}$ (at intermediate pressure)
(iii) $\dfrac{x}{m}$ = constant $\times p^{n}$ (at lower pressure)
(a) All are correct
(b) All are wrong
(c) (i) and (ii) are correct
(d) (iii) is correct

11. In the preparation of CaO from $CaCO_3$ using the equilibrium
$$CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$$
K_p is expressed as
$$\log K_p = 7.282 - \frac{8500}{T}$$
During complete decomposition of $CaCO_3$, the temperature in celcius to be used is
(a) 1167 (b) 894
(c) 8500 (d) 850

12. If the salts M_2X, QY_2 and PZ_3 have the same solubilities, their K_{sp} values are related as
(a) $K_{sp}(M_2K) = K_{sp}(QY_2) < K_{sp}(PZ_3)$
(b) $K_{sp}(M_2X) > K_{sp}(QY_2) = K_{sp}(PZ_3)$
(c) $K_{sp}(M_2X) > K_{sp}(QY_2) = K_{sp}(PZ_3)$
(d) $K_{sp}(M_2X) > K_{sp}(QY_2) > K_{sp}(PZ_3)$

13. The emf of the cell involving the following reaction

$$2Ag^+ + H_2 \longrightarrow 2Ag + 2H^+$$

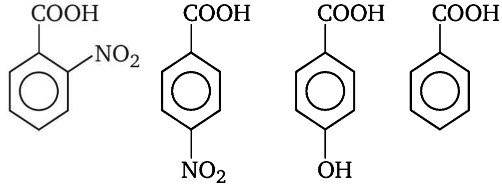
is 0.80 V. The standard oxidation potential of silver electrode is
(a) – 0.80 V (b) 0.80 V
(c) 0.40 V (d) – 0.40 V

14. In diborane (B_2H_6) there are
(a) three $3c - 2e^-$ bonds and three $2c - 2e^-$ bonds
(b) four $3c - 2e^-$ bonds and two $2c - 2e^-$ bonds
(c) two $3c - 2e^-$ bonds and four $2c - 2e^-$ bonds
(d) None of the above

15. The hybridisation states of $[Ni(CO)_4]$, $[Ni(CN)_4]^{2-}$ and $[NiCl_4]^{2-}$ species are respectively
(a) sp^3, sp^3, dsp^2 (b) dsp^2, sp^3, sp^3
(c) sp^3, dsp^2, dsp^2 (d) sp^3, dsp^2, sp^3

16. Which of the following Grignard reagents is suitable for the preparation of 3-methyl-2-butanol?
(a) 2-butanone + methyl magnesium bromide
(b) Acetone + ethyl magnesium bromide
(c) Acetaldehyde + isopropyl magnesium bromide
(d) Ethyl propionate + methyl magnesium bromide

17. Arrange the following acids in order of their increasing acidity.



[A] [B] [C] [D]

(a) $A < B < C < D$ (b) $B < C < A < D$
(c) $C < B < D < A$ (d) $C < D < B < A$

18. Which of the following are isoelectronic molecules?
(a) NO^+ and F_2 (b) Co and O_2^{2-}
(c) CO and NO^+ (d) O_2^{2-} and N_2

19. The following reagent is used for introducing a formyl group (—CHO) into the benzene ring
(a) CO + HCl
(b) HCN + HCl
(c) Both (a) and (b)
(d) None of the above

20. In the following sequence of reactions, the end product is

$$CaC_2 \xrightarrow{H_2O} A \xrightarrow{Hg^{2+}/H_2SO_4} B \xrightarrow{[O]} C \xrightarrow{Ca(OH)_2} D \xrightarrow{heat} E$$

(a) acetaldehyde (b) formaldehyde
(c) acetic acid (d) acetone

21. Arrange the following $CH_3CH_2CH_2Cl$ (I), $CH_3CH_2—CHCl—CH_3$ (II), $(CH_3)_2CHCH_2Cl$ (III) and $(CH_3)_3C—Cl$ (IV) in order of decreasing tendency towards $S_{N}2$ reactions
(a) I > III > II > IV (b) III > IV > II > I
(c) II > I > III > IV (d) IV > III > II > I

22. A carbonyl compound with molecular weight 86, does not reduce Fehling's solution, forms crystalline bisulphite derivatives and gives iodoform test. The possible compounds can be
(a) 2-pentanone and 3-pentanone
(b) 2-pentanone and 3-methyl-2-butanone
(c) 2-pentanone and pentanal
(d) 3-pentanone and 3-methyl-2-butanone

23. When propionic acid is treated with aqueous $NaHCO_3$, CO_2 is liberated. The 'C' of CO_2 comes from
(a) methyl group
(b) carboxylic acid group
(c) methylene group
(d) bicarbonate

24. The energy of an electron in the first Bohr orbit of H atom is –13.6 eV. The possible energy value of the excited state(s) for electrons in Bohr orbits of hydrogen is
(a) – 3.4 eV
(b) – 4.2 eV
(c) – 6.8 eV
(d) + 6.8 eV

25. The electrons identified by quantum numbers n and l (i) $n=4$; $l=1$, (ii) $n=4, l=0$; (iii) $n=3$, $l=2$; (iv) $n=3, l=1$ can be placed in order of increasing energy, from the lowest to highest as

(a) (iv) < (ii) < (iii) < (i)
(b) (ii) < (iv) < (i) < (iii)
(c) (i) < (iii) < (ii) < (iv)
(d) (iii) < (i) < (iv) < (ii)

26. One mole of an ideal gas expands at a constant temperature of 300 K from an initial volume of 10 L to a final volume of 20 L. The work done in expanding the gas is

(a) 750 J (b) 1726 J
(c) 1500 J (d) 3456 J

27. Assuming the salts to unionized in solution which of the following has the highest osmotic pressure?

(a) 1% CsCl (b) 1% RbCl
(c) 1% KCl (d) 1% NaCl

28. Which of the d-orbital is used in sp^3d hybridisation?

(a) d_{xy} (b) $d_{x^2-y^2}$
(c) d_{z^2} (d) d_{yz}

29. Formic acid can be distinguished from acetic acid by reacting with

(a) $NaHCO_3$
(b) dil. acidified $KMnO_4$ solution
(c) 2, 4-dinitrophenyl hydrazine
(d) Na metal

30. An alkyl cyanide forms an amide when it is treated with

(a) $H_2O + HCl$ (b) $NaOH + H_2O$
(c) $H_2O_2 + NaOH$ (d) $H_2SO_4 + H_2O$

31. A compound 'X' neither reacts with sodium, displacing hydrogen nor with phosphorus pentachloride to give hydrogen chloride. X reduces an alkaline solution of Cu (II) salt on gentle warming. The structure of X is

(a) primary alcohol
(b) secondary alcohol
(c) a ketone
(d) an aldehyde

32. The end product (B) in the following sequence of reactions is

$$CH_3Cl \xrightarrow{KCN} B \xrightarrow{H^+/H_2O} B$$

(a) CH_3COOH (b) $HCOOH$
(c) CH_3NH_2 (d) CH_3COCH_3

33. Given,

$$NH_3(g) + 3Cl_2(g) \rightleftharpoons NCl_3(g) + 3HCl(g) - \Delta H_1$$
$$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g) - \Delta H_2$$
$$H_2(g) + Cl_2(g) \rightleftharpoons 2HCl(g) + \Delta H_3$$

The heat of formation of $NCl_3(g)$ in terms of ΔH_1, ΔH_2 and ΔH_3 is

(a) $\Delta H_f = -\Delta H_1 + \frac{1}{2}\Delta H_2 - \frac{3}{2}\Delta H_3$
(b) $\Delta H_f = -\Delta H_1 + \frac{1}{2}\Delta H_2 + \frac{3}{2}\Delta H_3$
(c) $\Delta H_f = -\Delta H_1 - \frac{1}{2}\Delta H_2 - \frac{3}{2}\Delta H_3$
(d) None of the above

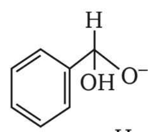
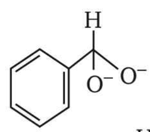
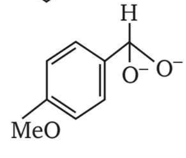
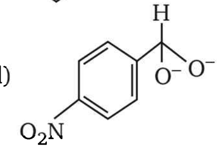
34. For the reaction,

$$N_2O_5 \longrightarrow 2NO_2 + \frac{1}{2}O_2$$
$$\frac{-d[N_2O_5]}{dt} = k_1[N_2O_5]$$
$$\frac{d[NO_2]}{dt} = k_2[N_2O_5]$$
$$\frac{d[O_2]}{dt} = k_3[N_2O_5]$$

The relation in between k_1, k_2 and k_3 is

(a) $2k_1 = k_2 = 4k_3$ (b) $k_1 = k_2 = k_3$
(c) $2k_1 = 4k_2 = k_3$ (d) None of these

35. In a Cannizzaro's reaction, the intermediate that will be the best hydride donor is

(a) 
(b) 
(c) 
(d) 

36. The gold numbers of A, B, C and D are 0.04, 0.0002, 10 and 25 respectively. The protective powers of A, B, C and D are in the order

(a) $A > B > C > D$ (b) $B > A > C > D$
(c) $D > C > B > A$ (d) $C > A > B > D$

37. When chlorine is passed through hot concentrated alkali solutions which one of the following is formed?
(a) [Tetraoxochloric (VII)]
(b) [Trioxochlorate (V)]
(c) Chloric (III) acid
(d) [Monooxochlorate (I)]

38. Which of the following has –O–O–linkage?
(a) $H_2S_2O_6$ (b) $H_2S_2O_8$
(c) $H_2S_2O_3$ (d) $H_2S_4O_6$

39. $KMnO_4$ gets reduced to
(a) K_2MnO_4 in neutral medium
(b) MnO_2 in acidic medium
(c) Mn^{2+} in alkaline medium
(d) MnO_2 in neutral medium

40. Which of the following is an outer orbital complex?
(a) $[Fe(CN)_6]^{4-}$ (b) $[FeF_6]^{3-}$
(c) $[Co(NH_3)_6]^{3+}$ (d) $[Co(CN)_6]^{3-}$

41. Which of the following has the largest number of isomers?
(a) $[Ru(NH_3)_4Cl_2]^+$ (b) $[Co(en)_2Cl_2]^+$
(c) $[Ir(PR_3)_2H(CO)]^{2+}$ (d) $[Co(NH_3)_5Cl]^{2+}$

42. In the following nuclear transmutation
$${}_{92}U^{238} + X \longrightarrow {}_{92}U^{239} \xrightarrow{-\beta} Y \xrightarrow{-\beta} {}_{94}Pu^{239}$$
X and Y respectively are
(a) ${}_0n^1$, ${}_{93}Np^{239}$ (b) ${}_0n^1$, ${}_{92}Np^{240}$
(c) γ, ${}_{93}Np^{239}$ (d) ${}_0H^1$, ${}_{92}Np^{239}$

43. Le-Chatelier's principle is not applicable to
(a) $Fe(s) + S(s) \rightleftharpoons FeS(s)$
(b) $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$
(c) $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$
(d) $N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$

44. For a concentrated solution of a weak electrolyte A_x, B_y, the degree of dissociation is given as
(a) $\alpha = \sqrt{\dfrac{K_{eq}}{C(x+y)}}$
(b) $\alpha = \sqrt{\dfrac{K_{eq}C}{(xy)}}$
(c) $\alpha = \left(\dfrac{K_{eq}}{(C^{x+y-1}) \cdot x^x y^y}\right)^{\frac{1}{(x+y)}}$
(d) $\alpha = \sqrt{\dfrac{K_{eq}}{xyC}}$

45. A fuel has the same knocking property as a mixture of 70% isooctane (2, 2, 4-trimethylpentane) and 30% *n*-heptane by volume. The octane number of the fuel is
(a) 100 (b) 70
(c) 50 (d) 40

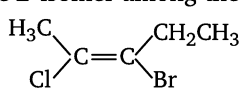
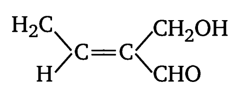
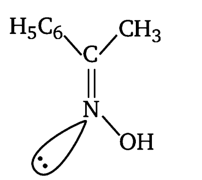
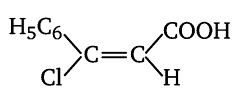
46. When Friedel-Crafts alkylation of benzene is carried out with *n*-propyl bromide, the major product is
(a) *n*-propyl benzene
(b) isopropyl benzene
(c) 2-ethyl benzene
(d) None of the above

47. Cumene $\xrightarrow[\text{(ii) } H_2O,\, H^+]{\text{(i) } O_2}$ (X) and (Y)

(X) and (Y) are respectively
(a) toluene, propene
(b) toluene, propylchloride
(c) phenol, acetone
(d) phenol, acetaldehyde

48. Hydrolysis of XeF_4 and XeF_6 with water gives
(a) $XeOF_4$ (b) XeO_2F_2
(c) XeO_3 (d) $XeOF_2$

49. Arrange the following carbocations in order of increasing stability
$(CH_3)_3C\overset{+}{C}H_2$ $[A]$, $(CH_3)_3\overset{+}{C}$ $[B]$,
$CH_3CH_3C^+H_2$ $[C]$, $CH_3\overset{+}{C}HCH_2CH_3$ $[D]$
(a) $D < C < B < A$ (b) $C < D < A < B$
(c) $A < C < D < B$ (d) $B < D < C < A$

50. The Z isomer among the following is
(a) 
(b) 
(c) 
(d) 

Mathematics

1. If $a_i > 0$ for $i = 1, 2, \ldots, n$ and $a_1 a_2 \ldots a_n = 1$, then minimum value of $(2+a_1)(2+a_2)\ldots(2+a_n)$ is

(a) $2^{3n/2}$ (b) $2^{n/2}$
(c) 2^{2n} (d) 2^n

2. The solution set of the inequality $4^{-x+\frac{1}{2}} - 7 \cdot (2^{-x}) - 4 < 0$ for $x \in R$ is

(a) $(-\infty, 2)$ (b) $(-2, \infty)$
(c) $(-\infty, \infty)$ (d) $(2, \infty)$

3. Let $a, b > 0$ satisfy $a^3 + b^3 = a - b$, then

(a) $a^2 + b^2 > 1$ (b) $a^2 + b^2 < 0$
(c) $a^2 + b^2 = 1$ (d) $a^2 + ab + b^2 < 1$

4. A fair coin is tossed 100 times. The probability of getting tails an odd number of time is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) 0 (d) 1

5. Let $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$, then $f(\theta)$

(a) ≤ 0 only for $\theta \leq 0$ (b) ≥ 0 for all real θ
(c) ≤ 0 for all real θ (d) ≥ 0 only for $\theta \geq 0$

6. In a ΔABC, if $\tan\frac{A}{2} = \frac{5}{6}$, and $\tan\frac{B}{2} = \frac{20}{37}$ the sides a, b, c of the triangle are in

(a) GP (b) HP
(c) AP (d) None of these

7. Total number of solutions of $\sin^4 x + \cos^4 x = \sin x \cos x$ is $[0, 2\pi]$ is equal to

(a) 2 (b) 4 (c) 6 (d) 8

8. If $a\sin^{-1} x - b\cos^{-1} x = c$, then $a\sin^{-1} x + b\cos^{-1} x$ is equal to

(a) $\frac{\pi ab + c(a-b)}{a+b}$ (b) 0
(c) $\frac{\pi ab - c(a-b)}{a+b}$ (d) $\frac{\pi}{2}$

9. If algebraic sum of distances of a variable line from points $(2, 0)$, $(0, 2)$ and $(-2, -2)$ is zero, then the line passes through the fixed point

(a) $(-1, -1)$ (b) $(1, 1)$
(c) $(2, 2)$ (d) $(0, 0)$

10. A line is drawn through the point $P(3, 11)$ to cut the circle $x^2 + y^2 = 9$ at point A and B. Then, $PA \cdot PB$ is equal to

(a) 205 (b) 9
(c) 139 (d) 121

11. The locus of the point of intersection of the tangents at the extremeties of a chord of the circle $x^2 + y^2 = a^2$ which touches the circle $x^2 + y^2 - 2ax = 0$ passes through the point

(a) $(a/2, 0)$ (b) $(0, a/2)$
(c) $(a, 0)$ (d) $(0, 0)$

12. If the lines joining the origin to the intersection of the line $y = mx + 2$ and the circle $x^2 + y^2 = 1$ are at right angles, then

(a) $m = \sqrt{3}$ (b) $m = \pm\sqrt{7}$
(c) $m = \sqrt{1}$ (d) $m = \sqrt{5}$

13. If the parabolas $y^2 = 4x$ and $x^2 = 32y$ intersect at $(16, 8)$ at an angle θ, then θ equals to

(a) $\tan^{-1} 5/3$ (b) $\tan^{-1} 4/5$
(c) $\tan^{-1} 3/5$ (d) $\pi/2$

14. If the normal at the point $P(\theta)$ to the ellipse $\frac{x^2}{14} + \frac{y^2}{5} = 1$ intersects it again at the point $Q(2\theta)$, then $\cos\theta$ equals to

(a) $-\frac{2}{3}$ (b) $\frac{2}{3}$
(c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

15. The eccentricity of the hyperbola with latus rectum 12 and semi-conjugate axis $2\sqrt{3}$, is

(a) 3 (b) $\sqrt{\frac{3}{2}}$
(c) $2\sqrt{3}$ (d) 2

16. The projections of a directed lines segment on the coordinate axes are 12, 4, 3. The direction cosines of the line are

(a) $\frac{12}{13}, \frac{4}{13}, \frac{3}{13}$ (b) $\frac{12}{13}, \frac{4}{13}, -\frac{3}{13}$
(c) $-\frac{12}{13}, \frac{4}{13}, \frac{3}{13}$ (d) $\frac{12}{13}, -\frac{4}{13}, -\frac{3}{13}$

17. The equation of the plane that contains the point $(1, -1, 2)$ and is perpendicular to each of the planes $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$ is

(a) $5x + 4y - z = 7$ (b) $5x - 4y + z = 7$
(c) $-5x + 4y - z = 7$ (d) $5x - 4y - z = 7$

18. If $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$, then I is equal to

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{6}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

19. If $I_1 = \int_0^{\pi/2} f(\sin 2x) \sin x \, dx$ and $I_2 = \int_0^{\pi/4} f(\cos 2x) \cos x \, dx$, then I_1 / I_2 is equal to

(a) 1 (b) $\sqrt{2}$
(c) $\frac{1}{\sqrt{2}}$ (d) 2

20. The area bounded by the two parabolas $y^2 = x$ and $x^2 = y$ is given by

(a) 1 (b) $\frac{2}{3}$
(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

21. If $f(x) = p|\sin x| + qe^{|x|} + r|x|^3$ and if $f(x)$ is differentiable at $x = 0$, then

(a) $p = 0, q = 0$ and $r = 0$
(b) $p + q = 0$ and r is any real number
(c) $p + q + r = 0$
(d) $-p + q - r = 0$

22. If $f(0) = 0, f'(0) = 3$, then $y'(0)$ will be equal to, where $y = f(f(f(f(f(x)))))$

(a) 0 (b) 3 (c) 3^4 (d) 3^5

23. If $f(x) = xe^{x(1-x)}$, then $f(x)$ is

(a) increasing on R
(b) decreasing on $\left[-\frac{1}{2}, 1\right]$
(c) increasing on $\left[-\frac{1}{2}, 1\right]$
(d) decreasing on R

24. The parabola $y^2 = 4x$ and the circle $x^2 + y^2 - 6x + 1 = 0$ will

(a) intersect at exactly one point
(b) touch each other at two distinct points
(c) touch each other at exactly one point
(d) intersect at two distinct points

25. If $f(x) = \frac{x^2 - 1}{x^2 + 1}$ for every real number x, then the minimum value of f

(a) -1 (b) does not exist
(c) 0 (d) 1

26. The equation of the common tangent to the parabola $y^2 = 8x$ and rectangular hyperbola $xy = -1$ is

(a) $x - y + 2 = 0$ (b) $9x - 3y + 2 = 0$
(c) $2x + y + 1 = 0$ (d) $x + 2y - 1 = 0$

27. Let A and B be any two events, then $P(A \cap B)$

(a) $P(A \cup B) - P(A^C) - P(B^C)$
(b) $P(A) + P(B^C)$
(c) $P(B) + P(A^C)$
(d) None of the above

28. The solution of $\frac{dy}{dx} = x + y, y(0) = 0$ is

(a) $y = -x - 1 + e^{-x}$ (b) $y = -x - 1 + e^x$
(c) $y = -x + 1 + e^x$ (d) $y = x + 1 + e^x$

29. Let $f = \{(0, -1), (-1, -3), (2, 3), (3, 5)\}$ be a function from Z to Z defined by $f(x) = ax + b$. Then

(a) $a = 1, b = -2$ (b) $a = 2, b = 1$
(c) $a = 2, b = -1$ (d) $a = 1, b = 2$

30. Which of the following result is valid?

(a) $(1 + x)^n > (1 + nx)$ for all natural number n
(b) $(1 + x)^n \geq (1 + nx)$ for all natural number n, where $x > -1$
(c) $(1 + x)^n \leq (1 + nx)$ for all natural number n
(d) $(1 + x)^n < (1 + nx)$ for all natural number n

31. If n is a natural number, then

(a) $1^2 + 2^2 + \ldots + n^2 < n^3 / 3$
(b) $1^2 + 2^2 + \ldots + n^2 = n^3 / 3$
(c) $1^2 + 2^2 + \ldots + n^2 > n^3$
(d) $1^2 + 2^2 + \ldots + n^2 > n^3 / 3$

32. Which of the following statements is true?

(a) $\sqrt{51}$ is a rational number
(b) each radius of a circle is a chord
(c) circle is a particular case of an ellipse
(d) the centre of a circle bisects each chord of the circle

33. If $n > 1$ and n divides $(n-1)! + 1$, then
(a) n is always even
(b) n has to be composite number
(c) n is divisible by exactly two primes
(d) n has to be a prime

34. If $\lim_{x \to 0} \frac{ae^x - b\cos x + ce^{-x}}{x \sin x} = 2$, then
(a) $a = 1, b = 2, c = 1$ (b) $a = 1, b = 1, c = 2$
(c) $a = 2, b = 1, c = 1$ (d) $a = b = c = 1$

35. The number of solutions of $z^3 + \bar{z} = 0$ is
(a) 2 (b) 4 (c) 5 (d) 3

36. Reflection of the line $\bar{a}z + a\bar{z} = 0$ in the real axis is
(a) $\bar{a}z - a\bar{z} = 0$ (b) $az + \bar{a}\bar{z} = 0$
(c) $\bar{a}z + a\bar{z} = 0$ (d) $az - \bar{a}\bar{z} = 0$

37. If both the roots of the equation $x^2 - 6ax + 2 - 2a + 9a^2 = 0$ exceed 3, then
(a) $a < \frac{1}{2}$ (b) $a > \frac{1}{2}$
(c) $a < 1$ (d) $a > \frac{11}{9}$

38. The number of real values of x which satisfy the equation $\left|\frac{x}{x-1}\right| |x| = \frac{x}{|x-1|}$ is
(a) 2 (b) 1
(c) infinite (d) zero

39. If $I_n = \int_0^{\pi/4} \tan^n x \, dx$, then $\lim_{x \to \infty} n[I_n + I_{n+2}]$ is equal to
(a) $\frac{1}{2}$ (b) 1 (c) ∞ (d) 0

40. If $H_n = 1 + \frac{1}{2} + \ldots + \frac{1}{n}$, then the value of $S_n = 1 + \frac{3}{2} + \frac{5}{3} + \ldots + \frac{2n-1}{n}$ is
(a) $H_n + 2n$ (b) $n - 1 + H_n$
(c) $H_n - 2n$ (d) $2n - H_n$

41. The value of x satisfying $\log_2(3x-2) = \log_{\frac{1}{2}} x$ is
(a) 1 (b) $-\frac{1}{3}$ (c) -1 (d) $\frac{1}{3}$

42. If $\log_3 2$, $\log_3(2^x - 5)$ and $\log_3\left(2^x - \frac{7}{2}\right)$ are in AP, then x is equal to
(a) 8 (b) -8 (c) 3 (d) -3

43. If ${}^{n-1}C_r = (k^2 - 3)({}^nC_{r+1})$, then k belongs to
(a) $(\sqrt{3}, 2)$ (b) $(-\infty, -2)$
(c) $[-\sqrt{3}, \sqrt{3}]$ (d) $(2, \infty)$

44. The number of positive integers n such that 2^n divides $n!$ is
(a) one (b) two
(c) infinite (d) zero

45. The expression ${}^nC_0 + 2 \cdot {}^nC_1 + 3 \cdot {}^nC_2 + \ldots + (n+1) \cdot {}^nC_n$ is equal to
(a) $(n+1)2^n$ (b) $2^n(n+2)$
(c) $(n+2)2^{n-1}$ (d) $(n+2)2^{n+1}$

46. If A and B are coefficients of x^n in the expansions of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, then B/A is equal to
(a) $\frac{1}{2}$ (b) 2
(c) 1 (d) $\frac{1}{n}$

47. If A and B are two square matrices of the same order and m is a positive integer, then $(A+B)^m = {}^mC_0A^m + {}^mC_1A^{m-1}B + {}^mC_2A^{m-2}B^2 + \ldots + {}^mC_mB^m$, if
(a) $AB = -BA$ (b) $A^m = 0, B^m = 0$
(c) $AB = 2BA$ (d) $AB = BA$

48. If the system of linear equations $x + 2y - 3z = 1$, $(p+2)z = 3$, $(2p+1)y + z = 2$ has no solution, then
(a) $p = +2$ (b) $p = -2$
(c) $p = -\frac{1}{2}$ (d) $p = 3$

49. If $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$, then the number of distinct real roots of this equation in the interval $-\pi/2 < x < \pi/2$ is
(a) 2 (b) 0 (c) 1 (d) 3

50. Let m be a positive integer and $0 \le r \le m$ the value of $\sum_{r=0}^{m} \begin{vmatrix} 2r-1 & {}^mC_r & 1 \\ m^2 - 1 & 2^m & m+1 \\ \sin^2 m & \cos^2 m & \tan^2 m \end{vmatrix}$ will be
(a) 2^m (b) $m + 1$
(c) $m^2 - 1$ (d) 0

Answers

Physics

1. (d)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (a)	**6.** (b)	**7.** (d)	**8.** (c)	**9.** (a)	**10.** (d)
11. (d)	**12.** (a)	**13.** (d)	**14.** (d)	**15.** (a)	**16.** (b)	**17.** (a)	**18.** (b)	**19.** (d)	**20.** (a)
21. (b)	**22.** (a)	**23.** (b)	**24.** (d)	**25.** (a)	**26.** (c)	**27.** (b)	**28.** (a)	**29.** (a)	**30.** (b)
31. (c)	**32.** (a)	**33.** (c)	**34.** (a)	**35.** (b)	**36.** (c)	**37.** (b)	**38.** (d)	**39.** (a)	**40.** (b)
41. (*)	**42.** (c)	**43.** (a)	**44.** (b)	**45.** (a)	**46.** (c)	**47.** (c)	**48.** (a)	**49.** (b)	**50.** (a)

Chemistry

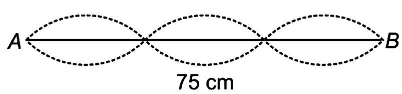
1. (b)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (d)	**9.** (d)	**10.** (c)
11. (b)	**12.** (a)	**13.** (b)	**14.** (c)	**15.** (d)	**16.** (a)	**17.** (d)	**18.** (c)	**19.** (c)	**20.** (d)
21. (a)	**22.** (b)	**23.** (d)	**24.** (a)	**25.** (a)	**26.** (b)	**27.** (d)	**28.** (c)	**29.** (b)	**30.** (a,d)
31. (d)	**32.** (a)	**33.** (d)	**34.** (a)	**35.** (d)	**36.** (b)	**37.** (b)	**38.** (b)	**39.** (d)	**40.** (b)
41. (b)	**42.** (a)	**43.** (a)	**44.** (c)	**45.** (b)	**46.** (b)	**47.** (c)	**48.** (c)	**49.** (c)	**50.** (a)

Mathematics

1. (b)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (b)	**6.** (d)	**7.** (a)	**8.** (a)	**9.** (d)	**10.** (d)
11. (a)	**12.** (b)	**13.** (c)	**14.** (a)	**15.** (d)	**16.** (a)	**17.** (d)	**18.** (b)	**19.** (b)	**20.** (c)
21. (b)	**22.** (d)	**23.** (c)	**24.** (d)	**25.** (a)	**26.** (a)	**27.** (d)	**28.** (b)	**29.** (c)	**30.** (b)
31. (d)	**32.** (c)	**33.** (d)	**34.** (a)	**35.** (b)	**36.** (b)	**37.** (d)	**38.** (b)	**39.** (b)	**40.** (d)
41. (a)	**42.** (c)	**43.** (a)	**44.** (b)	**45.** (c)	**46.** (a)	**47.** (d)	**48.** (b)	**49.** (a)	**50.** (d)

Hints & Solutions

Physics

1. 

$$\therefore \qquad n_p = \frac{P}{2l}\sqrt{\frac{T}{m}}$$

$$420 = \frac{P}{2l}\sqrt{\frac{T}{m}} \qquad \ldots\text{(i)}$$

$$315 = \frac{P-1}{2l}\sqrt{\frac{T}{m}} \qquad \ldots\text{(ii)}$$

Divided Eq. (i) by Eq. (ii), we get

$$\frac{420}{315} = \frac{P}{P-1} \Rightarrow P = 4$$

Now putting the value of P in Eq. (i)

$$420 = \frac{4}{2 \times 75 \times 10^{-2}}\sqrt{\frac{T}{m}}$$

$$105 \times 2 \times 75 \times 10^{-2} = \sqrt{\frac{T}{m}}$$

Lowest resonant frequency

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$n = \frac{1}{2 \times 75 \times 10^{-2}} \times 105 \times 2 \times 75 \times 10^{-2}$$

$$n = 105 \text{ Hz}$$

2. $W = 2.303\, nRT \log \frac{V_2}{V_1}$

$$= 2.303 \times 4 \times 8.31 \times 400 \log \frac{2V_1}{V_1}$$

$$= 2.303 \times 4 \times 8.31 \times 400 \times 0.3010$$

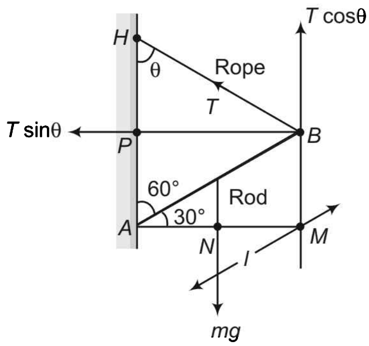
$$W = 9216.83 \text{ J}$$

Change in entropy

$$\Delta S = \frac{\Delta\theta}{T} = \frac{21220.13678}{400}$$

$$\Delta S = 23.04$$

3. Let the length of rod is l, taking moment about A



$$AN = \frac{l}{2}\cos 30°$$

$$AP = l\cos 60°$$

$$AM = l\cos 30°$$

Given $\quad T = \frac{mg}{2}$

$$T\cos\theta \times AM + T\sin\theta\, AP = mg \times AN$$

$$\frac{mg}{2} \times \cos\theta \times l \times \frac{\sqrt{3}}{2} + \frac{mg}{2} \times \sin\theta \times l \times \frac{1}{2}$$

$$= mg \times \frac{l}{2} \times \frac{\sqrt{3}}{2}$$

$$\sqrt{3}\cos\theta + \sin\theta = \sqrt{3}$$

$$\sqrt{3(1-\sin^2\theta)} = (\sqrt{3} - \sin\theta)$$

$$3(1-\sin^2\theta) = 3 + \sin^2\theta - 2\sqrt{3}\sin\theta$$

$$3 - 3\sin^2\theta = 3 - 2\sqrt{3}\sin\theta + \sin^2\theta$$

$$-4\sin^2\theta = -2\sqrt{3}\sin\theta$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\theta = 60°$$

4.

$$Y_1 = y\sin(\omega t + 0)$$

$$Y_2 = \frac{y}{2}\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$Y_3 = \frac{y}{3}\sin(\omega t + \pi)$$

According to superposition principle

$$Y = Y_1 + Y_2 + Y_3$$

$$= y\sin\omega t + \frac{y}{2}\cos\omega t + \frac{y}{3}\sin(\omega t + \pi)$$

$$= y\sin\omega t - \frac{y}{3}\sin\omega t + \frac{y}{2}\cos\omega t$$

$$= \frac{2y}{3}\sin\omega t + \frac{y}{2}\cos\omega t$$

$$= y\left(\frac{2}{3}\sin\omega t + \frac{1}{2}\cos\omega t\right)$$

$$= \frac{y}{6}(4\sin\omega t + 3\cos\omega t)$$

$$Y = \frac{4y}{6}\sin\omega t + \frac{3y}{6}\cos\omega t$$

$$A_1 = \frac{2y}{3} \text{ and } A_2 = \frac{y}{2}$$

$$A = \sqrt{A_1^2 + A_2^2}$$

$$= \sqrt{\frac{4y^2}{9} + \frac{y^2}{4}}$$

$$= y\sqrt{\frac{16+9}{36}} = \frac{y}{6} \times 5$$

$$= \frac{5y}{6} = 0.83y$$

5. Work done $\quad dW = pdV$

Given, $\quad p = aV^2$

and $\quad p = 10V^2$

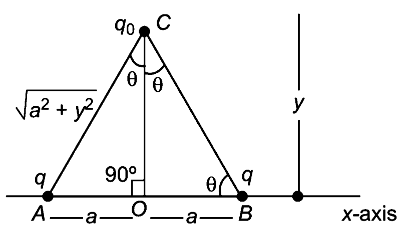
$\therefore \quad \int dW = \int_{V_1}^{V_2} 10V^2 dV$

$$= \int_1^2 10V^2 dV$$

$$= 10\left|\frac{V^3}{3}\right|_1^2$$

$$W = 23 \text{ J}$$

6.



Field at q_0

$$E = 2\left[\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(a^2+x^2)}\right]\cos\theta$$

Force at q_0

$$F = q_0E$$

$$F = 2\left[\frac{1}{4\pi\varepsilon_0}\cdot\frac{qq_0}{(a^2+x^2)}\right]\cos\theta$$

In AOC

$$\cos\theta = \frac{y}{\sqrt{a^2+y^2}}$$

$$F = 2\left[\frac{1}{4\pi\varepsilon_0}\cdot\frac{qq_0}{(a^2+y^2)}\right]\cdot\frac{y}{\sqrt{a^2+y^2}}$$

$$F = \frac{2qq_0y}{4\pi\varepsilon_0(a^2+y^2)^{3/2}}$$

For F to be maximum at q_0, $\frac{dF}{dy} = 0$

$$\frac{dF}{dy} = 0 = \frac{2qq_0}{4\pi\varepsilon_0}\left[\frac{(a^2+y^2)^{3/2} - 3(a^2+y^2)^{1/2}}{(a^2+y^2)^3}\right]$$

or $$y = \pm\frac{a}{\sqrt{2}}$$

7. If the particle comes to rest momentarily at a distance r from the fixed charge, then from conservation of energy, we have

$$\frac{1}{2}mu^2 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{a} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{r}$$

$$\frac{1}{2}\times 40\times 10^{-6}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{4\pi\varepsilon_0}\cdot Qq\left[\frac{1}{r}-\frac{1}{a}\right]$$

$$\left(\because u = 50\text{ cms}^{-1} = \frac{1}{2}\text{ ms}^{-1}\right)$$

$$= 9\times 10^9\times 10^{-8}\times 5\times 10^{-9}\left[\frac{1}{r}-\frac{1}{10/100}\right]$$

$$\frac{1}{2}\times 40\times 10^{-6}\times\frac{1}{2}\times\frac{1}{2} = 9\times 10^9\times 10^{-8}\times 5\times 10^{-9}\left[\frac{1}{r}-10\right]$$

$$\frac{1}{r} - 10 = \frac{100}{9}$$

$$\frac{1}{r} = \frac{100}{9} + 10 = \frac{190}{9}$$

$$r = 4.7\times 10^{-2}\text{m}$$

8. $$\frac{1}{R_1} = \frac{1}{8}+\frac{1}{8} \Rightarrow R_1 = 4$$

$$R_2 = 2 + 2 = 4\ \Omega$$

$$R_3 = 4 + 4 = 8\ \Omega$$

Similarly $\frac{1}{R_4} = \frac{1}{8}+\frac{1}{8} \Rightarrow R_4 = 4$

$\therefore$ Resultant resistance $R = 4 + 3 + 2 = 9\ \Omega$

$$V = I\times R$$

$$9 = I\times 9$$

Main current $\quad I = 1\text{ A}$

The current through in $3\ \Omega$ of resistance is 1 A.

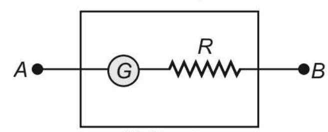
10. $$I_g = \frac{V}{G+R}$$

$$2\times 10^{-3} = \frac{0.2}{30+R}$$

$$30 + R = \frac{2\times 10^3}{20}$$

$$R = 100 - 30$$

$R = 70\ \Omega$ in series with galvanometer



Voltmeter

11. $$\frac{1}{2}mv^2 + \phi = E$$

$$= h\frac{c}{\lambda}$$

$$\frac{1}{2}mv^2 + 2\times 1.6\times 10^{-19} = 6.6\times 10^{-34}\times\frac{3\times 10^8}{450\times 10^{-9}}$$

$$mv^2 = \frac{6.6\times 10^{-34}\times 3\times 10^8\times 2}{450\times 10^{-9}} - 4\times 1.6\times 10^{-19}$$

$$\frac{1}{2}\times 1.9\times 10^{-31}v^2 = 8.8\times 10^{-19} - 4\times 1.6\times 10^{-19}$$

$$v^2 = \frac{4.8\times 10^{-19}}{1.9\times 10^{-31}}$$

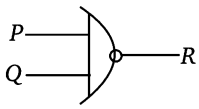
$$v = 1.58\times 10^6$$

12. The de-Broglie wavelength is given by

$$p = \frac{h}{\lambda}$$

$$p = \frac{2\pi\hbar}{\lambda} \qquad \left(\because \hbar = \frac{h}{2\pi}\right)$$

13. NAND



A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

15. $n = 1.5 \times 10^{13}$ Hz

$$p = \frac{h}{\lambda} = \frac{6.6 \times 10^{-34}}{c / n}$$

$$= \frac{6.6 \times 10^{-34} \times 1.5 \times 10^{13}}{3 \times 10^{8}}$$

$$= 3.3 \times 10^{29} \text{ kg-m/s}$$

16. $\dfrac{1.22 \times \lambda}{a} = \dfrac{x}{d}$

$$d = \frac{x \times a}{1.22 \times \lambda}$$

$$= \frac{1.4 \times 5 \times 10^{-3}}{1.22 \times 550 \times 10^{-9}}$$

$$= 10432 \text{ m}$$

$$= 10 \text{ km}$$

17. $E = \dfrac{hc}{\lambda} = 5.5 \times 1.6 \times 10^{-19}$

$$\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{5.5 \times 1.6 \times 10^{-19}}$$

$$\lambda = \frac{6 \times 3 \times 10^{-7}}{5 \times 1.6}$$

$$\lambda = \frac{9}{4} \times 10^{-7}$$

$$\lambda = 2.26 \times 10^{-7}$$

$$\lambda = 226 \text{ nm}$$

18. Given, $l = 50 \times 10^{-2}$ m, $Bv = 10$ mT/s

and $R = 1.1 \times 10^{-2}\ \Omega$

We know, $e = Bvl$

$$= 10 \times 50 \times 10^{-2} = 5 \text{ V}$$

$$P = \frac{e^2}{R} = \frac{25}{1.1 \times 10^{-2}}$$

$$= \frac{25}{11} \times 10^{3} = 2.27 \times 10^{3} \text{ W}$$

19. $u = 100$ m/s along x-axis

$\Rightarrow$ $\mathbf{u} = 100\mathbf{i}$ m/s

$B = 5 \cdot 0$ **T** along the z-axis

$\mathbf{B} = 5\mathbf{k}$

$E = Bv$

Electric field, $E = 5\hat{\mathbf{k}} \times 100\hat{\mathbf{i}}$

$\mathbf{E} = 500\hat{\mathbf{j}}$ V/m

20. Number of atom is 238 g sample of ${}_{92}U^{238}$

$= 6 \times 10^{23}$

decay constant, $\lambda = \dfrac{0.6931}{T}$

$$= \frac{0.6931}{1.5 \times 10^{17}}$$

Activity $= \lambda \cdot N$

$$= \frac{0.6931}{1.5 \times 10^{17}} \times 6 \times 10^{23}$$

$$= 2.8 \times 10^{6} \text{ s}^{-1}$$

21. $\rho = 0.50\ \Omega$-m

$$\sigma = e(n_e\mu_e + n_h\mu_h)$$

$$\because \quad \sigma = \frac{1}{\rho}$$

In intrinsic semi-conductor

$$n_e = n_h = n$$

$$\frac{1}{\rho} = e \times n\,(\mu_e + \mu_h)$$

$$\frac{1}{0.50} = 1.6 \times 10^{-19} \times n\,(0.39 + 0.11)$$

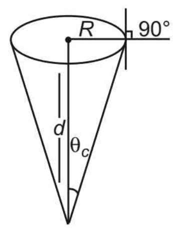
$$\frac{1 \times 10^{19}}{0.5 \times 0.5 \times 1.6} = n$$

$$\frac{100 \times 10^{19}}{25 \times 1.6} = n$$

$$\frac{40}{16}\times 10^{19} = n$$
$$n = 2.5\times 10^{19}/\text{m}^2$$

22. $$\Delta\lambda = \frac{v}{c}\lambda$$
$$\Rightarrow \quad \lambda = \frac{\Delta\lambda \cdot c}{v}$$
$$v = \Delta\lambda \cdot \frac{c}{\lambda}$$
$\because \Delta\lambda = 600.1 - 600 = 0.1$ nm
$$v = 0.1\times 10^{-9}\times \frac{3\times 10^{8}}{600\times 10^{-9}}$$
$$v = 50 \text{ km/s}$$

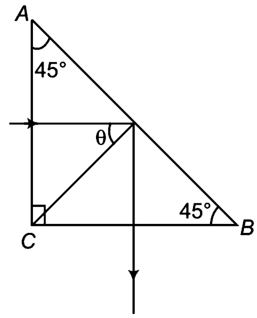
23. $$\frac{\sin\theta_c}{\sin 90°} = \frac{1}{\mu}$$



$$\sin\theta_c = \frac{1}{\mu}$$
$$\frac{R}{\sqrt{R^2+D^2}} = \frac{1}{\mu}$$
or $$\frac{R^2+D^2}{R^2} = \mu^2$$
or $$1+\frac{D^2}{R^2} = \mu^2$$
or $$\frac{D^2}{R^2} = \mu^2 - 1$$
or $$R = \frac{D}{\sqrt{\mu^2-1}}$$
$$= \frac{2\sqrt{7}}{\sqrt{\left(\frac{4}{3}\right)^2 - 1}} = \frac{2\sqrt{7}}{\sqrt{\frac{16}{9}-1}}$$
$$= 6 \text{ m}$$
Diameter of disc = 6m × 2 = 12 m

24. $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$\frac{1}{30} = (\mu - 1)\left(\frac{1}{\infty} + \frac{1}{10}\right)$$
$$\frac{10}{30} = (\mu - 1)$$
$$\mu = \frac{4}{3}$$

25.



For total internal reflection
$$\theta \geq C$$
Where C is the critical angle
$$\therefore \quad \sin\theta \geq \sin C$$
$$\sin\theta \geq \frac{1}{4}\mu \geq \frac{1}{\sin 45°}$$
$$\mu \geq \sqrt{2}$$

26. $$W_1 = \frac{D\lambda}{d}$$
$$W_2 = \frac{\lambda(D - 5\times 10^{-2})}{d}$$
$$W_1 - W_2 = 3\times 10^{-5} \text{ m}$$
$$\lambda\left(\frac{D}{d} - \frac{D}{d} + \frac{5\times 10^{-2}}{d}\right) = 3\times 10^{-5}$$
$$\lambda\left(\frac{5\times 10^{-2}}{10^{-3}}\right) = 3\times 10^{-5}$$
$$\lambda = \frac{3}{5}\times 10^{-6}$$
$$\lambda = 6000 \text{ Å}$$

28. Coefficient of performance
$$B = \frac{Q_2}{Q_1 - Q_2}$$
$$4.6 = \frac{35}{Q_1 - 35}$$
$$4.6Q_1 = 35 + 161$$
$$Q_1 = 42.6 \text{ kJ}$$

29. $V = 10$ volt
Modulating index = 80%
Peak voltage of modulating signal
$$= 10\times\frac{80}{100}$$
$$= 8 \text{ V}$$

30. $F = a + bt + \dfrac{1}{c + d \cdot x} + A\sin(\omega t + \phi)$

D of $F = D.0a = D$ of A

$$[MLT^{-2}] = a = A$$

Dimension of F = Dimension of $\dfrac{1}{c + d \cdot x}$

$$[MLT^{-2}] = \frac{1}{c + d \cdot x}$$

Since, $\quad c \equiv d \cdot x$

$$[MLT^{-2}] = \frac{1}{2c}$$

$$c = [M^{-1}L^{-1}T^{2}]$$

$$c \equiv d \cdot x$$

$$[M^{-1}L^{-1}T^{2}] = d[L]$$

$$d = [M^{-1}L^{-2}T^{2}]$$

$$A \cdot d = [MLT^{-2}] \cdot [M^{-1}L^{-2}T^{2}]$$

$$A \cdot d = [L^{-1}]$$

31. $\mathbf{A} = \mathbf{i} + \mathbf{j}, \mathbf{B} = \mathbf{i} + \mathbf{j} + c\hat{\mathbf{k}}$

$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$$

$$(\mathbf{i} + \mathbf{j}) \cdot (\mathbf{i} + \mathbf{j} + c\mathbf{k}) = \sqrt{2} \times \sqrt{2 + c^2} \times \frac{\sqrt{3}}{2}$$

$(\because \theta = 30^\circ)$

$$2 = \sqrt{2}\sqrt{2 + c^2} \times \frac{\sqrt{3}}{2}$$

$$4 = \sqrt{12 + 6c^2}$$

or $\quad 16 = 12 + 6c^2$

$$c^2 = \frac{4}{6} = \frac{2}{3}$$

$$c = \pm\sqrt{\frac{2}{3}}$$

32.

$$\mathbf{A} + \mathbf{B} = \mathbf{P} \quad \ldots(i)$$

$$\mathbf{A} - \mathbf{B} = \mathbf{Q} \quad \ldots(ii)$$

$$(\mathbf{A} + \mathbf{B}) \cdot (\mathbf{A} + \mathbf{B}) = P^2$$

$$(\mathbf{A} - \mathbf{B}) \cdot (\mathbf{A} - \mathbf{B}) = Q^2$$

$\Rightarrow \quad A^2 + 2AB\cos\theta + B^2 = P^2$

$\Rightarrow \quad A^2 - 2AB\cos\theta + B^2 = Q^2$

$\therefore \quad 2(A^2 + B^2) = (P^2 + Q^2)$

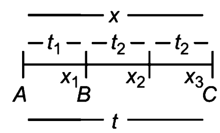
33. Distance = Total area covered by (v-t) graph

$$= \frac{1}{2} \times 1 \times 20 + 1 \times 20 + \frac{1}{2}(20 + 10) \times 1 + 1 \times 10$$

$$= 10 + 20 + 15 + 10$$

$$= 55 \text{ m}$$

35.



Time taken in the one third part of total distance coverd is t_1 and for remaining part is $2t_2$

$$t_1 = \frac{x/3}{5} = \frac{x}{15}$$

$$x_1 + x_2 = \frac{2}{3}x$$

$$3t_2 + 2t_2 = \frac{2}{3}x$$

$$5t_2 = \frac{2}{3}x$$

$$t_2 = \frac{2x}{15}$$

Total time $= t_1 + 2t_2$

$$= \frac{x}{15} + \frac{2x}{15}$$

$$t = \frac{3x}{15} = \frac{x}{5}$$

Average velocity

$$v = \frac{x}{\frac{x}{5}}$$

$$v = 5 \text{ m/s}$$

36. Given, $\mathbf{v} = p\mathbf{i} + q\mathbf{j}$

According to equation

$$2H = R$$

$$\frac{u^2\sin^2\theta}{2g} = \frac{u^2\sin 2\theta}{g}$$

$$\sin^2\theta = 2\sin\theta\cos\theta$$

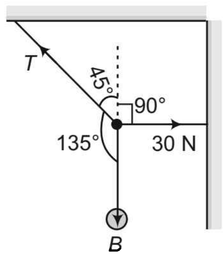
$$\tan\theta = 2$$

$$\tan\theta = \frac{q}{p}$$

$\therefore \quad 2 = \dfrac{q}{p}$

$$q = 2p$$

37.

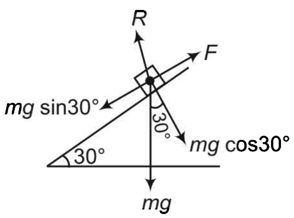


From Lami's theorem

$$\frac{B}{\sin 135°} = \frac{30}{\sin 135°}$$

$$B = 30 \text{ N}$$

38.



Given, $m = 1.2$ kg

$R = mg \cos 30°$

$= 1.2 \times 10 \times \frac{\sqrt{3}}{2}$

$R = (6\sqrt{3})$ N

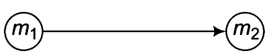
$f = mg \sin 30°$

$= 1.2 \times 10 \times \frac{1}{2}$

$F = 6$

39. $m_1 = 1$ kg, $m_2 = ?$

$u_1 = v$, $v_2 = 0$



After collision $v_1 = \frac{v}{4}$, $v_2 = ?$

From conservation of law

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$1 \times v + 0 = 1 \times \frac{v}{4} + m_2 \times v_2$$

$$v - \frac{v}{4} = m_2v_2$$

$$\frac{3v}{4} = m_2v_2 \quad \text{...(i)}$$

From the equation

$$v_1 - v_2 = -e(u_1 - u_2)$$

$$\frac{v}{4} - v_2 = -1(v - 0) \quad (\because e = 1)$$

$$\frac{v}{4} + v = v_2$$

$$\Rightarrow \quad v_2 = \frac{5v}{4} \quad \text{...(ii)}$$

From Eqs. (i) and (ii)

$$\frac{3v}{4} = m_2 \times \frac{5v}{4}$$

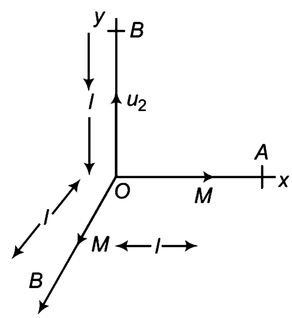
$$m_2 = 0.6 \text{ kg}$$

40. $I = mk^2$

where k is the radius of gyration.

Moment of inertia does not depend upon the torque.

41. Moment of inertia



$$I_{XZ} = M\left(\frac{l^2}{3}\right)$$

$$I_{YZ} = M\left(\frac{l^2}{3}\right)$$

From perpendicular axis theorem

$$I_Z = I_X + I_Y$$

$$= M\frac{l^2}{3} + M\frac{l^2}{3}$$

$$I_Z = \frac{2Ml^2}{3}$$

No option is correct

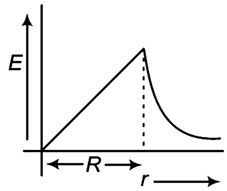
42. Let, a solid sphere of radius R, mass M and centre at O

$$I = \frac{GMr}{R^3}, \quad (r < R)$$

if $r > R$

$$I = \frac{GM}{r^2}$$

so



43. $\therefore \quad T = \dfrac{2\pi r^{3/2}}{(GM)^{1/2}}$

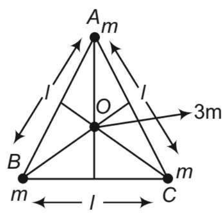
Taking square both side

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

Taking derivative of T^2 w.r.t. r^3

$$\frac{dT^2}{dr^3} = \frac{4\pi^2}{GM}$$

44.



As the gravitational force between any two particles is $F = \dfrac{Gmm}{a^2}$, the resultant force on each particle due to other two

$$F_R = \sqrt{F^2 + F^2 + 2F^2 \cos 60°}$$

$$F_R = \sqrt{3}\,\frac{Gm^2}{a^2}$$

$$AO = OB = OC = \frac{l\sqrt{3}}{2} \times \frac{2}{3} = \left(\frac{l}{\sqrt{3}}\right)$$

$$\therefore \qquad \frac{mv^2}{(l/\sqrt{3})} = \frac{\sqrt{3}\,Gm^2}{l^2}$$

$$v = \sqrt{\frac{Gm}{l}}$$

45. Kinetic energy = Potential energy

$$\frac{1}{2}m(kv_e)^2 = \frac{mgh}{1+\frac{h}{R}}$$

$$\because \qquad \frac{1}{2}mk^2 2gR = \frac{mgh}{1+\frac{h}{R}}$$

$$h = \frac{Rk^2}{1-k^2}$$

Distance of projectile from the surface = h

$\therefore$ Distance from the centre of earth

$$r = R + h$$

$$= R + \frac{Rk^2}{1-k^2}$$

$$r = \frac{R}{1-k^2}$$

46. The velocity of SHM

$$v = \omega\sqrt{a^2 - x^2}$$

According to question

$$v_1 = \omega\sqrt{a^2 - x_1^2} \qquad \ldots\text{(i)}$$

$$v_2 = \omega\sqrt{a^2 - x_2^2} \qquad \ldots\text{(ii)}$$

Dividing Eq. (i) by Eq. (ii)

$$\frac{v_1}{v_2} = \sqrt{\frac{a^2 - x_1^2}{a^2 - x_2^2}}$$

On both squaring

$$\frac{v_1^2}{v_2^2} = \frac{a^2 - x_1^2}{a^2 - x_2^2}$$

$$a^2 = \frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}$$

Put in Eq. (i)

$$v_1 = \omega\sqrt{a^2 - x_1^2}$$

$$v_1 = \frac{2\pi}{T}\sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2} - x_1^2}$$

$$T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$$

47. $5n'_1 - 2n_1 = 100$

n'_1 = Fundamental frequency of closed orgen pipe

n_1 = Fundamental frequency of open orgen pipe

$$\therefore \qquad n'_1 = \frac{v}{4l}$$

$$\therefore \qquad n_1 = \frac{v}{2l}$$

$$\frac{5v}{4l} - \frac{2v}{2l} = 100$$

or $$\frac{5V}{4l} - \frac{4V}{4l} = 100$$

or $$\frac{V}{4l} = 100$$

$$n'_1 = 100 \text{ Hz}$$

49. Given, $m = 1$ kg, $2r = 0.3$ m

and $c = 6 \times 10^{-3}$ N-m/rad

$$I = \frac{2}{5}mr^2$$

$$= \frac{2}{5} \times 1 \times \left(\frac{0.3}{2}\right)^2$$

$$= \frac{2}{5} \times \frac{0.09}{4}$$

$$I = 9 \times 10^{-3}$$

Time period of small oscillations

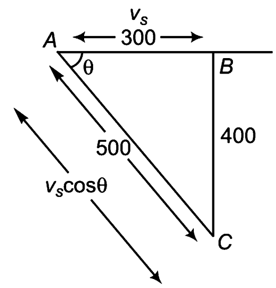
$$T = 2\pi\sqrt{\frac{I}{c}}$$

$$T = 2\pi\sqrt{\frac{9 \times 10^{-3}}{6 \times 10^{-3}}}$$

$$= 2 \times 3.14 \times \frac{1.732}{1.414}$$

$$T = 7.7 \text{ s}$$

50. $$v_s = 120 \text{ km/h} = \frac{100}{3} \text{ m/s}$$



$n = 640$ Hz and $v = 340$ m/s

$$n' = n\left(\frac{v}{v - v_s \cos\theta}\right)$$

$$= 640\left(\frac{340}{340 - \frac{100}{3} \times \frac{300}{500}}\right)$$

$$n' = 680 \text{ Hz}$$

Chemistry

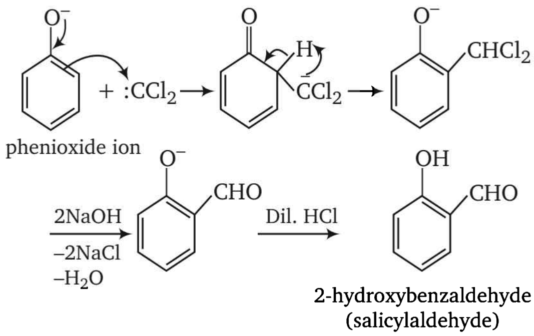
1. Reimer-Tiemann is an electrophilic substitution reaction and occurs through the following steps

   (i) Generation of electrophile

$$HO^- + CHCl_3 \rightleftharpoons H_2O + CCl_3^-$$

$$\longrightarrow \underset{\substack{\text{dichlorocarbene}\\ \text{(electrophile)}}}{:CCl_2} + Cl^-$$

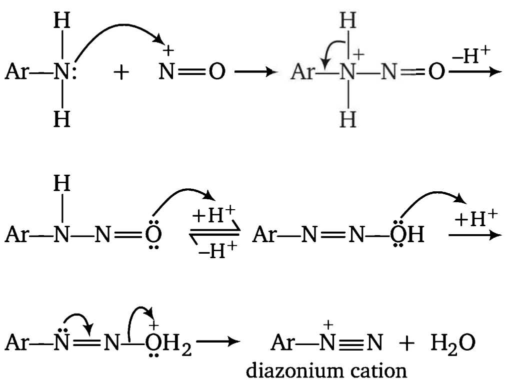
   (ii) Electrophilic substitution



2. The diazotization of amines is believed to occur the following mechanism

$$O{=}N{-}\ddot{\underset{\cdot\cdot}{O}}H + H^+ \rightleftharpoons O{=}N{-}\overset{+}{\underset{\cdot\cdot}{O}}H_2$$

$$\longrightarrow \underset{\substack{\text{nitrosonium}\\ \text{ion}}}{\overset{+}{N}{=}O} + H_2O$$



3. When blackish coloured compound MnO_2 is fused with KOH in presence of air, produces a dark green colored compound potassium manganate.

$$\underset{\substack{\text{blackish}\\ \text{colour}}}{MnO_2} + 4KOH + O_2 \xrightarrow{\text{Fuse}} \underset{\substack{\text{potassium mangnate}\\ \text{(green coloured)}}}{2K_2MnO_4} + 2H_2O$$

4. α-particle has 4 units of atomic mass and 2 units of positive charge, Thus, an α-emission reduces the atomic mass by 4 and atomic number by 2.

$${}_{92}U^{238} \xrightarrow{-{}_2\alpha^4 \text{ particle}} {}_{90}Th^{234}$$

5. As the ionisation enthalpy decreases down the group, therefore the non-metallic characters also decreases. Similarly, as the ionisation enthalpy increases along a period, consequently the non-metallic character also increases. Thus, the correct order of their non-metallic character is

$$F > N > C > B > Si$$

6. Nuclear reaction using α-particle (${}_2He^4$) as the bombarding particles

$${}_{95}Am^{241} + {}_2He^4 \longrightarrow {}_{97}Bk^{243} + 2\,{}_0n^1$$

7. Phosphorus pentoxide, P_4O_{10} has great affinity for water and gives orthophosphoric acid as final product.

   It is therefore named phosphoric anhydride.

$$P_4O_{10} \xrightarrow{2H_2O} \underset{\substack{\text{metaphosphoric}\\ \text{acid}}}{4HPO_3} \xrightarrow{2H_2O}$$

$$\underset{\substack{\text{pyrophosphoric}\\ \text{acid}}}{2H_4P_2O_7} \xrightarrow{2H_2O} \underset{\text{orthophosphoric acid}}{4H_3PO_4}$$

8. The limiting ionic conductance ($\lambda^{\infty}_{\text{cation}}$ or $\lambda^{\infty}_{\text{anion}}$) is directly proportional to the absolute ionic mobility of that ion ($\mu^{\infty}_{\text{cation}}$ or $\mu^{\infty}_{\text{anion}}$) *i.e.*,

$$\lambda^{\infty}_{\text{cation}} = \mu^{\infty}_{\text{cation}} \times \text{Faraday}$$

and $$\lambda^{\infty}_{\text{antion}} = \mu^{\infty}_{\text{anion}} \times \text{Faraday}$$

   where, Faraday = 96500 coulombs.

9. The effect of temperature on the rate constant was proposed by Arrhenius equation, the correct expression of which is given below:

$$\log_{10}\frac{k_2}{k_1} = \frac{E_a}{2.303R}\left(\frac{T_2 - T_1}{T_1 T_2}\right)$$

10. At lower pressure, the graph is nearly straight and sloping , *i.e.*,

$$\frac{x}{m} \propto p^1$$

or $$\frac{x}{m} = \text{constant} \times p^1$$

At high pressure, $\frac{x}{m}$ becomes independent of value of pressure.

$$\frac{x}{m} \propto p^0$$

or $$\frac{x}{m} = \text{constant}$$

In the intermediate range of pressure, $\frac{x}{m}$ will depend on p raised to powers between 1 and 0, *i.e.*, fractions.

$$\frac{x}{m} \propto p^{\frac{1}{n}}$$

or $$\frac{x}{m} = \text{constant} \times p^{\frac{1}{n}}$$

11. For the reaction,

$$CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$$

$$K_p = p_{CO_2}$$

[$\because$ Reaction takes place in open atmosphere,

$\therefore$ $p_{CO_2} = 1$]

or $K_p = 1$

$\because$ $$\log K_p = 7.282 - \frac{8500}{T}$$

$\therefore$ $$\log 1 = 7.282 - \frac{8500}{T}$$

(where, T = absolute temperature)

or $$0 = 7.282 - \frac{8500}{T}$$

or $$T = 1167.261\ \text{K}$$

$$= (1167.26 - 273)^\circ\text{C}$$

$$= 894.26^\circ\text{C}$$

$$\approx 894^\circ\text{C}$$

12. For M_2X;

$$M_2X \rightleftharpoons 2M^+ + X^{2-}$$

$\therefore$ $$K_{sp} = (2s)^2 \times s$$

$$= 4s^3$$

For QY_2;

$$QY_2 \rightleftharpoons Q^{2+} + 2Y^-$$

$\therefore$ $$K_{sp} = s \times (2s)^2 = 4s^3$$

For PZ_3;

$$PZ_3 \rightleftharpoons P^3 + 3Z^-$$

$\therefore$ $$K_{sp} = s \times (3s)^3 = 27s^4$$

Thus, K_{sp} is in order,

$$K_{sp}(M_2X) = K_{sp}(QY_2) < K_{sp}(PZ_3)$$

13. $2Ag^+ + H_2 \longrightarrow 2Ag + 2H^+$

By convention, the cell may be represented as,

$$H_2 \mid H^+ \mid Ag^+ \mid Ag$$

$\therefore$ $$E^\circ_{cell} = E^\circ_{Ag^+/Ag} - E^\circ_{H^+/H_2}$$

or $$E^\circ_{Ag^+/Ag} = E^\circ_{cell} + E^\circ_{H^+/H_2}$$

$$= 0.80 + 0 = 0.80\ \text{V}$$

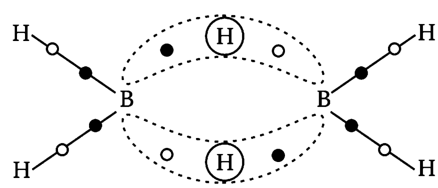
Thus, standard reduction potential of Ag electrode as,

$$E^\circ_{Ag^+/Ag} = 0.80\ \text{V}$$

and standard oxidation potential of Ag electrode as,

$$E^\circ_{Ag/Ag^+} = -0.80\ \text{V}$$

14. The structure of diborane is shown below

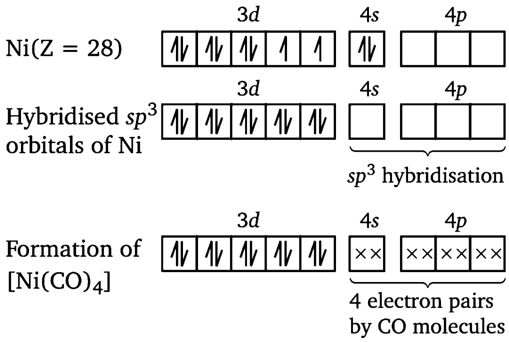


● Boron electron
○ Hydrogen electron

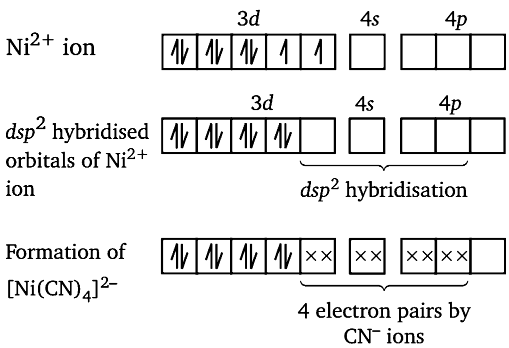
The four terminal B—H bonds are regular bonds ($2c - 2e^-$ bonds) but the two bridge B – H bonds are different and formed only by sharing of two electrons ($3c - 2e^-$ bonds).

15. In the formation of $[Ni(CO)_4]$, oxidation state of Ni is 0 and its electronic configuration is $[Ar]\ 3d^8 4s^2$.

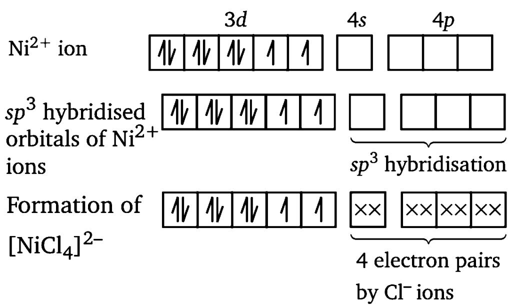
Hence, we have



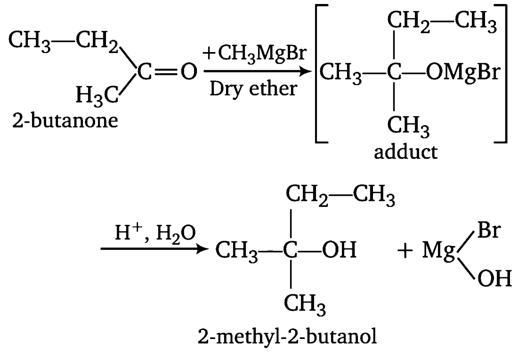
In $[Ni(CN)_4]^{2-}$, Ni is in +2 oxidation state and CN^- is a strong ligand and as it approaches the metal ion, the electrons must pair up.



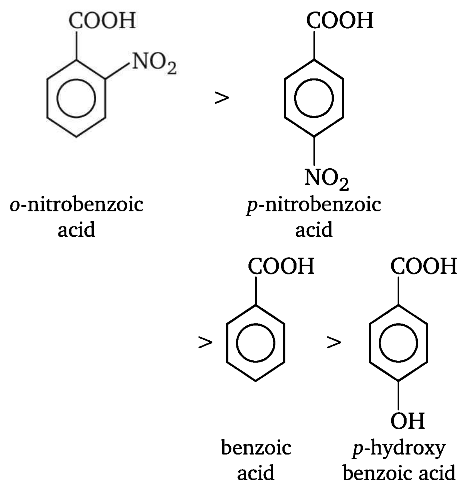
In $[NiCl_4]^{2-}$, Cl^- provides a weak field ligand. It is therefore, unable to pair up the unpaired electrons of the 3*d* orbitals.



16. Methyl magnesium bromide reacts with 2-butanone to form 2-methyl-2-butanol in two steps. The first step involves the nucleophilic attack of the Grignard reagent to the carbonyl group to form an adduct. The second step involves acidic hydrolysis of adduct to form alcohol.



17. Electron-donating substituents tend to decrease while electron-withdrawing substituents tend to increase the acidic strength of substituted benzoic acids relative to benzoic acids. However, due to *ortho*-effect, *o*-nitro-benzoic acid is the stronger acid then *p*-nitrobenzoic acid.



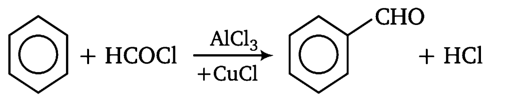
18. Number of electrons in $CO = 6 + 8 = 14$ electrons.

Number of electrons in $NO^+ = 7 + 8 - 1 = 14$ electrons.

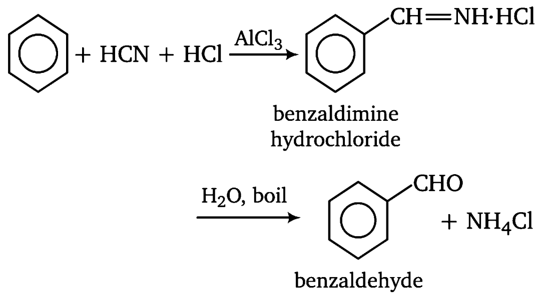
$\therefore$ CO and NO^+ are isoelectronic species.

19. **Gattermann-Koch formylation** When a mixture of CO and HCl gas is passed through a solution of benzene in the presence of anhydrous $AlCl_3$ and CuCl, benzaldehyde is formed.

$$CO + HCl \longrightarrow \underset{\text{formyl chloride}}{[HCOCl]}$$



If CO in this reaction is replaced by HCN, the reaction is called **Gattermann formylation**.



20. $\underset{\text{calcium carbide}}{CaC_2} \xrightarrow{H_2O} \underset{\text{acetylene }(A)}{HC\equiv CH} \xrightarrow[H_2SO_4]{Hg^{2+}} \underset{\text{acetaldehyde }(B)}{H_3C-\overset{\overset{\large O}{\|}}{C}-H}$

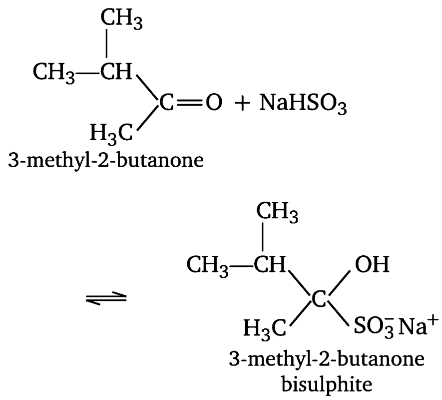
$\xrightarrow[\text{oxidation}]{[O]} \underset{\text{acetic acid }(C)}{CH_3-\overset{\overset{\large O}{\|}}{C}-OH} \xrightarrow{Ca(OH)_2} \underset{\text{calcium acetate }(D)}{(CH_3COO)_2Ca}$

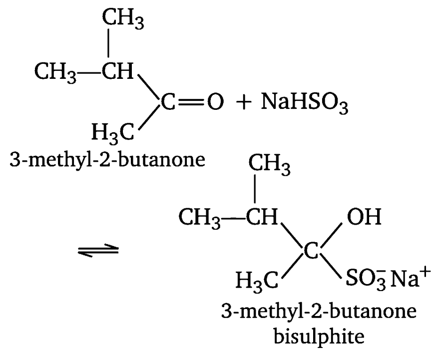
$\xrightarrow[\text{dry distillation}]{\text{heat}} \underset{\text{acetone }(E)}{CH_3-\overset{\overset{\large O}{\|}}{C}-CH_3}$

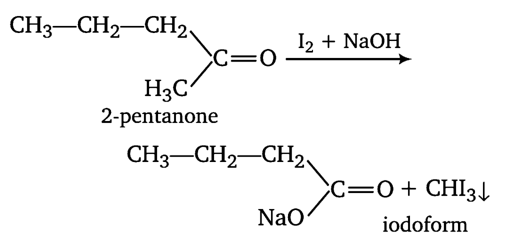
21. In S_{N^2} reactions, the attack of nucleophile occurs from the back side on the carbon atom carrying the halogen. Therefore, presence of bulky substituents on or near this carbon atom tends to hinder the approach of the nucleophile to the α-carbon due to steric hindrance and thus makes the reaction difficult to occur. Thus, the reactivity in S_{N^2} reactions follows the following order

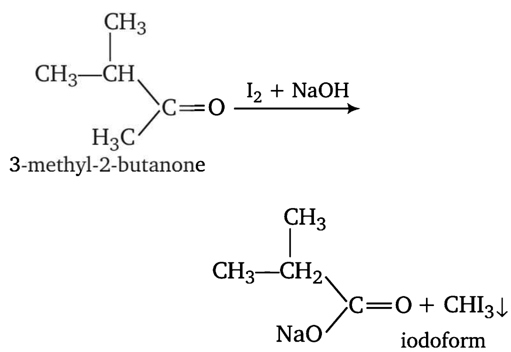
$CH_3CH_2CH_2Cl$ (I) > $(CH_3)_2CHCH_2Cl$ (III)
> $CH_3-CH_2-CHCl-CH_3$ (II)
> $(CH_3)_3C-Cl$ (IV).

22. Both 2-pentanone and 3-methyl -2-butanone have molecular weight 86, both do not reduce Fehling's solution, form crystalline bisulphite derivatives and give iodoform test, as have CH_3CO – group.

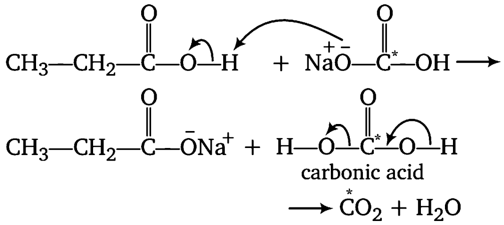








23. During the reaction of carboxylic acids with $NaHCO_3$, the CO_2 evolves comes from $NaHCO_3$ and not from carboxylic acids.



24. For H atom

$$E_n = \frac{E_1}{n^2}\,\text{eV} \quad (\text{where } E_1 = -13.6\,\text{eV})$$

For $n = 2$;

$$E_2 = -\frac{13.6}{2^2} = -3.4\,\text{eV}$$

25. The orbital hawing lower $(n+1)$ value has lower energy.

$\therefore$ (i) $n = 4, l = 1;\ n + l = 5$

(ii) $n = 4, l = 0;\ n + l = 4$

(iii) $n = 3, l = 2;\ n + l = 5$

(iv) $n = 3, l = 1;\ n + l = 4$

If two orbitals have same $(n + l)$ values, then the orbital with lower value of n has lower energy.

$\therefore$ Among (i) $n = 4, l = 1$ and (iii) $n = 3, l = 2$; (iii) has lower energy than (i).

Similarly, among (ii) $n = 4, l = 0$ and (iv) $n = 3, l = 1$; (iv) has lower energy than (ii).

Thus the order of increasing energy;

(iv) < (ii) < (iii) < (i)

26. Amount of work done in an isothermal expansion is given by

$$W = -2.303\,nRT \log\frac{V_2}{V_1}$$

$$= -2.303 \times 1 \times 8.314 \times 300 \times \log\frac{20}{10}$$

$$= 5744.14 \times 0.3010$$

$$= 1728.98\text{ J} \approx 1726\text{ J}$$

27. 1% CsCl means 1g of CsCl is dissolved in 1 L solution.

$$\therefore \text{ Moles of CsCl}(n) = \frac{w}{M} = \frac{1}{167.5}$$

$$= 0.0059\text{ moles.}$$

Moles of RbCl $(n) = \frac{1}{120.5} = 0.0082$ moles

Moles of KCl $(n) = \frac{1}{74.5} = 0.0134$ moles and

Moles of NaCl $(n) = \frac{1}{58.5} = 0.0170$ moles

$\because$ Osmotic pressure $(\pi) \propto n$ (number of moles)

$\therefore$ 1% NaCl solution has maximum osmotic pressure.

28. The hybridisation of one s, one d and three p-orbitals on a central atom gives rise to five sp^3d orbitals, three equatorial and two axial. This hybridisation uses one d_{z^2}, one s and all three of the p orbitals and shape of the molecule becomes trigonal bipyramidal.

29. Formic acid can be distinguished from acetic acid since it contains a hydrogen atom instead of methyl group. Therefore, formic acid acts as a reducing agent and decolorise acidified $KMnO_4$ solution.

$$\underset{}{HCOOH} + [O] \xrightarrow[]{\text{Acidified } KMnO_4} \underset{\text{carbonic acid}}{[HO—CO—OH]}$$

$$\longrightarrow CO_2 + H_2O$$

30. Amides are produced by alkyl cyanides by shaking them with cold conc. HCl or by dissolving the nitriles in conc. H_2SO_4 and then pouring them into water.

$$\underset{\text{alkyl cyanide}}{R—C{\equiv}N} + H_2O \xrightarrow{\text{Cold conc. HCl}} \underset{\text{amide}}{R—\overset{\overset{\displaystyle O}{\|}}{C}—NH_2}$$

$$\underset{\text{alkyl cyanide}}{R—C{\equiv}N} \xrightarrow[\text{and then pour in } H_2O]{\text{Dissolve in conc. } H_2SO_4} \underset{\text{amide}}{R—\overset{\overset{\displaystyle O}{\|}}{C}—NH_2}$$

31. Compound X cannot be primary or secondary alcohol because both react with sodium displacing hydrogen and PCl_5 to give hydrogen chloride.

$$2ROH + 2Na \longrightarrow 2RO^-Na^+ + H_2\uparrow$$

$$2ROH + PCl_5 \longrightarrow RCl + POCl_3 + \underset{\text{hydrogen chloride}}{HCl}$$

X is an aldehyde and not a ketone because it reduces Fehling's solution, *i.e.* alkaline solution of Cu (II) salt.

$$\underset{\text{aldehyde}}{RCHO} + \underset{\text{Fehling's solution}}{2Cu^{2+} + OH^-} \longrightarrow \underset{\text{carboxylate ion}}{RCOO^-} + \underset{\text{cuprous oxide (red ppt.)}}{Cu_2O\downarrow} + 3H_2O$$

32. $$CH_3Cl \xrightarrow{KCN} \underset{\substack{\text{alkyl cyanide}\\(A)}}{CH_3—C\equiv N} \xrightarrow{H^+/H_2O} \underset{\substack{\text{acetic acid}\\(B)}}{CH_3COOH}$$

33. $NH_3(g) + 3Cl_2(g) \rightleftharpoons NCl_3(g) + 3HCl(g);\quad -\Delta H_1 \ldots$(i)

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g);\quad -\Delta H_2\ldots$(ii)

$H_2(g) + Cl_2(g) \rightleftharpoons 2HCl(g);\quad +\Delta H_3\ldots$(iii)

Multiply Eq. (i) by 2 and Eq. (iii) by 3, add Eq. (i) to Eq. (ii) and subtract Eq. (iii) to get the heat of formation of $NCl_3(g)$;

$$N_2 + 3Cl_2 \rightleftharpoons 2NCl_3$$

$$2\Delta H_f = 2\times(-\Delta H_1) + (-\Delta H_2) - 3\times(+\Delta H_3)$$

or $\Delta H_f = -\Delta H_1 - \frac{1}{2}\Delta H_2 - \frac{3}{2}\Delta H_3$

34. The rate of reaction is given by

$$\frac{-d[N_2O_5]}{dt} = +\frac{1}{2}\frac{d[NO_2]}{dt} = +\frac{2d[O_2]}{dt}$$

Substituting the given values, we get

$$k_1[N_2O_5] = \frac{1}{2}k_2[N_2O_5] = 2k_3[N_2O_5]$$

or $\quad k_1 = \frac{k_2}{2} = 2k_3$

or $\quad 2k_1 = k_2 = 4k_3$

35. Carbonyl group which is linked to the electron-withdrawing group is relatively more positively charged and hence is attacked by OH^- more easily in Cannizzaro's reaction. Thus, the electron-with drawing nitro group facilitates the release of hydride ion form the intermediate.

36. Smaller the gold number of a protective colloid, the greater is its protective action

Gold number : 0.0002 0.04 10 25

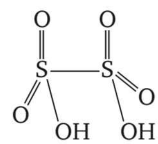
Protective powers : B > A > C > D

37. When Cl_2 is passed through hot concentrated alkali solutions, a mixture of chloride and chlorate is formed.

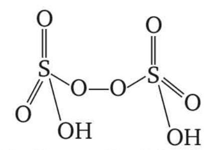
$$3\overset{0}{Cl_2} + 3NaOH\,(\text{excess}) \xrightarrow{\text{Hot}} 5Na\overset{-1}{Cl} + Na\overset{+5}{Cl}O_3 + 3H_2O$$

During these reactions, chlorine is simultaneously reduced to chloride ion (Cl^-) and is oxidised to trioxochlorate (V) ion (ClO_3^-).

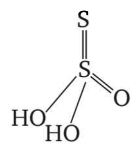
38.



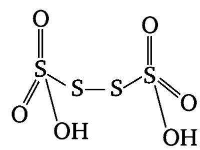
$H_2S_2O_6$ (Dithionic acid)
(O.N. of sulphur = +5)



$H_2S_2O_8$ (Peroxodisulphuric acid)
(O.N. of sulphur = +6)



$H_2S_2O_3$ (Thiosulphuric acid)
(O.N. of sulphur = +6, –2)



$H_2S_4O_6$ (Tetrathionic acid)
(O.N. of sulphur = +6, –1)

39. $KMnO_4$ is a powerful oxidising agent and gets reduced to

(i) MnO_2 in neutral medium.

$$2KMnO_4 + H_2O \longrightarrow 2KOH + 2MnO_2 + 3[O]$$

(ii) K_2MnO_4 in alkaline medium.

$$2KMnO_4 + 2KOH \longrightarrow 2K_2MnO_4 + H_2O + [O]$$

(iii) Mn^{2+} in acidic medium.

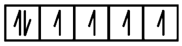
$$MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O$$

40. In $[FeF_6]^{4-}$ and $[Fe(CN)_6]^{4-}$, iron is in +2 oxidation state.

Fe^{2+} ion = $\overset{3d}{\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}}$

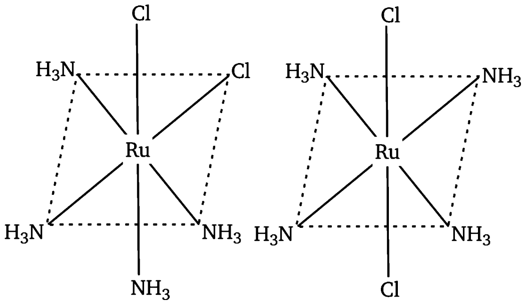
F^- is a weak ligand, hence 3d electrons do not pair up. As a result, $[FeF_6]^{4-}$ has four unpaired electrons and it forms outer orbital complex (sp^3d^2-hybridisation). Whereas, CN^- is a strong ligand, hence 3d electrons pair up. As a result, there is no unpaired electrons and it form inner orbital complex (d^2sp^3 hybridisation).

In $[Co(NH_3)_6]^{3+}$ and $[Co(CN)_6]^{3-}$, cobalt is in + 3 oxidation state.

Co^{3+} ion = 

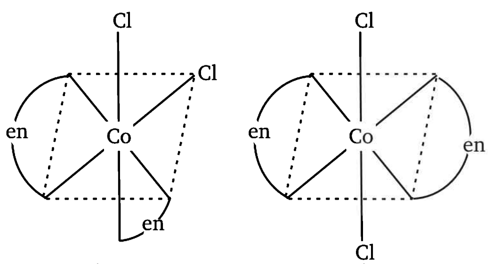
NH_3 and CN^-, both are strong field ligand, hence 3*d* electrons pair up and both form inner orbital complex (d^2sp^3 hybridisation) .

41. (a) $[Ru(NH_3)_4Cl_2]^+$ has two isomers– *cis* and *trans* isomers.



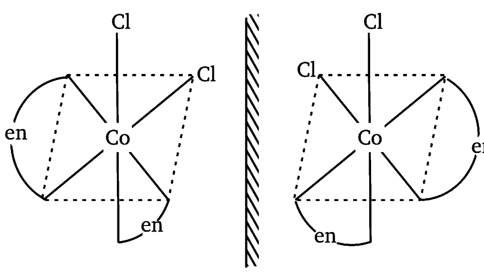
cis-isomer *trans*-isomer

(b) $[Co(en)_2Cl_2]^+$ shows both geometrical and optical isomerism.



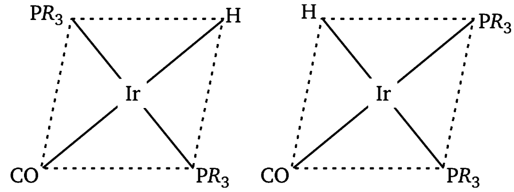
cis-form *trans*-form

cis–isomer is unsymmetrical, hence shows optical isomerism. It has three isomers.



mirror

(c) $[Ir(PR_3)H(CO)]^{2+}$ shows two geometrical isomers.



trans-isomers *cis*-isomers

(d) $[Co(NH_3)_5Cl]^{2+}$ does not show isomerism.

42. The elements neptunium ($Z = 93$) and plutonium ($Z = 94$) were first prepared in 1940 by bombarding U-238 with neutrons. In this process β-emission increases the atomic number by 1 with no change of atomic mass.

$$_{92}U^{238} + {}_0n^1 \longrightarrow \underset{\text{radioactive}}{_{92}U^{239}} \xrightarrow{\beta} \underset{\text{radioactive}}{_{93}Np^{239}} \xrightarrow{\beta} {}_{94}Pu^{239}$$

43. Le-Chatelier's principle is not applicable to solid-solid equilibrium.

$$Fe(s) + S(s) \rightleftharpoons FeS(s)$$

44. Dissociation of weak electrolyte A_xB_y as;

$$A_xB_y \rightleftharpoons xA^{y+} + yB^{x-}$$

At $t = 0$	C	0	0
At equal	$C(1-\alpha)$	$xC\alpha$	$yC\alpha$

Applying law of mass action

$$K_{eq} = \frac{[xC\alpha]^x [yC\alpha]^y}{C(1-\alpha)}$$

For weak electrolytes α <<< 1,

$\therefore$ $1 - \alpha = 1$

$\therefore$ $K_{eq}\,C = x^x y^y [C\alpha]^{x+y}$

or $\dfrac{K_{eq}}{C^{x+y-1}} = x^x y^y (\alpha)^{x+y}$

or $\alpha = \left(\dfrac{K_{eq}}{(C^{x+y-1})x^x y^y}\right)^{\frac{1}{x+y}}$

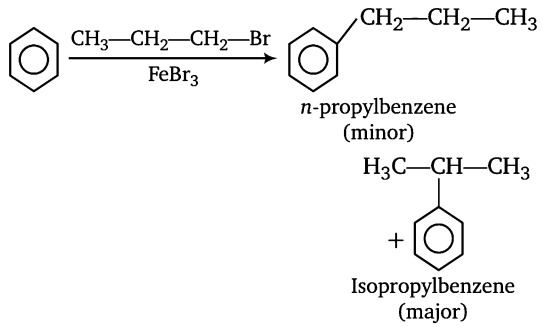
45. Octane number of a gasoline is defined as the percentage of *iso*-octane present in a mixture of *iso*-octane and *n*-heptane, which matches the fuel (gasoline) in knocking. Thus, a mixture of 70% *iso*-octane and 30% n-heptane has octane number 70.

46. During Friedel-craft alkylation of benzene with n-propyl bromide, the product will be isopropyl benzene (cumene) not n-propyl benzene because isopropylium ion is more stable (as it is 2° carbocation) than n-propylium ion (1° carbocation).

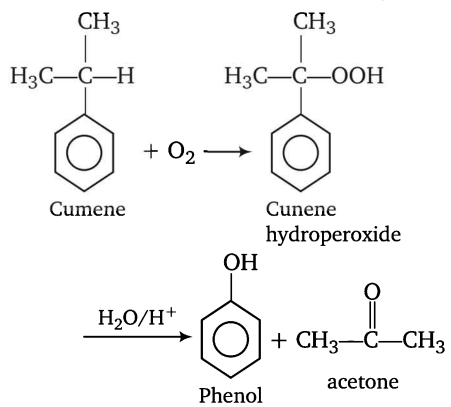
$$CH_3-CH_2-CH_2-Br \xrightarrow[-FeBr_4]{FeBr_3} CH_3-CH_2-\overset{\oplus}{C}H_2$$

$$\xrightarrow{H^- \text{ shifting}} CH_3-\overset{\oplus}{C}H-CH_3$$

Thus,



47.

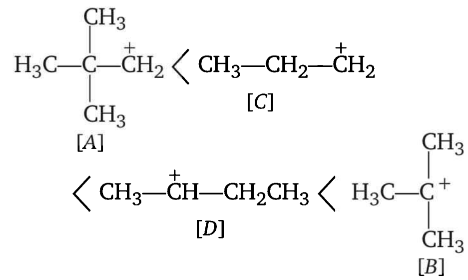


48. Hydrolysis of XeF_4 and XeF_6 with water gives various products but the common product is XeO_3, which is an explosive.

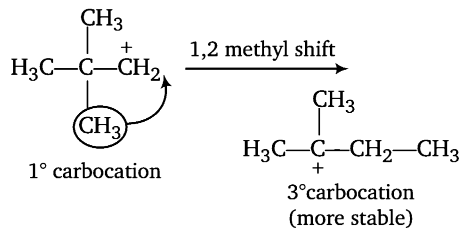
$$6XeF_4 + 12H_2O \longrightarrow 2XeO_3 + 24HF + 4Xe + 3O_2$$

$$XeF_6 + 3H_2O \longrightarrow XeO_3 + 6HF.$$

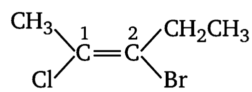
49. The order of increasing stability of carbocations is



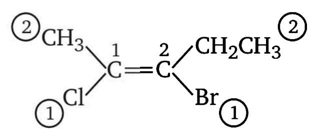
Carbocation [A] is least stable and converts to more stable 3° carbocation through rearrangement.



50.



Out of two groups, *i.e.* $-Cl$ and $-CH_3$ attached to the C-1 atom , the atomic number of Cl is greater than the C-atom of $-CH_3$ group. Hence, here Cl is given priority over $-CH_3$ group. Similarly, out of two groups attached to the C-2 atom *i.e.*, $-Br$ and $-CH_2CH_3$ the atomic number of Br is greater than the C atom of $-CH_2CH_3$ group. Thus, Br gets priority over C atom of $-CH_2CH_3$ group. Hence, the compound looks like:



It is very clear that the bulky groups are at the same side of the double bond, hence this is a Z-isomer.

Mathematics

1. Using AM $\geq$ GM

$\therefore \quad 2 + a_1 \geq \sqrt{2 \cdot a_1}$

$2 + a_2 \geq \sqrt{2 \cdot a_2}$

$\vdots \quad \vdots \quad \vdots$

$2 + a_n \geq \sqrt{2 \cdot a_n}$

$\therefore \quad (2 + a_1)(2 + a_2) + \ldots + (2 + a_n)$

$\geq \sqrt{(2 \cdot a_1)(2 \cdot a_2)\ldots(2 \cdot a_n)}$

$= 2^{n/2}\sqrt{a_1 \cdot a_2 \ldots a_n}$

$= 2^{n/2} \qquad (\because \ a_1 \cdot a_2 \ldots a_n = 1)$

2. Let $2^{-x} = t$

$\therefore \quad 2t^2 - 7t - 4 < 0$

$\Rightarrow \quad (2t + 1)(t - 4) < 0$

$\Rightarrow \quad -\frac{1}{2} < t < 4$

Since, 2^{-x} is always positive,

$\therefore \quad 0 < 2^{-x} < 4 \Rightarrow -2 < x < \infty$

3. Given, $a^3 + b^3 = a - b$

Let $\quad a = \frac{2}{3}, b = \frac{1}{3}$

Then, $\quad \left(\frac{2}{3}\right)^3 + \left(\frac{1}{3}\right)^3 = \frac{2}{3} - \frac{1}{3}$

$\Rightarrow \quad \frac{8}{27} + \frac{1}{27} = \frac{1}{3}$

$\Rightarrow \quad \frac{1}{3} = \frac{1}{3}$

Now, taking option (d),

$$a^2 + ab + b^2 = \frac{4}{9} + \frac{2}{9} + \frac{1}{9} = \frac{7}{9} < 1$$

4. Since, probability of getting tail in single coin is $p = \frac{1}{2}, q = \frac{1}{2}$

$\therefore$ Required probability $= P$(getting tail in one time)

$+ \ldots + P$ (getting tail in three times)

$+ P$ (getting tail in five times)

$+ P$ (getting tail in Ninety Nine times)

$$= {}^{100}C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{99} + {}^{100}C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{97}$$

$$+ {}^{100}C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{95} + \ldots + {}^{100}C_{99}\left(\frac{1}{2}\right)^{99}\left(\frac{1}{2}\right)^1$$

$$= \frac{1}{2^{100}}[{}^{100}C_1 + {}^{100}C_3 + {}^{100}C_5 + \ldots + {}^{100}C_{99}]$$

$$= \frac{1}{2^{100}} \times 2^{99} = \frac{1}{2}$$

5. Given, $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$

$= \sin\theta(2\sin 2\theta\cos\theta)$

$= \sin 2\theta \sin 2\theta = \sin^2 2\theta$

$= \frac{1}{2}(1 - \cos 4\theta)$

[since, $-1 \leq \cos 4\theta \leq 1$ for all real θ]

$\therefore f(\theta) \geq 0$ for all real θ.

6. Given, $\tan\frac{A}{2} = \frac{5}{6}, \tan\frac{B}{2} = \frac{20}{37}$

Now, $\tan\frac{A}{2}\tan\frac{B}{2} = \frac{5}{6} \times \frac{20}{37}$

$\Rightarrow \quad \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \times \sqrt{\frac{(s-a)(s-c)}{s(s-b)}} = \frac{50}{111}$

$\Rightarrow \quad \frac{s-c}{s} = \frac{50}{111} \Rightarrow 61s - 111c = 0$

$\Rightarrow \quad 61\left(\frac{a+b+c}{2}\right) - 111c = 0$

$\Rightarrow \quad 61a + 61b - 166c = 0$

Hence, option (d) is correct.

7. $\sin^4 x + \cos^4 x = \sin x \cos x$

$\Rightarrow (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cdot \cos^2 x$

$= \sin x \cdot \cos x$

$\Rightarrow \quad 1 - \frac{\sin^2 2x}{2} = \frac{\sin 2x}{2}$

$\Rightarrow \quad \sin^2 2x + \sin 2x - 2 = 0$

$\Rightarrow \quad (\sin 2x + 2)(\sin 2x - 1) = 0$

$\Rightarrow \quad \sin 2x = 1 \Rightarrow 2x = (4n + 1)\frac{\pi}{2}$

$\Rightarrow \quad x = (4n + 1)\frac{\pi}{4}$

$\Rightarrow \quad x = \frac{\pi}{4}, \frac{5\pi}{4}$

8. Given,

$a\sin^{-1}x - b\cos^{-1}x = c$ ...(i)

$$\Rightarrow \quad a\left(\frac{\pi}{2} - \cos^{-1}x\right) - b\cos^{-1}x = c$$

$$\Rightarrow \quad (a+b)\cos^{-1}x = \frac{a\pi}{2} - c$$

$$\Rightarrow \quad \cos^{-1}x = \frac{a\pi/2 - c}{a+b}$$

Again from Eq. (i)

$$a\sin^{-1}x - b\left(\frac{\pi}{2} - \sin^{-1}x\right) = c$$

$$\Rightarrow \quad (\sin^{-1}x)(a+b) = c + \frac{b\pi}{2}$$

$$\Rightarrow \quad \sin^{-1}x = \frac{c + b\pi/2}{a+b}$$

$$\therefore \quad a\sin^{-1}x + b\cos^{-1}x = \frac{a(c+b\pi/2)}{a+b} + \frac{b(a\pi/2 - c)}{a+b}$$

$$= \frac{c(a-b) + ab\pi}{a+b}$$

9. Let equation of line passing through $P(x_1, y_1)$ is

$$y - y_1 = m(x - x_1)$$

Since, the algebraic sum of the points (2, 0), (0, 2) and (–2, – 2) to the variable point is zero.

$$\therefore \quad \frac{-2m + mx_1 - y_1}{\sqrt{1+m^2}} + \frac{2 + mx_1 - y_1}{\sqrt{1+m^2}} + \frac{-2 + 2m + (mx_1 - y_1)}{\sqrt{1+m^2}} = 0$$

$$\Rightarrow \quad 3mx_1 - 3y_1 = 0 \Rightarrow y_1 = m(x_1)$$

Since, m is satisfied for every values of x

$$\therefore \quad y_1 = 0 \quad \Rightarrow \quad x_1 = 0$$

10. We know, if T is any point on the circle, then

$$PA \cdot PB = PT^2$$

$$\therefore \quad PT = \sqrt{3^2 + 11^2 - 9} = \sqrt{130 - 9}$$

$$= \sqrt{121}$$

$$\therefore \quad PA \cdot PB = (\sqrt{121})^2 = 121$$

11. Equation can be rewritten as

$$(x-a)^2 + y^2 = a^2$$

Any point on the circle is $(a + a\cos\theta, a\sin\theta)$

$\therefore$ Equation of tangent at $(a + a\cos\theta, a\sin\theta)$ is

$$x(a + a\cos\theta) + y(a\sin\theta) - a(x + a + a\cos\theta) = 0$$

$$\Rightarrow \quad ax\cos\theta + ay\sin\theta - a^2(1+\cos\theta) \quad \text{...(i)}$$

Equation of first circle is

$$x^2 + y^2 = a^2 \quad \text{...(ii)}$$

Let Eq. (i) meets the first circle at P and Q and the tangents at P and Q to the second circle intersected at (h, k), then Eq. (i) is the chord of contact of (h, k) with respect to the circle (ii) and thus equation is

$$hx + ky - a^2 = 0 \quad \text{...(iii)}$$

Eqs. (i) and (iii) represents the same line.

$$\therefore \quad \frac{h}{a\cos\theta} = \frac{k}{a\sin\theta} = \frac{a^2}{a^2(1+\cos\theta)}$$

$$\Rightarrow \quad \frac{h}{a} = \frac{\cos\theta}{1+\cos\theta}, \frac{k}{a} = \frac{\sin\theta}{1+\cos\theta}$$

$$\Rightarrow \quad \left(\frac{h}{a}\right)^2 + \left(\frac{k}{a}\right)^2 = \frac{\cos^2\theta + \sin^2\theta}{(1+\cos\theta)^2}$$

$$= \frac{1}{4\left[\cos^2\frac{\theta}{2}\right]^2}$$

$$\Rightarrow \quad \left(\frac{h}{a}\right)^2 + \left(\frac{k}{a}\right)^2 = \frac{1}{4}\left(1 + \tan^2\frac{\theta}{2}\right)^2 \quad \text{...(iv)}$$

$$\text{Now,} \quad \frac{k}{a} = \frac{\sin\theta}{1+\cos\theta} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$$

$$\Rightarrow \quad \frac{k}{a} = \tan\frac{\theta}{2}$$

$\therefore$ From Eq. (iv)

$$\left(\frac{h}{a}\right)^2 + \left(\frac{k}{a}\right)^2 = \frac{1}{4}\left(1 + \frac{k^2}{a^2}\right)^2$$

Hence, locus is

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{a}\right)^2 = \frac{1}{4}\left(1 + \frac{y^2}{a^2}\right)^2$$

Let $x = \frac{a}{2}, y = 0$

Then, $\left(\frac{a/2}{a}\right)^2 + 0^2 = \frac{1}{4}(1+0)^2$

$\Rightarrow \frac{1}{4} = \frac{1}{4}$

Hence, required point is $\left(\frac{a}{2}, 0\right)$.

12. Given equation of line $y = mx + 2$ or $\frac{y - mx}{2} = 1$ and circle $x^2 + y^2 = 1$

$\therefore$ Equation of the line joining the origin to the intersection of line and circle is

$$x^2 + y^2 - \left(\frac{y - mx}{2}\right)^2 = 0$$

$\Rightarrow 4(x^2 + y^2) - (y^2 + m^2x^2 - 2myx) = 0$

$\Rightarrow x^2(4 - m^2) + 3y^2 + 2mxy = 0$

Since, lines are at right angles.

$\therefore 4 - m^2 + 3 = 0 \quad (\because a + b = 0)$

$\Rightarrow m^2 = 7 \Rightarrow m = \pm\sqrt{7}$

13. Given curves are

$$y^2 = 4x$$

$\Rightarrow 2y\frac{dy}{dx} = 4 \Rightarrow \left(\frac{dy}{dx}\right)_{(16, 8)} = \frac{4}{16}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(16, 8)} \frac{1}{4} = m_1$ (say)

and $x^2 = 32y$

$\Rightarrow 2x = 32\frac{dy}{dx}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(16, 8)} = \frac{2 \times 16}{32}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(16, 8)} = 1 = m_2$ (say)

$\therefore$ Angle, between them, $\theta = \tan^{-1}\left(\frac{m_2 - m_1}{1 + m_1m_2}\right)$

$$= \tan^{-1}\left(\frac{1 - \frac{1}{4}}{1 + 1 \times \frac{1}{4}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3}{4}}{\frac{5}{4}}\right)$$

$$= \tan^{-1}\left(\frac{3}{5}\right)$$

14. Any point on the ellipse is

$$(\sqrt{14}\cos\theta, \sqrt{5}\sin\theta)$$

$\therefore$ Equation of normal at

$(\sqrt{14}\cos\theta, \sqrt{5}\sin\theta)$ is

$$\sqrt{14}x\sec\theta - \sqrt{5}y\ \mathrm{cosec}\,\theta = 9$$

Since, it passes through

$$(\sqrt{14}\cos 2\theta, \sqrt{5}\sin 2\theta)$$

$\therefore \sqrt{14}\sqrt{14}\cos 2\theta\sec\theta - \sqrt{5}\sqrt{5}\sin 2\theta\ \mathrm{cosec}\,\theta = 9$

$\Rightarrow 14\frac{\cos 2\theta}{\cos\theta} - 5\frac{\sin 2\theta}{\sin\theta} = 9$

$\Rightarrow 14(2\cos^2\theta - 1) - 10\cos^2\theta = 9\cos\theta$

$\Rightarrow 18\cos^2\theta - 9\cos\theta - 14 = 0$

$\Rightarrow (3\cos\theta + 2)(6\cos\theta - 7) = 0$

$\Rightarrow \cos\theta = -\frac{2}{3}, \cos\theta \neq \frac{7}{6}$

15. Let equation of hyperbola be,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Length of latusrectum, $\frac{2b^2}{a} = 12$

and length of semi-conjugate axis, $b = 2\sqrt{3}$

$\therefore \frac{2(2\sqrt{3})^2}{a} = 12$

$\Rightarrow a = 2$

$\therefore$ Eccentricity $e = \sqrt{1 + \frac{b^2}{a^2}}$

$$= \sqrt{1 + \frac{(2\sqrt{3})^2}{(2)^2}}$$

$$= \sqrt{1 + \frac{12}{4}} = 2$$

16. Now, $\sqrt{12^2 + 4^2 + 3^2} = \sqrt{144+16+9}$

$$= \sqrt{169} = 13$$

$\therefore$ Required direction cosines are

$$< \frac{12}{13}, \frac{4}{13}, \frac{3}{13} >$$

17. Let equation of any plane through $(1, -1, 2)$ is

$$a(x-1) + b(y+1) + c(z-2) = 0 \quad \ldots\text{(i)}$$

Since, above plane is perpendicular to the given planes.

$\therefore$ $$2a + 3b - 2c = 0 \quad \ldots\text{(ii)}$$

and $$a + 2b - 3c = 0 \quad \ldots\text{(iii)}$$

$\Rightarrow$ $$\frac{a}{-9+4} = \frac{b}{-(-6+2)} = \frac{c}{4-3}$$

$\Rightarrow$ $$\frac{a}{-5} = \frac{b}{4} = \frac{c}{1}$$

$\therefore$ From Eq. (i),

$$-5(x-1) + 4(y+1) + 1(z-2) = 0$$

$\Rightarrow$ $$5x - 4y - z = 7$$

18. Given, $I = \int_{\pi/6}^{\pi/3} \frac{dx}{\sqrt{1+\tan x}}$

$$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \ldots\text{(i)}$$

$$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{2} - x\right)} + \sqrt{\sin\left(\frac{\pi}{2} - x\right)}} dx$$

$$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \ldots\text{(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_{\pi/6}^{\pi/3} 1dx = [x]_{\pi/6}^{\pi/3}$$

$$= \left[\frac{\pi}{3} - \frac{\pi}{6}\right] = \frac{\pi}{6}$$

19. Given $I_1 = \int_0^{\pi/2} f(\sin 2x) \sin x dx$

$\Rightarrow$ $$I_1 = \int_0^{\pi/2} f(\sin 2x) \cos x dx$$

$$\left(\because \int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx\right]$$

$\Rightarrow$ $$2I_1 = \int_0^{\pi/2} f(\sin 2x)(\sin x + \cos x) dx$$

$$= \sqrt{2}\int_0^{\pi/2} f(\sin 2x) \cos\left(x - \frac{\pi}{4}\right) dx$$

Put $x - \frac{\pi}{4} = t \Rightarrow dx = dt$

$\therefore$ $$2I_1 = \sqrt{2}\int_{-\pi/4}^{\pi/4} f\left(\sin\left(\frac{\pi}{2} + 2t\right)\right) \cos t\, dt$$

$\therefore$ $$2I_1 = 2\sqrt{2}\int_0^{\pi/4} f(\cos 2t) \cos t\, dt$$

$\Rightarrow$ $$I_1 = \sqrt{2}\int_0^{\pi/4} f(\cos 2x) \cos x\, dx$$

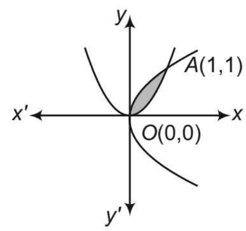
$\Rightarrow$ $$I_1 = \sqrt{2}\, I_2$$

20. The point of intersection of two parabolas $y^2 = x$ and $x^2 = y$ are $(0, 0)$ and $(1, 1)$.

$\therefore$ Required shaded area

$$= \int_0^1 (y_2 - y_1) dx$$

$$= \int_0^1 (\sqrt{x} - x^2)\, dx$$



$$= \left[\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right]_0^1$$

$$= \left[\frac{2}{3}(1) - \frac{1}{3}\right] = \frac{1}{3}$$

$$= 1 \text{ sq unit}$$

21. Given, $f(x) = p|\sin x| + qe^{|x|} + r|x|^3$

$$f(x) = \begin{cases} p\sin x + qe^x + rx^3 & , \ x \geq 0 \\ -p\sin x + qe^{-x} - rx^3 & , \ x < 0 \end{cases}$$

$$f'(x) = \begin{cases} p\cos x + qe^x + 3rx^2 & , \ x > 0 \\ -p\cos x - qe^{-x} - 3rx^2 & , \ x < 0 \end{cases}$$

Since, it is differentiable at $x = 0$

$\therefore$ LHD = RHD

$\Rightarrow p\cos(0) + qe^0 + 3r(0)^2$

$= -p\cos(0) - qe^{-0} - 3(0)^2 r \Rightarrow 2p + 2q = 0$

$\Rightarrow$ $$p + q = 0, \ r \in R$$

22. Let $f(x) = 3x$

$\Rightarrow \quad f'(x) = 3 \Rightarrow f'(0) = 3$, which is true.

$\therefore \quad f(f(f(f(f(x))))) = 3^5 x$

$\therefore \quad y = 3^5 x \Rightarrow y'(x) = 3^5$

$\Rightarrow \quad y'(0) = 3^5$

23. Given, $f(x) = xe^{x(1-x)}$

On differentiating w.r.t. x, we get

$$f'(x) = e^{x(1-x)} + xe^{x(1-x)} \times (1-2x)$$
$$= e^{x(1-x)}(1 + x - 2x^2)$$

Put $f'(x) = 0$

$\therefore \quad 2x^2 - x - 1 = 0$

$\Rightarrow \quad x = -\frac{1}{2}, 1$

Now, $f''(x) = e^{x(1-x)}(1-4x)$

$+ (1 + x - 2x^2)$

$e^{x(1-x)} \times (1-2x)$

$= e^{x(1-x)}[1 - 4x + (1 - x - 4x^2 + 4x^3)]$

$= e^{x(1-x)}[4x^3 - 4x^2 - 5x + 1]$

At $x = -\frac{1}{2}$

$$f''\left(-\frac{1}{2}\right) = e^{-\frac{1}{2}\left(1+\frac{1}{2}\right)}$$
$$\left[4\left(-\frac{1}{2}\right)^3 - 4\left(-\frac{1}{2}\right)^2 - 5\left(-\frac{1}{2}\right) + 1\right]$$

$= 2e^{-\frac{3}{4}} > 0$, minima

At $x = 1$

$f''(1) = e^{1(1-1)}[4(1)^3 - 4(1)^2 - 5(1) + 1]$

$= -4 < 0$, maxima

Hence, $f(x)$ is increasing on $\left[-\frac{1}{2}, 1\right]$ and

$f(x)$ is decreasing on $[1, \infty)$.

24. The intersection point of $y^2 = 4x$ and $x^2 + y^2 - 6x + 1 = 0$ is

$$x^2 + 4x - 6x + 1 = 0$$

$\Rightarrow \quad x^2 - 2x + 1 = 0$

$\Rightarrow \quad (x-1)^2 = 0$

$\Rightarrow \quad x = 1, \; y = \pm 2$

$\therefore$ Points are (1, 2), (1, – 2).

Hence, it intersect at two distinct points.

25. Given equation can be rewritten as

$$f(x) = 1 - \frac{2}{x^2 + 1}$$

Since, the maximum value of $\frac{2}{x^2+1}$ is 2.

$\therefore$ Minimum value of $f(x)$ is $1 - 2 = -1$

26. Any point on the rectangular hyperbola $xy = -1$ is $\left(t, -\frac{1}{t}\right)$.

$\therefore$ Equation of tangent at $\left(t, -\frac{1}{t}\right)$ is

$$xt - \frac{1}{t}y = -2 \Rightarrow y = xt^2 + 2t$$

which is also a tangent to the parabola

$\therefore \quad 2t = \frac{2}{t^2} \qquad \left(\because c = \frac{a}{m}\right)$

$\Rightarrow \quad t^3 = 1 \Rightarrow t = 1$

$\therefore \quad y = x + 2 \Rightarrow x - y + 2 = 0$

27. Hence option (d) is correct.

28. Given, $\frac{dy}{dx} = x + y, y(0) = 0$

$\Rightarrow \quad \frac{dy}{dx} - y = x$

$\therefore$ IF $= e^{-\int 1\,dx} = e^{-x}$

$\therefore$ Solution is

$$ye^{-x} = \int xe^{-x} dx$$
$$= -xe^{-x} + \int e^{-x} dx + C$$
$$= -xe^{-x} - e^{-x} + C$$

At (0, 0)

$$0e^{-0} = 0 - e^{-0} + C \Rightarrow C = 1$$

$\therefore$ Solution is

$$ye^{-x} = -xe^{-x} - e^{-x} + 1$$

$\Rightarrow \quad y = -x - 1 + e^x$

29. Given, $f(x) = ax + b$

$\therefore \quad f(0) = 0 + b \Rightarrow -1 = b$

and $f(-1) = -a + b \Rightarrow -3 = -a + b$

$\Rightarrow \quad -a = -3 + 1 \Rightarrow a = 2$

30. $(1+x)^n \geq (1+nx)$ is true for all natural number $x > -1$.

31. By taking option (d)

when $\quad n = 1$

$1 > 1/3$ true

when $\quad n = 2$

$5 \geq 8/3$ true

when $\quad n = 3$

$14 > 9$, true

when $\quad n = 4$

$30 > 21.33$, true

32. It is true that circle is a particular case of an ellipse.

33. For every prime values of n, n divides $(n-1)!+1$.

34. Given, $\lim_{x \to 0} \dfrac{ae^x - b\cos x + ce^{-x}}{x \sin x} = 2$

At $x = 0$ denominator is 0.

$\therefore$ Numerator should be zero.

$$\therefore \quad a - b + c = 0 \quad \ldots(i)$$

$$\therefore \quad 2 = \lim_{x \to 0} \frac{ae^x - b\cos x + ce^{-x}}{x \sin x} \left(\text{form } \frac{0}{0}\right)$$

$$\Rightarrow \quad 2 = \lim_{x \to 0} \frac{ae^x + b\sin x - ce^{-x}}{x\cos x + \sin x}$$

Here, at $x = 0$, donominator is zero, so numerator should be zero.

$$\therefore \quad a + 0 - c = 0$$

$$\Rightarrow \quad a = c \quad \ldots(ii)$$

Again $2 = \lim_{x \to 0} \dfrac{ae^x + b\sin x - ce^{-x}}{\sin x + x\cos x} \left(\text{form } \dfrac{0}{0}\right)$

$$\Rightarrow \quad 2 = \lim_{x \to 0} \frac{ae^x + b\cos x + ce^{-x}}{x\sin x + 2\cos x}$$

$$\Rightarrow \quad 2 = \frac{a+b+c}{2}$$

$$\Rightarrow \quad a + b + c = 4 \quad \ldots(iii)$$

On solving Eqs. (i), (ii) and (iii), we get

$$a = 1, b = 2 \text{ and } c = 1$$

35. Let $z = x + iy$

$\therefore z^3 + \bar{z} = 0$

$$\Rightarrow \quad (x+iy)^3 + (x - iy) = 0$$

$$\Rightarrow \quad x^3 - iy^3 + 3x^2yi - 3xy^2 + x - iy = 0$$

$$\Rightarrow x^3 - 3xy^2 + x + i(-y^3 - y + 3x^2y) = 0$$

$$\therefore \quad x^3 - 3y^2 + x = 0$$

$$\Rightarrow \quad x^2 - 3y^2 + 1 = 0 \quad \ldots(i)$$

$$\text{and} \quad y^3 + y - 3x^2y = 0$$

$$\Rightarrow \quad y^2 - 3x^2 + 1 = 0 \quad \ldots(ii)$$

On solving Eqs. (i) and (ii), we get

$$x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{2}}$$

$$\therefore \quad z = \pm\frac{1}{\sqrt{2}} \pm \frac{i}{\sqrt{2}}$$

Hence, four solutions exist.

36. Let $z = x + iy$

$\therefore \bar{a}(x+iy) + a(x-iy) = 0$

$$\Rightarrow \quad (\bar{a}+a)x + i(\bar{a}-a)y = 0$$

For reflection of real axis, we take '–' y-coordinate

$$\therefore \quad (\bar{a}+a)x - i(\bar{a}-a)y = 0$$

$$\Rightarrow \quad \bar{a}(x-iy) + a(x+iy) = 0$$

$$\Rightarrow \quad \bar{a}\bar{z} + az = 0$$

37. Condition for both the roots greater than 3 be

$$b^2 - 4ac > 0, 1 \cdot f(3) > 0, \frac{-b}{2a} > 3$$

Now, $\quad b^2 - 4ac > 0$

$$\Rightarrow \quad (-6a)^2 - 4 \times 1 \times (2 - 2a + 9a^2) > 0$$

$$\Rightarrow \quad 36a^2 - 4(2 - 2a + 9a^2) > 0$$

$$\Rightarrow \quad 4(2a - 2) > 0$$

$$\Rightarrow \quad a > 2 \quad \ldots(i)$$

$$f(3) > 0 \Rightarrow (3)^2 - 6a(3) + 2 - 2a + 9a^2 = 0$$

$$\Rightarrow \quad 9 - 18a + 2 - 2a + 9a^2 > 0$$

$$\Rightarrow \quad 9a^2 - 20a + 11 > 0$$

$$\Rightarrow \quad (9a - 11)(a - 1) > 0$$

$$a < 1 \text{ and } a > \frac{11}{9} \quad \ldots(ii)$$

$$\text{and} \quad \frac{-b}{2a} > 3,$$

$$\therefore \quad \frac{+6a}{2(1)} > 3 \Rightarrow a > 1 \quad \ldots(iii)$$

From Eqs. (i), (ii) and (iii), we get $a > 11/9$

38. Given $\left|\frac{x}{x-1}\right| + |x| = \frac{x}{|x-1|}$

(i) when $x > 1$,

$$\frac{x}{x-1} + x = \frac{x}{x-1}$$

$\Rightarrow x = 0$, does not exist

(ii) When $0 \le x < 1$

$$\frac{x}{1-x} + x = \frac{x}{1-x}$$

$$\Rightarrow \quad x = 0$$

(iii) when $-\infty < x < 0$

$$\therefore \quad \frac{-x}{-(x-1)} - x = \frac{x}{-(x-1)}$$

$$\Rightarrow \quad \frac{2x}{(x-1)} - x = 0$$

$$\Rightarrow \quad x\left[\frac{2-x+1}{x-1}\right] = 0$$

$x = 0$, $x = 3$, does not exist

Hence, only one solution exist.

39. Given, $I_n = \int_0^{\pi/4} \tan^2 x dx$

$$\Rightarrow \quad I_{n+2} = \int_0^{\pi/4} \tan^{n+2} x \, dx$$

$$\therefore I_n + I_{n+2} = \int_0^{\pi/4} \tan^n x(1 + \tan^2 x) \, dx$$

$$= \int_0^{\pi/4} \tan nx \sec^2 x \, dx$$

Put $\tan x = t \Rightarrow \sec^2 dx = dt$

$$\therefore \quad \int_0^1 t^n dt = \left[\frac{t^{n+1}}{n+1}\right]_0^1$$

$$= \frac{1}{n+1}$$

$$\therefore \lim_{n \to \infty} n[I_n + I_{n+2}] = \lim_{n \to \infty} \frac{1}{1 + 1/n}$$

$$= \frac{1}{1+0} = 1$$

40. Given, $H_n = 1 + \frac{1}{2} + \ldots + \frac{1}{n}$

Also, $S_n = 1 + \frac{3}{2} + \frac{5}{3} + \ldots + \frac{2n-1}{n}$

$$= 1 + \frac{(4-1)}{2} + \frac{(6-1)}{3} + \ldots + \frac{2n-1}{n}$$

$$= 1 + 2 - \frac{1}{2} + 2 - \frac{1}{3} + \ldots + 2 - \frac{1}{n}$$

$$= 1 + 2(n-1) - \left(\frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}\right)$$

$$= 1 + 2(n-1) - (H_n - 1)$$

$$= 2n - H_n$$

41. Given, $\log_2(3x - 2) = \log_{\frac{1}{2}} x$

$$\Rightarrow \quad \frac{\log(3x-2)}{\log 2} = \frac{\log x}{\log \frac{1}{2}} = \frac{\log x}{-\log 2}$$

$$\Rightarrow \quad \log(3x-2) = \log \frac{1}{x}$$

$$\Rightarrow \quad 3x^2 - 2x - 1 = 0$$

$$\Rightarrow \quad (x-1)\left(x + \frac{1}{3}\right) = 0$$

$$\Rightarrow \quad x = 1, \, x = -\frac{1}{3}$$

But $x = -\frac{1}{3}$ is not satisfied in $\log_{\frac{1}{2}} x$.

Hence, $x = 1$ is the required solution.

42. Since, it is in AP.

$$\therefore \quad \log_3(2^x - 5) = \frac{\log_3 2 + \log_3\left(2^x - \frac{7}{2}\right)}{2}$$

$$\Rightarrow \quad 2\log_3(2^x - 5) = \log_3 2\left(2^x - \frac{7}{2}\right)$$

$$\Rightarrow \quad (2^x - 5)^2 = 2 \cdot 2^x - 7$$

$$\text{Put} \quad x^2 = t$$

$$t^2 + 25 - 10t = 2t - 7$$

$$\Rightarrow \quad t^2 - 12t + 32 = 0$$

$$\Rightarrow \quad (t-8)(t-4) = 0$$

$$\Rightarrow \quad t = 4, 8$$

$$\Rightarrow \quad 2^x = 4, \, 2^x = 8$$

$$\Rightarrow \quad x = 2, \, x = 3$$

$$\Rightarrow \quad x = 3$$

$\because x = 2$ does not exist in $\log_3(2^x - 5)$

43. Given, ${}^{n-1}C_r = {}^nC_{r+1}(k^2-3)$

$$\Rightarrow \quad k^2 - 3 = \frac{{}^{n-1}C_r}{{}^nC_{r+1}} = \frac{r+1}{n}$$

As $0 \le r \le n-1$

$$\Rightarrow \quad 1 \le r+1 \le n$$

$$\Rightarrow \quad \frac{1}{n} \le \frac{r+1}{n} \le 1$$

$$\Rightarrow \quad \frac{1}{n} \le k^2 - 3 \le 1$$

$$\Rightarrow \quad 3 + \frac{1}{n} \le k^2 \le 4$$

$$\Rightarrow \quad \sqrt{3+\frac{1}{n}} \le k \le 2$$

As $n \to \infty$

$$\sqrt{3} < k \le 2$$

$$\therefore \quad k \in (\sqrt{3}, 2]$$

44. Let $E = \dfrac{2^n}{n!}$

When $n = 1$

$$E = \frac{2}{1!} = 2$$

When $n = 2$

$$E = \frac{2^2}{2!} = \frac{4}{2} = 2$$

when $n = 3$

$$E = \frac{2^3}{3!} = \frac{4}{3}$$

Hence, two values of n exist.

45. We know,

$$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots + {}^nC_nx^n$$

$$\Rightarrow x(1+x)^n = {}^nC_0x + {}^nC_1x^2 + {}^nC_2x^3 + \ldots + {}^nC_nx^{n+1}$$

On differentiating w.r.t. x, we get

$$(1+x)^n + nx(1+x)^{n-1} = {}^nC_0 + 2\cdot{}^nC_1x + 3\cdot{}^nC_2x^2 + \ldots + (n+1)\cdot{}^nC_nx^n$$

Put $x = 1$, we gt

$$2^n + 2^{n-1}\cdot n = {}^nC_1 + 2\cdot{}^nC_1 + 3\cdot{}^nC_2 + \ldots + (n+1)\cdot{}^nC_n$$

$$= (n+2)2^{n-1}$$

46. A = Coefficient of x^n in $(1+x)^{2n}$

$$= {}^{2n}C_n$$

and B = coefficient of x^n in $(1+x)^{2n-1}$

$$= {}^{2n-1}C_n$$

$$\therefore \quad \frac{B}{A} = \frac{{}^{2n-1}C_n}{{}^{2n}C_n}$$

$$= \frac{(2n-1)!}{(n-1)!\times n!} \times \frac{n!\times n!}{(2n)!}$$

$$= \frac{n}{2n} = \frac{1}{2}$$

47. Given, $(A+B)^m = {}^mC_0A^m + {}^mC_1A^{m-1}B + {}^mC_2A^{m-2}B^2 + \ldots + {}^mC_mB^m$

Put $m = 2$

$$(A+B)^2 = {}^2C_0A^2 + {}^2C_1AB + {}^2C_2B^2$$

$$\Rightarrow \quad A^2 + B^2 + AB + BA = A^2 + 2AB + B^2$$

$$\Rightarrow \quad AB + BA = 2AB$$

$$\Rightarrow \quad AB = BA$$

48. For non-zero solutions

$$\begin{vmatrix} 1 & 2 & -3 \\ 0 & 0 & p+2 \\ 0 & (2p+1) & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad -(p+2)[(2p+1)1 - 0] = 0$$

$$\Rightarrow \quad p = -2, -\frac{1}{2}$$

Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & p+2 \\ 0 & 2p+1 & 1 \end{bmatrix}$

$$C_{11} = -(p+2)(2p+1)$$
$$= -(2p^2+5p+2)$$

$C_{12} = 0, C_{13} = 0$

$C_{21} = -(6p+5), C_{22} = 1, C_{23} = -(2p+1)$

$C_{31} = 2p+4, C_{32} = -(p+2), C_{33} = 0$

$$\therefore \text{adj}(A) = \begin{bmatrix} -2p^2-5p-2 & -5-6p & 2p+4 \\ 0 & 1 & \\ 0 & -(2p+1) & 0 \end{bmatrix}$$

Now, $(\text{adj } A)B = \begin{bmatrix} -2p^2 - 19p - 9 \\ 3 - 2p - 4 \\ -6p - 3 \end{bmatrix}$

when $p = -2$

$(\text{adj } A)B = \begin{bmatrix} -8 + 38 - 9 \\ 3 + 4 - 4 \\ 12 - 3 \end{bmatrix} = \begin{bmatrix} 21 \\ 3 \\ 9 \end{bmatrix} \neq 0$ exist

when $p = -\frac{1}{2}$

$(\text{adj } (A))B = \begin{bmatrix} -2\left(\frac{1}{4}\right) + \frac{19}{2} - 9 \\ 3 - 2\left(-\frac{1}{2}\right) - 4 \\ -6\left(\frac{1}{2}\right) - 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, not exist

49. Let $\Delta = \begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix}$

Applying $R_1 \to R_1 + R_2 + R_3$ and taking common

$\Delta = (\sin x + \cos x + \cos x)\begin{vmatrix} 1 & 1 & 1 \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix}$

$= (\sin x + 2\cos x)\begin{vmatrix} 1 & 0 & 0 \\ \cos x & \sin x - \cos x & 0 \\ \cos x & 0 & \sin x - \cos x \end{vmatrix}$

$\Rightarrow 0 = (\sin x + 2\cos x)\, 1\, [(\sin x - \cos x)^2]$

$\Rightarrow \tan x = -2, \tan x = 1$

$\Rightarrow x = \frac{\pi}{4}, \tan^{-1}(-2)$

Hence, two values of x exist in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

50. $\Delta = \begin{vmatrix} \sum_{r=0}^{m} (2r - 1) & \sum_{r=0}^{m} {}^mC_r & \sum_{r=0}^{m} 1 \\ m^2 - 1 & 2^m & m + 1 \\ \sin^2 m & \cos^2 m & \tan^2 m \end{vmatrix}$

$= \begin{vmatrix} \frac{2m(m+1)}{2} - (m+1) & 2^m & m + 1 \\ m^2 - 1 & 2^m & m + 1 \\ \sin^2 m & \cos^2 m & \tan^2 m \end{vmatrix}$

$= \begin{vmatrix} m^2 - 1 & 2^m & m + 1 \\ m^2 - 1 & 2^m & m + 1 \\ \sin^2 m & \cos^2 m & \tan^2 m \end{vmatrix}$

$= 0$ ($\because R_1$ and R_2 are identicals)