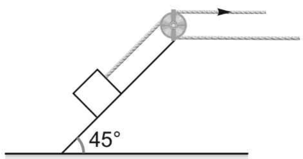
AMU

Engineering Entrance Exam

Solved Paper 2010

Physics

1. A block of mass 200 kg is being pulled up by men on an inclined plane at angle of 45° as shown in the figure. The coefficient of static friction is 0.5. Each man can only apply a maximum force of 500 N. Calculate the number of men required for the block to just start moving up the plane.



(a) 10 (b) 15
(c) 5 (d) 3

2. Two strings *A* and *B* are slightly out tune and produces beats of frequency 5 Hz. Increasing the tension in *B* reduces the beat frequency to 3 Hz. If the frequency of string *A* is 450 Hz, calculate the frequency of string *B*.

(a) 460 Hz (b) 455 Hz
(c) 445 Hz (d) 440 Hz

3. A resonance pipe is open at both ends and 30 cm of its length is in resonance with an external frequency 1.1 kHz. If the speed of sound is 330 m/s, which harmonic is in resonance?

(a) First (b) Second
(c) Third (d) Fourth

4. The SHM of a particle is given by

$x(t) = 5\cos\left(2\pi t + \frac{\pi}{4}\right)$ (in MKS units).

Calculate the displacement and the magnitude of acceleration of the particle at $t = 1.5$ s.

(a) -3.0 m, 100 m/s^2
(b) $+2.54$ m, 200 m/s^2
(c) -3.54 m, 140 m/s^2
(d) $+3.55$ m, 120 m/s^2

5. Calculate the ratio of rms speeds of oxygen gas molecules to that of hydrogen gas molecules kept at the same temperature.

(a) 1 : 4 (b) 1 : 8 (c) 1 : 2 (d) 1 : 6

6. The coefficient of volume expansion of a liquid is $49 \times 10^{-5}\ K^{-1}$. Calculate the fractional change in its density when the temperature is raised by 30°C.

(a) 7.5×10^{-3} (b) 3.0×10^{-3}
(c) 1.5×10^{-3} (d) 1.1×10^{-3}

7. Avalanche breakdown in a *p-n* junction diode is due to

(a) sudden shift of Fermi level
(b) increase in the width of forbidden gap
(c) sudden increase of impurity concentration
(d) cumulative effect of increased electron collision and creation of added electron hole pairs

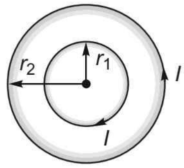
8. Any digital circuit can be realised by repetitive use of only

(a) NOT gate (b) OR gate
(c) AND gate (d) NOR gate

9. A solid sphere of mass 1 kg, radius 10 cm rolls down an inclined plane of height 7 m. The velocity of its centre as it reaches the ground level is

(a) 7 m/s (b) 10 m/s
(c) 15 m/s (d) 20 m/s

10. Two circular concentric loops of radii $r_1 = 20$ cm and $r_2 = 30$ cm are placed in the X-Y plane as shown in the figure. A current $I = 7$ A is flowing through them. The magnetic moment of this loop system is



(a) $+0.4\hat{\mathbf{k}}$(A-m^2) (b) $-1.5\hat{\mathbf{k}}$(A-m^2)
(c) $+1.1\hat{\mathbf{k}}$(A-m^2) (d) $+1.3\hat{\mathbf{j}}$(A-m^2)

11. In a Young's double slit experiment (slit distance d) monochromatic light of wavelength λ is used and the fringe pattern observed at a distance L from the slits. The angular position of the bright fringes are

(a) $\sin^{-1}\left(\frac{n\lambda}{d}\right)$ (b) $\sin^{-1}\left(\frac{\left(n+\frac{1}{2}\right)\lambda}{d}\right)$
(c) $\sin^{-1}\left(\frac{n\lambda}{L}\right)$ (d) $\sin^{-1}\left(\frac{\left(n+\frac{1}{2}\right)\lambda}{L}\right)$

12. Two energy levels of an electron in an atom are separated by 2.3 eV. The frequency of radiation emitted when the electrons go from higher to lower level is
(a) 6.95×10^{14} Hz (b) 3.68×10^{15} Hz
(c) 5.6×10^{14} Hz (d) 9.11×10^{15} Hz

13. What is the work function (in eV) of a substance if photoelectrons are just ejected for a monochromatic light of wavelength $\lambda = 3300$ Å?
(a) 3.75 (b) 3.25
(c) 1.63 (d) 0.75

14. The linear momentum of an electron, initially at rest, accelerated through a potential difference of 100 V is
(a) 9.1×10^{-24} (b) 6.5×10^{-24}
(c) 5.4×10^{-24} (d) 1.6×10^{-24}

15. The de-Broglie wavelength of a ball of mass 120 g moving at a speed of 20 m/s is
(a) 3.5×10^{-34} m (b) 2.8×10^{-34} m
(c) 1.2×10^{-34} m (d) 2.1×10^{-34} m

16. A square card of side length 1 mm is being seen through a magnifying lens of focal length 10 cm. The card is placed at a distance of 9 cm from the lens. The apparent area of the card through the lens is
(a) 1 cm^2 (b) 0.81 cm^2
(c) 0.27 cm^2 (d) 0.60 cm^2

17. An object moving at a speed of 5 m/s towards a concave mirror of focal length $f = 1$ m is at a distance of 9 m. The average speed of the image is
(a) $\frac{1}{5}$ m/s (b) $\frac{1}{10}$ m/s
(c) $\frac{5}{9}$ m/s (d) $\frac{4}{10}$ m/s

18. The magnetic field of an electromagnetic wave is given by

$B_y = 3 \times 10^{-7} \sin(10^3 x + 6.28 \times 10^{12} t)$.

The wavelength of the electromagnetic wave is
(a) 6.28 cm (b) 3.14 cm
(c) 0.63 cm (d) 0.32 cm

19. A 50 V AC is applied across an R-C (series) network. The rms voltage across the resistance is 40 V, then the potential across the capacitance would be
(a) 10 V (b) 20 V
(c) 30 V (d) 40 V

20. A pure inductive coil of 30 mH is connected to an AC source of 220 V, 50 Hz. The rms current in the coil is
(a) 50.35 A (b) 23.4 A
(c) 30.5 A (d) 12.3 A

21. A square loop of wire, side length 10 cm is placed at angle of 45° with a magnetic field that changes uniformly from 0.1 T to zero in 0.7 s. The induced current in the loop (its resistance is 1 Ω) is
(a) 1.0 mA (b) 2.5 mA
(c) 3.5 mA (d) 4.0 mA

22. The angle of dip at a certain place on earth is 60° and the magnitude of earth's horizontal component of magnetic field is 0.26 G. The magnetic field at the place on earth is
(a) 0.13 G (b) 0.26 G
(c) 0.52 G (d) 0.65 G

23. The dimensional formula for the magnetic field is
(a) $[MT^{-2}A^{-1}]$ (b) $[ML^2T^{-1}A^{-2}]$
(c) $[MT^{-2}A^{-2}]$ (d) $[MT^{-1}A^{-2}]$

24. The maximum velocity to which a proton can be accelerated in a cyclotron of 10 MHz frequency and radius 50 cm is
(a) 6.28×10^8 m/s (b) 3.14×10^8 m/s
(c) 6.28×10^7 m/s (d) 3.14×10^7 m/s

25. The radius of the path of an electron moving at a speed of 3×10^7 m/s perpendicular to a magnetic field 5×10^{-4} T is nearly
(a) 15 cm (b) 45 cm
(c) 27 cm (d) 34 cm

26. The resistance of the wire in the platinum resistance thermometer at ice point is $5\,\Omega$ and at steam point is $5.25\,\Omega$. When the thermometer is inserted in an unknown hot bath its resistance is found to be $5.5\,\Omega$. The temperature of the hot bath is
(a) 100°C (b) 200°C
(c) 300°C (d) 350°C

27. The density of copper is 9×10^3 kg/m^3 and its atomic mass is 63.5 u. Each copper atom provides one free electron. Estimate the number of free electrons per cubic metre in copper.
(a) 10^{19} (b) 10^{23}
(c) 10^{25} (d) 10^{29}

28. A conductor has been given a charge -3×10^{-7} by transferring electrons. Mass increase (in kg) of the conductor and the number of electrons added to the conductor are respectively
(a) 2×10^{-16} and 2×10^{31}
(b) 5×10^{-31} and 5×10^{19}
(c) 3×10^{-19} and 9×10^{16}
(d) 2×10^{-18} and 2×10^{12}

29. Under the action of a given coulombic force the acceleration of an electron is 2.5×10^{22} m/s^2. Then the magnitude of the acceleration of a proton under the action of same force is nearly
(a) 1.6×10^{-19} m/s^2 (b) 9.1×10^{31} m/s^2
(c) 1.5×10^{19} m/s^2 (d) 1.6×10^{27} m/s^2

30. An electron initially at rest fall a distance of 1.5 cm in a uniform electric field of magnitude 2×10^4 N/C. The time taken by the electron to fall this distance is
(a) 1.3×10^2 s (b) 2.1×10^{-12} s
(c) 1.6×10^{-10} s (d) 2.9×10^{-9} s

31. The constant of proportionality $\frac{1}{4\pi\varepsilon_0}$ in Coulomb's law has the following dimensions
(a) $C^{-2}N\,m^2$ (b) $C^2N^{-1}m^{-2}$
(c) $C^2N\,m^2$ (d) $C^{-2}N^{-1}m^{-2}$

32. The pressure on a swimmer 20 m below the surface of water at sea level is
(a) 1.0 atm (b) 2.0 atm
(c) 2.5 atm (d) 3.0 atm

33. The potential energy of 4-particles each of mass 1 kg placed at the four vertices of a square of side length 1 m is
(a) $+4.0\,G$ (b) $-7.5\,G$
(c) $-5.4\,G$ (d) $+6.3\,G$

34. Two masses 8 kg and 12 kg are connected at the two ends of a string that goes over a frictionless pulley. Calculate the acceleration of the masses and the tension in the string. Take $g = 10$ m/s^2
(a) 8 m/s^2, 144 N (b) 4 m/s^2, 112 N
(c) 6 m/s^2, 128 N (d) 2 m/s^2, 96 N

35. The backside of a truck is open and a box of 40 kg is placed 5 m away from the rear end. The coefficient of friction of the box with the surface of the truck is 0.15. The truck starts from rest with 2 m/s^2 acceleration. Calculate the distance covered by the truck when the box falls off.
(a) 20 m (b) 30 m
(c) 40 m (d) 50 m

36. The position of a particle x (in metre) at a time t second is given by the relation $r = (3t\,\hat{i} - t^2\,\hat{j} + 4\,\hat{k})$. Calculate the magnitude of velocity of the particle after 5 s.
(a) 3.55 m/s
(b) 5.03 m/s
(c) 8.75 m/s
(d) 10.44 m/s

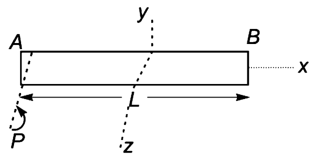
37. A monoatomic gas is kept at room temperature 300 K. Calculate the average kinetic energy of gas molecule (Use $k = 1.38 \times 10^{-23}$ MKS units)

(a) 0.138 eV
(b) 0.062 eV
(c) 0.039 eV
(d) 0.013 eV

38. A uniform magnetic field $B = 1.2$ mT is directed vertically upward throughout the volume of a laboratory chamber. A proton $(m_p = 1.67 \times 10^{-27}$ kg$)$ enters the laboratory horizontally from south to north. Calculate the magnitude of centripetal acceleration of the proton if its speed is 3×10^7 m/s.

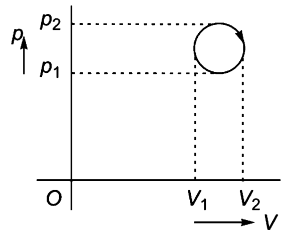
(a) 3.45×10^{12} m/s^2
(b) 1.67×10^{12} m/s^2
(c) 5.25×10^{12} m/s^2
(d) 2.75×10^{12} m/s^2

39. A rod of length L and mass M is rotating about an axis P perpendicular to the rod and parallel to z-axis, passing through one end A of the rod. The moment of inertia for rotation about this axis P is



(a) $\frac{1}{12}ML^2$ (b) $\frac{1}{4}ML^2$
(c) $\frac{1}{3}ML^2$ (d) $\frac{5}{12}ML^2$

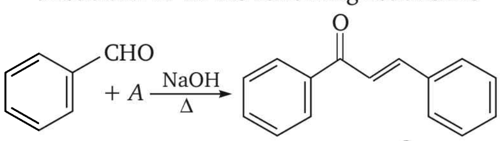
40. In the cyclic process shown in the p-V diagram calculate the work done.

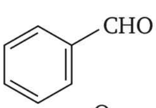


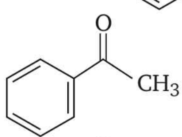
(a) $\pi\left(\frac{V_2 - V_1}{2}\right)^2$
(b) $\pi\left(\frac{p_2 - p_1}{2}\right)^2$
(c) $\frac{\pi}{4}(p_2 - p_1)(V_2 - V_1)$
(d) $\pi\,(p_2V_2 - p_1V_1)$

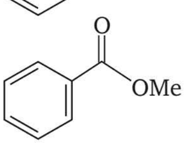
Chemistry

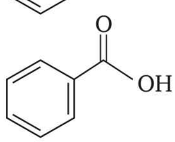
1. The reactant 'A' in the following reaction is



(a) 

(b) 

(c) 

(d) 

2. Compound which shows positive mesomeric effect

(a) $CH_2{=}CH{-}Cl$
(b) $C_6H_5{-}\overset{+}{N}{-}Me_3$
(c) $CH_2{=}CH{-}CH_2Cl$
(d) $C_6H_5{-}CH{=}CHCl$

3. Which of the following is an outer orbital complex?

(a) $[Cr(NH_3)_6]^{3+}$ (b) $[Ni(NH_3)_6]^{2+}$
(c) $[Fe(CN)_6]^{3-}$ (d) $[Mn(CN)_6]^{4-}$

4. 0.32 g of metal gave on treatment with an acid 112 mL of hydrogen at NTP. Calculate the equivalent weight of the metal.

(a) 58 (b) 32
(c) 11.2 (d) 24

5. A current of 0.5 A is passed for 30 min through a voltmeter containing $CuSO_4$ solution. Find the weight of Cu deposited.

(a) 3.18 g (b) 0.318 g
(c) 0.296 g (d) 0.150 g

6. Which of the following is not an artificial sweetener?
(a) Aspartame (b) Sucrolose
(c) Sucrose (d) Alitame

7. Gold number indicates
(a) protective action of lyophilic colloid
(b) charge on gold sol
(c) protective action of lyophobic colloid
(d) quantity of gold dissolved in a given sol

8. Which of the following will dissolve in excess of ammonia?
(a) AgI (b) AgBr
(c) AgCl (d) None of these

9. Who discovered the first noble gas compound?
(a) Neils Bohr (b) Neil Bartlett
(c) Neil Armstrong (d) William

10. Which of the following sets will have highest hydration enthalpy and highest ionic radius?
(a) Na and Li (b) Li and Rb
(c) K and Na (d) Cs and Na

11. What type of structure does $(NPCl_2)_4$ have?
(a) Linear (b) Hexagonal
(c) Cyclic (d) Polymeric

12. The pH of the solution produced by mixing equal volume of 2.0×10^{-3} M $HClO_4$ and 1.0×10^{-2} M $KClO_4$ is
(a) 2.7 (b) 2.3
(c) 3.0 (d) 1.0

13. Which of the following is non-existent?
(a) AlF_6^{3-} (b) CoF_6^{3-}
(c) BF_6^{3-} (d) SiF_6^{3-}

14. Which of the following is extracted from sea weeds?
(a) Quinine (b) Astatine
(c) Iodine (d) Germanium

15. How many hydrogen bonded water molecule(s) are associated with $CuSO_4 \cdot 5H_2O$?
(a) 1 (b) 2
(c) 3 (d) 4

16. The total number of atoms of all elements present in 1 mole of ammonium dichromate is
(a) 19 (b) 6.023×10^{23}
(c) 114.437×10^{23} (d) 84.322×10^{23}

17. What is the magnetic moment of Fe^{3+} ion in $[Fe(CN)_6]^{3-}$?
(a) 1.73 BM (b) 5.9 BM
(c) Diamagnetic (d) None of these

18. For the 19th electron of K the values of quantum number will be
(a) $4, 1, 0, +\frac{1}{2}$ (b) $4, 0, 0, +\frac{1}{2}$
(c) $3, 2, 0, +\frac{1}{2}$ (d) $3, 0, 0, +\frac{1}{2}$

19. Which of the following bases is not present in DNA?
(a) Alanine (b) Guanine
(c) Cytosine (d) Uracil

20. A student accidentally added conc. H_2SO_4 to potassium permanganate and it exploded due to the formation of an explosive which is
(a) MnO (b) Mn_2O_3
(c) Mn_2O_5 (d) Mn_2O_7

21. Which of the following is Vaska's compound ?
(a) $[Ni(PPh_3)_2Cl_2]$
(b) $[Rh(CO)_2Cl]_2$
(c) *trans*-$IrCl(CO)(PPh_3)_2$
(d) $IrCl(CO)_2(PPh_3)_2$

22. How many stereoisomers are possible in case of 3-chlorobutan-2-ol?
(a) 2 (b) 6
(c) 8 (d) 4

23. Which of the following has smallest number of molecules?
(a) 11.2 L of O_2 at NTP
(b) 8.0 g of O_2
(c) 0.1 mole of O_2
(d) 2.24×10^4 mL of O_2

24. Which of the following is not optically active ?
(a) Glycine (b) Tyrosine
(c) Lysine (d) Alanine

25. Which of the following is not a fat soluble vitamin?
(a) Vitamin A (b) Vitamin K
(c) Folic acid (d) Vitamin E

26. Which of the following exhibits square pyramidal geometry?
(a) XeF_6 (b) XeO_3
(c) BrF_5 (d) XeF_4

27. Which complex of Co^{2+} will have the weakest crystal field splitting?

(a) $[Co(CN)_6]^{4-}$ (b) $[CoCl_6]^{4-}$
(c) $[Co(en)_3]^{2+}$ (d) $[Co(H_2O)_6]^{2+}$

28. The ratio of rates of diffusion of hydrogen chloride and ammonia gases is

(a) 1 : 1.46 (b) 1 : 2.92
(c) 1.46 : 1 (d) 1 : 0.73

29. What shall be the pH of a weak acid of 10^{-3} M concentration which is only 10% ionized?

(a) 3 (b) 4
(c) 5 (d) 6

30. Which of the following is the major source of magnesium and is also a double salt?

(a) $MgCO_3$ (b) $Mg_2P_2O_7$
(c) $Mg\langle^{I}_{C_2H_5}$ (d) $KCl \cdot MgCl_2 \cdot 6H_2O$

31. Phosphine can be prepared by the reaction of water with

(a) calcium phosphide
(b) calcium hydride
(c) calcium dihydrogen phosphate
(d) calcium phosphate

32. The white ZnO turns yellow on heating because of

(a) Frenkel defect
(b) Metal excess defect
(c) Metal deficiency defect
(d) Schottky defect

33. Which of the processes is used in thermite welding?

(a) $TiO_2 + 4Na \longrightarrow Ti + 2Na_2O$
(b) $2Al + Fe_2O_3 \longrightarrow Al_2O_3 + 2Fe$
(c) $SnO_2 + 2C \longrightarrow Sn + 2CO$
(d) $Cr_2O_3 + 2Al \longrightarrow Al_2O_3 + 2Cr$

34. Which of the following has least tendency to undergo catenation?

(a) C (b) Si
(c) Ge (d) Sn

35. Methyl magnesium bromide on reaction with SO_2 followed by hydrolysis gives

(a) methyl sulphonic acid
(b) dithioacetic acid
(c) methane sulphinic acid
(d) ethane thiol

36. Which of the following configuration can undergo distortion?

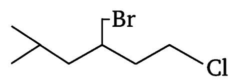
(a) $t_{2g}^{6}e_g^{1}$ (b) $t_{2g}^{6}e_g^{2}$
(c) $t_{2g}^{6}e_g^{4}$ (d) $t_{2g}^{6}e_g^{0}$

37. Order of the base strength of the compounds

(a) iv > ii > i > iii (b) iii > ii > iv > i
(c) ii > iii > iv > i (d) ii > iii > i > iv

38. Which of the following molecules does not have net dipole moment?

(a) $CH_3—Br$ (b) CH_2Cl_2
(c) HCOOH (d) $H_2C{=}CH_2$

39. IUPAC name  is

(a) 1,1-dimethyl-3-bromoethyl-5-chloropentane
(b) 3-bromomethyl-1-chloro-5-methylhexane
(c) 1-bromomethyl-2-chloroethyl-4-methylpentane
(d) 4-bromomethyl-1-chloro-6-methylheptane

40. In bakelite, the rings are joined to each other through

(a) $—CH_2—$ (b) $—O—$
(c) $—CH(OH)—$ (d) $—C(=O)—$

Mathematics

1. If A is a square matrix such that $A^2 = A$, then $(I - A)^3 + A$ is equal to

(a) A (b) $I - A$
(c) I (d) $3A$

2. For the equations $x + 2y + 3z = 1$, $2x + y + 3z = 2$, $5x + 5y + 9z = 4$

(a) there is only one solution
(b) there exists infinitely many solutions
(c) there is no solution
(d) None of the above

3. Let $f = \{(1, 1), (2, 4), (0, -2), (-1, -5)\}$ be a linear function from Z into Z. Then, $f(x)$ is

(a) $f(x) = 3x - 2$ (b) $f(x) = 6x - 8$
(c) $f(x) = 5x - 2$ (d) $f(x) = 7x + 2$

4. Let $f : R - \left\{\frac{5}{4}\right\} \to R$ be a function defined as $f(x) = \frac{5x}{4x + 5}$. The inverse of f is the map $g :$ Range $f \to R - \left\{\frac{5}{4}\right\}$ given by

(a) $g(y) = \frac{y}{5 - 4y}$ (b) $g(y) = \frac{5y}{5 + 4y}$
(c) $g(y) = \frac{5y}{5 - 4y}$ (d) None of these

5. Let * be a binary operation on the set Q of rational numbers defined by $a * b = \frac{ab}{4}$. The identity with respect to this operation is

(a) 1 (b) 2
(c) 3 (d) 4

6. Let $A = \{1, 0, 1, 2\}$, $B = \{4, 2, 0, 2\}$ and $f, g : A \to B$ be functions defined by $f(x) = x^2 - x$ and $g(x) = 2\left|x - \frac{1}{2}\right| - 1$. Then,

(a) $f = g$ (b) $f = 2g$
(c) $g = 2f$ (d) None of these

7. The complex number $z = \begin{vmatrix} 2 & 3+i & -3 \\ 3-i & 0 & -1+i \\ -3 & -1-i & 4 \end{vmatrix}$ is equal to

(a) $3 - 4i$ (b) $5 + 4i$
(c) $-5i$ (d) None of these

8. The number of solutions of the system of equations $\text{Re}(z^2) = 0, |z| = 2$ is

(a) 4 (b) 3
(c) 2 (d) 1

9. The angle of elevation of the top of a TV tower from three points A, B, C in a straight line in the horizontal plane through the foot of the tower are $\alpha, 2\alpha, 3\alpha$ respectively. If $AB = a$, the height of the tower is

(a) $a \tan\alpha$ (b) $a \sin\alpha$
(c) $a \sin 2\alpha$ (d) $a \sin 3\alpha$

10. The number of solutions of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$ is

(a) 0 (b) 1
(c) 2 (d) 3

11. If $2\tan^{-1}(\cos x) = \tan^{-1}(2\text{cosec } x)$, then x is equal to

(a) $\pi/2$ (b) $\pi/3$
(c) $\pi/4$ (d) $\pi/6$

12. If $\tan^{-1} 4x + \tan^{-1} 6x = \pi / 4$, then x is equal to

(a) $\frac{1}{12}$ (b) $\frac{1}{12}$ or $-\frac{1}{2}$
(c) $-\frac{1}{2}$ (d) None of these

13. The longest side of a triangle is 5 times the shortest side and the third side is 50 cm shorter than the longest side. If the perimeter of the triangle is at least 60 cm, the minimum length of the shortest side is

(a) 9 cm (b) 10 cm
(c) 11 cm (d) None of these

14. For $2 \le r \le n$, ${}^nC_r + 2 \cdot {}^nC_{r-1} + {}^nC_{r-2}$ is equal to

(a) ${}^{n+1}C_{r-1}$ (b) $2 \cdot {}^{n+1}C_{r+1}$
(c) $2 \cdot {}^{n+2}C_r$ (d) ${}^{n+2}C_r$

15. The coefficients of the $(r-1)$th, rth and $(r+1)$th terms in the expansion of $(x+1)^n$ are in the ratio 1 : 3 : 5. The pair (n, r) is

(a) (6, 3) (b) (7, 3)
(c) (5, 3) (d) (5, 1)

16. If $S_1 = a_2 + a_4 + a_6 + \ldots +$ upto 100 terms and $S_2 = a_1 + a_3 + a_5 + \ldots +$ upto 100 terms of a certain AP, then its common difference is
(a) $S_1 - S_2$ (b) $S_2 - S_1$
(c) $\frac{S_1 - S_2}{2}$ (d) None of these

17. If $\log_{10} 2, \log_{10}(2^x - 1)$ and $\log_{10}(2^x + 3)$ be three consecutive terms of an AP, then
(a) $x = 0$ (b) $x = 1$
(c) $x = \log_2 5$ (d) $x = \log_{10} 2$

18. In a GP $t_2 + t_5 = 216$ and $t_4 : t_6 = 1 : 4$ and all terms are integers, then its first term is
(a) 16 (b) 14
(c) 12 (d) None of these

19. If a, b, c, d and p are different real numbers such that
$$(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \le 0,$$
then a, b, c and d are in
(a) AP (b) GP
(c) HP (d) None of these

20. If a variate takes values $a, ar, ar^2, \ldots, ar^{n-1}$, then which of the following relations between means hold ?
(a) $A \cdot H = G^2$ (b) $\frac{A + H}{2} = G$
(c) $A > G > H$ (d) $A = G = H$

21. The condition that $x^3 - px^2 + qx - r = 0$ may have two of its roots equal to each other, but opposite in sign is
(a) $r = pq$ (b) $r = 2p^3 + pq$
(c) $r = p^2 q$ (d) None of these

22. The length L (in centimetre) of a copper rod is a linear function of its Celsius temperature C. In an experiment $L = 124.942$ when $C = 20$ and $L = 125.134$ when $C = 110$. The expression of L in terms of C is
(a) $L = \frac{0.192}{90}(C - 20) + 124.942$
(b) $L = \frac{0.192}{90}(C - 110) + 124.942$
(c) $L = \frac{192}{90}(C - 20) + 124.942$
(d) $L = \frac{192}{90}(C - 110) + 124.942$

23. C_1 is a circle with centre at the origin and radius equal to r and C_2 is a circle with centre at $(3r, 0)$ and radius equal to $2r$. The number of common tangents that can be drawn to the two circles is
(a) 1 (b) 2
(c) 3 (d) 4

24. Let $f(x, y) = 0$ be the equation of a circle. If $f(0, \lambda) = 0$ has equal roots $\lambda = 1, 1$ and $f(\lambda, 0)$ has roots $\lambda = \frac{1}{2}, 2$, then the centre of the circle is
(a) $\left(1, \frac{1}{2}\right)$ (b) $\left(\frac{5}{4}, 1\right)$
(c) $(5, 4)$ (d) $\left(\frac{1}{2}, 1\right)$

25. The line $x + y = 6$ is normal to the parabola $y^2 = 8x$ at the point
(a) (4, 2) (b) (2, 4)
(c) (2, 2) (d) (3, 3)

26. $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are three vectors of which every pair is non-collinear. If the vector $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ and $\vec{\mathbf{b}} + \vec{\mathbf{c}}$ are collinear with $\vec{\mathbf{c}}$ and $\vec{\mathbf{a}}$ respectively, then $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}$ is
(a) a unit vector
(b) the null vector
(c) equally inclined to $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$
(d) None of the above

27. A unit vector $\vec{\mathbf{a}}$ makes angles $\pi/4$ with $\hat{\mathbf{i}}$, $\pi/3$ with $\hat{\mathbf{j}}$ and an acute angle θ with $\hat{\mathbf{k}}$, then θ and $\vec{\mathbf{a}}$ are
(a) $\frac{\pi}{3}, \frac{\sqrt{2}\,\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}}{2}$ (b) $\frac{\pi}{3}, \frac{\sqrt{2}\,\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}}{2}$
(c) $\frac{\pi}{3}, \frac{\sqrt{2}\,\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}}{2}$ (d) $\frac{\pi}{3}, \frac{\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}}{2}$

28. Let $\vec{\mathbf{a}} = \hat{\mathbf{i}} - \hat{\mathbf{k}}, \vec{\mathbf{b}} = x\,\hat{\mathbf{i}} + \hat{\mathbf{j}} + (1 - x)\,\hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = y\,\hat{\mathbf{i}} + x\,\hat{\mathbf{j}} + (1 + x - y)\,\hat{\mathbf{k}}$. Then, $[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$ depends on
(a) only x (b) only y
(c) neither x nor y (d) both x and y

29. Equation of the plane through $(-1, -1, 1)$ which is parallel to $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 0$ is

(a) $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + 1 = 0$

(b) $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - 1 = 0$

(c) $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + 3 = 0$

(d) $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - 3 = 0$

30. The coordinates of a point on the line $\frac{x-1}{2} = \frac{y+1}{-3} = z$ at a distance $4\sqrt{14}$ from the point $(1, -1, 0)$ are

(a) $(9, -13, 4)$ (b) $(-9, 13, 4)$
(c) $(9, 13, -4)$ (d) None of these

31. The ratio in which the line segment joining the points $(4, 8, 10)$ and $(6, 10, -8)$ is divided by xy-plane is

(a) 5 : 4 externally
(b) 5 : 4 internally
(c) 3 : 2 externally
(d) None of these

32. $\lim_{x \to 1} \frac{x^{15} - 1}{x^{10} - 1}$ is equal to

(a) 2/3 (b) 3/2
(c) 1 (d) does not exist

33. The values of a and b such that the function defined by $f(x) = \begin{cases} 7, & \text{if } x \le 2 \\ ax + b, & \text{if } 2 < x < 9 \\ 21, & \text{if } x \ge 9 \end{cases}$ is a continuous function are

(a) $a = 3, b = 2$ (b) $a = 2, b = 3$
(c) $a = 7, b = 9$ (d) None of these

34. If $2^x + 2^y = 2^{x+y}$, then the value of $\frac{dy}{dx}$ at $x = y = 1$ is

(a) 0 (b) -1
(c) 1 (d) 2

35. A stone is dropped into a quiet lake and waves move in circles at the speed of 6 cm per second. At the instant, when the radius of the circular wave is 12 cm, the enclosed area is increasing at the rate of

(a) 120π cm^2/s (b) 130π cm^2/s
(c) 144π cm^2/s (d) None of these

36. For the function $f(x) = \frac{4}{3}x^3 - 8x^2 + 16x + 5$, $x = 2$ is a point of

(a) local maxima (b) local minima
(c) point of inflexion (d) None of these

37. $\int e^{x \log a} e^x \, dx$ is equal to

(a) $(ae)^x + C$ (b) $\frac{(ae)^x}{\log(ae)} + C$

(c) $\frac{e^x}{1 + \log a} + C$ (d) None of these

38. $\int e^x \left(\operatorname{cosec}^{-1} x + \frac{-1}{x\sqrt{x^2 - 1}} \right) dx$ is equal to

(a) $e^x \operatorname{cosec}^{-1} x + C$ (b) $e^x \sin^{-1} x + C$
(c) $e^x \sec^{-1} x + C$ (d) $e^x \cos^{-1} x + C$

39. $\int_{-1}^{1} \sin^5 x \cdot \cos^4 x \, dx$ is

(a) 0 (b) 1
(c) 2 (d) 3

40. The coefficient of x^3 in the expansion of e^{2x+3} as a series in powers of x is

(a) e^3 (b) $\frac{3}{4}e^3$

(c) $\frac{4}{3}e^3$ (d) None of these

41. The area of the region bounded by the line $y = 3x + 2$, the x-axis and the ordinates $x = -1$ and $x = 1$ is

(a) $\frac{13}{3}$ (b) $\frac{13}{4}$

(c) $\frac{13}{5}$ (d) $\frac{13}{6}$

42. The differential equation representing the family of curves $y = b\sin(x + a)$, where a, b are arbitrary constants is

(a) $\frac{d^2y}{dx^2} + y = 0$ (b) $\frac{d^2y}{dx^2} - y = 0$

(c) $\frac{dy}{dx} + y = 0$ (d) None of these

43. The general solution of the differential equation $ydx + (x + 2y^2)dy = 0$ is

(a) $xy + y^2 = C$ (b) $3xy + y^2 = C$
(c) $xy + y^3 = C$ (d) $3xy + 2y^3 = C$

44. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, then A^n is

(a) A

(b) $\begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$

(c) $\begin{bmatrix} 3^n & 3^n & 3^n \\ 3^n & 3^n & 3^n \\ 3^n & 3^n & 3^n \end{bmatrix}$

(d) None of the above

45. Value of $\begin{bmatrix} 2 & 4 & 6 \\ 2+3x & 4+3y & 6+3z \\ 2x & 2y & 2z \end{bmatrix}$ is

(a) 0 (b) 2
(c) 4 (d) 6

46. The value of $\begin{bmatrix} 2y+4 & 5y+7 & 8y+1 \\ 3y+5 & 6y+8 & 9y+2 \\ 4y+6 & 7y+9 & 10y+3 \end{bmatrix}$ is

(a) 2 (b) 3
(c) 5 (d) None of these

47. Coefficient of variations of two distributions are 55 and 65, and their standard deviations are 22 and 39 respectively. Their arithmetic means are respectively

(a) 15, 20 (b) 40, 60
(c) 30, 50 (d) None of these

48. A fair coin is tossed n number of times. If the probability of having at least one head is more than 90%, then n is greater than or equal to

(a) 2 (b) 3
(c) 4 (d) 5

49. Three cards are drawn successively without replacement from a pack of 52 well shuffled cards. The probability that first two cards are queens and the third card is a king, is

(a) $\frac{4}{52} \times \frac{4}{51} \times \frac{4}{50}$ (b) $\frac{4}{52} \times \frac{2}{51} \times \frac{1}{50}$
(c) $\frac{4}{52} \times \frac{3}{51} \times \frac{3}{50}$ (d) $\frac{4}{52} \times \frac{3}{51} \times \frac{4}{50}$

50. Bag Ist contains 3 red and 4 black balls, while another bag IInd contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be black. The probability that it was drawn from bag IInd is

(a) $\frac{7}{43}$ (b) $\frac{13}{43}$
(c) $\frac{21}{43}$ (d) None of these

51. For the binomial distribution $(p+q)^n$ whose mean is 20 and variance is 16, pair (n, p) is

(a) $\left(100, \frac{1}{5}\right)$ (b) $\left(100, \frac{2}{5}\right)$
(c) $\left(50, \frac{1}{5}\right)$ (d) $\left(50, \frac{2}{5}\right)$

52. The maximum value of $Z = 4x + y$ subject to the constraints, $x + y \le 50$, $3x + y \le 90$, $x \ge 0$, $y \ge 0$ is

(a) 40 (b) 130
(c) 120 (d) 50

53. If $f(x) = \frac{x-1}{x+1}$, then $f(2x)$ is

(a) $\frac{f(x)+1}{f(x)+3}$ (b) $\frac{3f(x)+1}{f(x)+3}$
(c) $\frac{f(x)+3}{f(x)+1}$ (d) $\frac{f(x)+3}{3f(x)+1}$

54. Complex number $z = \frac{i-1}{\cos(\pi/3) + i\sin(\pi/3)}$ in polar form is

(a) $r = \sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

(b) $r = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(c) $r = \sqrt{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

(d) None of the above

55. The number of solutions of equation

$$\sin^4\theta - 2\sin^2\theta - 1 = 0,$$

which lie between 0 and 2π is

(a) 0 (b) 1
(c) 2 (d) 4

56. The product r consecutive integers is divisible by

(a) $r!$ (b) $(r-1)!$
(c) $(r+1)!$ (d) None of these

57. The interior angles of a polygon are in arithmetic progression. The smallest angle is 120 and the common difference is 5. The number of sides of the polygon is

(a) 7 (b) 9
(c) 11 (d) 16

58. In a certain progression three consecutive terms are 30, 24, 20. The next term of the progression is

(a) 16 (b) $\frac{120}{7}$
(c) 18 (d) None of these

59. If x, y, z are three positive numbers, then the minimum value of $\frac{y+z}{x} + \frac{z+x}{y} + \frac{x+y}{z}$ is

(a) 1 (b) 2
(c) 3 (d) 6

60. A person standing at the junction (crossing) of two straight paths represented by the equations $x + y + 1 = 0$ and $x - y + 1 = 0$ wants to reach the path, whose equation is $6x - 7y + 8 = 0$ in least time. The equation of the path that he should follow is

(a) $7x + 6y + 7 = 0$ (b) $6x + 7y + 7 = 0$
(c) $7x + 6y + 4 = 0$ (d) $6x + 7y + 4 = 0$

61. If $ax^2 + 4xy + y^2 + ax + 3y + 2 = 0$ represents a parabola, then a is

(a) – 4 (b) 4
(c) 0 (d) 6

62. The position vector of a point R, which divides the line joining two points P and Q whose position vectors are $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $-\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ respectively, in the ratio 2 : 1 externally is

(a) $-3\hat{\mathbf{i}} - \hat{\mathbf{k}}$ (b) $3\hat{\mathbf{i}} + \hat{\mathbf{k}}$
(c) $2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ (d) None of these

63. Let $\vec{\mathbf{b}} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ and $\vec{\mathbf{c}}$ be two vectors perpendicular to each other xy-plane, then a vector in the same plane having projections 1 and 2 along $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$, respectively, is

(a) $\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ (b) $2\hat{\mathbf{i}} - \hat{\mathbf{j}}$
(c) $2\hat{\mathbf{i}} + \hat{\mathbf{j}}$ (d) None of these

64. The value of λ for which the lines $\frac{1-x}{3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{6-z}{7}$ are perpendicular to each other is

(a) – 1 (b) – 2
(c) 1 (d) 2

65. If the function $f(x)$ satisfies $\lim_{x \to 1} \frac{f(x) - 3}{x^2 - 1} = \pi$, then $\lim_{x \to 1} f(x)$ is

(a) 1 (b) 2
(c) 3 (d) π

66. If $f(x) = 3e^{x^2}$, then

$f'(x) - 2xf(x) + \frac{1}{3} f(0) - f'(0)$ is equal to

(a) 0 (b) 1
(c) $\frac{7}{3} e^{x^2}$ (d) None of these

67. A car starts from a point P at time $t = 0$ seconds and stops at point Q. The distance x, in metres, covered by it, in t seconds is given by $x = t^2\left(3 - \frac{2}{3}t\right)$. The time taken by it to reach Q in seconds is

(a) 1/2 (b) 3
(c) 1 (d) None of these

68. $\int e^x \left(\frac{1 + \sin x}{1 + \cos x}\right) dx$ is equal to

(a) $e^x \tan \frac{x}{2} + C$ (b) $e^x \cos \frac{x}{2} + C$
(c) $x^x \sin x + C$ (d) $e^x \cos x + C$

69. The sum of the series $\frac{1}{2 \cdot 3} + \frac{1}{4 \cdot 5} + \frac{1}{6 \cdot 7} + \ldots + \infty$ is equal to

(a) $\log(2e)$ (b) $\log(e/2)$
(c) $\log(4/e)$ (d) None of these

70. The differential equation representing the family of parabolas having vertex at origin and axis along positive direction of x-axis is

(a) $y^2 - 2xy \frac{dy}{dx} = 0$ (b) $y^2 + 2xy \frac{dy}{dx} = 0$
(c) $y^2 - 2xy \frac{d^2y}{dx^2} = 0$ (d) $y^2 + 2xy \frac{d^2y}{dx^2} = 0$

Answers

Physics

1. (c)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (a)	**6.** (*)	**7.** (d)	**8.** (d)	**9.** (b)	**10.** (c)
11. (a)	**12.** (c)	**13.** (a)	**14.** (c)	**15.** (b)	**16.** (a)	**17.** (a)	**18.** (c)	**19.** (c)	**20.** (b)
21. (a)	**22.** (c)	**23.** (a)	**24.** (d)	**25.** (d)	**26.** (b)	**27.** (d)	**28.** (d)	**29.** (c)	**30.** (d)
31. (a)	**32.** (d)	**33.** (c)	**34.** (d)	**35.** (a)	**36.** (d)	**37.** (c)	**38.** (a)	**39.** (c)	**40.** (c)

Chemistry

1. (b)	**2.** (a,d)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (c)	**7.** (a)	**8.** (c)	**9.** (b)	**10.** (b)
11. (c)	**12.** (c)	**13.** (c)	**14.** (c)	**15.** (a)	**16.** (c)	**17.** (a)	**18.** (b)	**19.** (d)	**20.** (d)
21. (c)	**22.** (d)	**23.** (c)	**24.** (a)	**25.** (c)	**26.** (c)	**27.** (b)	**28.** (a)	**29.** (b)	**30.** (d)
31. (a)	**32.** (b)	**33.** (b)	**34.** (d)	**35.** (c)	**36.** (a)	**37.** (c)	**38.** (d)	**39.** (b)	**40.** (a)

Mathematics

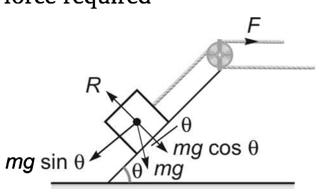
1. (c)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (d)	**6.** (*)	**7.** (d)	**8.** (c)	**9.** (c)	**10.** (d)
11. (c)	**12.** (b)	**13.** (b)	**14.** (d)	**15.** (b)	**16.** (d)	**17.** (c)	**18.** (c)	**19.** (b)	**20.** (a)
21. (a)	**22.** (a)	**23.** (c)	**24.** (b)	**25.** (b)	**26.** (c)	**27.** (a)	**28.** (c)	**29.** (b)	**30.** (a)
31. (b)	**32.** (b)	**33.** (b)	**34.** (b)	**35.** (c)	**36.** (c)	**37.** (b)	**38.** (a)	**39.** (a)	**40.** (c)
41. (a)	**42.** (a)	**43.** (d)	**44.** (b)	**45.** (a)	**46.** (d)	**47.** (b)	**48.** (c)	**49.** (d)	**50.** (c)
51. (a)	**52.** (c)	**53.** (b)	**54.** (a)	**55.** (a)	**56.** (a)	**57.** (b)	**58.** (b)	**59.** (d)	**60.** (a)
61. (b)	**62.** (a)	**63.** (b)	**64.** (b)	**65.** (c)	**66.** (b)	**67.** (b)	**68.** (a)	**69.** (b)	**70.** (a)

* No option is matching.

Hints & Solutions

Physics

1. Total force required



$$F = mg\sin\theta + f$$
$$= mg\sin\theta + \mu_s R$$
$$= mg\sin\theta + \mu_s mg\cos\theta$$
$$= mg[\sin\theta + \mu_s\cos\theta]$$
$$= 200\times 10[\sin 45^\circ + 0.5\cos 45^\circ]$$
$$F = \frac{200\times 10\times 3}{2\sqrt{2}}$$

The number of men required will be

$$\frac{200\times 10\times 3}{500\times 2\sqrt{2}} \approx 5$$

2. n_A = known frequency = 450 Hz, $n_B = ?$

$x = 5$ Hz which is decreasing after tension is increased (*ie*, $x\downarrow$)

Hence $\quad n_A\downarrow - n_B = x\downarrow \quad$ ...(i) correct

$n_B - n_A\downarrow = x\downarrow \quad$ ...(ii) wrong

$\Rightarrow \quad n_B = n_A - x = 450 - 5 = 445$ Hz

3. Fundamental frequency of open pipe

First harmonic $= n_1 = \dfrac{v}{2l} = \dfrac{330}{2\times 0.3} = 550$ Hz

Second harmonic $= 2\times n_1 = 1100$ Hz $= 1.1$ kHz

4. Displacement $x(t) = 5\cos\left(2\pi\times\dfrac{3}{2}+\dfrac{\pi}{4}\right)$

$$5\cos\left[\frac{13\pi}{4}\right] = -3.5\text{ m}$$

$$y = 5\cos\left(2\pi t + \frac{\pi}{4}\right)$$

$\therefore \quad v = -10\pi\sin\left(2\pi t + \dfrac{\pi}{4}\right)$

$\therefore \quad$ acceleration $= -20\pi^2\cos\left(2\pi t + \dfrac{\pi}{4}\right)$

$$= -20\pi^2\cos\left(2\pi\times\frac{3}{2}+\frac{\pi}{4}\right)$$

$$= 20\pi^2 \cos \frac{13\pi}{4}$$

$$= 140 \text{ m/s}^2$$

5. Root mean square velocity

$$v_{rms} \propto \frac{1}{\sqrt{M}}$$

so $$\frac{(v_{rms})_{O_2}}{(v_{rms})_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}}$$

$$= \sqrt{\frac{2}{32}} = \frac{1}{4}$$

6. Variation of density with temperature is given by

$$\rho' = \frac{\rho}{1 + \gamma\Delta\theta}$$

Fraction change is

$$\frac{\rho' - \rho}{\rho} = \left[\frac{1}{1 + 49 \times 10^{-5} \times 30} - 1\right]$$

$$= 1.5 \times 10^{-2}$$

7. At high reverse voltage, due to high electric field, the minority charge carriers, while crossing the junction acquire very high velocities. These by collision breaks down the covalent bonds generating more carriers. A chain reaction is established giving rise to high current. This mechanism is called avalanche breakdown.

8. NOR and NAND gates are universal gates. Any digital circuit can be realised by repetitive use of these (NOR and NAND) gates.

9. When a body of mass m and radius R rolls down on inclined plane of height h and angle of inclination θ, it loses potential energy. However, it acquires both linear and angular speeds. Velocity at the lowest point $v = \sqrt{\dfrac{2gh}{1 + \dfrac{K^2}{R^2}}}$

For solid sphere $\dfrac{K^2}{R^2} = \dfrac{2}{5}$

$$\therefore \quad v = \sqrt{\frac{2 \times 10 \times 7}{1 + \frac{2}{5}}} = 10 \text{ m/s}$$

10. Here, magnetic moment due to loop 1, $M_1 = iA_1$

$$= i\pi r_1^2$$

$$= 7 \times \pi (0.20)^2 = 0.28\pi$$

Similarly, magnetic moment due to loop 2,

$$M_2 = iA_2$$

$$= i\pi r_2^2$$

$$= 7 \times \pi(0.30)^2 = 0.63\pi$$

Net magnetic moment

$$= M_1 - M_2 = 0.63\pi - 0.28\pi = 1.1 \text{ Am}^2$$

11. For constructive interference

Path difference $\Delta = d \sin\theta = n\lambda$

$$\Rightarrow \quad \theta = \sin^{-1}\left[\frac{n\lambda}{d}\right]$$

12. Given, $E_2 - E_1 = 2.3$ eV

or $$\nu = \frac{E_2 - E_1}{h} = \frac{2.3 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$$= 0.55 \times 10^{15}$$

$$= 5.5 \times 10^{14} \text{ Hz}$$

13. Work function, $\phi = \dfrac{hc}{\lambda}$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{3300 \times 10^{-10}}$$

$$= 6.0 \times 10^{-19} \text{ J}$$

$$= \frac{6.0 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 3.75 \text{ eV}$$

14. de-Broglie wavelength of electron is given by

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

Substituting the value of E, we get

$$\lambda = \frac{h}{\sqrt{2meV}}$$

Here $m = 9.1 \times 10^{-31}$ kg; $e = 1.6 \times 10^{-19}$ C

and $h = 6.6 \times 10^{-34}$ Js

we get $$\lambda = \frac{12.27}{\sqrt{V}} \times 10^{-10} = \frac{12.27}{\sqrt{V}} \text{ Å}$$

The de-Broglie wavelength of electrons, when accelerated through a potential difference of 100 V will be

$$\lambda = \frac{12.27}{\sqrt{100}} = 1.227 \text{ Å}$$

Moreover, $$\lambda = \frac{h}{p}$$

$$\Rightarrow \quad p = \frac{6.6 \times 10^{-34}}{1.227 \times 10^{-10}} = 5.5 \times 10^{-24} \text{ kg-ms}^{-1}$$

15. de-Broglie wavelength, $\lambda = \frac{h}{mv}$

$$= \frac{6.6 \times 10^{-34}}{120 \times 10^{-3} \times 20}$$

$$= 2.75 \times 10^{-34} \text{m}$$

16. Focal length of converging lens $f = +10$ cm

$$u = -9 \text{ cm}$$

From lens formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

or $$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} + \frac{1}{(-9)}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{9}$$

or $$v = -90 \text{ cm}$$

Magnification, $m = \frac{v}{u} = \frac{-90}{-9} = 10$ m

$\therefore$ Apparent area of card through lens

$$= 10 \times 10 \times 1 \times 1 = 100 \text{ mm}^2$$

$$= 1 \text{ cm}^2$$

17. According to mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

Here $u = -9$ m and $f = -1$ m

$$\frac{1}{(-1)} - \frac{1}{(-9)} = \frac{1}{v}$$

$\Rightarrow$ $$v = \frac{-9}{8} \text{ m}$$

As the object moves at a constant speed of 5 m/s after 1 s the new position of image is

$$u' = -9\text{m} + 5\text{m} = -4\text{m}$$

$\therefore$ $$\frac{1}{(-1)} - \frac{1}{(-4)} = \frac{1}{v'}$$

$\Rightarrow$ $$v' = -\frac{4}{3} \text{ m}$$

The shift in the position of image in 1s is

$$v - v' = -\frac{9}{8} + \frac{4}{3} \approx \frac{1}{5}$$

$\therefore$ Average speed of image $= \frac{1}{5}$ m/s

18. Given, $B_y = 3 \times 10^{-7} \sin(10^3 x + 6.28 \times 10^{12} t)$

Comparing with the general equation

$$B_y = B_0 \sin(kx + \omega t)$$

we get $k = 10^3$

or $$\frac{2\pi}{\lambda} = 10^3 \qquad \left[k = \frac{2\pi}{\lambda}\right]$$

$\Rightarrow$ $$\lambda = \frac{2\pi}{10^3}$$

$$= 6.28 \times 10^{-3} \text{m}$$

$$= 0.63 \text{cm}$$

19. For an *R-C* circuit

Applied voltage, $V = \sqrt{V_R^2 + V_C^2}$

$\therefore$ $$50 = \sqrt{(40)^2 + V_C^2}$$

$\Rightarrow$ $$V_C = 30 \text{ V}$$

20. Given $L = 30$ mH

$$V_{\text{rms}} = 220 \text{ V}$$

$$f = 50 \text{ Hz}$$

Now, $$X_L = \omega L = 2\pi f L$$

$$= 2 \times 3.14 \times 50 \times 30 \times 10^{-3}$$

$$= 9.42 \Omega$$

The rms current in the coil is

$$i_{\text{rms}} = \frac{V_{\text{rms}}}{X_L} = \frac{220 \text{ V}}{9.42 \Omega} = 23.4 \text{ A}$$

21. Initial magnetic flux linked with the loop

$$\phi_1 = B_1 A_1 \cos\phi$$

$$= 0.1 \times (10 \times 10^{-2})^2 \cos 45°$$

$$= \frac{0.1 \times 10^{-2} \times 1}{\sqrt{2}} = \frac{10^{-3}}{\sqrt{2}}$$

Final magnetic flux linked with the loop $\phi_2 = 0$

Now, induced emf in the loop $e = \frac{-d\phi}{dt}$

$$= \frac{-\left[\frac{10^{-3}}{\sqrt{2}}\right]}{0.7} = 10^{-3} \text{V}$$

$\therefore$ Induced current $= \frac{e}{R} = \frac{10^{-3}}{1} = 1$ mA

22. Horizontal component of earth's magnetic field is the component of earth's magnetic field along the horizontal direction.

$$B_H = B \cos\delta$$

$$0.26 = B \cos 60°$$

$\Rightarrow$ $$B = \frac{0.26}{\cos 60°} = 0.52 \text{G}$$

23. Magnetic field $= \frac{\text{Force}}{\text{Charge} \times \text{velocity}}$

$$= \frac{[\text{MLT}^{-2}]}{[\text{AT}][\text{LT}^{-1}]} = [\text{MA}^{-1}\text{T}^{-2}]$$

24. The motion of proton in magnetic field will be circular.

$\therefore \quad v = 2\pi rf$

$= 2 \times 3.14 \times 50 \times 10^{-2} \times 10 \times 10^{6}$

$= 3.14 \times 10^{7}$ m/s

25. When a charged particle moves inside a uniform magnetic field then the radius of the circular path is

$r = \dfrac{mv}{Bq}$

$= \dfrac{9.1 \times 10^{-31} \times 3 \times 10^{7}}{5 \times 10^{-4} \times 1.6 \times 10^{-19}} = 0.34 \text{ m} = 34 \text{ cm}$

26. For resistance thermometers

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100°\text{C}$$

Here $R_t = 5.5\,\Omega$, $R_0 = 5\,\Omega$, $R_{100} = 5.25\,\Omega$

$\therefore \quad t = \dfrac{5.5 - 5}{5.25 - 5} \times 100$

$= \dfrac{0.5}{0.25} \times 100 = 200°\text{C}$

27. Density of copper,

$\rho = 9 \times 10^{3}\ \text{kg/m}^3 = 9 \times 10^{6}\text{g/m}^3$

Avogadro number, $N_A = 6.023 \times 10^{23}$

Mass of 1 mole of copper atom, $M = 63.5$ g

Thus, number of free electrons per volume is

$n = \dfrac{N_A}{M}\rho = \dfrac{6.023 \times 10^{23}}{63.5} \times 9 \times 10^{6}$

$= 8.5 \times 10^{28}\,\text{m}^{-3}$

28. Number of electrons added to the conductor

$n = \dfrac{q}{e}$

$= \dfrac{-3 \times 10^{-7}}{1.6 \times 10^{-19}} = 1.8 \times 10^{12}$

Mass increase of the conductor

$= 1.8 \times 10^{12} \times 9.1 \times 10^{-31}$

$\simeq 20 \times 10^{-19} = 2 \times 10^{-18}$

29. Coulombic force on electron, $F_e = a_e m_e$

Similarly, for proton $F_p = a_p m_p$

Here $\quad F_e = F_p$

$\therefore \quad a_e m_e = a_p m_p$

$\Rightarrow \quad a_p = \dfrac{a_e m_e}{m_p}$

$= \dfrac{(2.5 \times 10^{22})(9.1 \times 10^{-31})}{1.6 \times 10^{-27}}$

$= 1.42 \times 10^{19}$

$\approx 1.5 \times 10^{19}\ \text{m/s}^2$

30. As the electron is moving in a straight line and starts from rest.

Thus, from $s = ut + \frac{1}{2}at^2$

we get $\quad h = (0)t + \frac{1}{2}a_e t^2$

$t = \sqrt{\dfrac{2h}{a_e}}$

$= \sqrt{\dfrac{2hm_e}{eE}} \qquad \left[a_e = \dfrac{eE}{m_e}\right]$

$= \sqrt{\dfrac{2 \times 1.5 \times 10^{-2} \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \times 2 \times 10^{4}}}$

$= 2.9 \times 10^{-9}$ s

31. From Coulomb's law

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

$\Rightarrow \quad \left[\dfrac{1}{4\pi\varepsilon_0}\right] = \dfrac{[F \times r^2]}{[q]^2}$

$= \dfrac{[\text{newton}]\,[\text{metre}]^2}{[\text{coulomb}]^2}$

$= \text{Nm}^2\text{C}^{-2}$

32. Pressure at depth h

$= p_a + \rho g h$

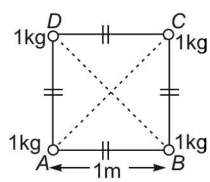
where p_a is atmospheric pressure

$= 1.01 \times 10^{5}\ \text{N/m}^2$

$\therefore \quad p_{\text{total}} = 1.01 \times 10^{5} + 10^{3} \times 10 \times 20$

$= 3.01 \times 10^{5}\,\text{Pa} = 3 \text{ atm}$

33. Here, $AB = BC = CD = DA = 1$ m



$BD = AC$

$= \sqrt{1^2 + 1^2}$

$= \sqrt{2}$ m

Total potential energy

$$+\left[\frac{-G\times1\times1}{DA}\right]+\left[\frac{-G\times1\times1}{BD}\right]+\left[\frac{-G\times1\times1}{AC}\right]$$

$$=4\times\left[\frac{-G\times1\times1}{1}\right]+2\left[\frac{-G\times1\times1}{\sqrt{2}}\right]=-5.4\,G$$

34. Equations of motion are

$m_1a=T-m_1g$

and $m_2a=m_2g-T$

$\Rightarrow$ $8a=T-8g$ ...(i)

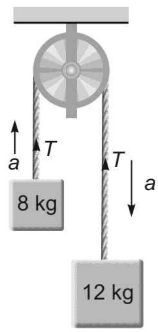
$12a=12g-T$ ... (ii)

From Eqs. (i) and (ii), we get

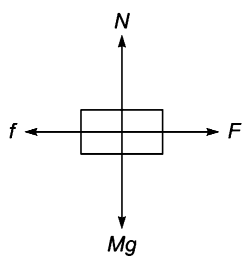
$$a=\frac{g}{5}=2\text{ m/s}^2$$

Substituting the value of a in Eq. (i)

we get $T=96$ N



35. The various forces acting on the block are as shown



As the truck moves in forward direction with acceleration 2 m/s^2, the box experiences a force F in backward direction,

$$F=ma$$

$=40\times2=80$ N in backward direction

Its motion will be opposed by force of friction

$$f=\mu N=\mu mg=0.15\times40\times10=60\text{ N}$$

The acceleration of the box relative to the truck toward the rear end is

$$a=\frac{F-f}{m}=\frac{80-60}{40}=0.5\text{ m/s}^2$$

It t be the time taken by box to fall off the truck

$$s=ut+\frac{1}{2}at^2$$

$$5=0+\frac{1}{2}\times0.5\times t^2$$

$$t=\sqrt{20}\text{ s}$$

During this time the distance covered by truck

$$x=0\times t+\frac{1}{2}\times2\times(\sqrt{20})^2$$

$$=20\text{ m}$$

36. Given, $\vec{\mathbf{r}}=3t\,\hat{\mathbf{i}}-t^2\,\hat{\mathbf{j}}+4\,\hat{\mathbf{k}}$

$$\therefore \quad \frac{d\vec{\mathbf{r}}}{dt}=\vec{\mathbf{v}}=3\hat{\mathbf{i}}-2t\,\hat{\mathbf{j}}$$

At $t=5$ s

$$\vec{\mathbf{v}}=3\hat{\mathbf{i}}-10\,\hat{\mathbf{j}}$$

$$v=\sqrt{(3)^2+(10)^2}=\sqrt{109}=10.44\text{ m/s}$$

37. Average kinetic energy of gas molecules

$$=\frac{3}{2}\times k_B\times T$$

$$KE_{av}=\frac{3}{2}\times1.38\times10^{-23}\times300=6.21\times10^{-21}$$

$$=\frac{6.21\times10^{-21}\text{ eV}}{1.6\times10^{-19}}=3.8\times10^{-2}$$

$$=0.038\text{ eV}$$

38. Centripetal acceleration $a_c=\dfrac{F_m}{m_p}$

$$=\frac{Bqv}{m_p}$$

$$=\frac{1.2\times10^{-3}\times1.6\times10^{-19}\times3\times10^7}{1.67\times10^{-27}}$$

$$=3.6\times10^{12}\text{ m/s}$$

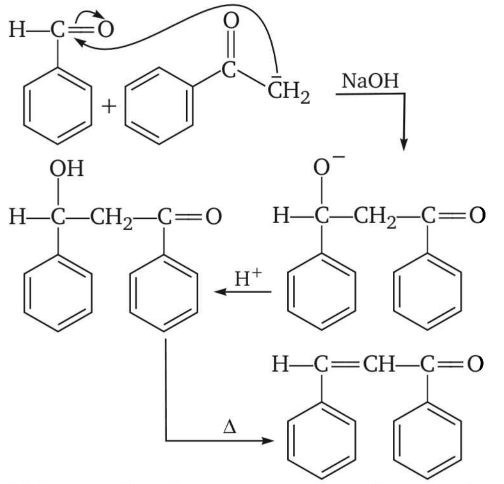
39. Moment of inertia of long uniform thin rod about an axis passing through its edge and perpendicular to the rod is $\dfrac{ML^2}{3}$.

40. Work done = area enclosed by the cycle consider cycle to be an ellipise,

$$\therefore \text{ Area}=\pi\left[\frac{p_2-p_1}{2}\right]\left[\frac{V_2-V_1}{2}\right]$$

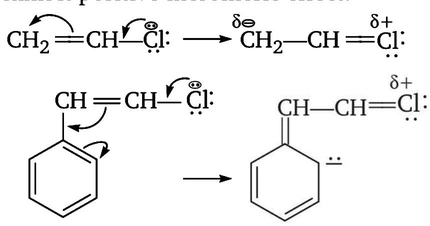
$$=\frac{\pi}{4}(p_2-p_1)(V_2-V_1)$$

Chemistry

1. 

This reaction is an example of cross-aldol condensation.

2. Compound in which lone pair of electrons are in conjugation with an unsaturated system, exhibit positive mesomeric effect.

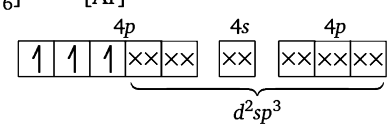


Thus, $CH_2{=}CH{-}Cl$ and $C_6H_5CH{=}CHCl$ both exhibit positive mesomeric effect.

3. (a) In $[Cr(NH_3)_6]^{3+}$, Cr is present as Cr^{3+}.

$Cr^{3+} = [Ar]\ 3d^3$

$[Cr(NH_3)_6]^{3+} = [Ar]$

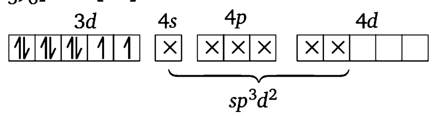


Since, $(n-1)d$ orbitals are used for hybridisation, it is an inner orbital complex.

(b) In $[Ni(NH_3)_6]^{2+}$, Ni is present as Ni^{2+}.

$Ni^{2+} = [Ar]\ 3d^8 4s^0$

$[Ni(NH_3)_6]^{2+} = [Ar]$

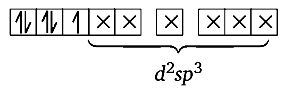


Since outer d (*ie, nd*) orbitals are used, it is an outer orbital complex.

(c) In $[Fe(CN)_6]^{3-}$, Fe is present as Fe^{3+}

$Fe^{3+} = [Ar]\ 3d^5$

$[Fe(CN)_6]^{3-} = [Ar]$

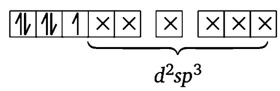


It is also an inner orbital complex.

(d) In $[Mn(CN)_6]^{4-}$, Mn is present as Mn^{2+}.

$Mn^{2+} = [Ar]\ 3d^5 4s^0$

$[Mn(CN)_6]^{4-} = [Ar]$



It is also an inner orbital complex.

4. Eq. weight of metal

$$= \frac{\text{weight of metal}}{\text{weight of hydrogen displaced}} \times 1.008$$

$$\text{weight of hydrogen liberated} = \frac{112}{22400} \times 2.016$$
$$= 1.008 \times 10^{-2}$$

$$\therefore \text{Eq. weight of metal} = \frac{0.32}{1.008 \times 10^{-2}} \times 1.008$$
$$= 32\,\text{g equiv}^{-1}$$

5. From the Faraday's first law, weight of Cu deposited,

$$w = \frac{63.5}{2 \times 96500} \times 0.5 \times 30 \times 60$$
$$= \frac{63.5 \times 0.5 \times 1800}{2 \times 96500}$$
$$= 0.296\ \text{g}$$

6. Sucrose is a carbohydrate and thus is a natural sweetener.

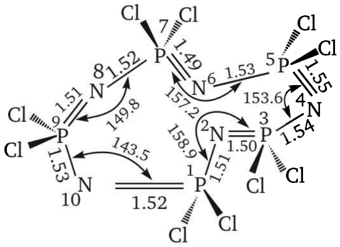
7. Gold number is the measure of protective power of lyophilic colloid. [Higher the value of gold number, lower will be the protective power.]

8. AgCl readily dissolves in excess of ammonia due to the formation of soluble complex $[Ag(NH_3)_2]Cl$.

9. Neil Bartlett discovered the first noble gas compound which is $XePtF_6$. He prepared it by the interaction of deep red vapours of PtF_6 and xenon at room temperature.

10. As the atomic/ionic size increases, hydration enthalpy decreases. On moving downwards in a group, since ionic/atomic radii increases, thus hydration enthalpy decreases. Hence, among alkali metals Li has the highest hydration enthalpy while Rb has the highest ionic radius.

11. $(NPCl_2)_4$ is a tetramer, not a polymer and has the following structure.



Cyclic structure of $(NPCl_2)_4$

12. $KClO_4$ being a neutral salt does not affect the nature of solution.

$$[HClO_4] = \frac{2.0 \times 10^{-3}}{2} = 1.0 \times 10^{-3} = [H^+]$$

$$pH = -\log[H^+] = -\log 1.0 \times 10^{-3}$$

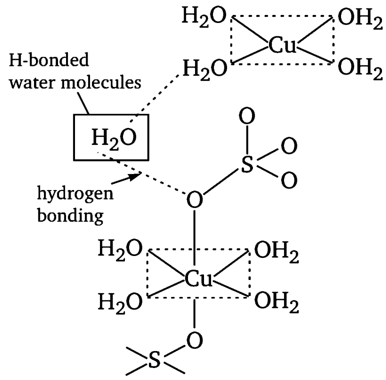
$$= 3$$

13. Boron because of the absence of *d* orbitals, can not extend its covalency up to 6. Its maximum covalency is four. Thus, BF_6^{3-} does not exist.

14. Sea weeds, caliche or crude chile salt petre are some important sources of iodine. Sea weeds of laminaria variety are generally used to extract iodine as their ash, called kelp, contains 0.4 –1.3% iodine as iodides.

15. The structure of complex $CuSO_4 \cdot 5H_2O$ or $Cu(H_2O)SO_4 4H_2O$ is as

In crystalline form, four water molecules are coordinated with Cu atom forming a square-planar geometry and the two O atoms of sulphate ion complete the distorted octahedron. The fifth water molecule is attached through H-bonding between one of the coordinated H_2O molecule and one of the sulphate ion.



16. In in 1 mole of $(NH_4)_2Cr_2O_7$, the number of molecules $= 6.023 \times 10^{23}$

$\therefore$ In 1 mole $(NH_4)_2Cr_2O_7$, the total number of atoms of all elements present

$$= 6.023 \times 10^{23} \times 19$$

$$= 114.43 \times 10^{23}$$

17. $Fe^{3+} = [Ar]\, 3d^5$

($\because$ CN^- being a strong field ligand causes pairing)

$\therefore$ Magnetic moment, $\mu = \sqrt{n(n+2)}$ BM

(where, n = number of unpaired electrons)

$$= \sqrt{1(1+2)} = \sqrt{3} = 1.73 \text{ B M}$$

18. The electronic configuration of potassium, K is

$$_{19}K = [Ar]\, 4s^1$$

The 19th electron enters in 4*s*-orbital.

For $4s^1$

$$n = 4$$

$$l = 0 \quad (\because \text{for } s \text{ orbital}, l = 0)$$

$$m = 0$$

$$s = +\frac{1}{2}$$

Thus, the values of quantum number for 19th electron of K will be 4, 0, 0, 1/2.

19. Uracil, a pyrimidine base is present only in RNA but not in DNA.

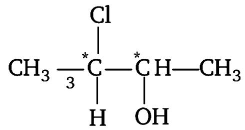
20. When concentrated sulphuric acid is added to potassium permanganate, a mild explosive manganese heptoxide is obtained.

$$2KMnO_4 + H_2SO_4 \rightarrow Mn_2O_7 + K_2SO_4 + H_2O$$

Note Mn_2O_7 is a powerful oxidising agent and combustible substance. Wood, cotton catches fire readily when comes in contact of it.

21. *trans*-chlorocarbonyl *bis* (triphenylphosphine) iridium (I) *ie, trans*-IrCl (CO) $(PPh_3)_2$ is called Vaska's compound.

22. The structure of 3-chlorobutan-2-ol is



(C^* = chiral carbon atom)

Since, 3-chlorobutane-2-ol contains 2 asymmetric (chiral) carbon atoms, thus, the number of optical isomers = $2^n = 2^2 = 4$

23. (a) Number of molecules in 11.2 L of O_2

$$= \frac{11.2}{22.4} \times 6.023 \times 10^{23}$$

$$= 0.5 \times N_A$$

(b) Number of molecules in 8.0 g of O_2

$$= \frac{8.0}{32} \times 6.023 \times 10^{23}$$

$$= 0.25 \times N_A$$

(c) Number of molecules in 0.1 mole of O_2

$$= 0.1 \times 6.023 \times 10^{23}$$

$$= 0.1 \times N_A$$

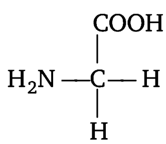
(d) Number of molecules in 2.24×10^4 mL of O_2

$$= \frac{2.24 \times 10^4}{22400} \times 6.023 \times 10^{23}$$

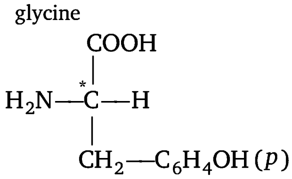
$$= N_A$$

Thus, 0.1 mole of O_2 contains smallest number of molecules.

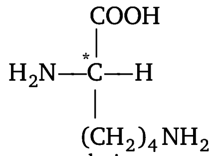
24. The structures of the given amino acids are as

(a) 

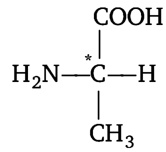
glycine

(b) 

tyrosine

(c) 

lysine

(d) 

alanine

(C^* = asymmetric carbon atom)

Thus, glycine because of the absence of asymmetric carbon atom is optically inactive.

25. Vitamin A, D, E and K are fat soluble while remaining are water soluble.

26. (a) $XeF_6 \Rightarrow 6bp + 1lp$, thus distorted octahedral geometry.

(b) $XeO_3 \Rightarrow 3bp + 1lp$, thus pyramidal geometry.

(c) $BrF_5 \Rightarrow 5bp + 1lp$, thus square pyramidal geometry.

(d) $XeF_4 \Rightarrow 4bp + 2lp$ thus square planar geometry.

27. Crystal field splitting depends upon the field produced by the ligand and charge of the metal ion.

In all the given complexes, charge of metal ion *ie,* Co is +2, thus, crystal field splitting depends only upon the strength of ligand.

The order of field strength of the ligands is

$$Cl^- < H_2O < \text{en} < CN^-$$

Thus, $[CoCl_6]^{4-}$ will have the weakest crystal field splitting.

28. According to Graham's law,

rate of diffusion, $r \propto \frac{1}{\sqrt{M}}$

$$\frac{r_{HCl}}{r_{NH_3}} = \sqrt{\frac{M_{NH_3}}{M_{HCl}}}$$

$$= \sqrt{\frac{17}{36.5}}$$

$$= \sqrt{\frac{1}{2.15}}$$

$$= 1 : 1.46$$

29. $[H^+] = \sqrt{K_a \cdot C} = \alpha \cdot C \; [\because K_a = \alpha^2 \cdot C]$

$$= \frac{10}{100} \times 10^{-3}$$

$$= 10^{-4}$$

$$pH = -\log[H^+]$$

$$= -\log 10^{-4}$$

$$= +4$$

30. Carnallite, $KCl \cdot MgCl_2 \cdot 6H_2O$ is a double salt and is a major source of magnesium.

31. Calcium phosphide, when reacts with water, produces phosphine (PH_3).

$$Ca_3P_2 + 6H_2O \longrightarrow 3Ca(OH)_2 + 2PH_3$$

32. ZnO, on heating, loses oxygen and thus, the zinc metal becomes in excess in ZnO crystal. Hence, the formula of ZnO becomes $Zn_{1+x}O$. The excess Zn^{2+} ions move to interstitial sites and the electrons to neighbouring interstitial sites. The electrons are responsible for the yellow colour of ZnO.

$$ZnO \xrightarrow{\Delta} Zn^{2+} + \frac{1}{2}O_2 + 2e^-$$

Thus, yellow colour of ZnO is due to metal excess defect.

33. A mixture of Al and Fe_2O_3 in the ratio of 1 : 3 is called thermite and is used in thermite welding.

Thus, $2Al + Fe_2O_3 \longrightarrow Al_2O_3 + 2Fe$ is used in thermite welding.

34. The catenation power depends upon the strength of element-element bond. On moving down a group of the periodic table, size in increases. Due to which element-element bond strength and thus, the catenation power decreases. Thus, the order of catenation power of given elements is

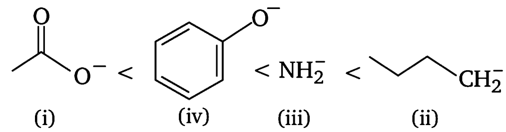
$$C > Si > Ge > Sn$$

35. $$\underset{\text{grignard reagent}}{CH_3MgBr} + O{=}S{=}O \longrightarrow BrMgO{-}\underset{\underset{CH_3}{|}}{S}{=}O \xrightarrow[-BrMgOH]{H_2O} \underset{\text{methane sulphinic acid}}{CH_3SO(OH)}$$

36. According to Jahn-Teller theorem, any non-linear molecular system in a degenerate electronic state is unstable and readily undergoes distortion to give a system of lower symmetry and lower energy by removing the degeneracy. This effect is more important for odd number occupancy of the *eg*. level. Thus, $t_{2g}^6 e_g^1$ readily undergoes distortion.

37. As the electronegativity of electron donating atom decreases, base strength increases.

The order of electronegativity is

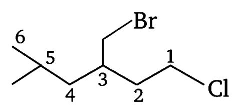


$$C < N < O$$

But CH_3COO^- and $C_6H_5O^-$ both have O as donor atom, thus their basicity is decided by resonance. CH_3COO^- being more resonance stabilised, is less basic. Thus, the order of base strength of the given compounds is

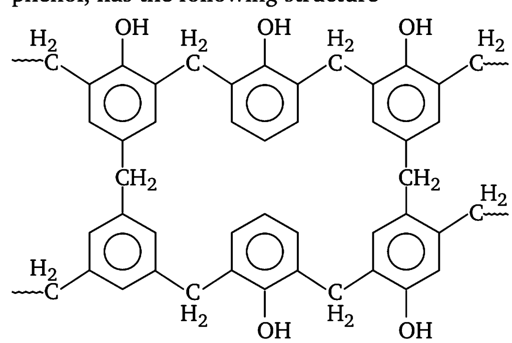
38. Symmetrical molecules have zero dipole moment. Thus, dipole moment of $CH_2{=}CH_2$ is zero.

39.



3-bromomethyl-1-chloro-5-methylhexane

40. Bakelite, a condensation polymer of HCHO and phenol, has the following structure



Thus, the rings are joined together through $-CH_2$ units.

Mathematics

1. Given that, $A^2 = A$

Then, $(I - A)^3 + A$

$= (I)^3 + (-A)^3 + 3(I)(-A)^2 + 3(-A)(I)^2 + A$

$= I - (A)^3 + 3A^2 - 3A + A$

$\because I^3 = I, IA = A$

$= I - A(A)^2 + 3(A)^2 - 3A + A$

$= I - A(A) + 3A - 3A + A \qquad (\because A^2 = A)$

$= I - A^2 + A$

$= (I - A) + A \qquad (\because A^2 = A)$

2. Given system of equation is

$$x + 2y + 3z = 1$$
$$2x + y + 3z = 2$$
$$5x + 5y + 9z = 4$$

The augmented matrix

$$[A : B] = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 1 & 3 & 2 \\ 5 & 5 & 9 & 4 \end{bmatrix}$$

Apply $\begin{cases} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 5R_1 \end{cases} \sim \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -3 & -3 & 0 \\ 0 & -5 & -6 & -1 \end{bmatrix}$

Apply $R_3 \to R_3 - \frac{5}{3}R_2 \sim \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -3 & -3 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix}$

Here, rank of $[A : B]$ = rank of $[A]$

So, the system is consistent.

But, rank of $[A]$ = number of unknowns.

Hence, the system have only one solution.

3. $f = \{(1, 1), (2, 4), (0, -2), (-1, -5)\}$

Let the linear function is,

$y = mx + c \qquad \ldots(i)$

Then, at (1, 1), $\quad 1 = m + c \qquad \ldots(ii)$

at $(0, -2)$, $-2 = c$,

then $m = 3$

Hence, the linear expression becomes

$$y = f(x) = 3x - 2$$

4. $f : R - \left\{\frac{5}{4}\right\} \to R$ and $f(x) = \frac{5x}{4x + 5}$

Let $\quad y = \frac{5x}{4x + 5}$

$\Rightarrow \quad 4xy + 5y = 5x \Rightarrow \quad x(5 - 4y) = 5y$

$\Rightarrow \quad x = \frac{5y}{(5 - 4y)} = f^{-1}(y)$

$\Rightarrow f^{-1}(x) = \frac{5x}{5 - 4x}$ or $g(y) = \frac{5y}{5 - 4y}$

which is the inverse of f is the map g.

Range $f \to R - \left\{\frac{5}{4}\right\}$.

5. Given that,

* be a binary operation on the set of rational numbers

and $\quad a * b = \frac{ab}{4}$

By inspection, we get '4' is the identity elements with respect to '*' operation.

Let $\quad a * e = a = e * a$,

where is identity element.

$\therefore \quad a * e = \frac{ae}{4} = a$

$\Rightarrow \quad ae - 4a = 0 \Rightarrow \quad a(e - 4) = 0 \Rightarrow e = 4 \neq 0$

$O \notin Q$

Hence, identity element $(e) = 4$

6. We have $\quad A = \{1, 0, 1, 2\}$

$B = \{4, 2, 0, 2\}$

and $\quad f, g : A \to B$

Now, $f(x) = x^2 - x \Rightarrow \quad f(0) = 0 - 0 = 0$

and $\quad f(1) = 1 - 1 = 0 \qquad \ldots(i)$

and $\quad f(2) = 2^2 - 2 = 2$

Also, $\quad g(x) = 2\left|x - \frac{1}{2}\right| - 1$

$\Rightarrow \quad g(0) = 2\left|\frac{-1}{2}\right| - 1 = 1 - 1 = 0$

and $\quad g(1) = 2\left|1 - \frac{1}{2}\right| - 1 = 1 - 1 = 0 \qquad \ldots(ii)$

and $\quad g(2) = 2\left|2 - \frac{1}{2}\right| - 1 = \frac{2\,3}{2} - 1 = 2$

Hence from equation (i) and (ii), we can say that $f(x) = g(x)$

$\Rightarrow \quad f = g$

7. $z = \begin{vmatrix} 2 & 3 + i & -3 \\ 3 - i & 0 & -1 + i \\ -3 & -1 - i & 4 \end{vmatrix}$

Expand w.r.t. 'R_1'

$= 2\{(1+i)(i-1)\} - (3+i)\{4(3-i) + 3(i-1)\}$
$+ 3\{(1+i)(3-i)\}$

$= 2\{(i^2 - 1)\} - (3+i)\{12 - 4i + 3i - 3\}$
$+ 3\{3 + 3i - i - i^2\}$

$= 2(-1-1) - (3+i)(-i+9) + 3(2i+4)$

$\Rightarrow \quad -4 - \{-3i + 27 - i^2 + 9i\} + 6i + 12$

$\Rightarrow \quad -4 - 6i - 28 + 6i + 12 = -20$ (Purely Real)

8. $\text{Re}(z^2) = 0, |z| = 2$

Let $z = x + iy$

$z^2 = (x+iy)^2 = x^2 + i^2y^2 + 2ixy$

$z^2 = (x^2 - y^2) + i(2xy)$

$\Rightarrow \text{Re}(z^2) = \text{Re}\{(x^2 - y^2) + i(2xy)\}$

$\text{Re}(z^2) = x^2 - y^2 \quad \ldots\text{(i)}$

$|z| = \sqrt{x^2 + y^2} = 2$

$\Rightarrow \quad x^2 + y^2 = 4 \quad \ldots\text{(ii)}$

Given, $\text{Re}(z^2) = 0$

$x^2 - y^2 = 0 \quad \ldots\text{(iii)}$

On adding Eqs. (ii) and (iii)

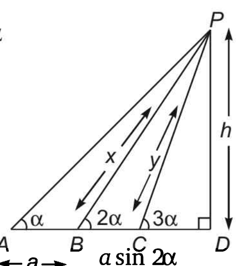
$2x^2 = 4 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

and $y = \pm\sqrt{2}$

Hence, $z = \pm\sqrt{2}(1+i)$

9. In ΔAPB, $\angle ABP = \pi - 2\alpha$

$\angle APB = \alpha$



In ΔBCP, $\angle BCP = \pi - 3\alpha$

$\angle BPC = \alpha$

By sine Law in ΔABP

$$\frac{a}{\sin\alpha} = \frac{x}{\sin\alpha}$$

$\Rightarrow x = a$

Again, sine law in ΔBPC

$$\frac{x}{\sin(\pi - 3\alpha)} = \frac{y}{\sin 2\alpha} \Rightarrow y = \frac{a\sin 2\alpha}{\sin 3\alpha}$$

Again sine law in ΔPCD,

$$\frac{h}{\sin 3\alpha} = \frac{y}{\sin 90^\circ}$$

$$\Rightarrow \quad h = y\cdot\sin 3\alpha = \frac{a\sin 2\alpha}{\sin 3\alpha}\cdot\sin 3\alpha$$

$\Rightarrow \quad h = a\sin 2\alpha$

10. Given, $\tan x + \sec x = 2\cos x$

$$\frac{(1+\sin x)}{\cos x} = 2\cos x$$

$1 + \sin x = 2\cos^2 x$

$1 + \sin x = 2(1 - \sin^2 x)$

$1 + \sin x = 2 - 2\sin^2 x$

$2\sin^2 x + \sin x - 1 = 0$

$2\sin^2 x + 2\sin x - \sin x - 1 = 0$

$2\sin x(\sin x + 1) - 1(\sin x + 1) = 0$

$(\sin x + 1)(2\sin x - 1) = 0$

$\sin x = -1, 1/2$

$\sin x = \sin 270^\circ, \sin 30^\circ, \sin 150^\circ$

$\Rightarrow \quad x = \pi/6, 5\pi/6, 3\pi/2$

Hence there are three solutions of given equation in $[0, 2\pi]$

11. Given, $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

$\Rightarrow \quad 2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

$$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1 - \cos^2 x}\right) = \tan^{-1}(2\operatorname{cosec} x)$$

$$\Rightarrow \quad \frac{2\cos x}{\sin^2 x} = \frac{2}{\sin x}$$

$\Rightarrow \quad \sin^2 x - \sin x\cdot\cos x = 0$

$\Rightarrow \quad \sin x(\sin x - \cos x) = 0$

$\Rightarrow \quad \sin x = 0$ or $\tan x = 1$

$\Rightarrow \quad x = 0$ or $x = \pi/4$

Here, $x = \pi/4$ satisfies the given equation.

So, $x \neq 0$

12. $\tan^{-1}(4x) + \tan^{-1}(6x) = \pi/4$

$$\Rightarrow \quad \tan^{-1}\left(\frac{4x + 6x}{1 - 24x^2}\right) = \pi/4$$

$$\Rightarrow \quad \frac{10x}{1 - 24x^2} = \tan(\pi/4) = 1$$

$\Rightarrow \quad x\cdot 10 = 1 - 24x^2$

$\Rightarrow \quad 24x^2 + 10x - 1 = 0$

$\Rightarrow \quad 24x^2 + 12x - 2x - 1 = 0$

$\Rightarrow \quad 12x(2x+1) - 1(2x+1) = 0$

$\Rightarrow \quad (2x+1)(12x-1) = 0$

$\Rightarrow \quad x = -1/2$ or $1/12$

13. Let the longest side of triangle is a.

The shortest side of triangle is c.

And the third side of triangle is b.

Then, according to question

$a = 5c$ and $b = 5c - 50$

Given, perimeter of triangle = 60

$\Rightarrow \quad a+b+c=60$

$\Rightarrow \quad 5c+5c-50+c=60$

$\Rightarrow \quad 11c=110 \Rightarrow \quad c=10$

Hence, the minimum length of the shortest side of triangle = 10 cm.

14. Given, $2 \le r \le n$,

$\because$ We know that, ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$${}^nC_r + 2\cdot{}^nC_{r-1} + {}^nC_{r-2}$$

Now, $({}^nC_r + {}^nC_{r-1}) + ({}^nC_{r-1} + {}^nC_{r-2})$

$$= ({}^{n+1}C_r + {}^{n+1}C_{r-1}) = {}^{n+2}C_r$$

15. Coefficient of $(r-1)$th term in $(1+x)^n = {}^nC_{r-2}$

Coefficient of rth term in $(1+x)^n = {}^nC_{r-1}$

Coefficient of $(r+1)$th term in $(1+x)^n = {}^nC_r$

Given, $\frac{{}^nC_{r-2}}{{}^nC_{r-1}} = \frac{1}{3} \Rightarrow 3\cdot{}^nC_{r-2} = {}^nC_{r-1}$

$\Rightarrow \quad 3\cdot\frac{n!}{r-2!\,n-r+2!} = \frac{n!}{r-1!\,n-r+1!}$

$\Rightarrow \quad \frac{3}{(n-r+2)} = \frac{1}{(r-1)}$

$\Rightarrow \quad 3r-3 = n-r+2$

$\Rightarrow \quad 4r-n=5 \quad \ldots(i)$

and $\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{3}{5} \Rightarrow 5\cdot{}^nC_{r-1} = 3\cdot{}^nC_r$

$5\cdot\frac{n!}{r-1!\,n-r+1!} = 3\cdot\frac{n!}{r!\,n-r!}$

$\Rightarrow \quad \frac{5}{(n-r+1)} = \frac{3}{r}$

$\Rightarrow \quad 5r = 3n-3r+3$

$\Rightarrow \quad 8x-3n=3 \quad \ldots(ii)$

Eq. (i) $\times$ 3 – Eq.(ii).

$$\begin{array}{r} 12r-3n=15 \\ -\,8r-3n=3 \\ \hline 4r=12 \end{array}$$

$(r=3)$

from (i) $\quad 12-n=5$

$(n=7)$

Hence, $(n, r) = (7, 3)$ option (b)

16. Given that

$S_1 = a_2 + a_4 + a_6 + \ldots +$ upto 100 terms $\quad \ldots$(i)

$S_2 = a_1 + a_3 + a_5 + \ldots +$ upto 100 terms $\quad \ldots$(ii)

Let d be the common difference of their AP then

$d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots \quad \ldots$(iii)

Now, subtracting equation (ii) from Eq. (i), we get

$S_1 - S_2 = (a_2 - a_1) + (a_4 - a_3) + (a_6 - a_5) + \ldots$ upto 100 terms

$\Rightarrow \quad S_1 - S_2 = d + d + d + \ldots$ 100 time

using equation (iii)

$\Rightarrow \quad S_1 - S_2 = 100d \Rightarrow d = \frac{S_1 - S_2}{100}$

$\therefore$ option (d) is correct

17. Given, $\log_{10} 2, \log_{10}(2^x-1), \log_{10}(2^x+3)$ are in AP.

Then , $\log_{10}(2^x-1) - \log_{10} 2$

$$= \log_{10}(2^x+3) - \log_{10}(2^x-1)$$

$$\log_{10}\left(\frac{2^x-1}{2}\right) = \log_{10}\left(\frac{2^x+3}{2^x-1}\right)$$

$$(2^x-1)^2 = 2(2^x+3)$$

$$2^{2x}+1-2\cdot2^x = 2\cdot2^x+6$$

$$2^{2x}-4.2x-5=0$$

$$2^{2x}-5\cdot2^x+2^x-5=0$$

$$2^x(2^x-5)+1(2^x-5)=0$$

$$(2^x+1)(2^x-5)=0$$

$\Rightarrow \quad 2^x = 5 \quad (\because 2^x \ne -1)$

$\Rightarrow \quad x = \log_2 5$

18. In a GP, $t_2 + t_5 = 216 \quad \ldots$(i)

$t_4 : t_6 = 1:4 \quad \ldots$(ii)

Let the first term of GP = a

and common ratio = r

From Eq. (ii),

$ar^3 : ar^5 = 1:4 \Rightarrow 1:r^2 = 1:4 \Rightarrow r=2$

From Eq. (i),

$$ar + ar^4 = 216$$

$\Rightarrow \quad a(2+16) = 216$

$\Rightarrow \quad a\cdot18 = 216 \Rightarrow \quad a=12$

19. We have, $(a^2+b^2+c^2)p^2 - 2p(ab+bc+cd) + (b^2+c^2+d^2) \le 0$

$\Rightarrow \quad (ap-b)^2 + (bp-c)^2 + (cp-d)^2 \le 0, \quad \ldots$(i)

but $\quad (ap-b)^2 + (bp-c)^2 + (cp-d)^2 \ge 0 \quad \ldots$(ii)

($\because$ sum of squares can't be ≤ 0)

$\therefore$ From Eqs. (i) and (ii),

$$(ap-b)^2 + (bp-c)^2 + (cp-d)^2 = 0$$

$\Rightarrow \quad ap-b = 0 = bp-c = cp-d$

$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c} = p \Rightarrow a, b, c, d$ are in GP.

20. Given series $a, ar, ar^2, \ldots, ar^{n-1}$

Then, arithmatic mean of first two terms

$$A = \frac{a + ar}{2} \Rightarrow A = \frac{a(1+r)}{2}$$

Harmonic mean of first two terms

$$H = \frac{2 \cdot a \cdot ar}{a + ar} = \frac{2ar}{(1+r)}$$

and geometric mean of first two terms

$$G = \sqrt{a \cdot ar}$$

$$G^2 = a^2 \cdot r \quad \ldots(i)$$

Now, $$AH = \frac{a(1+r)}{2} \cdot \frac{2ar}{(1+r)} = a^2 r \quad \ldots(ii)$$

From Eqs. (i) and (ii),

$$G^2 = A \cdot H$$

21. The given cubic Equation is

$$x^3 - px^2 + qx - r = 0$$

Given condition, two roots are equal, but opposite in sign.

ie, $(\alpha, -\alpha, \beta)$

Sum of roots $= \alpha - \alpha + \beta = p \Rightarrow \beta = p$

'β' is the roots of given cubic equation, so it satisfieds

$$p^3 - p^3 + Pq - r = 0 \Rightarrow pq = r$$

22. Let thelinear function in c is

$$L = ac + b \quad \ldots(i)$$

when, $L = 124.942$ and $c = 20$

$$124.942 = 20a + b \quad \ldots(ii)$$

when, $L = 125.134$ and $c = 110$

$$125.134 = 110.\, a + b \quad \ldots(iii)$$

Eq. (iii) and Eqs. (ii)

$$0.192 = 90.\, a$$

$$\left\{a = \frac{0.192}{90}\right\}$$

from Eqs. (iii)

$$b = 125.1345 - \frac{(0.192)}{9} \times 11$$

$$\{b = 124.942\}$$

Hence, $$\left\{L = \frac{0.192}{90} \times C + 124.942\right\}$$

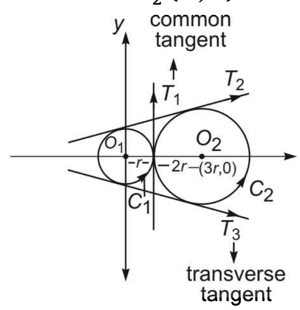
this expression satisfied option (a)

ie, $$L = \frac{0.192}{90} \times (C - 20) + 124.942$$

At $C = 20$ then $L = 124.942$

23. O_1 (0, 0) = centre of circle

O_2 (3r, 0)



$$O_1O_2 = \sqrt{(3r - 0)^2 + (0 - 0)^2} = 3r$$

Also, we have $r_1 = r \Rightarrow r_2 = 2r$

So, $O_1O_2 = r_1 + r_2 \Rightarrow O_1O_2 = r_1 + r_2$

Hence, there are three common tangents of two circles C_1 and C_2.

24. Given, $f(x, y) = 0$ is equation of circle.

Let $f(x, y) \equiv x^2 + y^2 + 2gx + 2fy + c = 0 \quad \ldots(i)$

Also, given $f(0, \lambda) = 0$

has roots $\lambda = 1, 1$

Then, $f(0, \lambda) = \lambda^2 + 2f\lambda + c = 0$

Put $\lambda = 1$, $\quad 2f + c = -1 \quad \ldots(ii)$

and given $f(\lambda, 0) = 0$ have roots $\left(\frac{1}{2}, 2\right)$.

Then, $\quad \lambda^2 + 2g\lambda + c = 0$

Put $\quad \lambda = 2,\ 4g + c = -4 \quad \ldots(iii)$

Put $\quad \lambda = 1/2;\ g + c = -1/4 \quad \ldots(iv)$

Solve Eqs. (iii) and (iv), we get

$$g = -5/4 \text{ and } c = 1$$

From Eq. (ii), $f = -1$

Hence, centre of circle $f(x, y)$ is $\Rightarrow (-g, -f)$ $= (5/4, 1)$

25. Given line, $x + y = 6$ and parabola $y^2 = 8x$

Now, differentiating the prabola

$$2y\frac{dy}{dx} = 8 \Rightarrow \frac{dy}{dx} = \frac{4}{y}$$

Now, normal's slope of parabola at (x_1, y_1) is

$$\frac{dx}{dy} = -\frac{y_1}{4}$$

Eq. of normal of parabola at (x_1, y_1) is

$$(y - y_1) = \left(-\frac{y_1}{4}\right)(x - x_1)$$

$$4y - 4y_1 = -xy_1 + x_1y_1$$

$$xy_1 + 4y - (4y_1 + x_1y_1) = 0 \quad \ldots(i)$$

but the line $x + y - 6 = 0$ also, a normal to a parabola. So, both lines are parallel to each other.

Then, $\frac{y_1}{1} = \frac{4}{1} = \frac{4y_1 + x_1y_1}{6}$

Taking first two parts, we get $y_1 = 4$

Taking last two parts $4y_1 + x_1y_1 = 24$

Put $y_1 = 4, 16 + 4x_1 = 24$

$\Rightarrow 4x_1 = 8 \Rightarrow x_1 = 2$

Hence, the line $x + y = 6$ is normal to parabola $y^2 = 8x$ at (2, 4).

26. Given $\vec{a}, \vec{b}, \vec{c}$ vector, every pair is non-collinear.

ie, $\left.\begin{array}{l} \vec{a} \times \vec{b} \neq 0 \\ \vec{b} \times \vec{c} \neq 0 \\ \vec{c} \times \vec{a} \neq 0 \end{array}\right\}$

Now, $(\vec{a} + \vec{b})$ collinear with $\vec{c}$.

Then, $(\vec{a} + \vec{b}) \times \vec{c} = 0$

$\vec{a} \times \vec{c} + \vec{b} \times \vec{c} = 0$

$-\vec{c} \times \vec{a} + \vec{b} \times \vec{c} = 0$ ...(i)

and $(\vec{b} + \vec{c})$ collinear with $\vec{a}$

Then, $(\vec{b} + \vec{c}) \times \vec{a} = 0$

$(\vec{b} \times \vec{a}) + (\vec{c} \times \vec{a}) = 0$ ...(ii)

On adding Eqs. (i) and (ii), we get

$(\vec{b} \times \vec{a}) + (\vec{b} \times \vec{c}) = 0$

$\Rightarrow (\vec{a} \times \vec{b}) + (\vec{c} \times \vec{b}) = 0$

$\Rightarrow (\vec{a} + \vec{c}) \times \vec{b} = 0$

$\Rightarrow (\vec{a} + \vec{b} + \vec{c}) \times \vec{b} - (\vec{b} \times \vec{b}) = 0$ $(\because \vec{b} \times \vec{b} = 0)$

$\Rightarrow (\vec{a} + \vec{b} + \vec{c}) \times \vec{b} = 0$ ...(iii)

Similarly, $(\vec{a} + \vec{b} + \vec{c}) \times \vec{a} = 0$ ...(iv)

$(\vec{a} + \vec{b} + \vec{c}) \times \vec{c} = 0$ ...(v)

From Eqs. (iii), (iv) and (v), we observe that the vector $(\vec{a} + \vec{b} + \vec{c})$ is equally inclined to $\vec{a}, \vec{b}, \vec{c}$.

27. Let the unit vector is $\hat{a} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}} = \frac{\vec{a}}{|\vec{a}|}$

Now, from question, $\hat{a} \cdot \hat{i} = |\hat{a}||\hat{i}| \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{x}{|\vec{a}|} = \frac{1}{\sqrt{2}}$

$\Rightarrow 2x^2 = x^2 + y^2 + z^2$

$\Rightarrow x^2 = y^2 + z^2$ ...(i)

and $\hat{a} \cdot \hat{j} = |\hat{a}||\hat{j}| \cos\frac{\pi}{3} = \frac{1}{2}$

$\Rightarrow \frac{y}{|\vec{a}|} = \frac{1}{2} \Rightarrow 4y^2 = x^2 + y^2 + z^2$

$\Rightarrow 3y^2 = x^2 + z^2$ ...(ii)

and $\hat{a} \cdot \hat{k} = |\hat{a}||\hat{k}| \cos\theta$

$(\because \theta \to$ Acute angle)

$\Rightarrow \frac{z}{|\vec{a}|} = \cos\theta$

$\Rightarrow \frac{z^2}{(x^2 + y^2 + z^2)} = \cos^2\theta$ ...(iii)

From Eqs. (i) and (ii), we get

$x = \pm\sqrt{2}y = \pm\sqrt{2}z$

$\Rightarrow \frac{x^2}{2} = \frac{y^2}{1} = \frac{z^2}{1} = \frac{x^2 + y^2 + z^2}{4}$,

$\frac{z^2}{x^2 + y^2 + z^2} = \cos^2\theta = \frac{1}{4}$

$\Rightarrow \theta = \pi/3$

and $\hat{a} = (\sqrt{2}\hat{i} + \hat{j} + \hat{k})/2$

28. If $\vec{a} = \hat{i} - \hat{k}, \vec{b} = x\hat{i} + \hat{j} + (1 - x)\hat{k}$

$\vec{c} = y\hat{i} + x\hat{j} + (1 + x - y)\hat{k}$

Then, $[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1 - x \\ y & x & (1 + x - y) \end{vmatrix}$

Expand w.r.t. 'R_1'

$= (1 + x - y) - x(1 - x) - (x^2 - y)$

$= 1 + x - y - x + x^2 - x^2 + y = 1$

which is independent from x and y.

$\Rightarrow$ Depends on neither x nor y.

29. Given, point $A = (-1, -1, 1)$

Its position vector is

$\vec{a} = -\hat{i} - \hat{j} + \hat{k}$

and equation of parallel plane,

$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 0$

Here, $\vec{n} = (\hat{i} + \hat{j} + \hat{k})$,

Then, equation of plane passing through $\vec{a}$ and parallel to $\vec{r} \cdot \vec{n} = 0$ is

$$\vec{\mathbf{r}}\cdot\vec{\mathbf{n}} = \vec{\mathbf{a}}\cdot\vec{\mathbf{n}}$$

$$\vec{\mathbf{r}}\cdot(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = (-\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 1$$

$$\Rightarrow \quad \vec{\mathbf{r}}\cdot(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - 1 = 0$$

30. The given equation of line is

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z-0}{1} = \text{'}r\text{' (say)} \quad \ldots\text{(i)}$$

which passes through the point (1, −1, 0) and having DR's are 2, −3, 1.

Let $(2r+1, -3r-1, r)$ be the point on the line (i) at distance $4\sqrt{14}$ from the point (1, −1, 0), then

$$\{(2r+1)-1\}^2 + \{(-3r-1)+1\}^2 + \{r-0\}^2 = 16\times 14$$

$$(2r)^2 + (-3r)^2 + r^2 = 16\times 14$$

$$4r^2 + 9r^2 + r^2 = 16\times 14$$

$$14r^2 = 16\times 14$$

$$r = 4$$

Hence, the point is {9, −13, 4}.

31. Let the given line segment is divided by xy-plane at point P in the ratio $\lambda : 1$.

Then, section formula

$$P = \left\{\frac{6\lambda+4}{\lambda+1}, \frac{10\lambda+8}{\lambda+1}, \frac{-8\lambda+10}{\lambda+1}\right\}$$

Since, $P = \left\{\frac{6\lambda+4}{\lambda+1}, \frac{10\lambda+8}{\lambda+1}, \frac{-8\lambda+10}{\lambda+1}\right\}$

divided by xy-plane, so $z = 0$

$$\Rightarrow \quad \frac{-8\lambda+10}{\lambda+1} = 0$$

$$\Rightarrow \quad \lambda = \frac{5}{4}$$

Since, 'λ' is positive.

So, the line segment divided by xy-plane in the ratio 5 : 4 internally.

32. $\lim\limits_{x\to 1} \frac{x^{15}-1}{x^{10}-1} \qquad \left(\text{form}\frac{0}{0}\right)$

Use L' Hospital rule

$$\lim_{x\to 1}\frac{15x^4-0}{10x^9-0} = \lim_{x\to 1}\frac{3}{2}x^5$$

$$= 3/2(1)^5 = (3/2)$$

33. Given, $f(x) = \begin{cases} 7 & , \text{ if } x \le 2 \\ ax+b & , \text{ if } 2 < x < 9 \\ 21 & , \text{ if } x \ge 9 \end{cases}$

continuous function.

At $x = 2$,

RHL $f(2+h) = \lim\limits_{h\to 0} a(2+h) + b$
$= 7$

LHL $f(2-h) = \lim\limits_{h\to 0} a(2-h) + b$
$= 2a + b$

$f(2) = 7$

Since, $f(x)$ is continuous at $x = 2$, then

$$f(2) = \text{LHL} = \text{RHL}$$

$$2a + b = 7 \quad \ldots\text{(i)}$$

At $x = 9$,

RHL $f(9+h) = \lim\limits_{h\to 0} 21 = 21$

LHL $f(9-h) = \lim\limits_{h\to 0} a(9-h) + b$
$= 9a + b$

$f(9) = 21$

Since, $f(x)$ is continuous at $x = 9$, then

$$f(a) = \text{LHL} = \text{RHL}$$

$$9a + b = 21 \quad \ldots\text{(ii)}$$

Suibtracting Eq. (ii) from Eqs. (i)

$$7a = 14 \Rightarrow a = 2$$

from Eq. (i), $b = 3$

34. $2^x + 2^y = 2^{x+y}$

On differentiating w.r.t x,

$$2^x \log 2 + 2^y \log 2\frac{dy}{dx} = 2^{(x+y)}\cdot\log 2\left\{1 + \frac{dy}{dx}\right\}$$

$$\Rightarrow \quad 2^x + 2^y\frac{dy}{dx} = 2^{(x+y)} + 2^{(x+y)}\frac{dy}{dx}$$

$$\Rightarrow \quad (2^x - 2^x\cdot 2^y) = (2^x\cdot 2^y - 2^y)\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{(2^x\cdot 2^y - 2^x)}{(2^x\cdot 2^y - 2^y)}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{\text{at }(1,1)} = -1$$

35. Given that,

Velocity, $\frac{dr}{dt} = 6$ cm

Also, given radius $r = 12$ cm

Area = Area of circle (waves)

$$A = \pi r^2$$

On differential thing w.r.t. 't'

$$\frac{dA}{dt} = \frac{d}{dt}(\pi r^2) = \frac{d}{dr}(\pi r^2)\left(\frac{dr}{dt}\right)$$

$$\frac{dA}{dt} = 2\pi r\left(\frac{dr}{dt}\right)$$

$$\frac{dA}{dt} = 2(\pi)(12)(6)$$

$$\frac{dA}{dt} = 144\pi \text{ cm}^2/s$$

Increasing area $= \frac{dA}{dt} = 144\pi \text{ cm}^2/s$

36. $f(x)=\frac{4}{3}x^3-8x^2+16x+5$ ...(i)

There will be a point of inflexion at point $x=2$, if $\left(\frac{d^2y}{dx^2}\right)_{\text{at }(x=2)}=0$, but $\frac{d^3y}{dx^3}\neq 0$

From Eq. (i),

$$f'(x)=4x^2-16x+16$$
$$f''(x)=8x-16$$
$$f''(2)=16-16=0$$

and $f'''(x)=8\neq 0$

Hence, at $x=2$, $f(x)$ shown point of inflexion.

37. $\int e^{x\log a}\cdot e^x dx$

$$I=\int a^x\cdot e^x dx \quad \text{...(i)}$$

$$\Rightarrow I=\left[e^x\cdot\frac{a^x}{\log_e a}-\int e^x\cdot\frac{a^x}{\log_e a}dx\right]$$

$$\Rightarrow I=\frac{e^x\cdot a^x}{\log_e a}-\frac{1}{\log_e a}\int e^x\cdot a^x\cdot dx$$

$$\Rightarrow I=\frac{e^x\cdot a^x}{\log_e a}-\frac{1}{\log_e a}\cdot I \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \left(\frac{1+\log_e a}{\log_e a}\right)I=\frac{e^x\cdot a^x}{\log_e a}$$

$$\Rightarrow (\log_e e+\log_e a)I=e^x\cdot a^x$$

$$\Rightarrow I=\frac{(ea)^x}{\log(ae)}+c$$

38. $\int e^x\left(\text{cosec}^{-1}-x+\frac{-1}{x\sqrt{x^2-1}}\right)dx$

$$=\int e^x\,\text{cosec}^{-1}x\,dx-\int\frac{e^x}{x\sqrt{x^2-1}}dx$$

$$=\left[\text{cosec}^{-1}x\cdot e^x-\int\frac{-1}{x\sqrt{x^2-1}}\cdot e^x dx\right]$$
$$-\int\frac{ex}{x\sqrt{x^2-1}}dx$$

$$=e^x\cdot\text{cosec}^{-1}x+\int\frac{e^x}{x\sqrt{x^2-1}}dx$$
$$-\int\frac{e^x}{\sqrt{x^2-1}}\cdot x\,dx$$

$$=e^x\cdot\text{cosec}^{-1}x+c$$

39. $\int_{-1}^{1}\sin^5 x\cdot\cos^4 x\,dx$

Here, $f(x)=\sin^5 x\cdot\cos^4 x$

$\Rightarrow f(-x)=-\sin^5 x\cdot\cos^4 x$

$\Rightarrow f(-x)=-f(x)$

Since, $f(x)$ is an odd function.

So, by definite integral property

$$\int_{-a}^{a}f(x)\,dx=0,\text{ if } f(x) \text{ is odd}$$

40. Given, $f(x)=e^{2x+3}$

$f'(x)=2\cdot e^{2x+3}$, $f'(0)=2\cdot e^3$

$f''(x)=4\cdot e^{2x+3}$, $f''(0)=4\cdot e^3$

$f'''(x)=8\cdot e^{2x+3}$, $f'''(0)=8\cdot e^3$

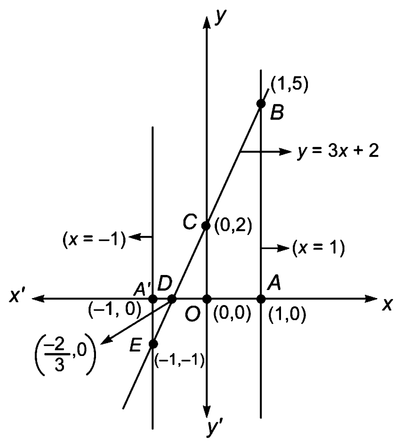
By Maclaurin's series,

$$f(e^{2x+3})=1+\frac{x\cdot f'(0)}{1!}+\frac{x^2f''(0)}{2!}+\frac{x^3f'''(0)}{3!}+\dots$$
$$=1+\frac{x\cdot(2e^3)}{1!}+\frac{x^2(4e^3)}{2!}+\frac{x^3\cdot(8e^3)}{3!}+\dots$$

Hence, the coefficient x^3 in the expansion of e^{2x+3}

$$=\frac{8e^3}{3!}=\frac{8e^3}{6}=\frac{4}{3}e^3$$

41. Given line $y=3x+2$ and the ordinates are $x=-1$ to $x=1$



Area of $A'ED=\int_{-1}^{-2/3}(3x+2)\,dx$

$$=\left[\frac{3x^2}{2}+2x\right]_{-1}^{-2/3}$$
$$=\left[\frac{2}{3}-\frac{4}{3}\right]-\left[\frac{-1}{2}\right]$$
$$=\left(\frac{-1}{6}\right)=\left(\frac{1}{6}\right)\text{ (numerically)}$$

Area of $ADB=\int_{-2/3}^{1}(3x+2)\,dx$

$$=\left[\frac{3x^2}{2}+2x\right]_{-2/3}^{1}$$

$$= \left(\frac{7}{2}\right) - \left(\frac{2}{3} - \frac{4}{3}\right)$$

$$= \frac{25}{6}$$

Hence, total area $= \left(\frac{1}{6} + \frac{25}{6}\right) = \frac{26}{6}$

$$= \frac{13}{3}$$

42. Given family of curves

$$y = b\sin(x + a) \qquad \ldots(\text{i})$$

On differentiating w.r.t. 'x'

$$\frac{dy}{dx} = b\cos(x + a)$$

Again, differentiating w.r.t. 'x'

$$\frac{d^2y}{dx^2} = -b\sin(x + a)$$

From Eq. (i),

$$\frac{d^2y}{dx^2} = -y$$

$$\frac{d^2y}{dx^2} + y = 0$$

43. Given differential equation

$$ydx + (x + 2y^2)dy = 0$$

$$ydx + xdy + 2y^2dy = 0$$

$$\Rightarrow \quad \int d(xy) + 2\int y^2 dy = 0$$

On integrating,

$$\Rightarrow \quad xy + 2y^3/3 = c/3$$

$$\Rightarrow \quad 3xy + 2y^3 = c$$

44. $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$$A \cdot A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+1 & 1+1+1 & 1+1+1 \\ 1+1+1 & 1+1+1 & 1+1+1 \\ 1+1+1 & 1+1+1 & 1+1+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3^1 & 3^1 & 3^1 \\ 3^1 & 3^1 & 3^1 \\ 3^1 & 3^1 & 3^1 \end{bmatrix} = \begin{bmatrix} 3^{(2-1)} & 3^{(2-1)} & 3^{(2-1)} \\ 3^{(2-1)} & 3^{(2-1)} & 3^{(2-1)} \\ 3^{(2-1)} & 3^{(2-1)} & 3^{(2-1)} \end{bmatrix}$$

$$A \cdot A^2 = \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix}\begin{bmatrix} 3^1 & 3^1 & 3^1 \\ 3^1 & 3^1 & 3^1 \\ 3^1 & 3^1 & 3^1 \end{bmatrix} \qquad (\because 3^0 = 1)$$

$$= \begin{bmatrix} 3+3+3 & 3+3+3 & 3+3+3 \\ 3+3+3 & 3+3+3 & 3+3+3 \\ 3+3+3 & 3+3+3 & 3+3+3 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 3^2 & 3^2 & 3^2 \\ 3^2 & 3^2 & 3^2 \\ 3^2 & 3^2 & 3^2 \end{bmatrix}$$

$$= \begin{bmatrix} 3^{(3-1)} & 3^{(3-1)} & 3^{(3-1)} \\ 3^{(3-1)} & 3^{(3-1)} & 3^{(3-1)} \\ 3^{(3-1)} & 3^{(3-1)} & 3^{(3-1)} \end{bmatrix}$$

Similarly,

$$A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$$

45. $\begin{bmatrix} 2 & 4 & 6 \\ 2+3x & 4+3y & 6+3z \\ 2x & 2y & 2z \end{bmatrix}$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 2 & 4 & 6 \\ 2x & 2y & 2z \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 3x & 3y & 3z \\ 0 & 0 & 0 \end{bmatrix} \qquad (\because R_1 \approx R_2)$$

$$\because R_1 = R_3 = 0$$

$= 0 + 0 = 0$

46. $\begin{bmatrix} 2y+4 & 5y+7 & 8y+1 \\ 3y+5 & 6y+8 & 9y+2 \\ 4y+6 & 7y+9 & 10y+3 \end{bmatrix}$

Put $y = 0$

$$\sim \begin{bmatrix} 4 & 7 & 1 \\ 5 & 8 & 2 \\ 6 & 9 & 3 \end{bmatrix}$$

Apply $C_2 \to C_2 - C_1$ and $C_1 \to C_1 - C_3$

$$\sim \begin{bmatrix} 3 & 3 & 1 \\ 3 & 3 & 2 \\ 3 & 3 & 3 \end{bmatrix} \qquad (\because C_1 \text{ and } C_2 \text{ are parallel})$$

$= 0$

47. Given that,coefficient of variations of two distributions are

$$c \cdot v_1 = 55, \quad c \cdot v_2 = 65$$

and $\quad \sigma_1 = 22, \quad \sigma_2 = 39$

Then, arithmetic mean $\bar{x}_1 = \frac{\sigma_1}{c \cdot v_1} \times 100$

$$= \frac{22}{55} \times 100$$

$$= \frac{2}{5} \times 100$$

$= 40$

and arithmetic mean $\bar{x}_2 = \frac{\sigma_2}{c \cdot v_2} \times 100$

$= \frac{39}{65} \times 100$

$= \frac{39}{13} \times 20$

$= 60$

48. Since, the probability of getting at least one head in 'n' tosses $= 1 - \left(\frac{1}{2}\right)^n$, therefore

$$1 - \left(\frac{1}{2}\right)^n \geq 0.9$$

$\Rightarrow \quad \left(\frac{1}{2}\right)^n \leq 0.1$

$\Rightarrow \quad 2^n \geq 10$

$\Rightarrow \quad n \geq 4$

Hence, the least value of 'n' is 4.

49. Probability when first two cards are queens out of 52 $= \frac{^4C_1}{^{52}C_1} \times \frac{^3C_1}{^{51}C_1}$

Probability when third card is king out of 50

$= \frac{^4C_1}{^{50}C_1}$

Hence, the required probability

$$= \frac{^4C_1 \times {^3C_1} \times {^4C_1}}{^{52}C_1 \times {^{51}C_1} \times {^{50}C_1}}$$

$$= \frac{4}{52} \times \frac{3}{51} \times \frac{4}{50}$$

50. The probability of black ball drawn from bag Ist

$$= \frac{4}{3+4} = \frac{4}{7}$$

The probability of black ball drawn from bag IInd $= \frac{6}{5+6} = \frac{6}{11}$

Total probability $= \frac{4}{7} + \frac{6}{11} = \left(\frac{86}{77}\right)$

So, required probability

$$= \frac{(6/11)}{(86/77)}$$

$$= \frac{6 \times 77}{86 \times 11}$$

$$= \frac{21}{43}$$

51. Given that, $(p+q)^n$, Mean = 20

Variance = 16, $(n, p) = ?$

In binomial distribution,

Mean $= np = 20$ ...(i)

Variance $= npq = 16$ ...(ii)

and $p = 1 - q$ ...(iii)

$\Rightarrow \quad q = 4/5$ (using Eqs. (i)and (ii))

From Eq. (iii),

$$p = 1/5$$

From Eq. (i),

$$\frac{n}{5} = 20 \Rightarrow n = 100$$

subject to $(n, p) = (100, 1/5)$

52. Given LPP, max $Z = 4x + y$

So, $x + y \leq 50$

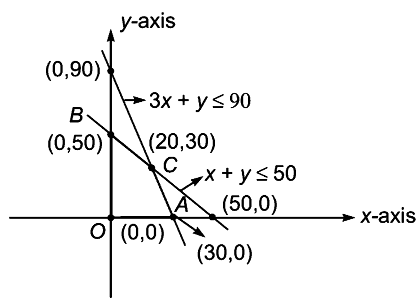
$3x + y \leq 90$

$x \geq 0, y \geq 0$

By **Graphical method**

Equations	Points
$x + y = 50$	(0, 50), (50, 0)
$3x + y = 90$	(0, 90), (30, 0)

Let 20 unit = 1 sq



Convex polygon is $OABC$.

By corner **Point Method**,

$A(30, 0)$ $4.30 + 1.0 = 120$

$B(0, 50)$ $4.0 + 1.50 = 50$

$C(20, 30)$ $4.20 + 1.30 = 110$

of $z = 120$

53. $f(x) = \frac{x-1}{x+1}$

Then, $f(2x) = \frac{2x-1}{2x+1} = \frac{4x-2}{4x+2} = \frac{3x-3+x+1}{x-1+3x+3}$

$$= \frac{3(x-1)+(x+1)}{(x-1)+3(x+1)} = \frac{3\left(\frac{x-1}{x+1}\right)+1}{\left(\frac{x-1}{x+1}\right)+3}$$

$$= \frac{3f(x)+1}{f(x)+3}$$

54. $z = \dfrac{i-1}{\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)}$

$$= \frac{i-1}{\left(\frac{1+i\sqrt{3}}{2}\right)} = \frac{2(i-1)(1-i\sqrt{3})}{(1-i^2\cdot 3)}$$

$$= \frac{2}{4}(i-1-i^2\sqrt{3}+i\sqrt{3})$$

$$= \frac{1}{2}(\sqrt{3}-1+i\sqrt{3}+i)$$

$$\Rightarrow \quad \frac{(\sqrt{3}-1)}{2} + \frac{i(\sqrt{3}+1)}{2} \quad \ldots\text{(i)}$$

Let $\quad r\cos\theta = \frac{\sqrt{3}-1}{2} \quad \ldots\text{(ii)}$

$$r\sin\theta = \frac{\sqrt{3}+1}{2} \quad \ldots\text{(iii)}$$

On squaring and adding equations (ii)and (iii)

$$r^2 = \frac{1}{4}\{3+1-2\sqrt{3}+3+1+2\sqrt{3}\}$$

$$r^2 = 2, \Rightarrow r = \sqrt{2}$$

From on dividing equation (iii) from Eqs. (ii)

$$\tan\theta = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\tan\theta = \frac{(\sqrt{3}+1)^2}{3-1} = \frac{3+1+2\sqrt{3}}{2}$$

$$\tan\theta = 2+\sqrt{3}$$

$$\theta = \frac{7\pi}{12}$$

Principal value of 'θ' $= \pi - \frac{7\pi}{12} = \frac{5\pi}{12}$

Hence, the polar form is

$$r = \sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$$

55. Given equation is

$$\sin^4\theta - 2\sin^2\theta - 1 = 0$$

$$\Rightarrow \quad (\sin^2\theta - 1)^2 = 2$$

$$\Rightarrow \quad \sin^2\theta = 1 \pm \sqrt{2}$$

$$\Rightarrow \quad \sin^2\theta = 1+\sqrt{2} > 2$$

$$\text{or} \quad \sin^2\theta = 1-\sqrt{2} < 0$$

Both are not possible, since $-1 \le \sin\theta \le 1$.
Hence, number of solutions of the given equation is 0.

56. The product 'r' consecutive integers is divisible by

$$r! = r\cdot(r-1)\cdot(r-2)\ldots 3\cdot 2\cdot 1$$

57. Let there be n-sides of the polygon, then the sum of its interior angles is given by

$$S_n = (2n-4)\text{ right angle}$$

$$= (n-2)\times 180° \quad \ldots\text{(i)}$$

Since, the interior angles form an A.P. with first term $a = 120°$ and common difference $d = 5°$.

$$\therefore \quad S_n = \frac{n}{2}[2\times 120° + (n-1)5°] \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii),

$$(n-2)\times 180° = \frac{n}{2}[2\times 120° + (n-1)\times 5°]$$

$$\Rightarrow \quad (n-2)\times 360 = n(5n+235)$$

$$\Rightarrow \quad n^2 - 25n + 144 = 0$$

$$\Rightarrow \quad (n-16)(n-9) = 0$$

$$\Rightarrow \quad n = 16 \quad \text{or} \quad n = 9$$

But, when $n = 16$ the last angle

$$a_n = a + (n-1)d$$

$$= 120° + (16-1)\times 5 = 195°$$

which is not possible.

Hence, $\quad n = 9$

58. Given terms is 30, 24, 20, here we observe that, $\frac{1}{30}, \frac{1}{24}, \frac{1}{20}$ are in AP

The common difference

$$d = \frac{1}{24} - \frac{1}{30} = \frac{1}{120}$$

and first term $a = \frac{1}{30}$

Then, the fourth term is $T_4 = a + 3d$

$$= \frac{1}{30} + 3\cdot\frac{1}{120}$$

$$= \frac{1}{30} + \frac{1}{40} = \frac{7}{120}$$

Hence, the next term of progression 30, 24, 20 is $\left(\frac{120}{7}\right)$.

59. Given expression $\frac{y+z}{x} + \frac{z+x}{y} + \frac{x+y}{z}$

$$= \left(\frac{y+z}{x}\right) + \left(\frac{z+x}{y}\right) + \left(\frac{x+y}{z}\right) + 3 - 3$$

$$= \left(\frac{y+z}{x}+1\right) + \left(\frac{z+x}{y}+1\right) + \left(\frac{x+y}{z}+1\right) - 3$$

$$= (x+y+z)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right) - 3 \quad \ldots\text{(i)}$$

We know that,

$$\text{AM} \ge \text{HM}$$

$$= \frac{(x+y+z)}{3} \ge \frac{3}{\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)}$$

$$= (x+y+z)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right) \ge 9$$

The minimum value of $(x+y+z)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ is 9.

From Eq. (i), minimum value of

$$(x+y+z)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)-3 \text{ is}$$
$$=9-3$$
$$=6$$

60. A person standing the intersection point of two straight paths represented by

$$x+y+1=0, \quad x-y+1=0$$

and their intersection point is $P(-1, 0)$. Since the person wants to reach the path whose equation $6x-7y+8=0$ in a least time, for this, he choose the perpendicular path from that path to intersection point P.

Let the path is $7x+6y+\lambda=0$ ...(i)

It passes through 'P' $(-1, 0)$

$$7(-1)+6(0)+\lambda=0$$
$$\Rightarrow \quad \lambda=7$$

Hence, the required path is, $7x+6y+7=0$.

61. Given, $ax^2+4xy+y^2+ax+3y+2=0$

Since, the given equation represent a parabola, then it satisfies, $H^2=AB$ ...(i)

On comparing with,

$$Ax^2+2Hxy+By^2+2gx+2fy+c=0$$

Then, $\quad A=a, B=1$ and $H=2$

From Eq. (i),

$$(2)^2=(a)(1)$$
$$a=4$$

62. Let $\vec{\mathbf{a}}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $\vec{\mathbf{b}}=-\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$

Now, the position vector of a point (R) which divides the line joining two points, whose position vector are $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ in the ratio 2 : 1 externally is

$$=\left(\frac{2\cdot\vec{\mathbf{b}}-1\cdot\vec{\mathbf{a}}}{2-1}\right)$$
$$=\left(\frac{-2\hat{\mathbf{i}}+2\hat{\mathbf{j}}-2\hat{\mathbf{k}}-\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}}{1}\right)=(-3\hat{\mathbf{i}}-\hat{\mathbf{k}})$$

63. Let $\vec{\mathbf{c}}=x\hat{\mathbf{i}}+y\hat{\mathbf{j}}$, then

$$\vec{\mathbf{b}}\perp\vec{\mathbf{c}}\Rightarrow\vec{\mathbf{b}}\cdot\vec{\mathbf{c}}=4x+3y=0$$
$$\Rightarrow \quad \frac{x}{3}=\frac{y}{-4}=\lambda$$
$$\Rightarrow \quad x=3\lambda, y=-4\lambda$$
$$\therefore \quad \vec{\mathbf{c}}=\lambda(3\hat{\mathbf{i}}-4\hat{\mathbf{j}})$$

Let the required vector be $\vec{\mathbf{a}}=a_1\hat{\mathbf{i}}+a_2\hat{\mathbf{j}}$, then the projections of $\vec{\mathbf{a}}$ on $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are $\frac{\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}}{|\vec{\mathbf{b}}|}$ and $\frac{\vec{\mathbf{a}}\cdot\vec{\mathbf{c}}}{|\vec{\mathbf{c}}|}$ respectively.

$$\therefore \quad \frac{\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}}{|\vec{\mathbf{b}}|}=1 \text{ and } \frac{\vec{\mathbf{a}}\cdot\vec{\mathbf{c}}}{|\vec{\mathbf{c}}|}=2 \quad \text{(given)}$$
$$\Rightarrow \quad 4a_1+3a_2=5$$
$$\text{and} \quad 3a_1-4a_2=10$$
$$\Rightarrow \quad a_1=2, a_2=-1$$

Hence, the required vector $=2\hat{\mathbf{i}}-\hat{\mathbf{j}}$

64. Given, $\frac{1-x}{3}=\frac{y-2}{2\lambda}=\frac{z-3}{2}$ ...(i)

and $\frac{x-1}{3\lambda}=\frac{y-1}{1}=\frac{6-z}{7}$ ...(ii)

Eqs. (i) and (ii), can be written as

$$\frac{x-1}{-3}=\frac{y-2}{2\lambda}=\frac{z-3}{2}$$
$$\text{and} \quad \frac{x-1}{3\lambda}=\frac{y-1}{1}=\frac{z-6}{-7}$$

Since, both are perpendicular to each other, then

$$(-3)(3\lambda)+(2\lambda)(1)+(2)(-7)=0$$
$$\Rightarrow \quad -9\lambda+2\lambda-14=0$$
$$\Rightarrow \quad 7\lambda=-14$$
$$\Rightarrow \quad \lambda=-2$$

65. Given, $\lim_{x\to1}\frac{f(x)-3}{x^2-1}=\pi,$ ...(i)

We have, $f(x)-3=\left\{\frac{f(x)-3}{x^2-1}\right\}\cdot(x^2-1)$

Taking limit $x\to1$ on both sides,

$$\lim_{x\to1}\{f(x)-3\}=\lim_{x\to1}\left\{\frac{f(x)-3}{x^2-1}\right\}\cdot(x^2-1)$$
$$\lim_{x\to1}\{f(x)-3\}=\pi\cdot\lim_{x\to1}(x^2-1) \quad \text{[from Eq. (i)]}$$
$$\lim_{x\to1}f(x)=3+\pi\cdot(1-1)$$
$$=3+\pi\cdot0$$
$$=3$$

66. Given, $f(x)=3e^{x^2}$

$$\Rightarrow \quad f'(x)=3x^2\cdot2x=6xe^{x^2}$$
$$f(0)=3e^0=3$$
$$\text{and} \quad f'(0)=6(0)e^0=0$$

Then, $\quad f'(x)-2xf(x)+\frac{1}{3}f(0)-f'(0)$

$= 6x \cdot e^{x^2} - 6x \cdot e^{x^2} + 1 - 0$

$= 1$

67. Given that, $x = t^2\left(3 - \frac{2}{3}t\right)$

$$x = 3t^2 - \frac{2}{3}t^3 \quad \ldots(\text{i})$$

Velocity $= \frac{dx}{dt} = 6t - 2t^2 \quad \ldots(\text{ii})$

Velocity at Q

$$\frac{dx}{dt} = 0,\ 6t - 2t^2 = 0$$

$\Rightarrow \quad 2t(3 - t) = 0$

$\Rightarrow \quad t = 3\text{ s}$

68. $I = \int e^x\left(\frac{1 + \sin x}{1 + \cos x}\right) dx$

$I = \int e^x\left(\frac{1 + \sin x}{2\cos^2 x/2}\right) dx$

$I = \int \left(\frac{e^x}{2}\sec^2\frac{x}{2} + e^x \tan\frac{x}{2}\right) dx$

$I = \frac{1}{2}\int e^x \sec^2\frac{x}{2}\, dx + \int e^x \tan\frac{x}{2}\, dx$

$I = \frac{1}{2}\int e^x \sec^2\frac{x}{2}\, dx + \left\{e^x \tan\frac{x}{2} - \frac{1}{2}\int e^x \sec^2\frac{x}{2}\, dx\right\}$

$I = \frac{1}{2}\int e^x \sec^2\frac{x}{2}\, dx + e^x \tan\frac{x}{2} - \frac{1}{2}\int e^x \sec^2\frac{x}{2}\, dx$

$I = e^x \tan\frac{x}{2} + c$

69. $\frac{1}{2\cdot 3} + \frac{1}{4\cdot 5} + \frac{1}{6\cdot 7} + \ldots + \infty$

....

....

Here, $\quad T_n = \frac{1}{2n\cdot(2n+1)}$

$T_n = \frac{1}{2n} - \frac{1}{2n+1}$

$T_1 = \frac{1}{2} - \frac{1}{3}$

$T_2 = \frac{1}{4} - \frac{1}{5}$

$T_3 = \frac{1}{6} - \frac{1}{7} \ldots \infty$

Adding all,

$(T_1 + T_2 + T_3 + \ldots + T_\infty)$

$= \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \ldots\right) - \left(\frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \ldots\right) \quad \ldots(\text{i})$

We know that,

$$\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots \infty$$

From Eq. (i),

$(T_1 + T_2 + \ldots + T_\infty) = -\left(-\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots \infty\right)$

$= -\left\{\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots \infty\right) - 1\right\}$

$= -\{\log_e 2 - \log_e e\}$

$= -\log(2/e)$

$= \log(e/2)$

70. The equation of family of parabola having vertex at origin and axis along positive direction of axis of x is given as

$$y^2 = 4ax \quad \ldots(\text{i})$$

$\Rightarrow \quad 2y\frac{dy}{dx} = 4a$

$\Rightarrow \quad 2y\frac{dy}{dx} = \frac{y^2}{x} \quad$ [using Eq. (i)]

$\Rightarrow \quad \frac{y^2}{x} - 2y\frac{dy}{dx} = 0$

$\Rightarrow \quad y^2 - 2xy\frac{dy}{dx} = 0$