AMU
Engineering Entrance Exam
Solved Paper 2009

PHYSICS

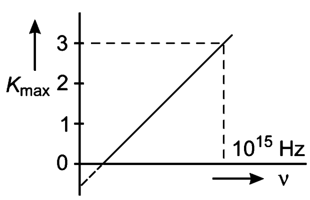
1. Consider Fraunhofer diffraction pattern obtained with a single slit at normal incidence. At the angular position of first diffraction minimum, the phase difference between the wavelets from the opposite edges of the slit is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) π (d) 2π

2. Which of the following lines of the H-atom spectrum belongs to the Balmer series?

(a) 1025 Å (b) 1218 Å
(c) 4861 Å (d) 18751 Å

3. Figure represents a graph of kinetic energy of most energetic photoelectrons, K_{max} (in eV), and frequency (ν) for a metal used as cathode in photoelectric experiment. The threshold frequency of light for the photoelectric emission from the metal is



(a) 1×10^{14} Hz (b) 1.5×10^{14} Hz
(c) 2.1×10^{14} Hz (d) 2.7×10^{14} Hz

4. Using the following data:

Mass of hydrogen atom = 1.00783 u

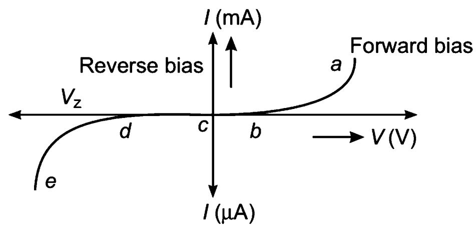
Mass of neutron = 1.00867 u

Mass of nitrogen atom $({}_7N^{14}) = 14.00307$ u

The calculated value of the binding energy of the nucleus of the nitrogen atom $({}_7N^{14})$ is close to

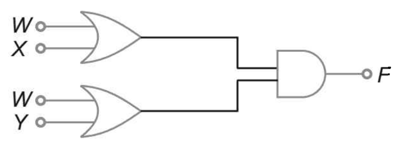
(a) 56 MeV (b) 98 MeV
(c) 104 MeV (d) 112 MeV

5. The graph given below represents the *I-V* characteristics of a zener diode. Which part of the characteristics curve is most relevant for its operation as a voltage regulator?



(a) *ab* (b) *bc*
(c) *cd* (d) *de*

6. The diagram of a logic circuit is given below.



The output *F* of the circuit is given by

(a) $W \cdot (X + Y)$ (b) $W \cdot (X \cdot Y)$
(c) $W + (X \cdot Y)$ (d) $W + (X + Y)$

7. A quantity *X* is given by $\varepsilon_0 L \frac{\Delta V}{\Delta t}$, where ε_0 is the permittivity of free space, *L* is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for *X* is the same as that of

(a) electrical resistance
(b) electric charge
(c) electric voltage
(d) electric current

8. Displacement (x) of a particle is related to time (t) as

$$x = at + bt^2 - ct^3$$

where *a*, *b* and *c* are constants of the motion. The velocity of the particle when its acceleration is zero is given by

(a) $a + \frac{b^2}{c}$ (b) $a + \frac{b^2}{2c}$
(c) $a + \frac{b^2}{3c}$ (d) $a + \frac{b^2}{4c}$

9. A body is thrown vertically up with a velocity u. It passes three points A, B and C in its upward journey with velocities $\frac{u}{2}, \frac{u}{3}$ and $\frac{u}{4}$ respectively. The ratio of the separations between points A and B and between B and C, *ie*, $\frac{AB}{BC}$ is
(a) 1 (b) 2
(c) $\frac{10}{7}$ (d) $\frac{20}{7}$

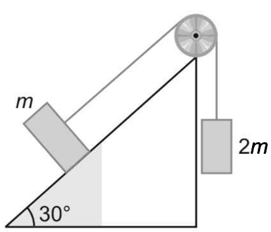
10. A body moves from a position
$\vec{\mathbf{r}}_1 = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}})$ m to a position
$\vec{\mathbf{r}}_2 = (3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$ m under the influence of a constant force $\vec{\mathbf{F}} = (4\hat{\mathbf{i}} + \hat{\mathbf{j}} + 6\hat{\mathbf{k}})$ N. The work done by the force is
(a) 57 J (b) 58 J
(c) 59 J (d) 60 J

11. A particle moves in the *x-y* plane under the influence of a force such that its linear momentum is

$$\vec{\mathbf{p}}(t) = A[\hat{\mathbf{i}} \cos(kt) - \hat{\mathbf{j}} \sin(kt)]$$

where A and k are constants. The angle between the force and momentum is
(a) 0° (b) 30°
(c) 45° (d) 90°

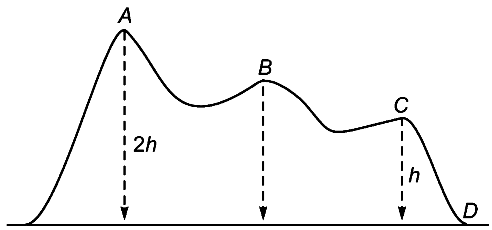
12. Two blocks of masses m and $2m$ are connected by a light string passing over a frictionless pulley. As shown in the figure, the mass m is placed on a smooth inclined plane of inclination 30° and $2m$ hangs vertically. If the system is released, the blocks move with an acceleration equal to



(a) $\frac{g}{4}$ (b) $\frac{g}{3}$
(c) $\frac{g}{2}$ (d) g

13. Identify the wrong statement.
(a) The electrical potential energy of a system of two protons shall increase if the separation between the two is decreased.
(b) The electrical potential energy of a proton-electron system will increase if the separation between the two is decreased.
(c) The electrical potential energy of a proton-electron system will will increase if the separation between the two is increased.
(d) The electrical potential energy of system of two electrons shall increase if the separation between the two is decreased.

14. A small roller coaster starts at point A with a speed u on a curved track as shown in the figure.



The friction between the roller coaster and the track is negligible and it always remains in contact with the track. The speed of roller coaster at point D on the track will be
(a) $(u^2 + gh)^{1/2}$ (b) $(u^2 + 2gh)^{1/2}$
(c) $(u^2 + 4gh)^{1/2}$ (d) u

15. A particle is moving in the *x-y* plane with a constant velocity along a line parallel to the *x*-axis away from the origin. The magnitude of its angular momentum about the origin
(a) is zero (b) remains constant
(c) goes on increasing (d) goes on decreasing

16. Two particles A and B, initially at rest, move towards each other under a mutual force of attraction. At the instant when the speed of A is v and that of B is $2v$, the speed of the centre of mass of the system is
(a) zero (b) v
(c) $1.5v$ (d) $3v$

17. A geostationary satellite is orbiting the earth at a height of $6R$ above the surface of the earth; R being the radius of the earth. What will be the time period of another satellite at a height $2.5R$ from the surface of the earth?
(a) $6\sqrt{2}$ h (b) $6\sqrt{2.5}$ h
(c) $6\sqrt{3}$ h (d) 12 h

18. $\vec{F}_{pe}$ represents electrical force on proton due to electron and $\vec{F}_{ep}$ on electron due to proton in a hydrogen atom. Similarly, $\vec{F}'_{pe}$ represents the gravitational force on proton due to electron and $\vec{F}'_{ep}$ the corresponding force on electron due to proton. Which of the following is not true?

(a) $\vec{F}_{pe} + \vec{F}_{ep} = 0$

(b) $\vec{F}'_{pe} + \vec{F}'_{ep} = 0$

(c) $\vec{F}_{pe} + \vec{F}'_{pe} + \vec{F}_{ep} + \vec{F}'_{ep} = 0$

(d) $\vec{F}_{pe} + \vec{F}'_{pe} = 0$

19. Two uniform brass rods A and B of length l and $2l$ and radii $2r$ and r respectively are heated to the same temperature. The ratio of the increase in the volume of A to that of B is

(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) 1 : 4

20. Water flows steadily through a horizontal pipe of a variable cross-section. If the pressure of water is p at a point where the velocity of flow is v, what is the pressure at another point where the velocity of flow is $2v$, ρ being the density of water?

(a) $p + 2\rho v^2$ (b) $p - 2\rho v^2$
(c) $p + \frac{3}{2}\rho v^2$ (d) $p - \frac{3}{2}\rho v^2$

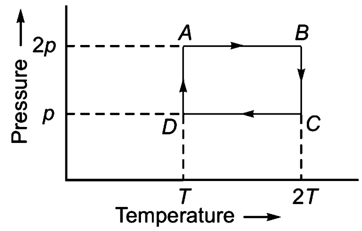
21. A vessel contains oil (density 0.8 g cm^{-3}) over mercury (density 13.6 g cm^{-3}). A homogenous sphere floats with half volume immersed in mercury and the other half in oil. The density of the material of the sphere in g cm^{-3} is

(a) 12.8 (b) 7.2
(c) 6.4 (d) 3.3

22. A steel wire of cross-sectional area 3×10^{-6} m^2 can withstand a maximum strain of 10^{-3}. Young's modulus of steel is 2×10^{11} Nm^{-2}. The maximum mass the wire can hold is
(Take $g = 10$ ms^{-2})

(a) 40 kg (b) 60 kg
(c) 80 kg (d) 100 kg

23. One mole of an ideal gas having initial volume V, pressure $2p$ and temperature T undergoes a cyclic process $ABCDA'$ as shown below.



The net work done in the complete cycle is

(a) zero (b) $\frac{1}{2} RT \ln 2$
(c) $RT \ln 2$ (d) $\frac{3}{2} RT \ln 2$

24. When two moles of oxygen is heated from 0°C–10°C at constant volume, its internal energy changes by 420 J. What is the molar specific heat of oxygen at constant volume?

(a) 5.75 JK^{-1} mol^{-1} (b) 10.5 JK^{-1} mol^{-1}
(c) 21 JK^{-1} mol^{-1} (d) 42 JK^{-1} mol^{-1}

25. A vessel contains 32 g of O_2 at a temperature T. The pressure of the gas is p. An identical vessel containing 4 g of H_2 at a temperature $2T$ has a pressure of

(a) $8p$ (b) $4p$
(c) p (d) $\frac{p}{8}$

26. A tuning fork produces 4 beats s^{-1} when sounded with a sonometer wire of vibrating length 48 cm. It produces 4 beats s^{-1} also when the vibrating length is 50 cm. What is the frequency of the tuning fork?

(a) 196 Hz (b) 284 Hz
(c) 375 Hz (d) 460 Hz

27. The displacement y of a particle is given by $y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000t)$. This expression may be considered to be a result of the superposition of how many simple harmonic motions?

(a) 2 (b) 3
(c) 4 (d) 5

28. A progressive wave in a medium is represented by the equation $y = 0.1 \sin\left(10\pi t - \frac{5}{11}\pi x\right)$ where y and x are in cm and t in second. The wavelength and velocity of the wave is

(a) $\frac{5}{11}$ m, 31.4 cm s^{-1} (b) 4.4 m, 22 cm s^{-1}
(c) 2.2 m, 11 cm s^{-1} (d) $\frac{11}{5}$ m, 22 cm s^{-1}

29. Identify the wrong statement.

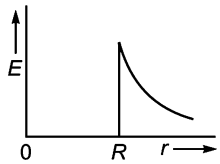
(a) In an electric field two equipotential surfaces can never intersect.

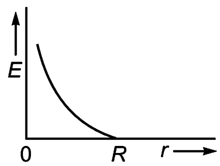
(b) A charged particle free to move in an electric field shall always move in the direction of $\vec{\mathbf{E}}$.

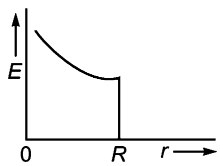
(c) Electric field at the surface of a charged conductor is always normal to the surface.

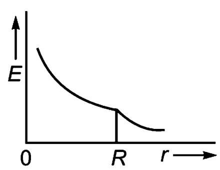
(d) The electric potential decrease along a line of force in an electric field.

30. A metallic spherical shell of radius R has a charge $-Q$ on it. A point charge $+Q$ is placed at the centre of the shell. Which of the graphs shown below may correctly represent the variation of the electric field E with distance r from the centre of the shell?

(a) 

(b) 

(c) 

(d) 

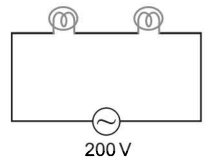
31. Two positive point charges of 12 μC and 5 μC, are placed 10 cm apart in air. The work needed to bring them 4 cm closer is

(a) 2.4 J (b) 3.6 J
(c) 4.8 J (d) 6.0 J

32. Two parallel plate capacitors of capacitances C and $2C$ are connected in parallel and charged to a potential difference V_0. The battery is then disconnected and the region between the plates of the capacitor C is completely filled with a material of dielectric constant 2. The potential difference across the capacitors now becomes

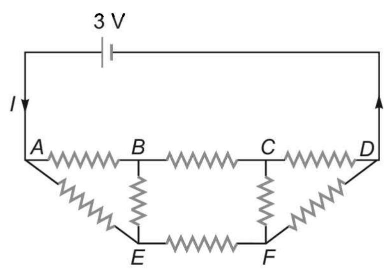
(a) $\frac{F_0}{4}$ (b) $\frac{V_0}{2}$
(c) $\frac{3V_0}{4}$ (d) V_0

33. Two bulbs marked 200 V–100 W and 200 V–200 W are joined in series and connected to a power supply of 200 V. The total power consumed by the two will be near to



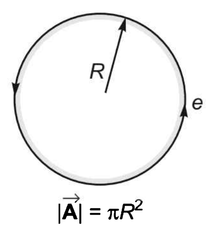
(a) 35 W (b) 66 W
(c) 100 W (d) 300 W

34. Figure shows a network of eight resistors, each equal to 2 Ω, connected to a 3 V battery of negligible internal resistance. The current I in the circuit is



(a) 0.25 A (b) 0.50 A
(c) 0.75 A (d) 1.0 A

35. An electron is moving in an orbit of radius R with a time period T as shown in the figure. The magnetic moment produced may be given by



(a) $\vec{\mathbf{M}} = \frac{2\pi|e|\vec{\mathbf{A}}}{T}$

(b) $\vec{\mathbf{M}} = -\frac{2\pi|e|\vec{\mathbf{A}}}{T}$

(c) $\vec{\mathbf{M}} = \frac{|e|\vec{\mathbf{A}}}{T}$

(d) $\vec{\mathbf{M}} = -\frac{|e|\vec{\mathbf{A}}}{T}$

$|e|$ represents the magnitude of the electron charge.

36. A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 ms^{-1}, at right angles to the horizontal component of the earth's magnetic field of strength 0.30×10^{-4} Wb m^{-2}. The instantaneous value of the induced potential gradient in the wire, from west to east, is

(a) $+1.5 \times 10^{-3}$ Vm^{-1} (b) -1.5×10^{-3} Vm^{-1}

(c) $+1.5 \times 10^{-4}$ Vm^{-1} (d) -1.5×10^{-4} Vm^{-1}

37. A uniformly wound solenoid coil of self-inductance 1.8×10^{-4} H and resistance 6 Ω is broken up into two identical coils. These identical coils are then connected in parallel across a 12 V battery of negligible resistance. The time constant for the current in the circuit is

(a) 0.1×10^{-4} s (b) 0.2×10^{-4} s

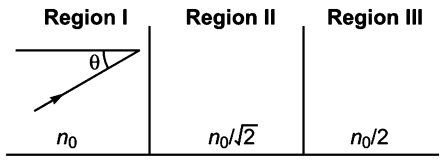
(c) 0.3×10^{-4} s (d) 0.4×10^{-4} s

38. An *LC* circuit contains a 20 mH inductor and a 50 μF capacitor with an initial charge of 10 mC. The resistance of the circuit is negligible. Let the instant the circuit is closed be $t = 0$. At what time is the energy stored completely magnetic?

(a) $t = 0$ (b) $t = 1.54$ ms

(c) $t = 3.14$ ms (d) $t = 6.28$ ms

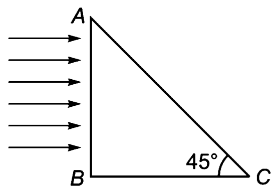
39. A beam of light is travelling from region II to region III (see the figure). The refractive index in region I, II and III are n_0, $\frac{n_0}{\sqrt{2}}$ and $\frac{n_0}{2}$ respectively. The angle of incidence θ for which the beam just misses entering region III is



(a) 30° (b) 45°

(c) 60° (d) $\sin^{-1}(\sqrt{2})$

40. A beam of light consisting of red, green and blue colours is incident on a right-angled prism *ABC*. The refractive indices of the material of the prism for the above red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. The colour/colours transmitted through the face *AC* of the prism will be



(a) red only (b) red and green

(c) all the three (d) None of these

CHEMISTRY

1. In XeF_6, oxidation state and state of hybridisation of Xe and shape of the molecule are, respectively

(a) +6, sp^3d^3, distorted octahedral

(b) +4, sp^3d^2, square planar

(c) +6, sp^3, pyramidal

(d) +6, sp^3d^2, square pyramidal

2. The following species will not exhibit disproportionation reaction

(a) ClO^- (b) ClO_2^-

(c) ClO_3^- (d) ClO_4^-

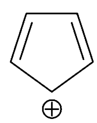
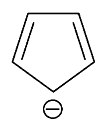
3. Relative stabilities of the following carbocations will be in the order

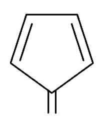
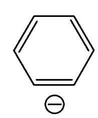
$\overset{\oplus}{C}H_3$ (*A*) $CH_3\overset{\oplus}{C}H_2$ (*B*) $\overset{\oplus}{C}H_2OCH_3$ (*C*)

(a) $C > B > A$ (b) $C < B < A$

(c) $B > C > A$ (d) $C > A > B$

4. Which of the following species is aromatic?

(a)  (b) 

(c)  (d) 

5. Benzalkonium chloride is a

(a) cationic surfactant and antiseptic

(b) anionic surfactant and soluble in most of organic solvents

(c) cationic surfactant and insoluble in most of organic solvents

(d) cationic surfactant and antimalarial

6. Which factor/s will increase the reactivity of $>C=O$ group?

(i) Presence of a group with positive inductive effect
(ii) Presence of a group with negative inductive effect
(iii) Presence of large alkyl group
(a) only (i)
(b) only (ii)
(c) (i) and (iii)
(d) (ii) and (iii)

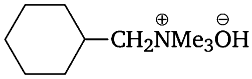
7. In the following reaction,

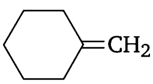
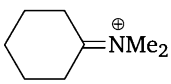
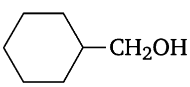
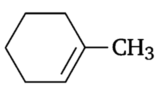
$$RCH_2CH{=}CH_2 + ICl \longrightarrow [A]$$

Markownikoff's product [A] is

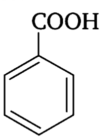
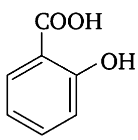
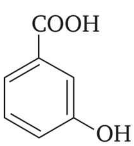
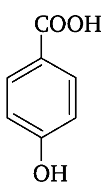
(a) $RCH_2CH(Cl)—CH_2I$ (b) $RCH_2CH(I)—CH_2Cl$
(c) $RCH—C(I){=}CH_2$ (d) $RCH{=}CH—CH_2I$

8. Thermal decomposition of

 gives

(a)  (b) 
(c)  (d) 

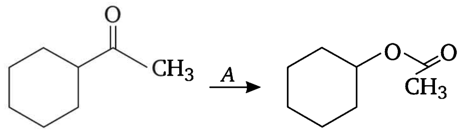
9. Which of the following aromatic acids is most acidic?

(a)  (b) 
(c)  (d) 

10. In the preparation of chlorobenzene from aniline, the most suitable reagent is
(a) chlorine in the presence of ultraviolet light
(b) chlorine in the presence of $AlCl_3$
(c) nitrous acid followed by heating with Cu_2Cl_2
(d) HCl and Cu_2Cl_2

11. Comparing basic strength of NH_3, CH_3NH_2 and $C_6H_5NH_2$ it may be concluded that
(a) basic strength remains unaffected
(b) basic strength of alkyl amines is lowest
(c) basic strength of aryl amines is lowest
(d) basic strength of NH_3 is highest

12. The most suitable reagent A, for the reaction



is/are
(a) O_3
(b) H_2O_2
(c) $NaOH\text{-}H_2O_2$
(d) *m*-chloroperbenzoic acid

13. Mammals' fats are hydrolysed to release fatty acids by
(a) amylase (b) lactase
(c) lipase (d) insulin

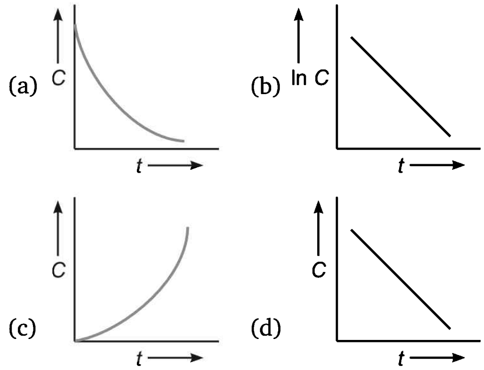
14. Which of the following represents *neo*-pentyl alcohol?
(a) $CH_3CH(CH_3)CH_2CH_2OH$
(b) $(CH_3)_3C—CH_2OH$
(c) $CH_3(CH_2)_3OH$
(d) $CH_3CH_2CH(OH)C_2H_5$

15. The most reactive compound towards electrophilic nitration is
(a) toluene
(b) benzene
(c) benzoic acid
(d) nitrobenzene

16. Arrange the following compounds in order of their decreasing reactivity with an electrophile, $E^{\oplus}$.
(A) Chlorobenzene,
(B) 2,4-dinitrochlorobenzene,
(C) *p*-nitrochlorobenzene
(a) $C > B > A$ (b) $B > C > A$
(c) $A > C > B$ (d) $A > B > C$

17. Sodium chloride is soluble in water but not in benzene because
(a) $\Delta H_{hydration} < \Delta H_{\text{lattice energy in water}}$ and $\Delta H_{hydration} > \Delta H_{\text{lattice energy in benzene}}$
(b) $\Delta H_{hydration} > \Delta H_{\text{lattice energy in water}}$ and $\Delta H_{hydration} < \Delta H_{\text{lattice energy in benzene}}$
(c) $\Delta H_{hydration} = \Delta H_{\text{lattice energy in water}}$ and $\Delta H_{hydration} < \Delta H_{\text{lattice energy in benzene}}$
(d) $\Delta H_{hydration} < \Delta H_{\text{lattice energy in water}}$ and $\Delta H_{hydration} = \Delta H_{\text{lattice energy in benzene}}$

18. The plot between concentration *versus* time for a zero order reaction is represented by



(c) sp^3d and has a trigonal bipyramidal structure
(d) sp^3d^2 and has an octahedral structure

19. Which of the following reactions cannot be a base for electrochemical cell?
(a) $H_2 + O_2 \longrightarrow H_2O$
(b) $AgNO_3 + Zn \longrightarrow Zn(NO_3)_2 + Ag$
(c) $AgNO_3 + NaCl \longrightarrow AgCl\downarrow + NaNO_2$
(d) $KMnO_4 + FeSO_4 + H_2SO_4 \longrightarrow K_2SO_4 + Fe_2(SO_4)_3 + MnSO_4 + H_2O$

20. The strength of 10 volume of H_2O_2 solution is
(a) 10 (b) 68
(c) 60.70 (d) 30.36

21. Which of the following species is non-linear?
(a) ICl_2^- (b) I_3^-
(c) N_3^- (d) ClO_2^-

22. For the reaction, $2A + B \rightarrow C + D$ the order of reaction is
(a) one with respect to $[B]$
(b) two with respect to $[A]$
(c) three
(d) cannot be predicted

23. The basic structural unit in silicates is
(a) SiO_2 (b) $[Si_2O_7]^{2-}$
(c) SiO_4 tetrahedron (d) $[Si_2O_5]^{2-}$

24. In which of the following reactions, H_2O_2 is acting as a reducing agent?
(a) $SO_2 + H_2O_2 \longrightarrow H_2SO_4$
(b) $2KI + H_2O_2 \longrightarrow 2KOH + I_2$
(c) $PbS + 4H_2O_2 \longrightarrow PbSO_4 + 4H_2O$
(d) $Ag_2O + H_2O_2 \longrightarrow 2Ag + H_2O + O_2$

25. Which of the following oxides is most acidic in nature?
(a) BeO (b) MgO
(c) CaO (d) BaO

26. The state of hybridisation of S in SF_4 is
(a) sp^3 and has a lone pair of electron
(b) sp^2 and has tetrahedral structure

27. If two moles of glucose are oxidised in the body through respiration, the number of moles of ATP produced are
(a) 19 (b) 38
(c) 57 (d) 76

28. The potential of the cell for the reaction,
$$M(s) + 2H^+(1\ M) \longrightarrow H_2(g)(1\ atm) + M^{2+}(0.1\ M)$$
is 1.500 V. The standard reduction potential for $M^{2+}/M(s)$ couple is
(a) 0.1470 V (b) 1.470 V
(c) 14.70 V (d) None of these

29. The element with atomic number 117 if discovered would be placed in
(a) noble gas family
(b) alkali family
(c) alkaline earth family
(d) halogen family

30. van't Hoff factor of *aq.* K_2SO_4 at infinite dilution has value euqal to
(a) 1 (b) 2
(c) 3 (d) between 2 and 3

31. Which set of characteristics of ZnS crystal is correct?
(a) Coordination number (4 : 4): *ccp*; Zn^{2+} ion in the alternate tetrahedral voids
(b) Coordination number (6 : 6); *hcp*; Zn^{2+} ion in all tetrahedral voids
(c) Coordination number (6 : 4); *hcp*; Zn^{2+} ion in all octahedral voids
(d) Coordination number (4 : 4); *ccp*; Zn^{2+} ion in all tetrahedral voids

32. When a radioactive substance is kept in vacuum, the rate of its disintergration per second
(a) increases considerably
(b) is not affected
(c) suffers a slight decrease
(d) increases only if the products are gaseous

33. An aqueous solution whose pH is zero will be called as
(a) acidic (b) basic
(c) neutral (d) amphoteric

34. The bond angle and % of *d*-character in SF_6 are
(a) 120°, 20% (b) 90°, 33%
(c) 109°, 25% (d) 90°, 25%

35. Which of the following species will be diamagnetic?
(a) $[Fe(CN)_6]^{4-}$ (b) $[FeF_6]^{3-}$
(c) $[Co(C_2O_4)_3]^{3-}$ (d) $[CoF_6]^{3-}$

36. One component of a solution follows Raoult's law over the entire range $0 \le x_1 \le 1$. The second component must follow Raoult's law in the range when x_2 is
(a) close to zero (b) close to 1
(c) $0 \le x_2 \le 0.5$ (d) $0 \le x_2 \le 1$

37. Select wrong statement.
(a) If a very small amount of $AlCl_3$ is added to gold sol, coagulation occurs, but if a large quantity of $AlCl_3$ is added, there is no coagulation.
(b) Organic ions are more strongly adsorbed on charged surfaces in comparison to inorganic ions.
(c) Both emulsifier and peptising agents stabilise colloids but their actions are different.
(d) Colloidal solutions are thermodynamically stable.

38. An adiabatic process occurs in
(a) open system
(b) closed system
(c) isolated system
(d) in all the given systems

39. Approximate relationship between dissociation constant of water (K) and ionic product of water (K_w) is
(a) $K_w = K$ (b) $K_w = 55.6 \times K$
(c) $K_w = 18 \times K$ (d) $K_w = 14 \times K$

40. For the reaction at 298 K

$$A(g) + B(g) \longrightarrow C(g)$$

$\Delta E = -5$ cal and $\Delta S = -10$ cal K^{-1}
(a) $\Delta G = +2612$ cal (b) $\Delta G = -2612$ cal
(c) $\Delta G = +261.2$ cal (d) $\Delta G = -261.2$ cal

MATHEMATICS

1. In an ellipse, if the lines joining focus to the extremities of the minor axis form an equilateral triangle with the minor axis, then the eccentricity of the ellipse is
(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}}{4}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{\frac{2}{3}}$

2. If the planes $x = cy + bz$, $y = az + cx$, $z = bx + ay$ pass through a line, then $a^2 + b^2 + c^2 + 2abc$ is
(a) 0 (b) 1
(c) 2 (d) 3

3. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z + \cos^{-1} t = 4\pi$, then the value of $x^2 + y^2 + z^2 + t^2$ is
(a) $xy + zy + zt$ (b) $1 - 2xyzt$
(c) 4 (d) 6

4. Four dice are rolled. The number of possible outcomes in which at least one dice shows 2 is
(a) 625 (b) 671
(c) 1023 (d) 1296

5. If $f(x+y) = f(x)f(y)$ for all x and y and if $f(5) = 2$ and $f'(0) = 3$, then $f'(5)$ is
(a) 0 (b) 2
(c) 5 (d) 6

6. The equation of the curve satisfying the differential equation $y_2(x^2+1) = 2xy_1$ passing through the point (0, 1) and having slope of tangent at $x = 0$ as 3 is
(a) $y = x^3 + 3x + 1$ (b) $y = x^3 - 3x + 1$
(c) $y = x^2 + 3x + 1$ (d) $y = x^2 - 3x + 1$

7. For the function

$$f(x) = \lim_{n \to \infty} \frac{\log(2+x) - x^{2n} \sin x}{1 + x^{2n}},$$

which of the following is true?
(a) $\lim_{x \to 1^-} f(x)$ does not exist
(b) $\lim_{x \to 1^+} f(x)$ does not exist
(c) Both limits exist and $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$
(d) Both limits exist and $\lim_{x \to 1^-} f(x) \ne \lim_{x \to 1^+} f(x)$

8. The curve $y - e^{xy} + x = 0$ has a vertical tangent at the point
(a) (1, 1) (b) (1, 0)
(c) (0, 1) (d) None of these

9. If a hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with the major and minor axes of the ellipse and product of their eccentricities be 1, then the equation of hyperbola is

(a) $\frac{x^2}{9}-\frac{y^2}{25}=1$ (b) $\frac{x^2}{9}-\frac{y^2}{16}=1$

(c) $\frac{x^2}{16}-\frac{y^2}{25}=1$ (d) None of these

10. If p, q, r are positive and are in AP, then roots of the quadratic equation $px^2+qx+r=0$ are complex for

(a) $\left|\frac{r}{p}-7\right|\geq 4\sqrt{3}$ (b) $\left|\frac{p}{r}-7\right|<4\sqrt{3}$

(c) all p and r (d) no p and r

11. For any two sets A and B, if $A\cap X=B\cap X=\phi$ and $A\cup X=B\cup X$ for some set X, then

(a) $A-B=A\cap B$ (b) $A=B$

(c) $B-A=A\cap B$ (d) None of these

12. If the coefficient of variation of a distribution is 45% and the mean is 12, then its standard deviation is

(a) 5.2 (b) 5.3

(c) 5.4 (d) None of these

13. The largest term in the expansion of $(4+2x)^{49}$ where $x=\frac{1}{3}$ is

(a) 3rd (b) 5th

(c) 8th (d) None of these

14. The curve described parametrically by $x=t^2+t$ and $y=t^2-t$ represents

(a) a pair of straight lines

(b) an ellipse

(c) a parabola

(d) a hyperbola

15. Let r be a relation from R (set of real numbers) to R defined by $r=\{(a,b)\,|\,a,b\in R$ and $a-b+\sqrt{3}$ is an irrational number$\}$. The relation r is

(a) an equivalence relation

(b) reflexive only

(c) symmetric only

(d) transitive only

16. The set $C=\{z: z\bar{z}+a\bar{z}+\bar{a}z+b=0, b\in R$ and $b<|a|^2\}$ is

(a) a finite set (b) an infinite set

(c) an empty set (d) None of these

17. For $\frac{|x-1|}{x+2}<1$, x lies in the interval

(a) $(-\infty,-2)\cup\left(-\frac{1}{2},\infty\right)$

(b) $(-\infty,1)\cup[2,3]$

(c) $(-\infty,-4)$

(d) $\left[-\frac{1}{2},1\right]$

18. If $a, b, c>0$ and if $abc=1$, then the value of $a+b+c+ab+bc+ca$ lies in the interval

(a) $(\infty,-6]$ (b) $(-6,0)$

(c) $(0,6)$ (d) $(6,\infty)$

19. If P is a point (x,y) on the line $y=-3x$ such that P and the point $(3,4)$ are on the opposite sides of the line $3x-4y-8=0$, then

(a) $x>\frac{8}{15}, y<-\frac{8}{5}$ (b) $x>\frac{8}{5}, y<-\frac{8}{15}$

(c) $x=\frac{8}{15}, y=-\frac{8}{5}$ (d) None of these

20. In a sequence of 21 terms, the first 11 terms are in AP with common difference 2 and the last 11 terms are in GP with common ratio 2. If the middle term of AP be equal to the middle term of the GP, then the middle term of the entire sequence is

(a) $-\frac{10}{31}$ (b) $\frac{10}{31}$

(c) $\frac{32}{31}$ (d) $-\frac{31}{32}$

21. If n is an integer and if $\begin{vmatrix} x^n & x^{n+2} & x^{n+3} \\ y^n & y^{n+2} & y^{n+3} \\ z^n & z^{n+2} & z^{n+3} \end{vmatrix}$

$$=(x-y)(y-z)(z-x)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$$

then n equals

(a) 1 (b) –1

(c) 2 (d) None of these

22. A person draws a card from a pack of playing cards, replaces it and shuffles the pack. He continues doing this until he draws a spade. The chance that he will fail the first two times is

(a) $\frac{9}{64}$ (b) $\frac{1}{64}$

(c) $\frac{1}{16}$ (d) $\frac{9}{16}$

23. A particle is acted on by a force of 6 unit in the direction $9\hat{\mathbf{i}}+6\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and is displaced from the point $3\hat{\mathbf{i}}+4\hat{\mathbf{j}}-15\hat{\mathbf{k}}$ to the point $7\hat{\mathbf{i}}-6\hat{\mathbf{j}}+8\hat{\mathbf{k}}$. The work done is

(a) 18 (b) 15
(c) 12 (d) 9

24. A variable line through the point $\left(\frac{1}{5}, \frac{1}{5}\right)$ cuts the coordinate axes in the points A and B. If the point P divides AB internally in the ratio 3 : 1, then the locus of P is
(a) $3y + x = 20xy$
(b) $y + 3x = 20xy$
(c) $x + y = 20xy$
(d) $3x + 3y = 20xy$

25. The maximum value of $3\cos x + 4\sin x + 5$ is
(a) 5 (b) 6
(c) 7 (d) None of these

26. The number of positive integers satisfying the inequality ${}^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \le 50$ is
(a) 9 (b) 8
(c) 7 (d) 6

27. The distance of the point on $y = x^4 + 3x^2 + 2x$ which is nearest to the line $y = 2x - 1$ is
(a) $\frac{4}{\sqrt{5}}$ (b) $\frac{3}{\sqrt{5}}$
(c) $\frac{2}{\sqrt{5}}$ (d) $\frac{1}{\sqrt{5}}$

28. Let $f : R \to R$ be a differentiable function such that $f(3) = 3$, $f'(3) = \frac{1}{2}$.
Then, the value of $\lim_{x \to 3} \frac{\int_3^{f(x)} 2t^3\, dt}{x - 3}$ is
(a) 25 (b) 26
(c) 27 (d) None of these

29. If $f(x)$ be continuous function such that the area bounded by the curve $y = f(x)$, the x-axis and the lines $x = a$ and $x = 0$ is $\frac{a^2}{2} + \frac{a}{2}\sin a + \frac{\pi}{2}\cos a$. Value of $f\left(\frac{\pi}{2}\right)$ is
(a) $\frac{1}{2}$ (b) $\frac{a}{2}$
(c) $\frac{a^2}{2}$ (d) $\frac{\pi}{2}$

30. A curve through (1, 0) and satisfying the differential equation $(1 + y^2)\, dx - xy\, dy = 0$ is
(a) a circle (b) a parabola
(c) an ellipse (d) a hyperbola

31. If $f'(x) = g(x)$ and $g'(x) = -f(x)$ for all x and $f(2) = 4 = f'(2)$, then $f^2(4) + g^2(4)$ is
(a) 8 (b) 16
(c) 32 (d) 64

32. Equation $\cos 2x + 7 = a(2 - \sin x)$ can have a real solution for
(a) all values of a (b) $a \in [2, 6]$
(c) $a \in (-\infty, 2)$ (d) $a \in (0, \infty)$

33. Let $n(A) = 4$ and $n(B) = 6$. The number of one to one functions from A to B is
(a) 24 (b) 60
(c) 120 (d) 360

34. The sum of the series
$1 + \frac{1}{3}\cdot\frac{1}{4} + \frac{1}{5}\cdot\frac{1}{4^2} + \frac{1}{7}\cdot\frac{1}{4^3} + \ldots \infty$ is
(a) $\log_e 1$ (b) $\log_e 2$
(c) $\log_e 3$ (d) $\log_e 4$

35. If x^{2r} occurs in $\left(x + \frac{2}{x^2}\right)^n$, then $n - 2r$ must be of the form
(a) $3k - 1$ (b) $3k$
(c) $3k + 1$ (d) $3k + 2$

36. The equation of the circle which cuts orthogonally the circle $x^2 + y^2 - 6x + 4y - 3 = 0$, passes through (3, 0) and touches the axis of y is
(a) $x^2 + y^2 + 6x - 6y + 9 = 0$
(b) $x^2 + y^2 - 6x + 6y - 9 = 0$
(c) $x^2 + y^2 - 6x - 6y + 9 = 0$
(d) None of the above

37. Let a relation R in the set N of natural numbers be defined as $(x, y) \Leftrightarrow x^2 - 4xy + 3y^2 = 0\ \forall\ x, y \in N$. The relation R is
(a) reflexive
(b) symmetric
(c) transitive
(d) an equivalence relation

38. If x, y, z are three consecutive positive integers, then
$$\log_e \sqrt{x} + \log_e \sqrt{z} + \left(\frac{1}{2xz + 1}\right) + \frac{1}{3}\left(\frac{1}{2xz + 1}\right)^3 + \frac{1}{5}\left(\frac{1}{2xz + 1}\right)^5 + \ldots \text{ is}$$
(a) $\log_e \sqrt{y}$ (b) $\log_e y$
(c) $\log_e y^2$ (d) None of these

39. Solution set of inequality $\log_e \frac{x - 2}{x - 3}$ is
(a) $(2, \infty)$ (b) $(-\infty, 2)$
(c) $(-\infty, \infty)$ (d) $(3, \infty)$

40. Let z and w be two complex numbers such that $|z| \le 1, |w| \le 1$ and $|z + iw| = |z - i\overline{w}| = 2$. Then, z is equal to

(a) 1 or i (b) i or $-i$
(c) 1 or -1 (d) i or -1

41. If the system of equations

$$ax + ay - z = 0$$
$$bx - y + bz = 0$$
and $$-x + cy + cz = 0$$

has a non-trivial solution, then the value of

$$\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c} \text{ is}$$

(a) 0 (b) 1
(c) 2 (d) 3

42. A determinant of second order is made with the elements 0, 1. What is the probability that the determinant is positive?

(a) $\frac{7}{12}$ (b) $\frac{11}{12}$
(c) $\frac{3}{16}$ (d) $\frac{15}{16}$

43. If $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$ and $|\vec{\mathbf{a}}| = 7, |\vec{\mathbf{b}}| = 3, |\vec{\mathbf{c}}| = 5$, then angle between $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

44. A non-zero vector $\vec{\mathbf{a}}$ is parallel to the line of intersection of the plane determined by vectors $\hat{\mathbf{i}}, \hat{\mathbf{i}} - \hat{\mathbf{j}}$ and the plane determined by the vectors $\hat{\mathbf{i}} + \hat{\mathbf{j}}, \hat{\mathbf{i}} - \hat{\mathbf{k}}$. The angle between $\vec{\mathbf{a}}$ and $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$
(c) $\frac{\pi}{4}$ (d) None of these

45. Number of solutions of $|x - 1| = \cos x$ is

(a) 2 (b) 3
(c) 4 (d) None of these

46. If the slope of one of the lines represented by $ax^2 + 2hxy + by^2 = 0$ be the square of the other, then $\frac{a+b}{h} + \frac{8h^2}{ab}$ is

(a) 3 (b) 4
(c) 5 (d) 6

47. The value of $\left[(0.16)^{\log_{0.25}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \infty\right)}\right]^{1/2}$ is

(a) 1 (b) -1
(c) 0 (d) None of these

48. If P_m stands for ${}^mP_m$, then $1 + 1P_1 + 2P_2 + 3P_3 + \dots + n \cdot P_n$ is equal to

(a) $n!$ (b) $(n+3)!$
(c) $(n+2)!$ (d) $(n+1)!$

49. If $y = f\left(\frac{3x + \pi}{5x + 4}\right)$ and $f'(x) = \tan^2 x$, then $\frac{dy}{dx}$ at $x = 0$ is

(a) $\frac{12 + 5\pi}{16}$ (b) $\frac{12 - 5\pi}{16}$
(c) $\frac{5 + 12\pi}{16}$ (d) $\frac{5 - 12\pi}{16}$

50. Limit of $\int_0^x \left[\frac{1}{\sqrt{1+t^2}} - \frac{1}{1+t}\right] dt$ as $x \to \infty$ is

(a) $\log_2 e$ (b) $\log_e 2$
(c) $\log_2\left(\frac{1}{e}\right)$ (d) $\log_{1/e} 2$

51. The maximum value of $z = 10x + 6y$ subject to constraints $x \ge 0, y \ge 0, x + y \le 12, 2x + y \le 20$ is

(a) 72 (b) 80
(c) 104 (d) 110

52. If $2f(x^2) + 3f\left(\frac{1}{x^2}\right) = x^2 - 1$ for all $x \in R - \{0\}$, then $f(x^4)$ is

(a) $\frac{(1 - x^4)(2x^4 + 3)}{5x^4}$ (b) $\frac{(1 + x^4)(2x^4 - 3)}{5x^4}$
(c) $\frac{(1 - x^4)(2x^4 - 3)}{5x^4}$ (d) None of these

53. If $\int (\log x)^2 \, dx = x[f(x)]^2 + Ax[f(x) - 1] + c$, then

(a) $f(x) = \log x, A = 2$
(b) $f(x) = \log x, A = -2$
(c) $f(x) = -\log x, A = 2$
(d) $f(x) = -\log x, A = -2$

54. Let a and b be two integers such that $10a + b = 5$ and $P(x) = x + ax + b$. The integer n such that $P(10) \cdot P(11) = P(n)$ is

(a) 15 (b) 65
(c) 115 (d) 165

55. The mean age of a combined group of men and women is 25 yr. If the mean age of the group of men is 26 and that of the group of women is 21, then the percentage of men and women in the group is

(a) 46, 60 (b) 80, 20
(c) 20, 80 (d) 60, 40

56. A variable chord is drawn through the origin to the circle $x^2 + y^2 - 2ax = 0$. The locus of centre of circle drawn on this chord as diameter is

(a) $x^2 + y^2 + ax = 0$ (b) $x^2 + y^2 + ay = 0$
(c) $x^2 + y^2 - ax = 0$ (d) $x^2 + y^2 - ay = 0$

57. Let R and C denote the set of real numbers and complex numbers respectively. The function $f : C \to R$ defined by $f(z) = |z|$ is

(a) one to one
(b) onto
(c) bijective
(d) neither one to one nor onto

58. If x is complex, the expression $\dfrac{x^2 + 34x - 71}{x^2 + 2x - 7}$ takes all which lie in the interval (a, b) where

(a) $a = -1, b = 1$ (b) $a = 1, b = -1$
(c) $a = 5, b = 9$ (d) $a = 9, b = 5$

59. If $a_1, a_2, a_3, \ldots, a_n$ be an AP of non-zero terms, then $\dfrac{1}{a_1a_2} + \dfrac{1}{a_2a_3} + \ldots + \dfrac{1}{a_{n-1}a_n}$ is equal to

(a) $\dfrac{n-1}{a_1a_n}$ (b) $\dfrac{n}{a_1a_n}$
(c) $\dfrac{n+1}{a_1a_n}$ (d) None of these

60. If B is an invertible matrix and A is a matrix, then

(a) rank (BA) = rank (A)
(b) rank $(BA) \geq$ rank (B)
(c) rank $(BA) >$ rank (A)
(d) rank $(BA) >$ rank (B)

61. A and B are two events such that $P(A) = 0.3$ and $P(A \cup B) = 0.8$. If A and B be independent events, then $P(B)$ is

(a) $\dfrac{3}{7}$ (b) $\dfrac{5}{7}$
(c) $\dfrac{6}{7}$ (d) None of these

62. If $\vec{\mathbf{u}}_1$ and $\vec{\mathbf{u}}_2$ be vectors of unit length and θ be the angle between them, then $\dfrac{1}{2}|\vec{\mathbf{u}}_2 - \vec{\mathbf{u}}_1|$ is

(a) $\sin\theta$ (b) $\sin\dfrac{\theta}{2}$
(c) $\cos\theta$ (d) $\cos\dfrac{\theta}{2}$

63. The image of the point (1, 2, 3) by the plane $x + y + z + 3 = 0$ is

(a) $(-5, 4, -3)$ (b) $(-5, -4, -3)$
(c) $(5, -4, 3)$ (d) $(5, 4, 3)$

64. If $P = \sin^2\theta + \cos^4\theta$, then for all θ

(a) $1 \leq P \leq 2$ (b) $\dfrac{3}{4} \leq P \leq 1$
(c) $\dfrac{1}{2} \leq P \leq \dfrac{3}{4}$ (d) $\dfrac{1}{4} \leq P \leq \dfrac{1}{2}$

65. The straight lines L_1, L_2, L_3 are parallel and lie in the same plane. A total of m points are taken on L_1, n points on L_2, k points on L_3. The maximum number of triangles formed with vertices at these points are

(a) ${}^{m+n+k}C_3$
(b) ${}^{m+n+k}C_3 - {}^mC_3 - {}^nC_3$
(c) ${}^{m+n+k}C_3 + {}^mC_3 + {}^nC_3$
(d) None of the above

66. If $y = \cos^{-1}(\cos x)$, then $\dfrac{dy}{dx}$ is

(a) 1 in the whole plane
(b) -1 in the whole plane
(c) 1 in the 2nd and 3rd quadrants of the plane
(d) -1 in the 3rd and 4th quadrants of the plane

67. The differential equation representing the family of curves $y^2 = 2c(x + c^{2/3})$, where c is a positive parameter, is of

(a) order 3, degree 3 (b) order 2 degree 4
(c) order 1, degree 5 (d) order 5, degree 1

68. The range of function $f(x) = {}^{7-x}P_{x-3}$ is

(a) $\{1, 2, 3\ 4\}$ (b) $\{3, 4, 5, 6\}$
(c) $\{0, 1, 2, 3, 4, 5\}$ (d) $\{1, 2, 3\}$

69. If $I = \left|\displaystyle\int_2^5 \frac{\sin x\, dx}{(1 + x^2)}\right|$, then

(a) $I \geq \dfrac{1}{4}$
(b) I lies in the interval $\left(\dfrac{1}{4}, \dfrac{1}{5}\right)$
(c) I lies in the interval $\left(\dfrac{1}{5}, \dfrac{1}{6}\right)$
(d) $I \leq \dfrac{3}{10}$

70. If $ax^2 + bx + c = 0$ and $2x^2 + 3x + 4 = 0$ have a common root where $a, b, c \in N$ (set of natural numbers), the least value of $a + b + c$ is

(a) 13 (b) 11
(c) 7 (d) 9

Answers

❒ PHYSICS

1. (d)	**2.** (c)	**3.** (d)	**4.** (c)	**5.** (d)	**6.** (c)	**7.** (d)	**8.** (c)	**9.** (d)	**10.** (a)
11. (d)	**12.** (c)	**13.** (c)	**14.** (d)	**15.** (b)	**16.** (a)	**17.** (a)	**18.** (d)	**19.** (c)	**20.** (d)
21. (b)	**22.** (b)	**23.** (c)	**24.** (c)	**25.** (b)	**26.** (a)	**27.** (b)	**28.** (b)	**29.** (b)	**30.** (b)
31. (b)	**32.** (c)	**33.** (b)	**34.** (d)	**35.** (b)	**36.** (a)	**37.** (c)	**38.** (b)	**39.** (a)	**40.** (a)

❒ CHEMISTRY

1. (a)	**2.** (d)	**3.** (a)	**4.** (b)	**5.** (a)	**6.** (b)	**7.** (a)	**8.** (a)	**9.** (b)	**10.** (c)
11. (c)	**12.** (d)	**13.** (c)	**14.** (b)	**15.** (a)	**16.** (c)	**17.** (b)	**18.** (a)	**19.** (d)	**20.** (d)
21. (d)	**22.** (d)	**23.** (c)	**24.** (d)	**25.** (a)	**26.** (c)	**27.** (d)	**28.** (b)	**29.** (d)	**30.** (c)
31. (a)	**32.** (b)	**33.** (a)	**34.** (b)	**35.** (a)	**36.** (d)	**37.** (a)	**38.** (c)	**39.** (b)	**40.** (*)

❒ MATHEMATICS

1. (a)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (d)	**6.** (a)	**7.** (d)	**8.** (b)	**9.** (b)	**10.** (b)
11. (b)	**12.** (c)	**13.** (c)	**14.** (c)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (d)	**19.** (a)	**20.** (a)
21. (b)	**22.** (d)	**23.** (c)	**24.** (b)	**25.** (d)	**26.** (c)	**27.** (d)	**28.** (c)	**29.** (a)	**30.** (d)
31. (c)	**32.** (b)	**33.** (d)	**34.** (c)	**35.** (b)	**36.** (c)	**37.** (a)	**38.** (b)	**39.** (b,d)	**40.** (c)
41. (c)	**42.** (c)	**43.** (a)	**44.** (c)	**45.** (a)	**46.** (d)	**47.** (d)	**48.** (d)	**49.** (b)	**50.** (b)
51. (c)	**52.** (a)	**53.** (b)	**54.** (a)	**55.** (b)	**56.** (c)	**57.** (d)	**58.** (c)	**59.** (a)	**60.** (a)
61. (b)	**62.** (b)	**63.** (b)	**64.** (b)	**65.** (d)	**66.** (d)	**67.** (c)	**68.** (d)	**69.** (d)	**70.** (d)

Note : * No option is correct.

Solutions

PHYSICS

1. The angular position of first diffraction minimum is

$$\sin\theta = \frac{\lambda}{a}$$

The phase difference

$$\phi = \frac{2\pi}{\lambda}\times\Delta x$$

$$\phi = \frac{2\pi}{\lambda}\times\lambda$$

$$\phi = 2\pi$$

2. The wavelengths of different members of Balmer series are given by

$$\frac{1}{\lambda} = R_H\left[\frac{1}{2^2}-\frac{1}{n_i^2}\right], \text{ where } n_i = 3, 4, 5, \ldots$$

The first member of Balmer series (H_α) corresponds to $n_i = 3$. It has maximum energy and hence the longest wavelength.

Therefore, wavelength of H_α line (or longest wavelength)

$$\frac{1}{\lambda_1} = R_H\left[\frac{1}{2^2}-\frac{1}{3^2}\right]$$

$$= 1.097\times10^7\left(\frac{5}{36}\right)$$

or $\lambda_1 = \dfrac{36}{5\times1.097\times10^7} = 6.563\times10^{-7}$ m

$$n = 6563\ \text{Å}$$

The wavelength of the Balmer series limit corresponds to $n_i = \infty$ and has got shortest wavelength.

Therefore, wavelength of Balmer series limit is given by

$$\frac{1}{\lambda_\infty} = R_H\left[\frac{1}{2^2}-\frac{1}{\infty^2}\right] = 1.097\times10^7\times\frac{1}{4}$$

or $\lambda_\infty = \dfrac{4}{1.097\times10^7} = 3.646\times10^{-7}$ m

$$= 3646\ \text{Å}$$

Only 4861 Å is between the first and last line of the Balmer series.

3. Given, $K_{max} = 3$ eV

$$h = 4.125 \times 10^{-15} \text{ eV}$$

$$h\nu = K_{max} + W$$

$$4.125 \times 10^{-15} \times 10^{15} = 3 + W$$

or $$4.125 = 3 + W$$

or $$W = 1.125$$

The threshold frequency

$$\nu_0 = \frac{W}{h}$$

$$\nu_0 = \frac{1.125}{4.125 \times 10^{-15}}$$

$$\nu_0 = \frac{1.125 \times 10^{15}}{4.125}$$

$$\nu_0 = 2.72 \times 10^{14} \text{ Hz}$$

4. The binding energy of a nucleus may be defined as the energy equivalent to the mass defect of the nucleus.

If Δm is mass defect then according to Einstein's mass energy relation.

Binding energy

$$= \Delta m \cdot c^2 = [\{Zm_p + (A - Z)m_n\} - M]c^2$$

$$= (7 \times 1.00783 + 7 \times 1.00867 - 14.00307)\ uc^2$$

or $$\text{BE} = 0.1124 \times 931.5 \text{ MeV}$$

or $$\text{BE} = 104.6$$

5. When reverse bias is increased the electric field across the junction also increases. At some stage the electric field becomes so high that it breaks the covalent bonds creating electron-hole pairs. This mechanism is known as zener breakdown. In breakdown region for a long range of load (R_L) the voltage remains the same though the current may be large.

6. The output $F = (W + X)(W + Y)$

$$= W + (X \cdot Y)$$

7. Dimensions of $\varepsilon_0 = [\text{M}^{-1}\text{L}^{-3}\text{T}^4\text{A}^2]$

Dimensions of $L = [\text{L}]$

Dimensions of $\Delta V = [\text{ML}^2\text{T}^{-3}\text{A}^{-1}]$

Dimensions of $\Delta t = [\text{T}]$

As $$X = \varepsilon_0 L \frac{\Delta V}{\Delta t}$$

Dimensions of X

$$= \frac{[\text{M}^{-1}\text{L}^{-3}\text{T}^4\text{A}^2][\text{L}][\text{ML}^2\text{T}^{-3}\text{A}^{-1}]}{[\text{T}]}$$

$$= [\text{A}]$$

8. Displacement $x = at + bt^2 - ct^3$

Velocity, $v = \frac{dx}{dt} = a + 2bt - 3ct^2$ ...(i)

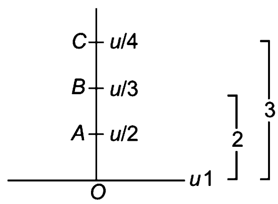
and acceleration, $a = \frac{d^2x}{dt^2} = 2b - 6ct$ ...(ii)

When acceleration $a = 0$; $t = \frac{b}{3c}$

Substituting the value of t in Eq. (i), we get

$$v = a + \frac{b^2}{3c}$$

9.



The equation of motion

$$\left(\frac{u}{2}\right)^2 = u^2 - 2g(AO)$$

$$2g \times AO = u^2 - \frac{u^2}{4} = \frac{3u^2}{4}$$

$$= \frac{3u^2}{8g}$$

When particle will reach at point B

$$\left(\frac{u}{3}\right)^2 = u^2 - 2g(OB)$$

$$OB = \frac{8u^2}{18g}$$

When particle will reach at point C

$$\left(\frac{u}{4}\right)^2 = u^2 - 2g(OC)$$

$$OC = \frac{15u^2}{32g}$$

$$AB = OB - OA = \frac{u^2}{g}\left[\frac{8}{18} - \frac{3}{8}\right] = \frac{5u^2}{72g}$$

$$BC = OC - OB = \frac{u^2}{g}\left[\frac{15}{32} - \frac{8}{18}\right] = \frac{7u^2}{288g}$$

The ratio, $$\frac{AB}{BC} = \frac{20}{7}$$

10. Given, $\vec{\mathbf{r}}_1 = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

and $\vec{\mathbf{r}}_2 = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

Now, $\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 = 1\hat{\mathbf{i}} - 1\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$

and $\vec{\mathbf{F}} = 4\hat{\mathbf{i}} + \hat{\mathbf{j}} + 6\hat{\mathbf{k}}$

$\therefore$ Work done $= \vec{\mathbf{F}} \cdot \vec{\mathbf{r}}$

$$W = (4\hat{\mathbf{i}} + \hat{\mathbf{j}} + 6\hat{\mathbf{k}})(\hat{\mathbf{i}} - \hat{\mathbf{j}} + 9\hat{\mathbf{k}})$$
$$= 4 - 1 + 54 = 57 \text{ J}$$

11. Given, $\vec{\mathbf{p}}(t) = A[\hat{\mathbf{i}} \cos(kt) - \hat{\mathbf{j}} \sin(kt)]$

As $\vec{\mathbf{F}} = \dfrac{d\vec{\mathbf{p}}}{dt}$

$$\vec{\mathbf{F}} = \frac{d}{dt}[A\{\hat{\mathbf{i}} \cos(kt) - \hat{\mathbf{j}} \sin(kt)\}]$$
$$= Ak[-\hat{\mathbf{i}} \sin(kt) - \hat{\mathbf{j}} \cos(kt)]$$

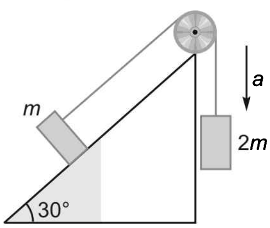
Now, $\vec{\mathbf{F}} \cdot \vec{\mathbf{p}} = A^2 k(-\cos kt \sin kt + \sin kt \cos kt)$

$$= 0$$

$\therefore$ The momentum and force are perpendicular to each other.

12. As $2m > m$

The acceleration of the two block system will be as shown in figure.



The equation for block of mass $2m$ is

$$2mg - T = 2ma \quad \ldots\text{(i)}$$

Similarly, for block of mass m

$$T - mg \sin 30° = ma \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we have

$$2mg - \frac{mg}{2} = 3\,ma$$

$\Rightarrow$ $a = \dfrac{g}{2}$

13. Potential energy $U = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r}$

or $U \propto \dfrac{1}{r}$

When r decreases U increases and *vice-versa*. Moreover, potential energy as well as force are positive, if there is repulsion between the particles and negative if there is attraction.

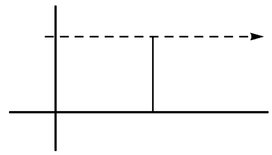
So, option (c) is incorrect.

14. There is no loss of energy. Therefore, the final velocity is the same as the initial velocity.

Hence, the speed of roller coaster at point D is u.

15. Angular momentum

= (Linear momentum) × (perpendicular distance to line of motion from the axis)



or angular momentum is moment of momentum. Here, the angle goes on decreasing from 90° but the perpendicular distance to the line of motion remains constant. Therefore, angular momentum is also constant (linear momentum $p = mv$ is constant).

16. As initially both the particles were at rest therefore, velocity of centre of mass was zero and there is no external force on the system so speed of centre of mass remains constant, *ie*, it should be equal to zero.

17. According to Kepler's law of periods

$$T^2 \propto a^3 \quad [a = \text{semi-major axis}]$$

Here, in case I a is $7R$ as satellite is $6R$ above the earth and for a geostationary satellite $T = 24$ h

$\therefore$ $$(24)^2 \propto (7R)^3 \quad \ldots\text{(i)}$$

Similarly for case II

$$T^2 \propto (3.5R)^3 \quad \ldots\text{(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{(24)^2}{T^2} = \frac{(7R)^3}{(3.5R)^3}$$

$\Rightarrow$ $$T^2 = \frac{(24)^2}{8}$$

or $$T = 6\sqrt{2} \text{ h}$$

18. In vector form of Coulomb's law proves that the forces $\vec{\mathbf{F}}_{12}$ and $\vec{\mathbf{F}}_{21}$ are equal and opposite.

or $$\vec{\mathbf{F}}_{21} = -\vec{\mathbf{F}}_{12}$$

$\therefore$ $$\vec{\mathbf{F}}_{pe} = -\vec{\mathbf{F}}_{ep}$$

$$\vec{\mathbf{F}}'_{pe} = -\vec{\mathbf{F}}'_{ep}$$

and $\vec{\mathbf{F}}_{pe} + \vec{\mathbf{F}}_{ep} = -\vec{\mathbf{F}}'_{ep} - \vec{\mathbf{F}}'_{pe}$

So, option (d) is incorrect.

19. For brass rod A

$$\text{Volume } V_1 = \pi(2r)^2 \times l \quad \dots(i)$$

For volume expansion,

$$V_1' = V_1(1 + \gamma\Delta t)$$

$$\Rightarrow \quad V_1' - V_1 \propto V_1$$

or $$\Delta V_1 \propto V_1 \quad \dots(ii)$$

Similarly, for brass rod B

Volume $$V_2 = \pi(r)^2 \times 2l \quad \dots(iii)$$

and $$\Delta V_2 \propto V_2 \quad \dots(iv)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{V_1}{V_2} = \frac{\pi 4r^2 l}{\pi r^2 2l} = \frac{2}{1}$$

From Eqs. (ii) and (iv),

$$\frac{\Delta V_1}{\Delta V_2} = \frac{2}{1}$$

20. According to Bernoulli's theorem the total energy (pressure energy, potential energy and kinetic energy) per unit volume or mass of an incompressible and non-viscous fluid in steady flow through a pipe remains constant throughout the flow, provided there is no source or sink of the fluid along the length of the pipe. Mathematically for unit volume of liquid flowing through a pipe.

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$$= \text{constant}$$

Here, $h_1 = h_2$, $v_1 = v$, $p_1 = p$ and $v_2 = 2v$

$$p + \frac{1}{2}\rho v^2 = p_2 + \frac{1}{2}\rho(4v^2)$$

or $$p_2 = p - \frac{3}{2}\rho v^2$$

21. Weight of the body

$$= \text{Weight of liquids displaced}$$

$$V \times d \times g = \frac{V}{2} \times 0.8 \times g + \frac{V}{2} \times 13.6 \times g$$

$$d = \frac{0.8}{2} + \frac{13.6}{2} = 0.4 + 6.8$$

$$= 7.2 \text{ g cm}^{-3}$$

22. Young's modulus $Y = \dfrac{\text{Stress}}{\text{Strain}}$

$$= \frac{\frac{F}{A}}{\text{Strain}}$$

or $$Y = \frac{mg}{A \times \text{strain}}$$

or $$m = \frac{Y \times A \times \text{strain}}{g}$$

$$= \frac{2 \times 10^{11} \times 10^{-3} \times 3 \times 10^{-6}}{10}$$

$$= 60 \text{ kg}$$

23. Work done $\Delta W = p\Delta V$

At constant pressure

$$\Delta W = p(V_f - V_i) = nR(T_f - T_i)$$

At constant temperature

$$\Delta W = nRT \ln\left(\frac{V_f}{V_i}\right) = nRT \ln\left(\frac{p_i}{p_f}\right)$$

$$\therefore \quad \Delta W_{AB} = 1 \times R \times (2T - T) = RT$$

$$\Delta W_{BC} = 1 \times R \times 2T \ln\frac{2p}{p} = 2RT \ln 2$$

$$\Delta W_{CD} = 1 \times R \times (T - 2T) = -RT$$

$$\Delta W_{DA} = 1 \times R \times T \ln\left(\frac{p}{2p}\right) = RT \ln\left(\frac{1}{2}\right)$$

Net work done in the complete cycle is

$$\Delta W = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CD} + \Delta W_{DA}$$

$$= RT + 2RT \ln 2 - RT + RT \ln\left(\frac{1}{2}\right)$$

$$= 2RT \ln 2 + RT \ln 1 - RT \ln 2$$

$$= 2RT \ln 2 - RT \ln 2 \quad (\because \ln 1 = 0)$$

$$= RT \ln 2$$

24. For isochoric process, internal energy

$$\Delta U = nC_V\Delta T = 420 \text{ J}$$

Molar specific heat

$$C_V = \frac{\Delta U}{n\Delta T}$$

$$= \frac{420}{2 \times 10} = 21 \text{ JK}^{-1} \text{ mol}^{-1}$$

25. For first vessel number of moles

$$n_1 = \frac{m_1}{M_1} = \frac{32}{32} = 1$$

Volume = V, Temperature = T

$\therefore \quad p_1 V = RT \quad \ldots(i)$

For second vessel

Number of moles = $n_2 = \frac{m_2}{M_2} = \frac{4}{2} = 2$

Volume = V, Temperature = $2T$

$\therefore \quad p_2 V = 2R(2T) \quad \ldots(ii)$

From Eqs. (i) and (ii),

$$p_2 = 4p_1 = 4p$$

26. The frequency, when a sonometer wire of vibrating length is 48 cm.

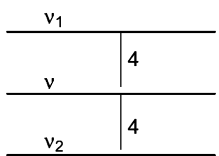
$$\nu_1 = \frac{c}{2 \times l_1} = \frac{c}{2 \times 0.48} = \frac{c}{0.96}$$

The frequency, when a sonometer wire of vibrating length is 50 cm.

$$\nu_2 = \frac{c}{2 \times l_2} = \frac{c}{2 \times 0.50} = \frac{c}{1.00}$$

$$\nu_1 - \nu_2 = 8$$

$$\nu_1 = \nu_2 + 8$$



$$\therefore \quad \frac{\nu_1}{\nu_2} = \frac{1.00}{0.96}$$

$$\nu_1 = \frac{1.00}{0.96} \nu_2$$

$$\because \quad \nu_2 + 8 = \nu_2 \times \frac{100}{96}$$

$$\nu_2 = 192$$

The frequency of the tuning fork.

$$\nu = \nu_2 + 4 = 192 + 4 = 196 \text{ Hz}$$

27. Given, $y = 4\cos^2 \frac{t}{2} \sin(1000t)$

$$= 2(1 + \cos t)\sin(1000t)$$

$$\left[1 + \cos\theta = 2\cos^2 \frac{\theta}{2}\right]$$

$$= 2\sin 1000t + 2\cos t \sin 1000t$$

$$= 2\sin 1000t + \sin(1001)t + \sin(999)t$$

Therefore, it consists of 3 SHM's.

28. The progressive wave given is

$$y = 0.1\sin\left(10\pi t - \frac{5}{11}\pi x\right)$$

Comparing it with general equation of progressive wave

$$y = a\sin(\omega t - kx)$$

we get $\quad k = \frac{5\pi}{11}$

or $\quad \frac{2\pi}{\lambda} = \frac{5\pi}{11}$

$\Rightarrow \quad \lambda = \frac{22}{5} = 4.4$ cm

Moreover, $\quad \omega t = 10\pi t$

or $\quad 2\pi\nu t = 10\pi t$

$\therefore \quad \nu = \frac{10\pi}{2\pi} = 5$ Hz

$\therefore$ Velocity $v = \lambda\nu = 22 \text{ cm s}^{-1}$

29. A positively charged particle free to move in electric field will move in the direction of electric field whereas a negatively charged particle will move in opposite direction of the field.

31. Potential energy of charges q_1 and q_2, r distance apart

$$= \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$$

For $r = 0.1$ m,

$$U_1 = \frac{1}{4\pi\varepsilon_0} \frac{12 \times 10^{-6} \times 5 \times 10^{-6}}{0.1}$$

$$= \frac{9 \times 10^9 \times 60 \times 10^{-12}}{0.1} = 5.4 \text{ J}$$

For $r = 0.06$ m,

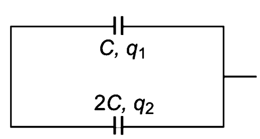
$$U_2 = \frac{9 \times 10^9 \times 60 \times 10^{-12}}{0.06} = 9 \text{ J}$$

$\therefore$ Work done = (9 − 5.4) J = 3.6 J

32. The charge

$$q_1 = CV_0$$

or $\quad V_0 = \frac{q_1}{C} \quad \ldots(i)$



$\therefore$ Capacitors are in parallel, in parallel V_0 is same for all capacitors.

$\therefore$ for second capacitor $V_0 = \frac{q_2}{2C} \quad \ldots(ii)$

From Eqs. (i) and (ii),

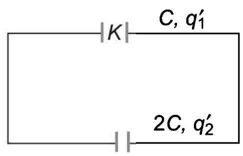
$$q_2 = 2q_1 \quad \ldots(iii)$$

After disconnecting the battery, the region between the plates of the capacitor C is completely filled with a material of dielectric constant ($K = 2$).

Then, $V_1 = \frac{q_1}{CK} = \frac{q_1}{2C}$

and $V_2 = \frac{q_2}{2C} = \frac{2q_1}{2C} = \frac{q_1}{C}$ [from Eq. (iii)]

Charge will flow from 2 to 1 till



$$\frac{q'_2}{2C} = \frac{q'_1}{KC}$$

$$\frac{q'_2}{2C} = \frac{q'_1}{2C}$$

ie, $q'_1 = q'_2$

Earlier potential $V_0 = \frac{q_1}{C}$

Now it is $V_0 = \frac{q'_1}{2C}$

Now, $q_1 + q_2 = 3q_1$ [from Eq. (iii)]

and $q'_1 + q'_2 = 3q_1$

or $2q'_1 = 3q_1$

or $= q'_1 = \frac{3q_1}{2}$

$\therefore$ New potential $\frac{q'_1}{2C} = \frac{3q_1}{4C}$

$$V = \frac{3V_0}{4} \quad [\because q_1 = V_0 C]$$

33. The power of Ist bulb

$$P_1 = \frac{V^2}{R_1} = \frac{(200)^2}{R_1}$$

$$100 = \frac{(200)^2}{R_1}$$

or $R_1 = \frac{200 \times 200}{100} = 400\ \Omega$

The power of IInd bulb

$$P_2 = \frac{(200)^2}{R_2}$$

or $200 = \frac{(200)^2}{R_2}$

or $R_2 = \frac{200 \times 200}{200} = 200\ \Omega$

The bulbs are joined in series.

So, $R = R_1 + R_2$

$= 400 + 200 = 600\ \Omega$

The total power

$$P = \frac{V^2}{R} = \frac{(200)^2}{600}$$

$$P = 66.7\ \text{W}$$

34. The resistance *AB*, *BC* and *CD* in series. The total resistance is

$$R_1 = 2 + 2 + 2 = 6\ \Omega$$

The resistance *AE*, *EF* and *FD* in series. The total resistance is

$$R_2 = 2 + 2 + 2 = 6\ \Omega$$

The resistance *BE* and *CF* are ineffective.

$\therefore$ R_1 and R_2 are in parallel

$\because$ The total resistance,

$$R = \frac{6 \times 6}{6 + 6} = 3\ \Omega$$

The current in the circuit,

$$I = \frac{V}{R}$$

$$I = \frac{3}{3} = 1.0\ \text{A}$$

35. Magnitude of the magnetic moment

$$M = I \cdot A \quad \begin{bmatrix}\text{where } I \text{ is the current} \\ \text{and } A \text{ in the area}\end{bmatrix}$$

The current produced in one revolution

$$I = ev = e\frac{2\pi}{T}$$

$\therefore$Magnetic moment $= \frac{2\pi}{T}|e|A$

As the electron is flowing in the anticlockwise direction. The current is flowing in the clockwise direction.

$\therefore$ $M = -\frac{2\pi}{T}|e|A$

36. Given, $B = 0.30 \times 10^{-4}\ \text{Wbm}^{-2}$, $l = 10$ m

and $v = 5.0\ \text{ms}^{-1}$

The induced potential gradient

$$V = Bvl$$

$$V = -0.30 \times 10^{-4} \times 5 \times 10$$

$$V = -1.5 \times 10^{-3}\ \text{Vm}^{-1}$$

From west to east,

$$V = +1.5 \times 10^{-3}\ \text{Vm}^{-1}$$

37. Given, self inductance, $L = 1.8 \times 10^{-4}$ H

Resistance, $R = 6\ \Omega$

When self inductance and resistance is broken up into identical coils.

then, self inductance of each coil

$$= \frac{1.8 \times 10^{-4}}{2}\ \text{H}$$

Resistance of each coil

$$= \frac{6\ \Omega}{2} = 3\ \Omega$$

Coil are then connected in parallel

$$\therefore \quad L' = \frac{\frac{1.8}{2} \times 10^{-4} \times \frac{1.8}{2} \times 10^{-4}}{\frac{1.8}{2} \times 10^{-4} + \frac{1.8}{2} \times 10^{-4}}$$

$$= 0.45 \times 10^{-4}\ \text{H}$$

and $R' = \frac{3 \times 3}{3 + 3} = 1.5\ \Omega$

Time constant $= \frac{L'}{R'}$

$$= \frac{0.45 \times 10^{-4}}{1.5} = 0.3 \times 10^{-4}\ \text{s}$$

38. Given, $L = 20$ mH $= 20 \times 10^{-3}$ H

$$C = 50\ \mu\text{F} = 50 \times 10^{-6}\ \text{F}$$

For LC circuit the frequency,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

or $T = 2\pi\sqrt{LC}$ $\left[\therefore T = \frac{1}{f}\right]$

At time $t = \frac{T}{4}$, energy stored is completely magnetic

The time, $t = \frac{T}{4}$

$$t = \frac{2\pi\sqrt{LC}}{4}$$

or $t = \frac{2\pi\sqrt{20 \times 10^{-3} \times 50 \times 10^{-6}}}{4}$

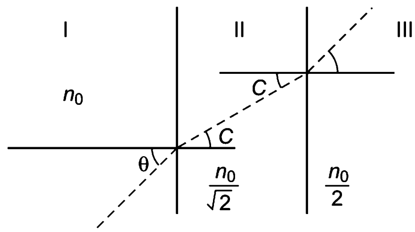
or $t = \frac{3.14\sqrt{10^{-6}}}{2}$

or $t = \frac{3.14 \times 10^{-3}}{2}$

or $t = 1.57 \times 10^{-3}$ s

$= 1.57$ ms

39.



The critical angle for region II and III

$$\sin C = \frac{\mu_{III}}{\mu_{II}} = \frac{\frac{n_0}{2}}{\frac{n_0}{\sqrt{2}}} = \frac{1}{\sqrt{2}}$$

ie $\angle C = 45°$

The ray, if incident at 45° at the interface of II and III it will be totally internally reflected.

Now, from Snell's law in region I and II.

$$n_0 \sin\theta = \frac{n_0}{\sqrt{2}} \sin C$$

or $\sin\theta = \frac{1}{\sqrt{2}} \times \sin 45°$

or $\sin\theta = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

or $\sin\theta = \frac{1}{2}$

or $\theta = 30°$

40. The refractive index $\mu = \frac{1}{\sin\theta_C}$

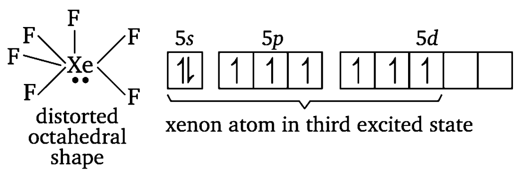
$$\mu = \frac{1}{\sin 45°} = \sqrt{2} = 1.414$$

Because the refractive index for green is 1.44 and blue is 1.47.

So, red alone will be transmitted.

CHEMISTRY

1. In XeF_6, the oxidation state of Xe is +6. The shape of XeF_6 should be pentagonal bipyramid due to sp^3d^3 hybridization but due to the presence of one lone pair at one *trans* position its shape becomes distorted octahedral.



2. In disproportionation reaction, compounds are simultaneously formed that contain a given element in a more oxidised and more reduced state than the initial one. In ClO_4^- oxidation number of Cl is +7 and it cannot increase it further, so ClO_4^- will not get oxidised and so, will not undergo disproportionation reaction.

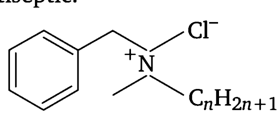
3. The dispersal of the charge stabilizes the carbocation. More the number of alkyl groups; the greater the dispersal of positive charge and therefore, more the stability of carbocation, thus $C_2H_5^+ > CH_3^+$. $O—CH_3$ is also an electron donating group, thus it will increase the stability of carbocation, hence, the correct order of stability is $C > B > A$.

4. A compound is said to be aromatic if it meets of the following criteria.

(i) The rings of the compound should be planar.

(ii) The cyclic system must contain $(4\pi + 2)$ π-electrons. Only option (b) contains 6π-electron, so it is aromatic.

5. Benzalkonium chloride, also known as alkyl dimethyl benzyl ammonium chloride is nitrogeneous. Cationic surfactant and it is used as a antiseptic.



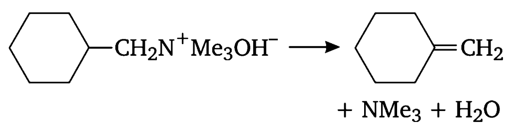
$n = 8, 10, 12, 14, 16, 18$

6. Carbonyl carbon becomes more reactive toward nucleophilic addition depending upon the magnitude of the positive charge on the carbonyl carbon atom. The introduction of negative inductive effect showing group ($-I$ effect) increases the reactivity while introduction of alkyl group ($+I$ effect) decreases the reactivity. So, large alkyl group decreases the reactivity of $>C=O$.

7. **Markownikoff's addition** The negative part of the unsymmetrical reagents adds to a less hydrogenated (more substituted) carbon atom of the double bond. In ICl, Cl is more electronegative. So, it will take negative charge, *ie*, I^+Cl^-. So, the product is

$$RCH_2CH{=}CH_2 + \overset{+}{I}\overset{-}{Cl} \longrightarrow R—CH_2—\underset{\underset{Cl}{|}}{CH}—CH_2I$$

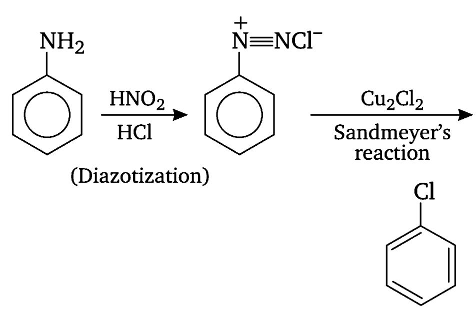
8. The formation of the alkene in an elimination reaction is called Hofmann elimination (Thermal decomposition). Elimination of hydrogen occurs from the β-carbon. So,



9. Due to resonance, the carbonyl group of benzoic acid is coplanar with the ring. If the electron withdrawing substituent (*ie*, $-I$ showing) is present at *ortho* position, it prevents the coplanarity and thus, the resonance. Hence, makes the acid more stronger.

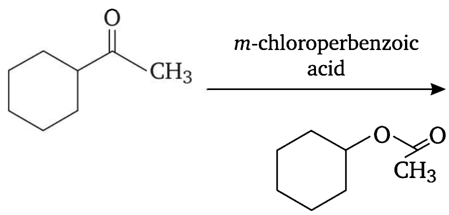
Thus, among the given acids, *ortho* hydroxy benzoic acid is the most acidic.

10.



11. Presence of methyl group increases the electron density on nitrogen. So, increases the basicity. Aniline is weaker base than the primary aliphatic amines and this may be explained by resonance. The lone pair of N is involved in resonance, thus not available for donation. That's why basic strength of aryl amines (aniline) is lowest.

12. Baeyer-Villiger oxidation involves transformation of a ketone into ester by reaction with a peracid. The net change is the insertion of an oxygen atom beween the carbonyl carbon and an adjacent carbon of the ketone. So, it is an example of Baeyer-Villiger oxidation, the most suitable reagent is *m*-chloroperbenzoic acid.



13. Fats and lipids are hydrolysed by lipase.

14. $(CH_3)_3C—CH_2OH$ is *neo*-pentyl alcohol.

15. Electrophilic nitration is possible when ring is activated. Electron withdrawing groups like $—NO_2$, —COOH, etc., deactivates the benzene nucleus while electron releasing groups like, $—CH_3$, $—C_2H_5$, etc., activates the benzene nucleus. Thus, toluene, due to the presence of electron releasing CH_3 group, is most reactive towards electrophilic nitration.

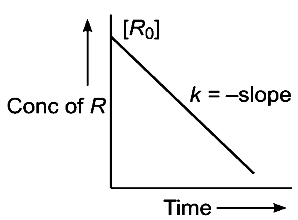
16. Chlorobenzene has only one deactivating group, *ie*, —Cl. In 2,4-dinitrochlorobenzene three deactivating groups, *ie*, two $—NO_2$ and one —Cl are present and *p*-nitrochlorobenzene two deactivating groups, *ie*, one NO_2 and one Cl are present. So, the order of reactivity is $A > C > B$.

17. For a compound to be soluble, the hydration energy must be greater than the lattice energy. Since, NaCl is soluble in water but insoluble in benzene.

$$\Delta H_{\text{hydration}} > \Delta H_{\text{lattice energy in water}}$$

and $$\Delta H_{\text{hydration}} < \Delta H_{\text{lattice energy in benzene}}$$

18.



[Variation in the concentration *Vs* time plot for a zero order reaction]

19. Electrochemical cell are based upon the reaction between various electrolytes. The reaction given in option (d) does not involve electrolytes, so it cannot be a base for electrochemical cell.

20. 10 volume = 1 volume of H_2O_2 gives 10 volume of O_2 at NTP.

$$\underset{2(2+32)=68\text{ g}}{2H_2O_2} \longrightarrow \underset{22400\text{ mL at NTP}}{2H_2O} + O_2$$

At NTP

$\because$ 22400 mL of O_2 is obtained from

$= 68$ g H_2O_2

$\therefore$ 10 mL of O_2 is obtained from

$$= \frac{68 \times 10}{22400} = 0.03035\text{ g } H_2O_2$$

1 mL of H_2O_2 solution contains

$= 0.03035$ g H_2O_2

100 mL of H_2O_2 solution contains

$= 0.03035 \times 100$

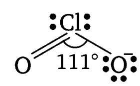
$= 3.035$ g H_2O_2

$\therefore$ Strength of 10 volume H_2O_2

$= 3.035 \times 10$

$= 30.35$ g/L

21. ICl_2^-, I_3^-, N_3^- are linear but ClO_2^- is angular due to sp^3 hybridisation of Cl atom.



So, ClO_2^- is non-linear.

22. Order of reaction is an experimentally determined quantity and thus, cannot be predicted from the given equation.

23. The basic structural unit in silicates is SiO_4 tetrahedron. In SiO_4^{4-} unit, silicon atom is bonded to four oxide ions tetrahedrally.

24. H_2O_2 is acting as a reducing agent in the reaction that involve increase in the oxidation state of oxygen of H_2O_2 (*ie*, in which H_2O_2 is being oxidised).

Reduction (oxidising agent)

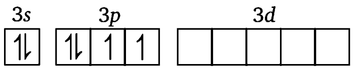
$$\overset{+1}{Ag_2O} + H_2\overset{-1}{O_2} \longrightarrow 2\overset{0}{Ag} + H_2\overset{-2}{O} + \overset{0}{O_2}$$

Oxidation (reducing agent)

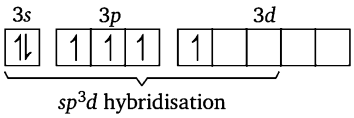
25. BeO is most acidic in nature amongst the given choices because acidity of oxides increases with decrease in electropositive character of central atom.

26. S in SF_4 possesses trigonal bipyramidal structure with sp^3d hybridisation.

S in ground state



S in excited state



27. 1 mole of glucose is oxidised to give 38 moles of ATP, So, 2 moles will give $= 2 \times 38 = 76$ moles of ATP.

28. Given, product = 0.1 M and reactant = 1 M

$$E_{cell} = E^\circ_{cell} - \frac{0.0591}{2} \log \frac{[\text{Products}]}{[\text{Reactants}]}$$

$$1.50 = E^\circ_{cell} - 0.02955 \log \left[\frac{0.1}{1}\right]$$

$$E^\circ_{cell} = 1.470 \text{ V}$$

$$E^\circ_{cell} = E^\circ_{H^+/H_2} - E^\circ_{M/M^{2+}}$$

$$E^\circ_{M/M^{2+}} = -1.470$$

So, $E^\circ_{M^{2+}/M} = 1.470 \text{ V}$

29. $117 = [\text{Rn}]\ 5f^{14}, 6d^{10}, 7s^2 7p^5$

Since, the last electron enters in p-orbital, it will be a p-block element and its group number

$= 5 + 2 = 7$ (VIIA)

So, the element would be placed in halogen family.

30. $K_2SO_4 \longrightarrow 2K^+ + SO_4^{2-}$

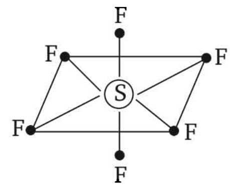
It gives 3 ions, hence, the van't Hoff factor = 3.

31. ZnS has zinc blende type structure (*ie*, ccp structure). The S^{2-} ions are present at the corners of the cube and at the centre of each face. Zinc ions occupy half of the tetrahedral sites. Each zinc ion is surrounded by four sulphide ions which are disposed towards the corner of regular tetrahedron. Similarly, S^{2-} ion is surrounded by four Zn^{2+} ions.

32. Rate of disintegration is not affected by environmental conditions.

33. pH = 0 means $[H^+] = 10^0 = 1$ M. Hence, solution is strongly acidic.

34. SF_6 has octahedral geometry, sp^3d^2 hybridisation and bond angle is 90°.



% of d-character

$$= \frac{2\ (\text{no. of } d\text{-orbitals})}{6(\text{total hybridised orbitals})} \times 100$$

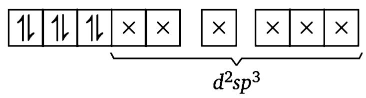
= 33%

So, SF_6 are bond angle = 90°

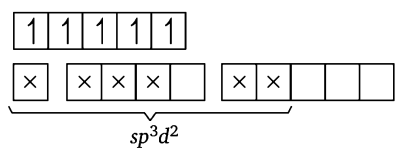
and d-character = 33%

35. $[Fe(CN)_6]^{4-}$ (strong field) $\longrightarrow Fe^{2+} \rightarrow$

$1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^6$

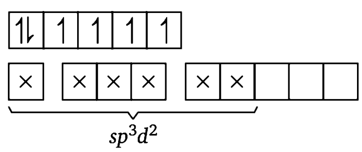


$[FeF_6]^{3-}$ (weak field) $\longrightarrow Fe^{3+} \rightarrow 1s^2 2s^2 2p^6 3s^2 3p^6 3d^5$

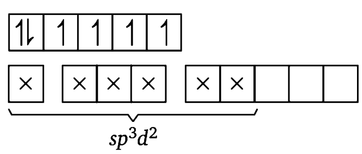


$[Co(C_2O_4)_3]^{3-}$ (weak field) $\longrightarrow Co^{3+} \rightarrow$

$1s^2 2s^2 2p^6 3s^2 3p^6 3d^6$



$[CoF_6]^3$ (weak field) $\longrightarrow Co^{3+} \rightarrow 1s^2 2s^2 2p^6 3s^2 3p^6 3d^6$



Hence, $[Fe(CN)_6]^{4-}$ due to the absence of unpaired electrons is diamagnetic.

36. Ideal solution obeys Raoult's law at every range of concentration. So, the second component must follow. Raoult's law in the range. When x_2 is $0 \le x_2 \le 1$.

37. The phenomenon of the precipitation of a colloidal solution by the addition of the excess of an electrolyte is called coagulation. When oppositely charged sols are mixed in almost equal proportions, their charges are neutralised. So, statement (a) is wrong.

38. In an adiabatic process, no exchange of heat takes place between the system and surroundings, *ie*, $dQ = 0$. Such a condition exists when the system is thermally isolated.

39. Dissociation constant

$H_2O \rightleftharpoons H^+ + OH^- : [H^+] = [OH^-] = 1 \times 10^{-7}$ M

and $[H_2O] = 1$ g/mL $= 1000$ gL^{-1}

$$= \frac{1000}{18} \text{ mol L}^{-1} = 55.56 \text{ M}$$

$$K = \frac{[H^+][OH^-]}{[H_2O]} = \frac{10^{-14}}{55.6}$$

$$K_w = 1 \times 10^{-14}$$

So, $K_w = 55.6 \times K$

40. Given, $\Delta E = -5$ cal, $\Delta S = -10$ cal K^{-1}

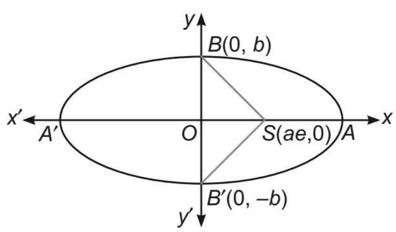
$$A(g) + B(g) \longrightarrow C(g)$$

$$\Delta H = \Delta E + \Delta nRT$$
$$= -5 - 1 \times 2.0 \times 298 = -601$$
$$\Delta G = \Delta H - T\Delta S$$
$$= -601 - 298 \times (-10) = 2379 \text{ cal}$$

So, none of the given options is correct.

MATHEMATICS

1. Since, $BB'S$ is an equilateral triangle.



$\therefore \quad BS = BB'$

$\Rightarrow \sqrt{(ae-0)^2 + (0-b)^2} = \sqrt{0 + (2b)^2}$

$\Rightarrow \quad a^2e^2 + b^2 = 4b^2 \quad \Rightarrow \quad b^2 = \frac{a^2e^2}{3}$

$\because \quad e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{e^2}{3}}$

$\Rightarrow \quad \frac{4e^2}{3} = 1 \quad \Rightarrow \quad e = \frac{\sqrt{3}}{2}$

2. Since, the given planes

$$x - cy - bz = 0$$
$$cx - y + az = 0$$

and $\quad bx + ay - z = 0$

passes through a line.

$$\therefore \quad \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$\Rightarrow 1(1 - a^2) + c(-c - ab) - b(ac + b) = 0$

$\Rightarrow \quad 1 - a^2 - c^2 - abc - abc - b^2 = 0$

$\Rightarrow \quad a^2 + b^2 + c^2 + 2abc = 1$

3. Given,

$\cos^{-1} x + \cos^{-1} y + \cos^{-1} z + \cos^{-1} t = 4\pi$

Which is possible only when

$\cos^{-1} x = \cos^{-1} y = \cos^{-1} z = \cos^{-1} t = \pi$

[$\because$ Domain of $\cos^{-1} x$ is $[0, \pi]$]

$\Rightarrow \quad x = y = z = t = \cos \pi = -1$

$\therefore \quad x^2 + y^2 + z^2 + t^2$

$= (-1)^2 + (-1)^2 + (-1)^2 + (-1)^2$

$= 4$

4. The total number of ways $= 6^4 = 1296$

$\therefore$ Required number of ways

$= 1296 -$ (none of the number shows 2)

$= 1296 - 5^4$

$= 671$

5. $\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\therefore \quad f'(5) = \lim_{h \to 0} \frac{f(5)f(h) - f(5)}{h}$

$\Rightarrow \quad f'(5) = \lim_{h \to 0} f(5) \frac{[f(h) - 1]}{h}$ ...(i)

Since, $f(x+y) = f(x)f(y)$

$\Rightarrow \quad f(0+0) = f(0)f(0)$

$\Rightarrow \quad f(0)\{f(0) - 1\} = 0 \Rightarrow f(0) = 1$

$\therefore$ From Eq. (i),

$$f'(5) = \lim_{h \to 0} f(5)\left[\frac{f(h) - f(0)}{h - 0}\right]$$

$\Rightarrow \quad f'(5) = f(5)f'(0)$

$= 2 \times 3 = 6$

6. Given, $\frac{d^2y}{dx^2}(x^2+1) = 2x\frac{dy}{dx}$

$$\Rightarrow \quad \frac{\frac{d^2y}{dx^2}}{\frac{dy}{dx}} = \frac{2x}{x^2+1}$$

On integrating both sides, we get

$$\log\frac{dy}{dx} = \log(x^2+1) + \log c$$

$\Rightarrow \quad \frac{dy}{dx} = c(x^2+1)$ ...(i)

As at $x = 0$, $\frac{dy}{dx} = 3$

$\therefore \quad 3 = c(0+1) \Rightarrow c = 3$

$\therefore$ From Eq. (i),

$$\frac{dy}{dx} = 3(x^2+1)$$

$\Rightarrow \quad dy = 3(x^2+1)dx$

Again integrating both sides, we get

$$y = 3\left(\frac{x^3}{3} + x\right) + c_1$$

At point (0, 1)

$$1 = 3(0+0) + c_1 \Rightarrow c_1 = 1$$

$\therefore \quad y = 3\left(\frac{x^3}{3} + x\right) + 1$

$\Rightarrow \quad y = x^3 + 3x + 1$

7. We know, $\lim_{n \to \infty} x^{2n} = \begin{cases} \infty, & \text{if } x^2 > 1 \\ 1, & \text{if } x^2 = 1 \\ 0, & \text{if } x^2 < 1 \end{cases}$

Given, $f(x) = \lim_{n \to \infty} \frac{\log(2+x) - x^{2n}\sin x}{1 + x^{2n}}$

For $x^2 = 1$, $f(x) = \lim_{n \to \infty} \frac{\log 3 - \sin 1}{2}$

$= \frac{1}{2}(\log 3 - \sin 1)$

For $x^2 < 1$,

$$f(x) = \log(2+x)$$

For $x^2 > 1$,

$$f(x) = \lim_{n \to \infty} \frac{\left(\frac{1}{x^{2n}}\right)\log(2+x) - \sin x}{\left(1 + \frac{1}{x^{2n}}\right)}$$

$= -\sin x$

$$\therefore \quad f(x) = \begin{cases} \log(2+x), & x^2 < 1 \\ \frac{1}{2}(\log 3 - \sin 1), & x = 1 \\ -\sin x, & x^2 > 1 \end{cases}$$

$\therefore \quad \lim_{x \to 1^-} f(x) = \lim_{h \to 0} \log(2 + 1 - h)$

$= \log 3$

$\lim_{x \to 1^+} f(x) = \lim_{h \to 0}[-\sin(1+h)]$

$= -\sin 1$

It is clear that both limits exist and $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$

8. Given, curve is $y - e^{xy} + x = 0$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} - e^{xy}\left(y + x\frac{dy}{dx}\right) + 1 = 0$$

$\Rightarrow \quad \frac{dy}{dx}(1 - xe^{xy}) = ye^{xy} - 1$

For vertical tangent,

$$\frac{dx}{dy} = 0$$

$\Rightarrow \quad \frac{1 - xe^{xy}}{ye^{xy} - 1} = 0$

$\Rightarrow \quad 1 - xe^{xy} = 0 \Rightarrow 1 = xe^{xy}$

$\Rightarrow \quad x = 1$ and $y = 0$

9. Let the equation of hyperbola be

$$\frac{x^2}{a_1^2} - \frac{y^2}{b_1^2} = 1 \quad \text{...(i)}$$

Given ellipse is

$$\frac{x^2}{25}+\frac{y^2}{16}=1$$

Here, $a^2=25, b^2=16$

$$\therefore \quad e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{16}{25}}$$

$$=\frac{3}{5}$$

$\therefore$ Foci of an ellipse $f(+ae, 0)$, *ie* $f(+3, 0)$

Also, $f(3, 0)$ passes through Eq. (i)

$$\therefore \quad \frac{9}{a_1^2}-0=1 \Rightarrow a_1^2=9$$

Also, $ee'=1$, where e' is the eccentricity of a hyperbola.

$$\therefore \quad \frac{3}{5}e'=1 \Rightarrow e'=\frac{5}{3}$$

Also, $\quad b_1^2=a_1^2(e_1^2-1)$

$$\Rightarrow \quad b_1^2=9\left(\frac{25}{9}-1\right)=16$$

Hence, required equation of a hyperbola is

$$\frac{x^2}{9}-\frac{y^2}{16}=1$$

10. Since, $2q=p+r$

Given that , $px^2+qx+r=0$ has complex roots.

$$\therefore \quad D<0 \Rightarrow q^2-4pr<0$$

$$\Rightarrow \quad \left(\frac{p+r}{2}\right)^2-4pr<0$$

$$\Rightarrow \quad p^2+r^2-14pr<0$$

$$\Rightarrow \quad \frac{p^2}{r^2}+1-\frac{14p}{r}<0$$

$$\Rightarrow \quad \left(\frac{p^2}{r^2}-\frac{14p}{r}+49\right)-48<0$$

$$\Rightarrow \quad \left(\frac{p}{r}-7\right)^2<48$$

$$\Rightarrow \quad \left|\frac{p}{r}-7\right|<4\sqrt{3}$$

11. Given, $A\cap X=B\cap X=\phi$

$\Rightarrow A$ and X, B and X are disjoint sets.

Also, $A\cup X=B\cup X \Rightarrow A=B$

12. Since, percentage of coefficient of variation

$$=\frac{\text{Standard deviation}}{\text{Mean}}\times 100$$

$$\therefore \quad 45=\frac{\sigma}{12}\times 100$$

$$\Rightarrow \quad \sigma=\frac{45\times 12}{100}=5.4$$

13. The general term in the expansion of $(4+2x)^{49}$ is

$$T_{r+1}={}^{49}C_r\,(4)^{49-r}\,(2x)^r$$

$$\text{Now,} \quad \frac{T_{r+1}}{T_r}=\frac{{}^{49}C_r}{{}^{49}C_{r-1}}\cdot\frac{(4)^{49-r}(2x)^r}{4^{50-r}(2x)^{r-1}}$$

$$=\frac{{}^{49}C_r}{{}^{49}C_{r-1}}\cdot(4)^{-1}(2x)^1$$

$$=\frac{50-r}{r}\cdot\frac{1}{4}\cdot 2x$$

At $x=\frac{1}{3}$,

$$\therefore \quad \frac{T_{r+1}}{T_r}=\frac{50-r}{r}\cdot\frac{1}{6}$$

As $\frac{T_{r+1}}{T_r}\geq 1$

$$\Rightarrow \quad \frac{50-r}{6r}\geq 1 \Rightarrow r\leq\frac{50}{7}$$

$$\Rightarrow \quad r=7.2, \quad ie \quad r=7$$

$$\therefore \quad T_{r+1}=T_8$$

14. Given, $x=t^2+t$ and $y=t^2-t$

On adding and subtracting, we get

$$\frac{x+y}{2}=t^2 \quad\text{and}\quad \frac{x-y}{2}=t$$

$$\Rightarrow \quad \frac{x+y}{2}=\left(\frac{x-y}{2}\right)^2$$

$$\Rightarrow \quad 2x+2y=x^2+y^2-2xy$$

$$\Rightarrow \quad x^2+y^2-2xy-2x-2y=0$$

Here, $a=b=1, h=-1, g=f=-1, c=0$

$$\text{Now,} \quad \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=\begin{vmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 0 \end{vmatrix}$$

$$=\begin{vmatrix} 1 & -1 & -1 \\ 0 & 0 & -2 \\ 0 & -2 & -1 \end{vmatrix}$$

$[R_2\to R_2+R_1$ and $R_3\to R_3+R_1]$

$=-4\neq 0$

Now, $h^2 - ab = (-1)^2 - (1)(1) = 0$

Hence, it represents a parabola.

15. Given, $r = \{(a, b) | a, b \in R$ and $a - b + \sqrt{3}$ is an irrational number$\}$

(i) **Reflexive**

$ara = a - a + \sqrt{3} = \sqrt{3}$ which is irrational number.

(ii) **Symmetric**

Now, $2r\sqrt{3} = 2 - \sqrt{3} + \sqrt{3} = 2$

which is not an irrational.

Also, $\sqrt{3}r2 = \sqrt{3} - 2 + \sqrt{3} = 2\sqrt{3} - 2$ which is an irrational.

$$2r\sqrt{3} \neq \sqrt{3}r2$$

which is not symmetric.

(iii) **Transitive**

Now, $\sqrt{3}r2$ and $2r4\sqrt{5}$, *ie*,

$$\sqrt{3} - 2 + \sqrt{3} + 2 - 4\sqrt{5} + \sqrt{3}$$
$$= 2\sqrt{3} - 4\sqrt{5} + \sqrt{3}$$
$$\neq \sqrt{3}\, r 4\sqrt{5}$$

$\therefore$ It is not transitive.

Hence, option (b) is correct.

16. Given, $C = \{z : z\bar{z} + a\bar{z} + \bar{a}z + b = 0,\ b \in R$ and $b < |a|^2\}$

Since, $z\bar{z} + a\bar{z} + \bar{a}z + b = 0,\ b \in R$ represents circle having centre at $-a$ and radius $\sqrt{|a|^2 - b}$.

Then z lies on the circle having infinite points.

Hence, C represents infinite sets.

17. Given, $\dfrac{|x-1|}{x+2} - 1 < 0$,

Case I : When $x < 1, |x - 1| = 1 - x$

$$\therefore \quad \frac{1-x}{x+2} - 1 < 0 \Rightarrow \frac{-2x-1}{x+2} < 0$$

$$\Rightarrow \frac{2x+1}{x+2} > 0 \Rightarrow x < -2 \text{ or } x > -\frac{1}{2}$$

But $x < 1$

$$\therefore \quad x \in (-\infty, -2) \cup \left(-\frac{1}{2}, 1\right) \quad \ldots\text{(i)}$$

Case II : When $x \geq 1, |x - 1| = x - 1$

$$\therefore \quad \frac{x-1}{x+2} - 1 < 0 \Rightarrow -\frac{3}{x+2} < 0$$

$$\Rightarrow \quad \frac{3}{x+2} > 0 \Rightarrow x > -2$$

But $x \geq 1$

$$\therefore \quad x \geq 1, \quad ie,\ x \in [1, \infty) \quad \ldots\text{(ii)}$$

$\therefore$ From Eqs. (i) and (ii), we get

$$x \in (-\infty, -2) \cup \left(-\frac{1}{2}, \infty\right)$$

18. Using, AM > GM

$$\therefore \quad \frac{a+b+c}{3} > \sqrt[3]{abc}$$

$$\Rightarrow \quad a + b + c > 3 \quad \ldots\text{(i)}$$

$[\because abc = 1$ given$]$

Also, GM > HM

$$\sqrt[3]{abc} > \frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}$$

$$\Rightarrow \quad (1)^{1/3} > \frac{3abc}{bc + ac + ab}$$

$$\Rightarrow \quad ab + bc + ac > 3 \quad \ldots\text{(ii)}$$

$\therefore$ From Eqs. (i) and (ii), we get

$$a + b + c + ab + bc + ac > 6$$

Hence, option (d) is correct.

19. Let $L_1 \equiv 3x - 4y - 8 = 0$

At point (3, 4),

$$L_1 \equiv 9 - 16 - 8 = -15 < 0$$

As point (x, y) and (3, 4) opposite sides of L_1

$$\therefore \quad 3x - 4y - 8 > 0 \quad \ldots\text{(i)}$$

$$\Rightarrow \quad 3x - 4(-3x) - 8 > 0 \quad [\because y = -3x]$$

$$\Rightarrow \quad 15x - 8 > 0 \Rightarrow x > \frac{8}{15}$$

Again from Eq. (i),

$$3\left(-\frac{y}{3}\right) - 4y - 8 > 0$$

$$\Rightarrow \quad -5y - 8 > 0 \Rightarrow y < -\frac{8}{5}$$

20. Since, the first 11 terms in AP

$$d = 2$$

$$\therefore \quad a_{11} = a + 10d = a + 20$$

The middle term of AP is

$$T_6 = a + 5d = a + 10$$

For the next 11 terms in GP,

$$r = 2$$

$\therefore$ The middle term of GP is $b(2)^5$ where b is the first term of a GP which is the last term of AP.

$$\therefore \quad b(2)^5 = (a + 20)32$$

According the given condition,

$$a + 10 = (a + 20)32$$

$\Rightarrow \quad 31a = 10 - 640 \quad \Rightarrow \quad a = -\dfrac{630}{31}$

$\therefore$ Middle term of entire sequence is 11th term.

$$\therefore \quad T_{11} = -\frac{630}{31} + 10 \times d$$

$$= -\frac{630}{31} + 10 \times 2 = -\frac{10}{31}$$

21. Now, $\begin{vmatrix} x^n & x^{n+2} & x^{n+3} \\ y^n & y^{n+2} & y^{n+3} \\ z^n & z^{n+2} & z^{n+3} \end{vmatrix}$

$$= x^n y^n z^n \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= x^n y^n z^n \begin{vmatrix} 1 & x^2 & x^3 \\ 0 & y^2 - x^2 & y^3 - x^3 \\ 0 & z^2 - x^2 & z^3 - x^3 \end{vmatrix}$$

$$= x^n y^n z^n [(y^2 - x^2)(z^3 - x^3) - (y^3 - x^3)(z^2 - x^2)]$$

$$= x^n y^n z^n (y - x)(z - x)[(y + x)(z^2 + x^2 + zx) - (z + x)(y^2 + x^2 + xy)]$$

$$= x^n y^n z^n (y - x)(z - x)(yz^2 + x^2y + xyz + xz^2 + x^3 + x^2z - (zy^2 + x^2z + xyz + xy^2 + x^3 + x^2y)]$$

$$= x^n y^n z^n (y - x)(z - x)(yz^2 + xz^2 - zy^2 - xy^2)$$

$$= x^n y^n z^n (x - y)(x - z)[x(z^2 - y^2) + yz(z - y)]$$

$$= x^n y^n z^n (x - y)(x - z)(z - y)(xy + yz + zx)$$

$$= x^{n+1} y^{n+1} z^{n+1} (x - y)(x - z)(z - y)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$$

It is comparing to the given result, we get

$$n + 1 = 0 \quad \Rightarrow \quad n = -1$$

22. Total number of cards = 52

Probability of getting spade $= \dfrac{13}{52} = \dfrac{1}{4}$

Probability of not getting spade $= 1 - \dfrac{1}{4} = \dfrac{3}{4}$

$\therefore$ Required probability $= \dfrac{3}{4} \cdot \dfrac{3}{4} = \dfrac{9}{16}$

23. Here, force $\vec{\mathbf{F}} = 6 \times \dfrac{(9\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}})}{\sqrt{81 + 36 + 4}}$

$$= \frac{6(9\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}})}{11}$$

Displacement vector $\vec{\mathbf{d}}$

$$= (7 - 3)\hat{\mathbf{i}} + (-6 - 4)\hat{\mathbf{j}} + (8 + 15)\hat{\mathbf{k}}$$

$$= 4\hat{\mathbf{i}} - 10\hat{\mathbf{j}} + 23\hat{\mathbf{k}}$$

$\therefore$ Work done $= \vec{\mathbf{F}} \cdot \vec{\mathbf{d}}$

$$= \frac{6}{11}(9\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \cdot (4\hat{\mathbf{i}} - 10\hat{\mathbf{j}} + 23\hat{\mathbf{k}})$$

$$= \frac{6}{11}(36 - 60 + 46)$$

$$= 12$$

24. Let the equation of line be

$$\frac{x}{a} + \frac{y}{b} = 1$$

Since, it passes through $\left(\dfrac{1}{5}, \dfrac{1}{5}\right)$

$$\therefore \quad \frac{1}{5a} + \frac{1}{5b} = 1$$

$$\Rightarrow \quad a + b = 5ab \quad \ldots\text{(i)}$$

Since, the point $P(x, y)$ divides AB joining $A(a, 0)$ and $B(0, b)$ internally in ratio $3:1$.

$\therefore \quad x = \dfrac{a}{4}, \; y = \dfrac{3b}{4} \Rightarrow a = 4x$ and $b = \dfrac{4y}{3}$

On putting the values of a and b in Eq. (i), we get

$$4x + \frac{4y}{3} = 5(4x)\left(\frac{4y}{3}\right)$$

$$\Rightarrow \quad 3x + y = 20xy$$

25. Let $f(x) = 3\cos x + 4\sin x + 5$

Since, $-\sqrt{3^2 + 4^2} \le 3\cos x + 4\sin x \le \sqrt{3^2 + 4^2}$

$$\Rightarrow \quad -5 \le 3\cos x + 4\sin x \le 5$$

$$\Rightarrow \quad -5 + 5 \le 3\cos x + 4\sin x + 5 \le 5 + 5$$

$$\Rightarrow \quad 0 \le f(x) \le 10$$

Hence, maximum value of $f(x)$ is 10.

Alternative

Let $\quad f(x) = 3\cos x + 4\sin x + 5$

$$\Rightarrow \quad f'(x) = -3\sin x + 4\cos x$$

$$\Rightarrow \quad f''(x) = -3\cos x - 4\sin x$$

Put $f'(x) = 0 \Rightarrow \tan x = \frac{4}{3}$

$\therefore \quad f''(x) \le 0$, maxima

$\therefore$ Maximum value of $f(x)$ at $\tan x = \frac{4}{3}$ is

$$f(x) = 3 \times \frac{3}{5} + 4 \times \frac{4}{5} + 5$$

$$= 10$$

26. Given, ${}^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \le 50$

$$\Rightarrow \quad \frac{(n+1)!}{3!(n-2)!} - \frac{(n+1)!}{2!(n-1)!} \le 50$$

$$\Rightarrow \quad \frac{(n+1)!}{3!}\left[\frac{1}{(n-2)!} - \frac{3}{(n-1)!}\right] \le 50$$

$$\Rightarrow \quad (n+1)!\left(\frac{n-1-3}{(n-1)!}\right) \le 300$$

$$\Rightarrow \quad (n+1)n(n-4) \le 300$$

For $n = 8$, it satisfy to the above inequality.

But $n = 1$ it does not satisfy the above inequality.

Hence, option (c) is correct.

27. Given, $y = x^4 + 3x^2 + 2x$

First we find out the equation of tangent on curve which is parallel to $y = 2x - 1$.

Now, $\quad \frac{dy}{dx} = 4x^3 + 6x + 2$

$\Rightarrow \; 4x^3 + 6x + 2 = 2 \Rightarrow x(4x^2 + 6) = 0$

$\Rightarrow \quad x = 0, y = 0$

$\therefore \quad \left(\frac{dy}{dx}\right)_{(0,0)} = 2$

$\therefore$ Equation of tangent is

$$y - 0 = 2(x - 0) \quad \Rightarrow \quad y = 2x$$

$\therefore$ Equation of normal at (0, 0) is

$$2y + x = 0$$

The intersection point of $2y + x = 0$ and $y = 2x - 1$ is $\left(\frac{2}{5}, -\frac{1}{5}\right)$.

$\therefore$ Required distance

$$= \sqrt{\left(\frac{2}{5} - 0\right)^2 + \left(-\frac{1}{5} - 0\right)^2}$$

$$= \sqrt{\frac{4}{25} + \frac{1}{25}} = \frac{1}{\sqrt{5}}$$

28. $$\lim_{x \to 3} \frac{\int_3^{f(x)} 2t^3 \, dt}{x - 3} = \lim_{x \to 3} \frac{2[f(x)]^3 \cdot f'(x)}{1}$$

$$= 2[f(3)]^3 \cdot f'(3) = 2 \times 3^3 \times \frac{1}{2}$$

$$= 27$$

29. According to the given condition,

$$\text{Area of curve} = \int_0^a f(x)\,dx$$

$$\Rightarrow \quad \frac{a^2}{2} + \frac{a}{2}\sin a + \frac{\pi}{2}\cos a = \int_0^a f(x)\,dx$$

On differentiating both sides w.r.t. a, we get

$$a + \frac{1}{2}\sin a + \frac{a}{2}\cos a - \frac{\pi}{2}\sin a = f(a)$$

$$\Rightarrow f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + \frac{1}{2}\sin\frac{\pi}{2} + \frac{\pi}{4}\cos\frac{\pi}{2} - \frac{\pi}{2}\sin\frac{\pi}{2}$$

$$\Rightarrow f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + \frac{1}{2} - \frac{\pi}{2}$$

$$\Rightarrow f\left(\frac{\pi}{2}\right) = \frac{1}{2}$$

30. Given, $(1 + y^2)\,dx - xy\,dy = 0$

$$\Rightarrow \quad \frac{y\,dy}{1 + y^2} = \frac{dx}{x}$$

On integrating both sides, we get

$$\frac{1}{2}\log(1 + y^2) = \log x + \log c$$

$$\Rightarrow \quad \sqrt{1 + y^2} = xc$$

Since, it is passing through (1, 0).

$\therefore \quad 1 = 1 \cdot c \quad \Rightarrow \quad c = 1$

$\therefore \quad \sqrt{1 + y^2} = x$

$\Rightarrow \quad 1 + y^2 = x^2 \quad \Rightarrow \quad x^2 - y^2 = 1$

which represents a hyperbola.

31. $$\frac{d}{dx}[f^2(x) + g^2(x)]$$

$$= 2[f(x)f'(x) + g(x)g'(x)]$$

$$= 2[f(x)g(x) + g(x)\{-f(x)\}]$$

$$= 0$$

$\Rightarrow \quad f^2(x) + g^2(x) = \text{constant}$

$\therefore \quad f^2(4) + g^2(4) = f^2(2) + g^2(2)$

$$= (4)^2 + (4)^2$$

$$= 32$$

32. Given, $\cos 2x + 7 = a(2 - \sin x)$

$\Rightarrow \quad 1 - 2\sin^2 x + 7 = 2a - a \sin x$

$\Rightarrow \quad 2\sin^2 x - a\sin x + (2a - 8) = 0$

$\therefore \quad \sin x = \dfrac{a \pm \sqrt{(-a)^2 - 8(2a-8)}}{2 \times 2}$

$= \dfrac{a \pm (a-8)}{4}$

For (+) sign,

$\sin x = \dfrac{a-4}{2}$

For (–) sign,

$\sin x = 2$ which is not possible.

We know $-1 \le \sin x \le 1$

$\therefore \quad -1 \le \dfrac{a-4}{2} \le 1$

$\Rightarrow \quad 2 \le a \le 6$

33. Given, $n(A) = 4$ and $n(B) = 6$

Here, $n(B) > n(A)$

Since, the function f is one-one and onto.

$\therefore$ Required number of ways

$= {}^6P_4$

$= \dfrac{6!}{2!}$

$= 360$

34. Since, $\log(1+x) - \log(1-x)$

$$= 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \infty\right]$$

Put $x = \dfrac{1}{2}$ on both sides, we get

$\log\left(\dfrac{3}{2}\right) - \log\left(\dfrac{1}{2}\right)$

$$= 2\left(\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{2^3} + \frac{1}{5}\cdot\frac{1}{2^5} + \dots \infty\right)$$

$\Rightarrow \quad \log 3 = 1 + \dfrac{1}{3}\cdot\dfrac{1}{4} + \dfrac{1}{5}\cdot\dfrac{1}{4^2} + \dots$

35. The general term of $\left(x + \dfrac{2}{x^2}\right)^n$ is

$$T_{R+1} = {}^nC_R (x)^{n-R}\left(\frac{2}{x^2}\right)^R$$

$$= {}^nC_R x^{n-3R} 2^R$$

For x^{2r} occurs, it means

$n - 3R = 2r$

$\Rightarrow \quad n - 2r = 3R$

Hence, $n - 2r$ is of the form $3k$.

36. Let the equation of circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \dots(i)$$

Since, circle (i) cuts the given circle orthogonally.

$\therefore \quad 2(-g)(3) + 2(-f)(-2) = c - 3$

$\Rightarrow \quad -6g + 4f = c - 3 \quad \dots(ii)$

Also, Eq. (i) passes through (3, 0).

$\therefore \quad 3^2 + 0^2 + 2g(3) + 2f(0) + c = 0$

$\Rightarrow \quad 6g + c + 9 = 0 \quad \dots(iii)$

As Eq. (i) touches y-axis.

$\therefore \quad |-f| = \sqrt{g^2 + f^2 - c}$

$\Rightarrow \quad g^2 = c \quad \dots(iv)$

From Eqs. (iii) and (iv), we get

$g = -3$ and $c = 9$

$\therefore$ From Eq. (ii),

$-6(-3) + 4f = 9 - 3 \quad \Rightarrow \quad f = -3$

$\therefore$ Required equation of circle is

$$x^2 + y^2 - 6x - 6y + 9 = 0$$

37. Given, $(x, y) \Leftrightarrow x^2 - 4xy + 3y^2 = 0$

or $\quad (x, y) \Leftrightarrow (x - y)(x - 3y) = 0$

(i) **Reflexive**

$xRx \Rightarrow (x - x)(x - 3x) = 0$

$\therefore$ It is reflexive.

(ii) **Symmetric**

Now, $\quad xRy \Leftrightarrow (x - y)(x - 3y) = 0$

and $\quad yRx \Leftrightarrow (y - x)(y - 3x) = 0$

$\Rightarrow \quad xRy \ne yRx$

$\therefore$ It is not symmetric.

Similarly, it is not transitive.

38. Let $x = n$, $y = n + 1$ and $z = n + 2$, where n is a positive integer.

$$\therefore \log_e \sqrt{x} + \log_e \sqrt{z} + \left(\frac{1}{2xz+1}\right) + \frac{1}{3}\left(\frac{1}{2xz+1}\right)^3 + \frac{1}{5}\left(\frac{1}{2xz+1}\right)^5 + \dots$$

$$= \log_e \sqrt{xz} + \frac{1}{2}\log_e\left(\frac{1 + \frac{1}{2xz+1}}{1 - \frac{1}{2xz+1}}\right)$$

$$= \log_e \sqrt{xz} + \frac{1}{2}\log_e\left(\frac{2xz+2}{2xz}\right)$$

$$= \log_e \sqrt{n(n+2)} + \log_e \sqrt{\frac{n(n+2)+1}{n(n+2)}}$$

$$= \log_e \sqrt{(n+1)^2} = \log_e (n+1)$$

$$= \log_e y$$

39. Let $f(x) = \log_e \dfrac{x-2}{x-3}$

$f(x)$ is defined either $(x-2) > 0$, $(x-3) > 0$ or $(x-2) < 0$, $(x-3) < 0$ or $x \neq 2, 3$

$\Rightarrow f(x)$ is defined either $x > 3$ or $x < 2$ or $x \neq 2, 3$.

ie, $x \in (-\infty, 2) \cup (3, \infty)$

40. Let $z = x_1 + iy_1$ and $w = x_2 + iy_2$

As $|z| \leq 1$ and $|w| \leq 1$

$\Rightarrow x_1^2 + y_1^2 \leq 1$ and $x_2^2 + y_2^2 \leq 1$

Now, $|z + iw| = |x_1 + iy_1 + i(x_2 + iy_2)| = 2$

$\Rightarrow (x_1 - y_2)^2 + (y_1 + x_2)^2 = 4$ ...(i)

and $|z - i\,\overline{w}| = |x_1 + iy_1 - i(x_2 - iy_2)| = 2$

$\Rightarrow (x_1 - y_2)^2 + (y_1 - x_2)^2 = 4$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$y_1 x_2 = 0$$

$\Rightarrow$ Either $y_1 = 0$ or $x_2 = 0$

When $y_1 = 0$, $x_1^2 \leq 1$

$\Rightarrow x = \pm 1$

$\therefore z = \pm 1 + i0$

41. Given systems have non-trivial solution.

$$\therefore \begin{vmatrix} a & a & -1 \\ b & -1 & b \\ -1 & c & c \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$\Rightarrow \begin{vmatrix} a & 0 & -(1+a) \\ b & -(1+b) & 0 \\ -1 & c+1 & c+1 \end{vmatrix} = 0$$

$$\Rightarrow -a(1+b)(1+c) - b[0 + (1+a)(1+c)] - 1[0 - (1+a)(1+b)] = 0$$

$$\Rightarrow -a(1+b)(1+c) - b(1+a)(1+c) + 1(1+a)(1+b) = 0$$

On dividing by $(1+a)(1+b)(1+c)$, we get

$$-\frac{a}{1+a} - \frac{b}{1+b} + \frac{1}{1+c} = 0$$

$$\Rightarrow -\frac{a}{1+a} + 1 - \frac{b}{1+b} + 1 + \frac{1}{1+c} = 2$$

$$\Rightarrow \frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c} = 2$$

42. Total number of ways = 16

The favourable ways are

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$$

$\therefore$ Required probability $= \dfrac{3}{16}$

43. Given, $\vec{\mathbf{a}} = -(\vec{\mathbf{b}} + \vec{\mathbf{c}})$

$\Rightarrow |\vec{\mathbf{a}}|^2 = |\vec{\mathbf{b}}|^2 + |\vec{\mathbf{c}}|^2 + 2\vec{\mathbf{b}} \cdot \vec{\mathbf{c}}$

$\Rightarrow 7^2 = 3^2 + 5^2 + 2[|\vec{\mathbf{b}}||\vec{\mathbf{c}}| \cos\theta]$

$\Rightarrow 15 = 2(3 \times 5)\cos\theta$

$\Rightarrow \cos\theta = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{3}$

44. Let $\vec{\mathbf{n}}_1$ and $\vec{\mathbf{n}}_2$ be the vectors normal to the plane determined by $\hat{\mathbf{i}}, \hat{\mathbf{i}} - \hat{\mathbf{j}}$ and $\hat{\mathbf{i}} + \hat{\mathbf{j}}, \hat{\mathbf{i}} - \hat{\mathbf{k}}$ respectively

$\therefore \vec{\mathbf{n}}_1 = \hat{\mathbf{i}} \times (\hat{\mathbf{i}} - \hat{\mathbf{j}}) = -\hat{\mathbf{k}}$

and $\vec{\mathbf{n}}_2 = (\hat{\mathbf{i}} + \hat{\mathbf{j}}) \times (\hat{\mathbf{i}} - \hat{\mathbf{k}}) = -\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

Since, $\vec{\mathbf{a}}$ is parallel to the line of intersection of the planes determined by the given planes.

$\therefore \vec{\mathbf{a}} \,||\, (\vec{\mathbf{n}}_1 \times \vec{\mathbf{n}}_2) \Rightarrow \vec{\mathbf{a}} = \lambda(\vec{\mathbf{n}}_1 \times \vec{\mathbf{n}}_2)$

$= \lambda(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

Let θ be the angle between $\vec{\mathbf{a}}$ and $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

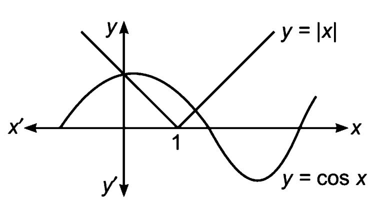
$$\therefore \cos\theta = \frac{\lambda(\hat{\mathbf{i}} + \hat{\mathbf{j}}) \cdot (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}})}{\sqrt{\lambda^2 + \lambda^2}\sqrt{1 + 4 + 4}}$$

$$= \frac{\lambda(1+2)}{\sqrt{2}\lambda \times 3} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

45. Let $y = |x - 1| = \cos x$

It is clear from the graph that two curves intersect two points.



Hence, number of solutions are 2.

46. Let the slopes be m, m^2.

$\therefore \quad m + m^2 = \frac{-2h}{b}$ and $mm^2 = \frac{a}{b}$

$\Rightarrow \quad m^3 = \left(\frac{a}{b}\right)$

Now, $\quad m(1+m) = \frac{-2h}{b}$

On cubing both sides, we get

$$m^3[1 + m^3 + 3m(1+m)] = \frac{-8h^3}{b^3}$$

$$\Rightarrow \quad \frac{a}{b}\left[1 + \frac{a}{b} + 3\left(\frac{-2h}{b}\right)\right] = \frac{-8h^3}{b^3}$$

$$\Rightarrow \quad \frac{b+a}{b} - \frac{6h}{b} = \frac{-8h^3}{ab^2}$$

$$\Rightarrow \quad b + a + \frac{8h^3}{ab} = 6h$$

$$\Rightarrow \quad \frac{b+a}{h} + \frac{8h^2}{ab} = 6$$

47. $\left[(0.16)^{\log_{0.25}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\dots\right)}\right]^{1/2}$

$$= \left[(0.16)^{\log_{0.25}\left(\frac{\frac{1}{3}}{1-\frac{1}{3}}\right)}\right]^{1/2}$$

$$= [_{(0.16)}\log_{(0.5)^2} 0.5]^{1/2}$$

$$= [(0.16)^{1/2}]^{1/2} = (0.4)^{1/2}$$

$$= \frac{2}{\sqrt{10}}$$

48. $1 + 1\cdot P_1 + 2\cdot P_2 + 3\cdot P_3 + \dots + n\cdot P_n$

$= 1 + 1\cdot(1!) + 2\cdot(2!) + \dots + n\cdot(n!)$

$= 1 + (2-1)1! + (3-1)2! + \dots + ((n+1)-1)n!$

$= 1 + 2! - 1! + 3! - 2! + \dots + (n+1)! - n!$

$= (n+1)!$

49. Given, $y = f\left(\frac{3x+\pi}{5x+4}\right)$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = f'\left(\frac{3x+\pi}{5x+4}\right)\frac{d}{dx}\left(\frac{3x+\pi}{5x+4}\right)$$

$$= f'\left(\frac{3x+\pi}{5x+4}\right)\cdot\frac{(5x+4)(3)-(3x+\pi)5}{(5x+4)^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = \tan^2\left(\frac{3x+\pi}{5x+4}\right)\cdot\frac{12-5\pi}{(5x+4)^2}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(x=0)} = \tan^2\left(\frac{\pi}{4}\right)\cdot\frac{12-5\pi}{4^2}$$

$$= \frac{12-5\pi}{16}$$

50. Let $f(x) = \int_0^x \left[\frac{1}{\sqrt{1+t^2}} - \frac{1}{1+t}\right]dt$

$$= \int_0^x \frac{1}{\sqrt{1+t^2}}\,dt - \int_0^x \frac{1}{1+t}\,dt$$

Let $\quad I_1 = \int_0^x \frac{1}{\sqrt{1+t^2}}\,dt$

Put $\quad t = \tan\theta \Rightarrow dt = \sec^2\theta\, d\theta$

$$\therefore \quad I_1 = \int_0^{\tan^{-1}x} \frac{\sec^2\theta}{\sec\theta}\,d\theta$$

$$= [\log(\sec\theta + \tan\theta)]_0^{\tan^{-1}x}$$

$$= \log(\sec\tan^{-1}x + x)$$

$$\therefore \ f(x) = \log(\sec\tan^{-1}x + x) - \log(1+x)$$

$$\therefore \ \lim_{x\to\infty} f(x) = \lim_{x\to\infty}\log\left(\frac{\sec\tan^{-1}x + x}{1+x}\right)$$

$$= \lim_{x\to\infty}\log\left(\frac{\sqrt{1+x^2}+x}{1+x}\right)$$

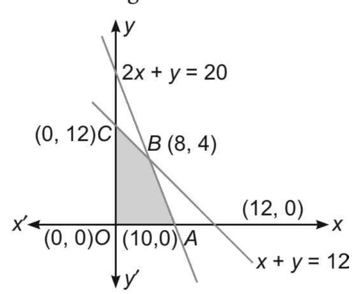
$$= \log\lim_{x\to\infty}\left(\frac{\sqrt{\frac{1}{x^2}+1}+1}{\frac{1}{x}+1}\right)$$

$$= \log 2$$

51. Given, constraints are

$x \ge 0,\ y \ge 0,\ x + y \le 12 + 2x + y \le 20$

The feasible reagion is $OABCO$



At $O(0,0)$, $z = 10(0) + 6(0) = 0$

At $A(10, 0)$, $z = 10(10) + 6(0) = 100$
At $B(8, 4)$, $z = 10(8) + 6(4) = 104$
At $C(0, 12)$, $z = 10(0) + 6(12) = 72$
Hence, maximum value of z is 104.

52. Given, $2f(x^2) + 3f\left(\frac{1}{x^2}\right) = x^2 - 1$ ...(i)

Replacing x by $\frac{1}{x}$, we get

$$2f\left(\frac{1}{x^2}\right) + 3f(x^2) = \frac{1}{x^2} - 1 \quad \text{...(ii)}$$

On multiplying Eq. (i) by 2, Eq. (ii) by 3 and subtracting Eq. (i) from Eq. (ii), we get

$$5f(x^2) = \frac{3}{x^2} - 1 - 2x^2$$

$$\Rightarrow \quad f(x^2) = \frac{1}{5x^2}(3 - x^2 - 2x^4)$$

$$\Rightarrow \quad f(x^4) = \frac{1}{5x^4}(3 - x^4 - 2x^8)$$

[replacing x by x^2]

$$= \frac{(1 - x^4)(2x^4 + 3)}{5x^4}$$

53. LHS $= \int (\log x)^2\, dx$

$$= x(\log x)^2 - \int x \cdot 2\log x \frac{1}{x}\, dx$$

$$= x(\log x)^2 - 2\left[x \log x - \int x \cdot \frac{1}{x}\, dx\right] + c$$

$$= x(\log x)^2 - 2[x \log x - x]$$

$$= x(\log x)^2 - 2x[\log x - 1] + c$$

But RHS is given by

$$x[f(x)]^2 + Ax[f(x) - 1] + c$$

$\therefore \quad f(x) = \log x \quad$ and $\quad A = -2$

54. Given, $P(x) = x + ax + b$

$\therefore \quad P(10) = 10 + 10a + b = 10 + 5 = 15$

and $\quad P(11) = 11 + 11a + b$

$= 11 + 5 + a = 16 + a$

$\because \quad P(10)P(11) = P(n)$

$\Rightarrow \quad 15(16 + a) = n + na + b$

$\Rightarrow \quad 240 + 15a = n + na + 5 - 10a$

$\Rightarrow \quad n + na - 25a - 235 = 0$

(a) When $n = 15$,

$15 + 15a - 25a - 235 = 0$

$\Rightarrow \quad a = -22$ and $b = 225$

(b) When $n = 65$,

$65 + 65a - 25a - 235 = 0$

$\Rightarrow a = -\frac{17}{4}$ which is not integer.

(c) When $n = 115$,

$115 + 115a - 25a - 235 = 0$

$\Rightarrow a = \frac{4}{3}$ which is not integer.

(d) When $n = 165$,

$165 + 165a - 25a - 235 = 0$

$\Rightarrow \quad a = \frac{1}{2}$ which is not integer.

Hence, option (a) is correct.

55. Let n_1 and n_2 be the number of men and women in a group.

According to the given condition,

$$\frac{n_1 \times 26 + n_2 \times 21}{n_1 + n_2} = 25$$

$\Rightarrow \quad 26n_1 + 21n_2 = 25n_1 + 25n_2$

$\Rightarrow \quad n_1 = 4n_2 \quad \Rightarrow \quad \frac{n_1}{n_2} = \frac{4}{1}$

$\Rightarrow \quad \frac{n_1}{n_2} = \frac{80}{20}$

Hence, option (b) is correct.

56. Given, equation of circle is

$$x^2 + y^2 - 2ax = 0$$

Let (h, k) be the mid point of the chord. Then, equation of chord is

$$T = S_1$$

$\therefore \quad xh + yk - a(x + h) = h^2 + k^2 - 2ah$

Also passes through (0, 0).

$\therefore \quad 0 + 0 - a(h) = h^2 + k^2 - 2ah$

$\Rightarrow \quad h^2 + k^2 - ah = 0$

$\therefore$ Locus of (h, k) is

$$x^2 + y^2 - ax = 0$$

57. Given $f : C \to R$ such that $f(z) = |z|$

We know modulus of z and $\bar{z}$ have same values, so $f(z)$ has many one.

Also, $|z|$ is always non-negative real numbers, so it is not onto function.

58. Let $y = \frac{x^2 + 34x - 71}{x^2 + 2x - 7}$

$\Rightarrow \quad x^2(y - 1) + x(2y - 34) + 71 - 7y = 0$

Since, x is complex number.

$\therefore \quad D < 0$

$\Rightarrow \quad (2y-34)^2 - 4(y-1)(71-7y) < 0$

$\Rightarrow \quad (y-17)^2 - (71y - 7y^2 - 71 + 7y) < 0$

$\Rightarrow \quad 8y^2 - 112y + 360 < 0$

$\Rightarrow \quad y^2 - 14y + 45 < 0$

$\Rightarrow \quad (y-9)(y-5) < 0$

$\Rightarrow \quad 5 < y < 9$

$\therefore \quad a = 5, b = 9$

59. Let $a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1} = d$

$$\therefore \quad \frac{1}{d}\left(\frac{d}{a_1a_2} + \frac{d}{a_2a_3} + \ldots + \frac{d}{a_{n-1}a_n}\right)$$

$$= \frac{1}{d}\left[\frac{a_2-a_1}{a_1a_2} + \frac{a_3-a_2}{a_2a_3} + \ldots + \frac{a_n-a_{n-1}}{a_{n-1}a_n}\right]$$

$$= \frac{1}{d}\left[\frac{1}{a_1} - \frac{1}{a_2} + \frac{1}{a_2} - \frac{1}{a_3} + \ldots + \frac{1}{a_{n-1}} - \frac{1}{a_n}\right]$$

$$= \frac{1}{d}\left[\frac{1}{a_1} - \frac{1}{a_n}\right] = \frac{1}{d}\left[\frac{a_n - a_1}{a_1a_n}\right]$$

$$= \frac{n-1}{a_1a_n}$$

60. Since, B is invertible, therefore B^{-1} exists.

Now, $\quad \text{rank}(A) = \text{rank}[(AB)B^{-1}]$

$\leq \text{rank}(AB)$

But $\quad \text{rank}(AB) \leq \text{rank}(A)$

$\therefore \quad \text{rank}(AB) = \text{rank}(A)$

61. Given, $P(A) = 0.3$ and $P(A \cup B) = 0.8$

Since, $P(A \cup B) = P(A) + P(B) - P(A)P(B)$

$\Rightarrow \quad 0.8 = 0.3 + P(B) - 0.3P(B)$

$\Rightarrow \quad 0.5 = 0.7P(B)$

$\Rightarrow \quad P(B) = \frac{5}{7}$

62. $$\left[\frac{1}{2}|\vec{\mathbf{u}}_2 - \vec{\mathbf{u}}_1|\right]^2 = \frac{1}{4}[|\vec{\mathbf{u}}_2|^2 + |\vec{\mathbf{u}}_1|^2 - 2\vec{\mathbf{u}}_2 \cdot \vec{\mathbf{u}}_1]$$

$$= \frac{1}{4}[1 + 1 - 2|\vec{\mathbf{u}}_2||\vec{\mathbf{u}}_1|\cos\theta]$$

$$= \frac{1}{4}[2 - 2\cos\theta]$$

$$= \sin^2\frac{\theta}{2}$$

$$\Rightarrow \quad \frac{1}{2}|\vec{\mathbf{u}}_2 - \vec{\mathbf{u}}_1| = \sin\frac{\theta}{2}$$

63. We know image (x, y, z) of a point (x_1, y_1, z_1) by the plane $ax + by + cz + d = 0$ is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$= -2 \cdot \frac{(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$$

Here, $(x_1, y_1, z_1) = (1, 2, 3)$ and plane

$$x + y + z + 3 = 0$$

$$\therefore \quad \frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$$

$$= -2 \cdot \frac{(1 \times 1 + 1 \times 2 + 1 \times 3 + 3)}{1^2 + 1^2 + 1^2}$$

$$\Rightarrow \quad \frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1} = -6$$

$\Rightarrow \quad x = -5, y = -4, z = -3$

Hence, required image point is $(-5, -4, -3)$.

64. Given, $P = \sin^2\theta + \cos^4\theta$

$$= \sin^2\theta + \cos^2\theta(1 - \sin^2\theta)$$

$$= \sin^2\theta + \cos^2\theta - \sin^2\theta\cos^2\theta$$

$$= 1 - \frac{\sin^2 2\theta}{4} = 1 - \frac{1}{8}(1 - \cos 4\theta)$$

$$= \frac{7}{8} + \frac{1}{8}\cos 4\theta$$

We know, $\quad -1 \leq \cos 4\theta \leq 1$

$$\Rightarrow \quad -\frac{1}{8} \leq \frac{\cos 4\theta}{8} \leq \frac{1}{8}$$

$$\Rightarrow \quad -\frac{1}{8} + \frac{7}{8} \leq \frac{7}{8} + \frac{\cos 4\theta}{8} \leq \frac{1}{8} + \frac{7}{8}$$

$$\Rightarrow \quad \frac{3}{4} \leq P \leq 1$$

65. Total number of points on a three lines are $m + n + k$.

$\therefore$ Maximum number of triangles

$$= {}^{m+n+k}C_3 - {}^{m}C_3 - {}^{n}C_3 - {}^{k}C_3$$

(subtract those triangles in which point on the same line)

66. Given, $y = \cos^{-1}\cos x$

For $\quad 0 \leq x \leq \pi, y = x$

$$\Rightarrow \quad \frac{dy}{dx} = 1$$

For $\pi < x \leq 2\pi$, $y = \cos^{-1}\cos(2\pi - x)$

$\Rightarrow \quad y = -x$

$$\Rightarrow \quad \frac{dy}{dx} = -1$$

Hence, option (d) is correct.

67. $y^2 = 2c(x + c^{2/3})$

$$\Rightarrow \quad 2y\frac{dy}{dx} = 2c \quad \Rightarrow \quad c = y\frac{dy}{dx}$$

$$\therefore \quad y^2 = 2y\frac{dy}{dx}\left(x + \left(y\frac{dy}{dx}\right)^{2/3}\right)$$

$$\Rightarrow \quad \left(\frac{y}{2\frac{dy}{dx}} - x\right) = \left(y\frac{dy}{dx}\right)^{2/3}$$

$$\Rightarrow \quad \left(y - 2x\frac{dy}{dx}\right)^3 = \left(2\frac{dy}{dx}\right)^3 \left(y\frac{dy}{dx}\right)^2$$

$$\Rightarrow \quad \left(y - 2x\frac{dy}{dx}\right)^3 = 8y^2\left(\frac{dy}{dx}\right)^5$$

Here, order = 1, degree = 5.

68. Given, $f(x) = {}^{7-x}P_{x-3}$

(i) $7 - x \geq 0 \Rightarrow x \leq 7$

(ii) $x - 3 \geq 0 \Rightarrow x \geq 3$

(iii) $7 - x \geq x - 3 \Rightarrow x \leq 5$

$\therefore \quad x \in \{3, 4, 5\}$

$\therefore \quad f(x) = \{{}^4P_0, {}^3P_1, {}^2P_2\}$

$= \{1, 3, 2\}$

69. Given, $I = \left|\int_2^5 \frac{\sin x\, dx}{(1 + x^2)}\right|$

$$\leq \int_2^5 \left|\frac{\sin x}{1 + x^2}\right| dx$$

$$\leq \int_2^5 \frac{1}{1 + x^2}\, dx \leq \int_2^5 \frac{1}{x^2}\, dx$$

$$= \left[\frac{x^{-1}}{-1}\right]_2^5 = -\left[\frac{1}{5} - \frac{1}{2}\right]$$

$$\Rightarrow \quad I \leq \frac{3}{10}$$

Hence, option (d) is the correct answer.

70. Let α be the common root of the given equation. Then

$a\alpha^2 + b\alpha + c = 0$ and $2\alpha^2 + 3\alpha + 4 = 0$

$\Rightarrow \quad \alpha^2(a - 2) + \alpha(b - 3) + c - 4 = 0$

$\Rightarrow a - 2 = 0, b - 3 = 0$ and $c - 4 = 0$

$\Rightarrow \quad a = 2, b = 3$ and $c = 4$

$\therefore \quad a + b + c = 2 + 3 + 4$

$= 9$