# Solved Paper 2008

# AMU

## Engineering Entrance Exam

### Physics

**1.** The energy ($E$), angular momentum ($L$) and universal gravitational constant ($G$) are chosen as fundamental quantities. The dimensions of universal gravitational constant in the dimensional formula of Planck's constant ($h$) is

(a) zero (b) −1
(c) 5/3 (d) 1

**2.** The component of vector $\vec{\mathbf{A}} = a_x \hat{\mathbf{i}} + a_y \hat{\mathbf{j}} + a_z \hat{\mathbf{k}}$ along the direction of $\hat{\mathbf{i}} - \hat{\mathbf{j}}$ is

(a) $a_x - a_y + a_z$ (b) $a_x - a_y$
(c) $(a_x - a_y)/\sqrt{2}$ (d) $(a_x + a_y + a_z)$

**3.** A body thrown vertically up to reach its maximum height in $t$ second. The total time from the time of projection to reach a point at half of its maximum height while returning (in second) is

(a) $\sqrt{2}t$ (b) $\left(1 + \frac{1}{\sqrt{2}}\right)t$
(c) $\frac{3t}{2}$ (d) $\frac{t}{\sqrt{2}}$

**4.** If a body is projected with an angle $\theta$ to the horizontal, then

(a) its velocity is always perpendicular to its acceleration
(b) its velocity becomes zero at its maximum height
(c) its velocity makes zero angle with the horizontal at its maximum height
(d) the body just before hitting the ground, the direction of velocity coincides with the acceleration

**5.** A river of salty water is flowing with a velocity 2 m/s. If the density of the water is 1.2 g/cc, then the kinetic energy of each cubic metre of water is

(a) 2.4 J (b) 24 J
(c) 2.4 kJ (d) 4.8 kJ

**6.** A ball is dropped from a height $h$ on a floor of coefficient of restitution $e$. The total distance covered by the ball just before second hit is

(a) $h(1 - 2e^2)$ (b) $h(1 + 2e^2)$
(c) $h(1 + e^2)$ (d) $he^2$

**7.** Two particles $A$ and $B$ initially at rest, move towards each other, under mutual force of attraction. At an instance when the speed of $A$ is $v$ and speed of $B$ is $2v$, the speed of centre of mass (CM) is

(a) zero (b) $v$ (c) $2.5v$ (d) $4v$

**8.** Starting from rest, the time taken by a body sliding down on a rough inclined plane at 45° with the horizontal is, twice the time taken to travel on a smooth plane of same inclination and same distance. Then the coefficient of kinetic friction is

(a) 0.25 (b) 0.33 (c) 0.50 (d) 0.75

**9.** A steel wire can withstand a load up to 2940 N. A load of 150 kg is suspended from a rigid support. The maximum angle through which the wire can be displaced from the mean position, so that the wire does not break when the load passes through the position of equilibrium, is

(a) 30° (b) 60° (c) 80° (d) 85°

**10.** The moment of inertia of a thin circular disc about an axis passing through its centre and perpendicular to its plane is $I$. Then, the moment of inertia of the disc about an axis parallel to its diameter and touching the edge of the rim is

(a) $I$ (b) $2I$ (c) $\frac{3}{2}I$ (d) $\frac{5}{2}I$

**11.** The orbit of geo-stationary satellite is circular, the time period of satellite depends on

(i) mass of the satellite
(ii) mass of the earth

(iii) radius of the orbit
(iv) height of the satellite from the surface of earth

Which of the following is correct ?
(a) (i) only
(b) (i) and (ii)
(c) (i), (ii) and (iii)
(d) (ii), (iii) and (iv)

**12.** A particle is executing simple harmonic motion with an amplitude $A$ and time period $T$. The displacement of the particles after $2T$ period from its initial position is
(a) $A$ (b) $4A$ (c) $8A$ (d) zero

**13.** A load of 1 kg weight is a attached to one end of a steel wire of area of cross-section $3\ mm^2$ and Young's modulus $10^{11}\ N/m^2$. The other end is suspended vertically from a hook on a wall, then the load is pulled horizontally and released. When the load passes through its lowest position the fractional change in length is ($g = 10\ m/s^2$)
(a) $0.3 \times 10^{-4}$ (b) $0.3 \times 10^{-3}$
(c) $0.3 \times 10^{3}$ (d) $0.3 \times 10^{4}$

**14.** The surface tension of soap solution is 0.03 N/m. The work done in blowing to form a soap bubble of surface area $40\ cm^2$, (in J), is
(a) $1.2 \times 10^{-4}$ (b) $2.4 \times 10^{-4}$
(c) $12 \times 10^{-4}$ (d) $24 \times 10^{-4}$

**15.** Two rain drops reach the earth with different terminal velocities having ratio 9 : 4. Then the ratio of their volumes is
(a) 3 : 2 (b) 4 : 9 (c) 9 : 4 (d) 27 : 8

**16.** One litre of oxygen at a pressure of 1 atm and two litres of nitrogen at a pressure of 0.5 atm, are introduced into a vessel of volume 1 L. If there is no change in temperature, the final pressure of the mixture of gas (in atm) is
(a) 1.5 (b) 2.5 (c) 2 (d) 4

**17.** There is some change in length when a 33000 N tensile force is applied on a steel rod of area of cross-section $10^{-3}\ m^2$. The change of temperature required to produce the same elongation, if the steel rod is heated, is (The modulus of elasticity is $3 \times 10^{11}\ N/m^2$ and the coefficient of linear expansion of steel is $1.1 \times 10^{-5}/°C$).
(a) 20°C (b) 15°C (c) 10°C (d) 0°C

**18.** In the adiabatic compression, the decrease in volume is associated with
(a) increase in temperature and decrease in pressure
(b) decrease in temperature and increase in pressure
(c) decrease in temperature and decrease in pressure
(d) increase in temperature and increase in pressure

**19.** Which of the following is true in the case of an adiabatic process, where $\gamma = C_p/C_V$ ?
(a) $p^{1-\gamma}T^{\gamma}$ = constant
(b) $p^{\gamma}T^{1-\gamma}$ = constant
(c) $pT^{\gamma}$ = constant
(d) $p^{\gamma}T$ = constant

**20.** Two slabs $A$ and $B$ of equal surface area are placed one over the other such that their surfaces are completely in contact. The thickness of slab $A$ is twice that of $B$. The coefficient of thermal conductivity of slab $A$ is twice that of $B$. The first surface of slab $A$ is maintained at 100°C, while the second surface of slab $B$ is maintained at 25°C. The temperature at the contact of their surfaces is
(a) 62.5°C (b) 45°C
(c) 55°C (d) 85°C

**21.** When a sound wave of wavelength $\lambda$ is propagating in a medium, the maximum velocity of the particle is equal to the wave velocity. The amplitude of wave is
(a) $\lambda$ (b) $\lambda/2$
(c) $\frac{\lambda}{2\pi}$ (d) $\frac{\lambda}{4\pi}$

**22.** A car is moving with a speed of 72 km/h towards a hill. Car blows horn at a distance of 1800 m from the hill. If echo is heard after 10 s, the speed of sound (in m/s) is
(a) 300 (b) 320
(c) 340 (d) 360

**23.** The refractive index of a material of a plano-concave lens is 5/3, the radius of curvature is 0.3 m. The focal length of the lens in air is
(a) – 0.45 m (b) – 0.6 m
(c) – 0.75 m (d) – 1.0 m

**24.** The Young's double slit experiment is performed with blue and with green light of wavelengths 4360 Å and 5460 Å respectively. If $x$ is the distance of 4th maximum from the central one, then

(a) $x$ (blue ) = $x$ (green)
(b) $x$ (blue) > $x$ (green)
(c) $x$ (blue) < $x$ (green)
(d) $\frac{x\text{ (blue)}}{x\text{ (green)}} \frac{5460}{4360}$

**25.** An achromatic combination of lenses produces

(a) images in black and white
(b) coloured images
(c) images unaffected by variation of refractive index with wavelength
(d) highly enlarged images are formed

**26.** In Fraunhofer diffraction experiment, $L$ is the distance between screen and the obstacle, $b$ is the size of obstacle and $\lambda$ is wavelength of incident light. The general condition for the applicability of Fraunhofer diffraction is

(a) $\frac{b^2}{L\lambda} >> 1$ (b) $\frac{b^2}{L\lambda} = 1$
(c) $\frac{b^2}{L\lambda} << 1$ (d) $\frac{b^2}{L\lambda} \neq 1$

**27.** With a standard rectangular bar magnet the time period of a vibration magnetometer is 4 s. The bar magnet is cut parallel to its length into four equal pieces. The time period of vibration magnetometer when one piece is used (in second) (bar magnet breadth is small) is

(a) 16 (b) 8 (c) 4 (d) 2

**28.** The magnetised wire of moment $M$ and length $l$ is bent in the form of semicircle of radius $r$. Then its magnetic moment is

(a) $\frac{2M}{\pi}$ (b) $2M$
(c) $\frac{M}{\pi}$ (d) zero

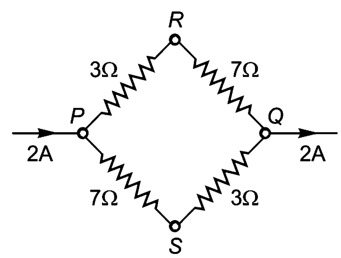
**29.** A charge of $1\,\mu$C is divided into two parts such that their charges are in the ratio of 2 : 3. These two charges are kept at a distance 1 m apart in vacuum. Then, the electric force between them (in N) is

(a) 0.216 (b) 0.00216
(c) 0.0216 (d) 2.16

**30.** Two charges $+q$ and $-q$ are kept apart. Then at any point on the right bisector of line joining the two charges

(a) the electric field strength is zero
(b) the electric potential is zero
(c) both electric potential and electric field strength are zero
(d) both electric potential and electric field strength are non-zero

**31.** A current of 2 A flows in an electric circuit as shown in figure. The potential difference $(V_R - V_S)$, in volts ($V_R$ and $V_S$ are potentials at $R$ and $S$ respectively) is




(a) −4 (b) +2 (c) +4 (d) −2

**32.** When a battery connected across a resistor of $16\,\Omega$, the voltage across the resistor is 12 V. When the same battery is connected across a resistor of $10\,\Omega$, voltage across it is 11 V. The internal resistance of the battery in ohm is

(a) $\frac{10}{7}$ (b) $\frac{20}{7}$ (c) $\frac{25}{7}$ (d) $\frac{30}{7}$

**33.** One junction of a certain thermoelectric couple is at a fixed temperature $T_r$ and the other junction is at temperature $T$. The thermo-electromotive force for this is expressed by $E = k(T - T_r)\left[T_0 - \frac{1}{2}(T + T_r)\right]$.

At temperature $T = \frac{1}{2}T_0$, the thermoelectric power is

(a) $\frac{1}{2}kT_0$ (b) $kT_0$
(c) $\frac{1}{2}kT_0^2$ (d) $\frac{1}{2}k(T_0 - T_r)^2$

**34.** In a galvanometer 5% of the total current in the circuit passes through it. If the resistance of the galvanometer is $G$, the shunt resistance $S$ connected to the galvanometer is

(a) $19G$ (b) $\frac{G}{19}$ (c) $20G$ (d) $\frac{G}{20}$

**35.** Two concentric coils of 10 turns each are placed in the same plane. Their radii are 20 cm and 40 cm and carry 0.2 and 0.3 A. current respectively in opposite directions. The magnetic induction (in T) at the centre is

(a) $\frac{3}{4}\mu_0$ (b) $\frac{5}{4}\mu_0$ (c) $\frac{7}{4}\mu_0$ (d) $\frac{9}{4}\mu_0$

**36.** The number of turns in primary and secondary coils of a transformer is 50 and 200 respectively. If the current in the primary coil is 4 A, then the current in the secondary coil is

(a) 1 A (b) 2 A (c) 4 A (d) 5 A

**37.** X-rays of wavelength 0.140 nm are scattered from a block of carbon. What will be the wavelengths of *X*-rays scattered at 90° ?

(a) 0.140 nm (b) 0.142 nm
(c) 0.144 nm (d) 0.146 nm

**38.** An X-ray tube produces a continuous spectrum of radiation with its shortest wavelength of $45 \times 10^{-2}$ Å. The maximum energy of a photon in the radiation in eV is

($h = 6.62 \times 10^{-34}$ J-s, $c = 3 \times 10^8$ m/s)

(a) 27,500 (b) 22,500
(c) 17,500 (d) 12,500

**39.** $F_{pp}$, $F_{nn}$ and $F_{np}$ are the nuclear forces between proton-proton, neutron-neutron and neutron-proton respectively. Then relation between them is

(a) $F_{pp} = F_{nn} \neq F_{np}$ (b) $F_{pp} \neq F_{nn} = F_{np}$
(c) $F_{pp} = F_{nn} = F_{np}$ (d) $F_{pp} \neq F_{nn} \neq F_{np}$

**40.** Which of the following statements is not correct when a junction diode is in forward bias ?

(a) The width of depletion region decreases.
(b) Free electrons on *n*-side will move towards the junction.
(c) Holes on *p*-side move towards the junction.
(d) Electron on *n*-side and holes on *p*-side will move away from junction.

## Chemistry

**41.** An electronic transition in hydrogen atom results in the formation of $H_\alpha$ line of hydrogen in Lyman series, the energies associated with the electron in each of the orbits involved in the transition (in kcal mol$^{-1}$) are

(a) −313.6, − 34.84 (b) −313.6, − 78.4
(c) −78.4, − 34.84 (d) −78.4, − 19.6

**42.** The velocities of two particles *A* and *B* are 0.05 and 0.02 ms$^{-1}$ respectively. The mass of *B* is five times the mass of *A*. The ratio of their de-Broglie's wavelength is

(a) 2 : 1 (b) 1 : 4
(c) 1 : 1 (d) 4 : 1

**43.** If the mass defect of $_5B^{11}$ is 0.081 u, its average binding energy (in MeV) is

(a) 8.60 (b) 6.85
(c) 5.60 (d) 5.86

**44.** The atomic numbers of elements *A*, *B*, *C* and *D* are $Z-1$, $Z$, $Z+1$ and $Z+2$, respectively. If '*B*' is a noble gas, choose the correct answers from the following statements

(1) '*A*' has higher electron affinity
(2) '*C*' exists in +2 oxidation state
(3) '*D*' is an alkaline earth metal

(a) (1) and (2) (b) (2) and (3)
(c) (1) and (3) (d) (1), (2) and (3)

**45.** The bond length of HCl molecule is 1.275 Å and its dipole moment is 1.03 D. The ionic character of the molecule (in percent) (charge of the electron = $4.8 \times 10^{-10}$ esu) is

(a) 100 (b) 67.3
(c) 33.66 (d) 16.83

**46.** Which one of the following is a correct set ?

(a) $H_2O$, $sp^3$, angular
(b) $BCl_3$, $sp^3$, angular
(c) $NH_4^+$, $dsp^2$, square planar
(d) $CH_4$, $dsp^2$, tetrahedral

**47.** Ammonium carbamate decomposes as

$NH_2COONH_4(s) \rightleftharpoons 2NH_3(g) + CO_2(g)$

For the reaction, $K_p = 2.9 \times 10^{-5}$ atm$^3$. If we start with 1 mole of the compound, the total pressure at equilibrium would be

(a) 0.766 atm (b) 0.0582 atm
(c) 0.0388 atm (d) 0.0194 atm

**48.** What is the temperature at which the kinetic energy of 0.3 moles of helium is equal to the kinetic energy of 0.4 moles of argon at 400 K ?
(a) 400 K (b) 873 K
(c) 533 K (d) 300 K

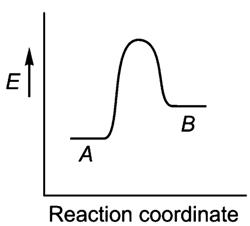
**49.** When 25 g of a non-volatile solute is dissolved in 100 g of water, the vapour pressure is lowered by $2.25 \times 10^{-1}$ mm. If the vapour pressure of water at 20°C is 17.5 mm, what is the molecular weight of the solute ?
(a) 206 (b) 302
(c) 350 (d) 276

**50.** 50 mL of $H_2O$ is added to 50 mL of $1 \times 10^{-3}$ M barium hydroxide solution. What is the pH of the resulting solution ?
(a) 3.0 (b) 3.3
(c) 11.0 (d) 11.7

**51.** **Assertion (A) :** The aqueous solution of $CH_3COONa$ is alkaline in nature.
**Reason (R) :** Acetate ion undergoes anionic hydrolysis
The correct answer is
(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is not true.
(d) (A) is not true but (R) is true.

**52.** When same quantity of electricity is passed through aqueous $AgNO_3$ and $H_2SO_4$ solutions connected in series, $5.04 \times 10^{-2}$ g of $H_2$ is liberated. What is the mass of silver (in grams) deposited ? (Eq. wts. of hydrogen = 1.008, silver = 108)
(a) 54 (b) 0.54
(c) 5.4 (d) 10.8

**53.** When electric current is passed through acidified water for 1930 s, 1120 mL of $H_2$ gas is collected (at STP) at the cathode. What is the current passed in amperes ?
(a) 0.05 (b) 0.50 (c) 5.0 (d) 50




**54.** For a crystal, the angle of diffraction ($2\theta$) is 90° and the second order line has a *d* value of 2.28 Å. The wavelength (in Å) of X-rays used for Bragg's diffraction is
(a) 1.612 (b) 2.00 (c) 2.28 (d) 4.00

**55.** In a 500 mL flask, the degree of dissociation of $PCl_5$ at equilibrium is 40% and the initial amount is 5 moles. The value of equilibrium constant in mol $L^{-1}$ for the decomposition of $PCl_5$ is
(a) 2.33 (b) 2.66 (c) 5.32 (d) 4.66

**56.** For a reversible reaction $A \rightleftharpoons B$, which one of the following statements is wrong from the given energy profile diagram ?
(a) Activation energy of forward reaction is greater than backward reaction
(b) The forward reaction is endothermic
(c) The threshold energy is less than that of activation energy
(d) The energy of activation of forward reaction is equal to the sum of heat of reaction and the energy of activation of backward reaction

**57.** Calculate $\Delta H$ in kJ for the following reaction
$$C(g) + O_2(g) \longrightarrow CO_2(g)$$
Given that,
$$H_2O(g) + C(g) \longrightarrow CO(g) + H_2(g); \quad \Delta H = +131 \text{ kJ}$$
$$CO(g) + \frac{1}{2}O_2(g) \longrightarrow CO_2(g); \quad \Delta H = -282 \text{ kJ}$$
$$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(g); \quad \Delta H = -242 \text{ kJ}$$
(a) −393 (b) +393
(c) +655 (d) −655

**58.** Which one of the following graphs represents Freundlich adsorption isotherm ?

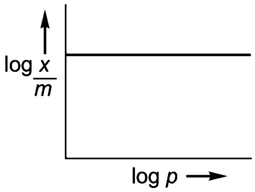
**59.** Which one of the following reactions represents the oxidising property of $H_2O_2$ ?
(a) $2KMnO_4 + 3H_2SO_4 + 5H_2O_2 \longrightarrow K_2SO_4 + 2MnSO_4 + 8H_2O + 5O_2$
(b) $2K_3[Fe(CN)_6] + 2KOH + H_2O_2 \longrightarrow 2K_4[Fe(CN)_6] + 2H_2O + O_2$
(c) $PbO_2 + H_2O_2 \longrightarrow PbO + H_2O + O_2$
(d) $2KI + H_2SO_4 + H_2O_2 \longrightarrow K_2SO_4 + I_2 + 2H_2O$

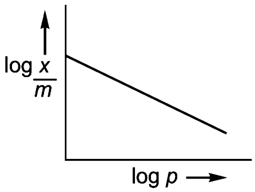
**60.** Which of the following statements are correct for alkali metal compounds ?

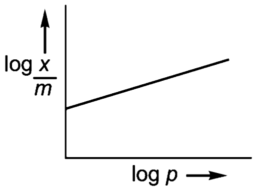
(i) Superoxides are paramagnetic in nature.

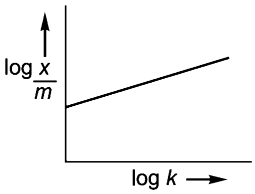
(ii) The basic strengths of hydroxides increases down the group.

(iii) The conductivity of chlorides in their aqueous solutions decreases down the group.

(a) 


(b) 


(c) 


(d) 


(iv) The basic nature of carbonates in aqueous solutions is due to cationic hydrolysis.

(a) (i), (ii) and (iii) only
(b) (i) and (ii) only
(c) (ii), (iii) and (iv) only
(d) (iii) and (iv) only

**61.** Boron halides behave as Lewis acids because of their ......... nature.

(a) proton donor
(b) covalent
(c) electron deficient
(d) ionising

**62.** Identify $B$ in the following reaction

$$H_4SiO_4 \xrightarrow[-H_2O]{1000\ °C} A \xrightarrow[\Delta]{\text{Carbon}} B + CO$$

(a) corundum (b) quartz
(c) silica (d) carborundum

**63.** The correct order of reducing abilities of hydrides of V group elements is

(a) $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$
(b) $NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$
(c) $NH_3 < PH_3 > AsH_3 > SbH_3 > BiH_3$
(d) $SbH_3 > BiH_3 > AsH_3 > NH_3 > PH_3$

**64.** The number of sigma and pi bonds in peroxodisulphuric acid are, respectively

(a) 9 and 4 (b) 11 and 4
(c) 4 and 8 (d) 4 and 9

**65.** Which one of the following reactions does not occur ?

(a) $F_2 + 2Cl^- \longrightarrow 2F^- + Cl_2$
(b) $Cl_2 + 2F^- \longrightarrow 2Cl^- + F_2$
(c) $Br_2 + 2I^- \longrightarrow 2Br^- + I_2$
(d) $Cl_2 + 2Br^- \longrightarrow 2Cl^- + Br_2$

**66.** The compound in which the number of $d\pi$- $p\pi$ bonds are equal to those present in $ClO_4^-$

(a) $XeF_4$ (b) $XeO_3$
(c) $XeO_4$ (d) $XeF_6$

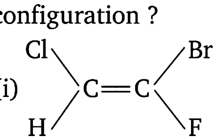
**67.** $[Co(NH_3)_5SO_4]$ Br and $[Co(NH_3)_5Br]$ $SO_4$ are a pair of ......... isomers.

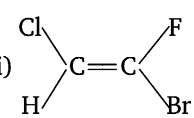
(a) ionisation (b) ligand
(c) coordination (d) hydrate

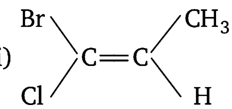
**68.** Among the following compounds, which one is not responsible for depletion of ozone layer ?

(a) $CH_4$ (b) $CFCl_3$
(c) NO (d) $Cl_2$

**69.** Which of the following compound(s) has 'Z' configuration ?

(i) 

(ii) 

(iii) 

(a) (i) only (b) (ii) only
(c) (iii) only (d) (i) and (iii)

**70.** According to Cahn-Ingold-Prelog sequence rules, the correct order of priority for the given groups is
(a) $—COOH > —CH_2OH > —OH > —CHO$
(b) $—COOH > —CHO > —CH_2OH > —OH$
(c) $—OH > —CH_2OH > —CHO > —COOH$
(d) $—OH > —COOH > —CHO > —CH_2OH$

**71.** What are $X$ and $Y$ respectively in the following reaction ?

$$Z\text{-product} \xleftarrow{Y} \text{2-butyne} \xrightarrow{X} E\text{-product}$$

(a) $Na/NH_3$(liq.) and $Pd/BaSO_4 + H_2$
(b) $Ni/140°C$ and $Pd/BaSO_4 + H_2$
(c) $Ni/140°C$ and $Na/NH_3$(liq.)
(d) $Pd/BaSO_4 + H_2$ and $Na/NH_3$(liq.)

**72.** In which of the following reactions, chlorine acts as an oxidising agent ?
(i) $CH_3CH_2OH + Cl_2 \longrightarrow CH_3CHO + HCl$
(ii) $CH_3CHO + Cl_2 \longrightarrow CCl_3 \cdot CHO + HCl$
(iii) $CH_4 + Cl_2 \xrightarrow{h\nu} CH_3Cl + HCl$
The correct answer is
(a) (i) only
(b) (ii) only
(c) (i) and (iii)
(d) (i), (ii) and (iii)

**73.** The correct order of reactivity of hydrogen halides with ethyl alcohol is
(a) $HF > HCl > HBr > HI$
(b) $HCl > HBr > HF > HI$
(c) $HBr > HCl > HI > HF$
(d) $HI > HBr > HCl > HF$

**74.** The IUPAC name of $C_2H_5—O—CH<\begin{smallmatrix}CH_3\\CH_3\end{smallmatrix}$ is
(a) ethoxy propane
(b) 1,1-dimethyl ether
(c) 2-ethoxy isopropane
(d) 2-ethoxy propane

**75.** Acetone on addition to methyl magnesium bromide forms a complex, which on decomposition with acid gives $X$ and $Mg(OH)Br$. Which one of the following is $X$ ?
(a) $CH_3OH$
(b) $(CH_3)_3COH$
(c) $(CH_3)_2CHOH$
(d) $CH_3CH_2OH$

**76.** Identify $A$ and $B$ in the following reaction

$$CH_3—CH_3 \xleftarrow{B} CH_3COOH \xrightarrow{A} CH_3CH_2OH$$

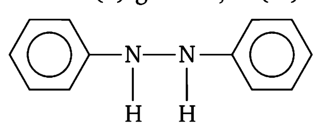
| | A | B |
|---|---|---|
| (a) | HI + red P | $LiAlH_4$ |
| (b) | Ni/Δ | $LiAlH_4$ |
| (c) | $LiAlH_4$ | HI + red P |
| (d) | Pd-$BaSO_4$ | Zn + HCl |

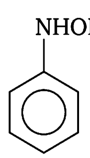
**77.** The structure of the compound formed, when nitrobenzene is reduced by lithium aluminium hydride ($LiAlH_4$) is

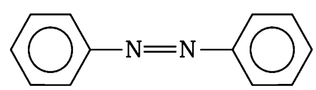
**78.** The energy released in an atomic bomb explosion is mainly due to
(a) release of electrons
(b) release of neutrons
(c) lesser mass of products than initial material
(d) greater mass of products than initial material

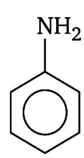
**79.** If $\overline{M}_w$ is the weight average molecular weight and $\overline{M}_n$ is the number average molecular weight of a polymer, the poly dispersity index (PDI) of the polymer is given by
(a) $\dfrac{\overline{M}_n}{\overline{M}_w}$  (b) $\dfrac{\overline{M}_w}{\overline{M}_n}$
(c) $\overline{M}_w \times \overline{M}_n$  (d) $\dfrac{1}{\overline{M}_w \times \overline{M}_n}$

**80.** Hydrolysis of sucrose with dilute aqueous sulphuric acid yields
(a) 1 : 1 D-(+)-glucose; D-(–)–fructose
(b) 1 : 2 D-(+)-glucose; D-(–)–fructose
(c) 1 : 1 D-(–)-glucose; D-(+)–fructose
(d) 1 : 2 D-(–)-glucose; D-(+)–fructose

(a) 


(b) 


(c) 


(d) 

## Mathematics

**81.** If $f: R \to C$ is defined by $f(x) = e^{2ix}$ for $x \in R$, then $f$ is (where $C$ denotes the set of all complex numbers)

(a) one-one
(b) onto
(c) one-one and onto
(d) neither one-one nor onto

**82.** If $f: R \to R$ and $g: R \to R$ are defined by $f(x) = |x|$ and $g(x) = [x - 3]$ for $x \in R$, then $\left\{g(f(x)): -\frac{8}{5} < x < \frac{8}{5}\right\}$ is equal to

(a) $\{0, 1\}$ (b) $\{1, 2\}$
(c) $\{-3, -2\}$ (d) $\{2, 3\}$

**83.** If $f: [-6, 6] \to R$ is defined by $f(x) = x^2 - 3$ for $x \in R$, then

$(fofof)(-1) + (fofof)(0) + (fofof)(1)$

is equal to

(a) $f(4\sqrt{2})$ (b) $f(3\sqrt{2})$
(c) $f(2\sqrt{2})$ (d) $f(\sqrt{2})$

**84.** Given that $a, b \in \{0, 1, 2, \ldots, 9\}$ with $a + b \neq 0$ and that $\left(a + \frac{b}{10}\right)^x = \left(\frac{a}{10} + \frac{b}{100}\right)^y = 1000.$ Then, $\frac{1}{x} - \frac{1}{y}$ is equal to

(a) 1 (b) 1/2 (c) 1/3 (d) 1/4

**85.** For any integer $n \geq 1$, the sum $\sum_{k=1}^{n} k(k + 2)$ is equal to

(a) $\frac{n(n+1)(n+2)}{6}$
(b) $\frac{n(n+1)(2n+1)}{6}$
(c) $\frac{n(n+1)(2n+7)}{6}$
(d) $\frac{n(n+1)(2n+9)}{6}$

**86.** 9 balls are to be placed in 9 boxes and 5 of the balls cannot fit into 3 small boxes. The number of ways of arranging one ball in each of the boxes is

(a) 18720 (b) 18270
(c) 17280 (d) 12780

**87.** If ${}^nP_r = 30240$ and ${}^nC_r = 252$, then the ordered pair $(n, r)$ is equal to

(a) (12, 6) (b) (10, 5)
(c) (9, 4) (d) (16, 7)

**88.** If $(1 + x + x^2 + x^3)^5 = \sum_{k=0}^{15} a_k x^k$, then $\sum_{k=0}^{7} a_{2k}$ is equal to

(a) 128 (b) 256 (c) 512 (d) 1024

**89.** If $\alpha = \frac{5}{2!3} + \frac{5 \cdot 7}{3!3^2} + \frac{5 \cdot 7 \cdot 9}{4!3^3} + \ldots,$ then $\alpha^2 + 4\alpha$ is equal to

(a) 21 (b) 23 (c) 25 (d) 27

**90.** $\sum_{k=1}^{\infty} \frac{1}{k!}\left(\sum_{n=1}^{k} 2^{n-1}\right)$ is equal to

(a) $e$ (b) $e^2 + e$
(c) $e^2$ (d) $e^2 - e$

**91.** $\frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 5} + \frac{1}{3 \cdot 7} + \frac{1}{4 \cdot 9} + \ldots$ is equal to

(a) $2\log_e 2 - 2$ (b) $2 - \log_e 2$
(c) $2\log_e 4$ (d) $\log_e 4$

**92.** If $\alpha + \beta = -2$ and $\alpha^3 + \beta^3 = -56$, then the quadratic equation whose roots are $\alpha$ and $\beta$ is

(a) $x^2 + 2x - 16 = 0$
(b) $x^2 + 2x + 15 = 0$
(c) $x^2 + 2x - 12 = 0$
(d) $x^2 + 2x - 8 = 0$

**93.** The cubic equation whose roots are thrice to each of the roots of $x^3 + 2x^2 - 4x + 1 = 0$ is

(a) $x^3 - 6x^2 + 36x + 27 = 0$
(b) $x^3 + 6x^2 + 36x + 27 = 0$
(c) $x^3 - 6x^2 - 36x + 27 = 0$
(d) $x^3 + 6x^2 - 36x + 27 = 0$

**94.** If $A = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}$ and $f(t) = t^2 - 3t + 7$, then $f(A) + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix}$ is equal to

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

**95.** The inverse of the matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 3 \\ 3 & 3 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 3 & 1 \\ 4 & 3 & 8 \\ 3 & 4 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 3 & 4 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

**96.** $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ is equal to

(a) 0 (b) $a+b+c$
(c) $(a+b+c)^2$ (d) $(a+b+c)^3$

**97.** The points in the set $\left\{ z \in C : \arg\left(\dfrac{z-2}{z-6i}\right) = \dfrac{\pi}{2} \right\}$ (where $C$ denotes the set of all complex numbers) lie on the curve which is a

(a) circle (b) pair of lines
(c) parabola (d) hyperbola

**98.** If $\omega$ is a complex cube root of unity, then $\sin\left\{ (\omega^{10} + \omega^{23})\,\pi - \dfrac{\pi}{4} \right\}$ is equal to

(a) $\dfrac{1}{\sqrt{2}}$ (b) $\dfrac{1}{2}$ (c) 1 (d) $\dfrac{\sqrt{3}}{2}$

**99.** If $m_1, m_2, m_3$ and $m_4$ respectively denote the moduli of the complex numbers $1+4i$, $3+i$, $1-i$ and $2-3i$, then the correct one, among the following is

(a) $m_1 < m_2 < m_3 < m_4$
(b) $m_4 < m_3 < m_2 < m_1$
(c) $m_3 < m_2 < m_4 < m_1$
(d) $m_3 < m_1 < m_2 < m_4$

**100.** $\sqrt{3}$ cosec $20° -$ sec $20°$ is equal to

(a) 2
(b) $2 \sin 20° \cdot \text{cosec } 40°$
(c) 4
(d) $4 \sin 20° \cdot \text{cosec } 40°$

**101.** If $A = 35°, B = 15°$ and $C = 40°$, then $\tan A \cdot \tan B + \tan B \cdot \tan C + \tan C \cdot \tan A$ is equal to

(a) 0 (b) 1 (c) 2 (d) 3

**102.** If $\tan\theta + \tan\left(\theta + \dfrac{\pi}{3}\right) + \tan\left(\theta + \dfrac{2\pi}{3}\right) = 3$, then which of the following is equal to 1 ?

(a) $\tan 2\theta$ (b) $\tan 3\theta$
(c) $\tan^2\theta$ (d) $\tan^3\theta$

**103.** If $\alpha + \beta + \gamma = 2\theta$, then $\cos\theta + \cos(\theta - \alpha) + \cos(\theta - \beta) + \cos(\theta - \gamma)$ is equal to

(a) $4 \sin\dfrac{\alpha}{2} \cdot \cos\dfrac{\beta}{2} \cdot \sin\dfrac{\gamma}{2}$
(b) $4 \cos\dfrac{\alpha}{2} \cdot \cos\dfrac{\beta}{2} \cdot \cos\dfrac{\gamma}{2}$
(c) $4 \sin\dfrac{\alpha}{2} \cdot \sin\dfrac{\beta}{2} \cdot \sin\dfrac{\gamma}{2}$
(d) $4 \sin\alpha \cdot \sin\beta \cdot \sin\gamma$

**104.** $\{x \in R : \cos 2x + 2\cos^2 x = 2\}$ is equal to

(a) $\left\{ 2n\pi + \dfrac{\pi}{3} : n \in Z \right\}$
(b) $\left\{ n\pi \pm \dfrac{\pi}{6} : n \in Z \right\}$
(c) $\left\{ n\pi + \dfrac{\pi}{3} : n \in Z \right\}$
(d) $\left\{ 2n\pi - \dfrac{\pi}{3} : n \in Z \right\}$

**105.** If $\sin^{-1}\left(\dfrac{3}{x}\right) + \sin^{-1}\left(\dfrac{4}{x}\right) = \dfrac{\pi}{2}$, then $x$ is equal to

(a) 3 (b) 5 (c) 7 (d) 11

**106.** In $\Delta ABC$, if $\dfrac{1}{b+c} + \dfrac{1}{c+a} = \dfrac{3}{a+b+c}$, then $C$ is equal to

(a) 90° (b) 60° (c) 45° (d) 30°

**107.** In a triangle, if $r_1 = 2r_2 = 3r_3$, then $\dfrac{a}{b} + \dfrac{b}{c} + \dfrac{c}{a}$ is equal to

(a) $\dfrac{75}{60}$ (b) $\dfrac{155}{60}$ (c) $\dfrac{176}{60}$ (d) $\dfrac{191}{60}$

**108.** From the top of a hill $h$ metres high the angles of depressions of the top and the bottom of a pillar are $\alpha$ and $\beta$ respectively. The height (in metres) of the pillar is

(a) $\dfrac{h(\tan\beta - \tan\alpha)}{\tan\beta}$
(b) $\dfrac{h(\tan\alpha - \tan\beta)}{\tan\alpha}$

(c) $\dfrac{h(\tan\beta + \tan\alpha)}{\tan\beta}$

(d) $\dfrac{h(\tan\beta + \tan\alpha)}{\tan\alpha}$

**109.** The position vectors of $P$ and $Q$ are respectively $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$. If R is a point on $\overrightarrow{\mathbf{PQ}}$ such that $\overrightarrow{\mathbf{PR}} = 5\,\overrightarrow{\mathbf{PQ}}$, then the position vector of $R$ is

(a) $5\vec{\mathbf{b}} - 4\vec{\mathbf{a}}$ (b) $5\vec{\mathbf{b}} + 4\vec{\mathbf{a}}$

(c) $4\vec{\mathbf{b}} - 5\vec{\mathbf{a}}$ (d) $4\vec{\mathbf{b}} + 5\vec{\mathbf{a}}$

**110.** If the points with position vectors $60\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$, $40\hat{\mathbf{i}} - 8\hat{\mathbf{j}}$ and $a\hat{\mathbf{i}} - 52\hat{\mathbf{j}}$ are collinear, then $a$ is equal to

(a) −40 (b) −20 (c) 20 (d) 40

**111.** If the position vectors of $A, B$ and $C$ are respectively $2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ and $3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$, then $\cos^2 A$ is equal to

(a) 0 (b) $\dfrac{6}{41}$ (c) $\dfrac{35}{41}$ (d) 1

**112.** Let $\vec{\mathbf{a}}$ be a unit vector, $\vec{\mathbf{b}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = \hat{\mathbf{i}} + 3\hat{\mathbf{k}}$. Then, maximum value of $[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$ is

(a) −1 (b) $\sqrt{10} + \sqrt{6}$

(c) $\sqrt{10} - \sqrt{6}$ (d) $\sqrt{59}$

**113.** If $A$ and $B$ are independent events of a random experiment such that $P(A \cap B) = \dfrac{1}{6}$ and $P(\bar{A} \cap \bar{B}) = \dfrac{1}{3}$, then $P(A)$ is equal to

(Here, $\vec{\mathbf{E}}$ is the complement of the event $E$)

(a) 1/4 (b) 1/3 (c) $\dfrac{1}{9}$ (d) $\dfrac{2}{3}$

**114.** Let $S$ be the sample space of the random experiment of throwing simultaneously two unbiased dice with six faces (numbered 1 to 6) and let $E_k = \{(a, b) \in S : ab = k\}$ for $k \geq 1$.

If $p_k + P(E_k)$ for $k \geq 1$, then the correct among the following, is

(a) $p_1 < p_{30} < p_4 < p_6$

(b) $p_{36} < p_6 < p_2 < p_4$

(c) $p_1 < p_{11} < p_4 < p_6$

(d) $p_{36} < p_{11} < p_6 < p_4$

**115.** For $k = 1, 2, 3$ the box $B_k$ contains $k$ red balls and $(k+1)$ white balls. Let $P(B_1) = \dfrac{1}{2}$, $P(B_2) = \dfrac{1}{3}$ and $P(B_3) = \dfrac{1}{6}$. A box is selected at random and a ball is drawn from it. If a red ball is drawn, then the probability that it has come from box $B_2$, is

(a) $\dfrac{35}{78}$ (b) $\dfrac{14}{39}$ (c) $\dfrac{10}{13}$ (d) $\dfrac{12}{13}$

**116.** The distribution of a random variable $X$ is given below

| $X = x$ | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $P(X = x)$ | 1/10 | $k$ | 1/5 | $2k$ | 3/10 | $k$ |

The value of $k$ is

(a) 1/10 (b) 2/10 (c) 3/10 (d) 7/10

**117.** If the sum of the distances of a point $P$ from two perpendicular lines in a plane is 1, then the locus of $P$ is a

(a) rhombus (b) circle

(c) straight line (d) pair of straight lines

**118.** The transformed equation of $3x^2 + 3y^2 + 2xy = 2$, when the coordinate axes are rotated through an angle of 45°, is

(a) $x^2 + 2y^2 = 1$ (b) $2x^2 + y^2 = 1$

(c) $x^2 + y^2 = 1$ (d) $x^2 + 3y^2 = 1$

**119.** If $l, m, n$ are in arithmetic progression, then the straight line $lx + my + n = 0$ will pass through the point

(a) (−1, 2) (b) (1, −2)

(c) (1, 2) (d) (2, 1)

**120.** The value of $k$ such that the lines $2x - 3y + k = 0$, $3x - 4y - 13 = 0$ and $8x - 11y - 33 = 0$ are concurrent, is

(a) 20 (b) −7 (c) 7 (d) −20

**121.** The value of $\lambda$ such that

$\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$

represents a pair of straight lines, is

(a) 1 (b) −1 (c) 2 (d) −2

**122.** A pair of perpendicular straight lines passes through the origin and also through the point of intersection of the curve $x^2 + y^2 = 4$ with $x + y = a$. The set containing the value of 'a' is

(a) {−2, 2} (b) {−3, 3}

(c) {−4, 4} (d) {−5, 5}

**123.** In $\Delta ABC$ the mid points of the sides $AB, BC$ and $CA$ are respectively $(l, 0, 0), (0, m, 0)$ and $(0, 0, n)$. Then, $\dfrac{AB^2 + BC^2 + CA^2}{l^2 + m^2 + n^2}$ is equal to

(a) 2 (b) 4 (c) 8 (d) 16

**124.** The angle between the lines whose direction cosines are $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2}\right)$ and $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{-\sqrt{3}}{2}\right)$, is

(a) $\pi$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

**125.** If the lines $2x - 3y = 5$ and $3x - 4y = 7$ are two diameters of a circle of radius 7, then the equation of the circle is

(a) $x^2 + y^2 + 2x - 4y - 47 = 0$

(b) $x^2 + y^2 = 49$

(c) $x^2 + y^2 - 2x + 2y - 47 = 0$

(d) $x^2 + y^2 = 17$

**126.** The inverse of the point (1, 2) with respect to the circle $x^2 + y^2 - 4x - 6y + 9 = 0$, is

(a) $\left(1, \frac{1}{2}\right)$ (b) (2, 1)

(c) (0, 1) (d) (1, 0)

**127.** If $\theta$ is the angle between the tangents from $(-1, 0)$ to the circle $x^2 + y^2 - 5x + 4y - 2 = 0$, then $\theta$ is equal to

(a) $2\tan^{-1}\left(\frac{7}{4}\right)$ (b) $\tan^{-1}\left(\frac{7}{4}\right)$

(c) $2\cot^{-1}\left(\frac{7}{4}\right)$ (d) $\cot^{-1}\left(\frac{7}{4}\right)$

**128.** If $2x + 3y + 12 = 0$ and $x - y + 4\lambda = 0$ are conjugate with respect to the parabola $y^2 = 8x$, then $\lambda$ is equal to

(a) 2 (b) –2 (c) 3 (d) –3

**129.** For an ellipse with eccentricity 1/2 the centre is at the origin. If one directrix is $x = 4$, then the equation of the ellipse is

(a) $3x^2 + 4y^2 = 1$ (b) $3x^2 + 4y^2 = 12$

(c) $4x^2 + 3y^2 = 1$ (d) $4x^2 + 3y^2 = 12$

**130.** The distance between the foci of the hyperbola $x^2 - 3y^2 - 4x - 6y - 11 = 0$ is

(a) 4 (b) 6 (c) 8 (d) 10

**131.** The radius of the circle with the polar equation $r^2 - 8r(\sqrt{3}\cos\theta + \sin\theta) + 15 = 0$ is

(a) 8 (b) 7 (c) 6 (d) 5

**132.** $\lim\limits_{x \to 0} \dfrac{(1 - e^x)\sin x}{x^2 + x^3}$ is equal to

(a) –1 (b) 0

(c) 1 (d) 2

**133.** If $f : R \to R$ is defined by $f(x) = [x - 3] + |x - 4|$ for $x \in R$, then $\lim\limits_{x \to 3^-} f(x)$ is equal to

(a) –2 (b) –1 (c) 0 (d) 1

**134.** If $f : R \to R$ is defined by

$$f(x) = \begin{cases} \dfrac{\cos 3x - \cos x}{x^2}, & \text{for } x \neq 0 \\ \lambda, & \text{for } x = 0 \end{cases}$$

and if $f$ is continuous at $x = 0$, then $\lambda$ is equal to

(a) –2 (b) –4 (c) –6 (d) –8

**135.** If $f(2) = 4$ and $f'(2) = 1$, then

$$\lim_{x \to 2} \frac{x f(2) - 2f(x)}{x - 2}$$

is equal to

(a) –2 (b) 1 (c) 2 (d) 3

**136.** If $x = a\left\{\cos\theta + \log\tan\left(\frac{\theta}{2}\right)\right\}$ and $y = a\sin\theta$, then $\frac{dy}{dx}$ is equal to

(a) $\cot\theta$ (b) $\tan\theta$

(c) $\sin\theta$ (d) $\cos\theta$

**137.** If $y = \sin(\log_e x)$, then $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx}$ is equal to

(a) $\sin(\log_e x)$ (b) $\cos(\log_e x)$

(c) $y^2$ (d) $-y$

**138.** The angle between the curves $y^2 = 4x + 4$ and $y^2 = 36(9 - x)$ is

(a) 30° (b) 45°

(c) 60° (d) 90°

**139.** If $m$ and $M$ respectively denote the minimum and maximum of $f(x) = (x - 1)^2 + 3$ for $x \in [-3, 1]$, then the ordered pair $(m, M)$ is equal to

(a) (–3, 19) (b) (3, 19)

(c) (–19, 3) (d) (–19, –3)

**140.** The length of the subtangent at (2, 2) to the curve $x^5 = 2y^4$ is

(a) $\frac{5}{2}$ (b) $\frac{8}{5}$

(c) $\frac{2}{5}$ (d) $\frac{5}{8}$

**141.** If $\int e^x \left(\frac{1-\sin x}{1-\cos x}\right) dx = f(x) +$ constant, then $f(x)$ is equal to

(a) $e^x \cot\left(\frac{x}{2}\right) + c$

(b) $e^{-x} \cot\left(\frac{x}{2}\right) + c$

(c) $-e^x \cot\left(\frac{x}{2}\right) + c$

(d) $-e^{-x} \cot\left(\frac{x}{2}\right) + c$

**142.** If $I_n = \int x^n \cdot e^{cx}\, dx$ for $n \geq 1$, then $c \cdot I_n + n \cdot I_{n-1}$ is equal to

(a) $x^n e^{cx}$ (b) $x^n$

(c) $e^{cx}$ (d) $x^n + e^{cx}$

**143.** If $\int e^x (1+x) \cdot \sec^2 (xe^x)\, dx$

$= f(x) +$ constant, then $f(x)$ is equal to

(a) $\cos(xe^x)$ (b) $\sin(xe^x)$

(c) $2\tan^{-1}(x)$ (d) $\tan(x\, e^x)$

**144.** $\int_0^1 x^{3/2} \sqrt{1-x}\, dx$ is equal to

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{9}$ (c) $\frac{\pi}{12}$ (d) $\frac{\pi}{16}$

**145.** $\int_{-\pi/2}^{\pi/2} \sin|x|\, dx$ is equal to

(a) 0 (b) 1

(c) 2 (d) $\pi$

**146.** The area (in sq unit) of the region bounded by the curves $2x = y^2 - 1$ and $x = 0$ is

(a) 1/3 (b) 2/3

(c) 1 (d) 2

**147.** The solution of the differential equation

$$\frac{dy}{dx} = \frac{xy + y}{xy + x} \text{ is}$$

(a) $x + y - \log\left(\frac{cy}{x}\right)$

(b) $x + y = \log(cxy)$

(c) $x - y - \log\left(\frac{cx}{y}\right)$

(d) $y - x = \log\left(\frac{cx}{y}\right)$

**148.** The solution of the differential equation

$$\frac{dy}{dx} = \frac{x - 2y + 1}{2x - 4y} \text{ is}$$

(a) $(x - 2y)^2 + 2x = c$

(b) $(x - 2y)^2 + x = c$

(c) $(x - 2y) + 2x^2 = c$

(d) $(x - 2y) + x^2 = c$

**149.** The solution of the differential equation

$$\frac{dy}{dx} - y \tan x = e^x \sec x \text{ is}$$

(a) $y = e^x \cos x + c$

(b) $y \cos x = e^x + c$

(c) $y = e^x \sin x + c$

(d) $y \sin x = e^x + c$

**150.** The solution of the differential equation

$$xy^2\, dy - (x^3 + y^3)\, dx = 0 \text{ is}$$

(a) $y^3 = 3x^3 + c$

(b) $y^3 = 3x^3 \log(cx)$

(c) $y^3 = 3x^3 + \log(cx)$

(d) $y^3 + 3x^3 = \log(cx)$

# Answers

## PHYSICS

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** (a) | **2.** (c) | **3.** (b) | **4.** (c) | **5.** (c) | **6.** (b) | **7.** (a) | **8.** (d) | **9.** (b) | **10.** (d) |
| **11.** (d) | **12.** (d) | **13.** (a) | **14.** (b) | **15.** (d) | **16.** (c) | **17.** (c) | **18.** (d) | **19.** (a) | **20.** (a) |
| **21.** (c) | **22.** (c) | **23.** (a) | **24.** (c) | **25.** (c) | **26.** (c) | **27.** (c) | **28.** (a) | **29.** (b) | **30.** (b) |
| **31.** (c) | **32.** (b) | **33.** (a) | **34.** (b) | **35.** (b) | **36.** (a) | **37.** (b) | **38.** (a) | **39.** (c) | **40.** (d) |

### ➠ CHEMISTRY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **41.** (b) | **42.** (a) | **43.** (b) | **44.** (c) | **45.** (d) | **46.** (a) | **47.** (b) | **48.** (c) | **49.** (c) | **50.** (c) |
| **51.** (a) | **52.** (c) | **53.** (c) | **54.** (a) | **55.** (b) | **56.** (c) | **57.** (a) | **58.** (c) | **59.** (d) | **60.** (b) |
| **61.** (c) | **62.** (d) | **63.** (a) | **64.** (b) | **65.** (b) | **66.** (b) | **67.** (a) | **68.** (a) | **69.** (d) | **70.** (d) |
| **71.** (a) | **72.** (d) | **73.** (d) | **74.** (d) | **75.** (b) | **76.** (c) | **77.** (c) | **78.** (c) | **79.** (b) | **80.** (a) |

### ➠ MATHEMATICS

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **81.** (d) | **82.** (c) | **83.** (a) | **84.** (c) | **85.** (c) | **86.** (c) | **87.** (b) | **88.** (c) | **89.** (b) | **90.** (d) |
| **91.** (b) | **92.** (d) | **93.** (d) | **94.** (b) | **95.** (d) | **96.** (d) | **97.** (a) | **98.** (a) | **99.** (c) | **100.** (c) |
| **101.** (b) | **102.** (b) | **103.** (b) | **104.** (b) | **105.** (b) | **106.** (b) | **107.** (d) | **108.** (a) | **109.** (a) | **110.** (a) |
| **111.** (c) | **112.** (d) | **113.** (b) | **114.** (a) | **115.** (b) | **116.** (a) | **117.** (a) | **118.** (b) | **119.** (b) | **120.** (b) |
| **121.** (c) | **122.** (a) | **123.** (c) | **124.** (c) | **125.** (c) | **126.** (c) | **127.** (a) | **128.** (d) | **129.** (b) | **130.** (c) |
| **131.** (b) | **132.** (a) | **133.** (c) | **134.** (b) | **135.** (c) | **136.** (b) | **137.** (d) | **138.** (d) | **139.** (b) | **140.** (b) |
| **141.** (c) | **142.** (a) | **143.** (d) | **144.** (d) | **145.** (c) | **146.** (b) | **147.** (d) | **148.** (a) | **149.** (b) | **150.** (b) |

# Hints & Explanations

## Physics

**1.** $h \propto G^x L^y E^z$

$$[ML^2T^{-1}] \propto [M^{-1}L^3T^{-2}]^x [ML^2T^{-1}]^y [ML^2T^{-2}]^z$$

$$[ML^2T^{-1}] = k[M^{-1}L^3T^{-2}]^x [ML^2T^{-1}]^y [ML^2T^{-2}]^z$$

Comparing the powers, we get

$$1 = -x + y + z \quad \dots\text{(i)}$$

$$2 = 3x + 2y + 2z \quad \dots\text{(ii)}$$

$$-1 = -2x - y - 2z \quad \dots\text{(iii)}$$

On solving Eqs. (i), (ii) and (iii), we get

$$x = 0$$

**2.** Let $\vec{\mathbf{B}} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$

Then component of vector $\vec{\mathbf{A}}$ along $\vec{\mathbf{B}}$

$$= \frac{\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}}{|\vec{\mathbf{B}}|}$$

$$= \frac{(a_x \hat{\mathbf{i}} + a_y \hat{\mathbf{j}} + a_z \hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}})}{|\hat{\mathbf{i}} - \hat{\mathbf{j}}|} = \frac{a_x - a_y}{\sqrt{2}}$$

**3.** The ball is thrown vertically upwards then according to equation of motion

$$(0)^2 - u^2 = -2gh \quad \dots\text{(i)}$$

and $$0 = u - gt \quad \dots\text{(ii)}$$

From Eqs. (i) and (ii),

$$h = \frac{gt^2}{2}$$

When the ball is falling downwards after reaching the maximum height

$$s = ut' + \frac{1}{2} g(t')^2$$

$$\frac{h}{2} = (0)t' + \frac{1}{2} g(t')^2$$

$$\Rightarrow \quad t' = \sqrt{\frac{h}{g}} = \frac{t}{\sqrt{2}}$$

Hence, the total time from the time of projection to reach a point at half of its maximum height while returning $= t + t'$

$$= t + \frac{t}{\sqrt{2}}$$

**4.** Direction of velocity is always tangent to the path so at the top of trajectory, it is in horizontal direction.

**5.** Given, velocity of river, $(v) = 2$ m/s

Density of water $\rho = 1.2$ g/cc

Mass of each cubic metre

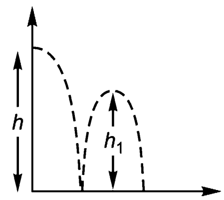
$$m = \frac{1.2 \times 10^{-3}}{(10^{-2})^3} = 1.2 \times 10^3 \text{ kg}$$

$$\therefore \quad \text{Kinetic energy} = \frac{1}{2} mv^2$$

$$= \frac{1}{2} \times 1.2 \times 10^3 \times (2)^2$$

$$= 2.4 \times 10^3 \text{ J} = 2.4 \text{ kJ}$$

**6.**




Total distance travelled by the ball before its second hit is

$$H = h + 2h_1$$
$$= h[1 + 2e^2] \qquad [\because h_1 = he^2]$$

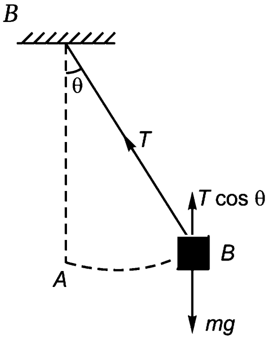
**7.** As initially both the particles were at rest therefore velocity of centre of mass was zero and there is no external force on the system so speed of centre of mass remains constant *ie,* it should be equal to zero.

**8.**
$$\mu = \tan\theta\left[1 - \frac{1}{n^2}\right]$$

Here, $\theta = 45°$ and $n = 2$

$$\therefore \quad \mu = \tan 45°\left[1 - \frac{1}{2^2}\right]$$
$$= 1 - \frac{1}{4} = \frac{3}{4} = 0.75$$

**9.** Now, at $B$




In equilibrium,

$$T\cos\theta = mg$$
$$\Rightarrow \quad \cos\theta = \frac{150 \times 9.8}{2940}$$
$$\Rightarrow \quad \cos\theta = 0.5$$
$$\Rightarrow \quad \theta = 60°$$

**10.** Moment of inertia of a circular disc about an axis passing through centre of gravity and perpendicular to its plane

$$I = \frac{1}{2}MR^2 \qquad \ldots\text{(i)}$$

From Eq. (i) $MR^2 = 2I$

Then, moment of inertia of disc about tangent in a plane $= \frac{5}{4}MR^2$

$$= \frac{5}{4}(2I) = \frac{5}{2}I$$

**11.** Time period of satellite

$T \propto \frac{1}{M^{1/2}}$, where $M$ is mass of earth.

$\propto (R + h)^{3/2}$ where $R$ is radius of the orbit, $h$ is the height of satellite from the earth's surface.

**12.** It is the least interval of time after which the periodic motion of a body repeats itself. Therefore, displacement will be zero.

**13.**
$$Y = \frac{mgl}{A\,\Delta l}$$
$$\Rightarrow \quad \frac{\Delta l}{l} = \frac{mg}{AY}$$
$$\therefore \quad \frac{\Delta l}{l} = \frac{1 \times 10}{3 \times 10^{-6} \times 10^{11}} = 0.3 \times 10^{-4}$$

**14.** In case of soap bubble

$$W = T \times 2 \times \Delta A$$
$$= 0.03 \times 2 \times 40 \times 10^{-4} = 2.4 \times 10^{-4}\ \text{J}$$

**15.** Terminal velocity, $v_T \propto r^2$

$$\text{or} \quad \frac{v_{T_1}}{v_{T_2}} = \frac{r_1^2}{r_2^2}$$
$$\therefore \quad \sqrt{\frac{9}{4}} = \frac{r_1}{r_2}$$
$$\text{or} \quad \frac{r_1}{r_2} = \frac{3}{2}$$
$$\therefore \quad v = \frac{4}{3}\pi r^3$$
$$\text{or} \quad \frac{v_1}{v_2} = \frac{r_1^3}{r_2^3} = \frac{27}{8}$$

**16.** Ideal gas equation is given by

$$pV = nRT \qquad \ldots\text{(i)}$$

For oxygen, $p = 1$ atm, $V = 1$ L, $n = n_O$

Therefore Eq. (i) becomes

$$\therefore \quad 1 \times 1 = n_{O_2}RT \quad \Rightarrow \quad n_{O_2} = \frac{1}{RT}$$

For nitrogen $p = 0.5$ atm, $V = 2$ L, $n = n_N$

$\therefore \quad 0.5 \times 2 = n_{N_2} RT$

$\Rightarrow \quad n_{N_2} = \dfrac{1}{RT}$

For mixture of gas

$$p_{mix} V_{mix} = n_{mix} RT$$

Here, $\quad n_{mix} = n_O + n_{N_2}$

$\therefore \quad \dfrac{p_{mix} V_{mix}}{RT} = \dfrac{1}{RT} + \dfrac{1}{RT}$

$\Rightarrow \quad p_{mix} V_{mix} = 2$

**17.** Modulus of elasticity $= \dfrac{\text{Force}}{\text{Area}} \times \dfrac{l}{\Delta l}$

$$3 \times 10^{11} = \frac{33000}{10^{-3}} \times \frac{l}{\Delta l}$$

$$\frac{\Delta l}{l} = \frac{33000}{10^{-3}} \times \frac{1}{3 \times 10^{11}}$$

$$= 11 \times 10^{-5}$$

Change in length, $\dfrac{\Delta l}{l} = \alpha \, \Delta T$

$$11 \times 10^{-5} = 1.1 \times 10^{-5} \times \Delta T$$

$\Rightarrow \quad \Delta T = 10$ K or $10°$C

**18.** In adiabatic compression temperature and hence, internal energy of the gas increases. In compression pressure will increase.

**19.** For adiabatic change equation of state is

$$pV^{\gamma} = \text{constant}$$

It can also be re-written as

$$TV^{\gamma - 1} = \text{constant} \left[ \text{as } p = \frac{nRT}{V} \right]$$

and $\quad p^{1-\gamma} T^{\gamma} = \text{constant} \left[ \text{as } V = \dfrac{nRT}{p} \right]$

**20.** The temperature at the contact of the surface

$$= \frac{K_1 d_2 \theta_1 + K_2 d_1 \theta_2}{K_1 d_2 + K_2 d_1}$$

$$= \frac{2K_2 d_2 \times 100 + 2d_2 \times K_2 \times 25}{2K_2 d_2 + K_2 2d_2}$$

$$= \frac{200 + 50}{4} = 62.5°\text{C}$$

**21.** $\quad v_{max} = v$

$\Rightarrow \quad A\omega = v \quad \Rightarrow \quad A \times 2\pi\nu = \nu\lambda$

or $\quad A = \dfrac{\lambda}{2\pi}$

**22.** The speed of the car is 72 km/h

$$= 72 \times \frac{5}{18} = 20 \text{ m/s}$$

The distance travelled by car in 10 s

$$= 10 \times 20 = 200 \text{ m}$$

Hence, the distance travelled by sound in reaching the hill and coming back to the moving driver

$= 1800 + (1800 - 200) = 3400$ m

So, the speed of sound $= \dfrac{3400}{10} = 340$ m/s

**23.** Lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

where, $R_2 = \infty$, $R_1 = 0.3$ m

$\therefore \quad \dfrac{1}{f} = \left(\dfrac{5}{3} - 1\right)\left(\dfrac{1}{0.3} - \dfrac{1}{\infty}\right)$

$\Rightarrow \quad \dfrac{1}{f} = \dfrac{2}{3} \times \dfrac{1}{0.3} \quad$ or $\quad f = 0.45$ m

**24.** Position of fourth maxima

$$x_0 = \frac{4D\lambda}{d} \quad \text{or} \quad x \propto \lambda$$

$\therefore \quad x$ (blue) $< x$ (green)

**25.** The image of an object in white light formed by a lens is usually coloured and blurred. This defect of image is called chromatic aberration and arises due to the fact that focal length of a lens is different for different colours. In case of two thin lenses in contact, the combination will be free from chromatic aberration. The lens combination which satisfies this condition are called achromatic lenses.

**26.** The general condition for Froun hofer diffraction is $\dfrac{b^2}{L\lambda} << 1$.

**27.** Time period of magnet, $T = 2\pi\sqrt{\dfrac{I}{MB}}$

When magnet is cut parallel to its length into four equal pieces. Then new

magnetic moment, $M' = \dfrac{M}{4}$

New moment of inertia, $I' = \dfrac{I}{4}$

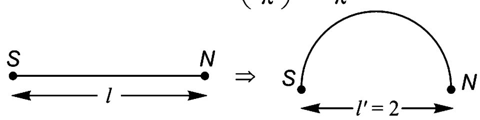
$\therefore \quad$ New time period, $T' = 2\pi\sqrt{\dfrac{I'}{M'B}}$

$\Rightarrow \quad T = T' = 4$ s

**28.** On bending a wire its pole strength remains unchanged whereas its magnetic moment changes.

New magnetic moment,

$$M' = m(2r) = m\left(\frac{2l}{\pi}\right) = \frac{3M}{\pi}$$



**29.** Ratio of charges = 2 : 3

$$\therefore \quad q_1 = \frac{2}{5} \times 1\,\mu\text{C}$$

and $$q_2 = \frac{3}{5} \times 1\,\mu\text{C}$$

Electrostatic force between the two charges

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

$$= \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 3 \times 10^{-6}}{5 \times 5 \times (1)^2}$$

$$= 2.16 \times 10^{-3}\text{ N}$$

**30.** At equitorial point

$$E_e = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

(directed from $+q$ to $-q$) and $V_e = 0$.

**31.** Current through each arm

$PRQ$ and $PSQ = 1$ A

$$V_P - V_R = 3\text{ V} \quad \text{...(i)}$$

$$V_P - V_S = 7\text{ V} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$V_R - V_S = +4\text{ V}$$

**32.** Here, $V < E$

$$\therefore \quad E = V + Ir$$

**For first case**

$$E = 12 + \frac{12}{16}r \quad \text{...(i)}$$

**For second case**

$$E = 11 + \frac{11}{10}r \quad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$12 + \frac{12}{16}r = 11 + \frac{11}{10}r$$

$$\Rightarrow \quad r = \frac{20}{7}\,\Omega$$

**33.** We know that thermoelectric power

$$S = \frac{dE}{dT}$$

Given, $E = K(T - T_r)\left[T_0 - \frac{1}{2}(T + T_r)\right]$

By differentiating the above equation w.r.t. $T$ and putting $T = \frac{1}{2}T_0$, we get $S = \frac{1}{2}kT_0$

**34.** Shunt of an ammeter,

$$S = \frac{I_g \times G}{I - I_g} = \frac{5 \times G}{100 - 5} = \frac{G}{19}$$

**35.** Two coils carry currents in opposite directions, hence net magnetic field at centre will be difference of the two fields.

*ie,* $$B_{\text{net}} = \frac{\mu_0}{4\pi} \cdot 2\pi N\left[\frac{i_1}{r_1} - \frac{i_2}{r_2}\right]$$

$$= \frac{10\,\mu_0}{2}\left[\frac{0.2}{0.2} - \frac{0.3}{0.4}\right] = \frac{5}{4}\mu_0$$

**36.** In a transformer

$$\therefore \quad \frac{N_P}{N_S} = \frac{I_S}{I_P}$$

$$\frac{50}{200} = \frac{I_S}{4}$$

$$\Rightarrow \quad I_S = 1\text{ A}$$

**37.** For $\phi = 90°$, $\cos\phi = 0$

So, $$\lambda' = \lambda + \frac{h}{m_e c}$$

$$= 0.140 \times 10^{-9} + \frac{6.63 \times 10^{-34}}{(9.1 \times 10^{-31})(3 \times 10^8)}$$

$$= (0.140 \times 10^{-9} + 2.4 \times 10^{-12})\text{ m}$$

$$= 0.142\text{ nm}$$

**38.** $$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{45 \times 10^{-12}}$$

$$= \frac{0.44 \times 10^{-14}}{1.6 \times 10^{-19}} = 0.275 \times 10^5\text{ eV}$$

**39.** Nuclear force between two particles is independent of charges of particle.

$$\Rightarrow \quad F_{pp} = F_{nn} = F_{np}$$

**40.** In forward biasing both electrons and protons move towards the junction and hence the width of depletion region decreases.

**41.** Energy of an electron in $n^{th}$ orbit,

$$E_n = -\frac{2\pi^2 k^2 m Z^2 e^4}{n^2 h^2}$$

On substituting the values of $k$, $m$, $e$ and $h$, we get

$$E_n = -\frac{2.172 \times 10^{-18} Z^2}{m^2} \text{ J atom}^{-1}$$

or $$= -\frac{1311.8 Z^2}{n^2} \text{ kJ mol}^{-1}$$

or $$= -\frac{313.52 Z^2}{n^2} \text{ kcal mol}^{-1}$$

[$\because$ 1 kcal = 4.184 kJ]

For H-atom, $Z = 1$

For Lyman series, $n_1 = 1$, $n_2 = 2$

Energy of electron in $n_1$ orbit

$$= -\frac{313.52 \times (1)^2}{(1)^2} \text{ kcal mol}^{-1}$$

$$= -313.52 \text{ kcal mol}^{-1} \approx -313.6 \text{ kcal mol}^{-1}$$

Energy of electron in $n_2$ orbit

$$= -\frac{313.52 \times (1)^2}{(2)^2} \text{ kcal mol}^{-1}$$

$$= -\frac{313.52}{4} \text{ kcal mol}^{-1} = -78.38 \text{ kcal mol}^{-1}$$

**42.** Given, velocity of particle $A = 0.05 \text{ ms}^{-1}$

Velocity of particle $B = 0.02 \text{ ms}^{-1}$

Let the mass of particle $A = x$

$\therefore$ The mass of particle $B = 5x$

de-Broglie's equation is

$$\lambda = \frac{h}{mv}$$

For particle $A$

$$\lambda_A = \frac{h}{x \times 0.05} \quad \text{...(i)}$$

For particle $B$

$$\lambda_B = \frac{h}{5x \times 0.02} \quad \text{...(ii)}$$

Eq (i)/(ii)

$$\frac{\lambda_A}{\lambda_B} = \frac{5x \times 0.02}{x \times 0.05}$$

$$\frac{\lambda_A}{\lambda_B} = \frac{2}{1} \text{ or } 2:1$$

**43.** Given, $\Delta m$ for ${}_5B^{11} = 0.081$ u

Number of nucleons = 11

Binding energy $= 931 \times \Delta m$ MeV

$= 931 \times 0.081$

$= 75.411$ MeV

Average binding energy

$$= \frac{\text{binding energy}}{\text{number of nucleons}}$$

$$= \frac{75.411}{11} = 6.85 \text{ MeV}$$

**44.** Given,

Atomic number of element $B = Z$

($\because$ noble gas $\therefore$ belong to zero group)

Atomic number of element $A = Z - 1$

(*ie*, halogens)

Atomic number of element $C = Z + 1$

(*ie*, group IA)

Atomic number of element $D = Z + 2$

(*ie*, group IIA)

$\because$ Element $B$ is a noble gas

$\therefore$ Element $A$ must be a halogen *ie*, have highest electron affinity.

and element $C$ must be an alkali metal and exist in +1 oxidation state.

and element $D$ must be an alkaline earth metal with +2 oxidation state.

**45.** Given,

observed dipole moment = 1.03 D

Bond length of HCl molecule, $d = 1.275$ Å

$= 1.275 \times 10^{-8}$ cm

Charge of electron, $e^- = 4.8 \times 10^{-10}$ esu

Percentage ionic character = ?

Theoretical value of dipole moment $= e \times d$

$= 4.8 \times 10^{-10} \times 1.275 \times 10^{-8}$ esu - cm

$= 6.12 \times 10^{-18}$ esu - cm = 6.12 D

Percentage ionic character

$$= \frac{\text{observed dipole moment}}{\text{theoretical value of dipole moment}} \times 100$$

$$= \frac{1.03}{6.12} \times 100 = 16.83\%$$

**46.**

| Molecule | *bp* + *lp* | Hybridisat-ion | Shape |
|---|---|---|---|
| $H_2O$ | 2 + 2 | $sp^3$ | angular |
| $BCl_3$ | 3 + 0 | $sp^2$ | trigonal planar |
| $NH_4^+$ | 4 + 0 | $sp^3$ | tetrahedral |
| $CH_4$ | 4 + 0 | $sp^3$ | tetrahedral |

**47.** $NH_2COONH_4(s) \rightleftharpoons 2NH_3(g) + CO_2(g)$

At equilibrium if partial pressure of $CO_2 = p$

then that of $NH_3 = 2p$

$$K_P = p^2_{NH_3} \times p_{CO_2} = (2p)^2 \times p = 4p^3$$

$$2.9 \times 10^{-5} \quad \text{or} \quad p^3 = 0.725 \times 10^{-5}$$

or $p^3 = 7.25 \times 10^{-6}$

or $p = 1.935 \times 10^{-2}$

Hence, total pressure $= 3p$

$$= 5.81 \times 10^{-2} = 0.0581 \text{ atm.}$$

**48.** Number of moles of helium = 0.3

Number of moles of argon = 0.4

We know that KE = $nRT$

$$\text{KE of helium} = 0.3 \times R \times T \quad \ldots(i)$$

$$\text{KE of argon} = 0.4 \times R \times 400 \quad \ldots(ii)$$

According to question

$$\text{KE of helium} = \text{KE of argon}$$

$$0.3 \times R \times T = 0.4 \times R \times 400$$

$$T = 533 \text{ K}$$

**49.** Given,

weight of non-volatile solute,

$$w = 25 \text{ g}$$

Weight of solvent, $W = 100$ g

Lowering of vapour pressure,

$$p° - p_s = 0.225 \text{ mm}$$

Vapour pressure of pure solvent,

$$p° = 17.5 \text{ mm}$$

Molecular weight of solvent ($H_2O$), $M = 18$ g

Molecular weight of solute, $m = ?$

According to Raoult's law

$$\frac{p° - p_s}{p°} = \frac{w \times M}{m \times W}$$

$$\frac{0.225}{17.5} = \frac{25 \times 18}{m \times 100}$$

$$m = \frac{25 \times 18 \times 17.5}{22.5} = 350 \text{ g}$$

**50.** In water, barium hydroxide is hydrolysed as follows.

$$Ba(OH)_2 \rightleftharpoons Ba^{2+} + 2OH^-$$

$$\text{conc. of } Ba^{2+} = 1 \times 10^{-3} \text{ M}$$

$$\text{conc. of } [OH^-] = 2 \times 1 \times 10^{-3} \text{ M}$$

$$= 2 \times 10^{-3} \text{ M}$$

$$\text{pOH} = -\log [OH^-]$$

$$= -\log (2 \times 10^{-3})$$

$$= 2.69$$

$$\text{pH} + \text{pOH} = 14$$

$$\text{pH} = 14 - \text{pOH}$$

$$= 14 - 2.69$$

$$= 11.3$$

$$\approx 11.0$$

**51.** $CH_3COONa + H_2O \longrightarrow CH_3COOH + NaOH$

The above process takes place in following steps

$$CH_3COONa \xrightleftharpoons[\text{in aqueous solution}]{\text{Ionisation}} CH_3COO^- + \underset{\text{strong base}}{Na^+}$$

$$CH_3COO^- + H_2O \longrightarrow CH_3COOH + OH^-$$

Acetate ion undergoes anionic hydrolysis and the resulting solution is slightly basic due to excess of $OH^-$ ions. Hence, both (A) and (R) are true and (R) is the correct explanation of (A).

**52.** Given, weight of hydrogen liberated

$$= 5.04 \times 10^{-2} \text{ g}$$

Eq. wt. of hydrogen = 1.008

Eq. wt. of silver = 108

Weight of silver deposited, $w = ?$

According to Faraday's second law of electrolysis

$$\frac{\text{weight of silver deposited}}{\text{weight of hydrogen liberated}} = \frac{\text{eq. wt. of silver}}{\text{eq. wt. of hydrogen}}$$

$$\frac{w}{5.04 \times 10^{-2}} = \frac{108}{1.008}$$

$$w = \frac{108 \times 5.04 \times 10^{-2}}{1.008} = 5.4 \text{ g}$$

**53.** Electrolysis of water takes place as follows

$$H_2O \rightleftharpoons \underset{\text{cathode}}{H^+} + \underset{\text{anode}}{OH^-}$$

**At anode**

$$OH^- \xrightarrow{\text{Oxidation}} OH + e^-$$

$$4OH \longrightarrow 2H_2O + O_2$$

**At cathode**

$$2H^+ + 2e^- \xrightarrow{\text{Reduction}} H_2$$

Given, time, $t = 1930$ s

Number of moles of hydrogen collected

$$= \frac{1120 \times 10^{-3}}{22.4} = 0.05 \text{ moles}$$

$\because$ 1 mole of hydrogen is deposited by = 2 moles of electrons

$\therefore$ 0.05 moles of hydrogen will be deposited by

$$= 2 \times 0.05$$
$$= 0.10 \text{ mole of electrons}$$

Charge, $Q = nF = 0.1 \times 96500$

Charge, $Q = it$

$$0.1 \times 96500 = i \times 1930$$

$$i = \frac{0.1 \times 96500}{1930} = 5.0 \text{ A}$$

**54.** Given, angle of diffraction $(2\theta) = 90°$

$$\theta = 45°$$

Distance between two planes, $d = 2.28$ Å

$n = 2$ [$\because$ second order diffraction]

Bragg's equation is

$$n\lambda = 2d \sin \theta$$
$$2 \times \lambda = 2 \times 2.28 \times \sin 45°$$
$$\lambda = 1.612$$

**55.**

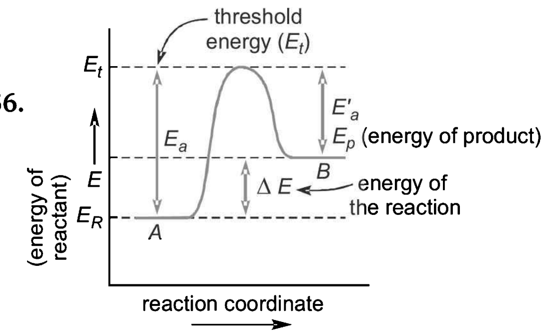
| $PCl_5$ | $\rightleftharpoons$ $PCl_3$ + | $Cl_2$ | |
|---|---|---|---|
| 5 | 0 | 0 | initial moles |
| $5(1-\alpha)$ | $5\alpha$ | $5\alpha$ | moles at equilibrium |
| $\frac{5(1-\alpha)}{0.5}$ | $\frac{5\alpha}{0.5}$ | $\frac{5\alpha}{0.5}$ | conc at equilibrium |

$$\alpha = 40\%$$
$$= 0.4$$

$$K_c = \frac{[PCl_3][Cl_2]}{[PCl_5]}$$

$$= \frac{\left(\frac{5 \times 0.4}{0.5}\right)\left(\frac{5 \times 0.4}{0.5}\right)}{\left(\frac{5 \times 0.6}{0.5}\right)} = \frac{16}{6} = 2.66 \text{ mol/L}$$

**56.**




where,

$E_a$ = activation energy of forward reaction

$E_a'$ = activation energy of backward reaction

The above energy profile diagram shows that

$$E_a > E_a'$$

The potential energy of the product is greater than that of the reactant, so the reaction is endothermic.

$$E_a = E_a' + \Delta E$$
$$E_t = E_a \text{ or } E_t > E_a'$$

**57.** Given,

$$H_2O(g) + C(g) \longrightarrow CO(g) + H_2(g); \quad \Delta H = 131 \text{ kJ} \quad \ldots(i)$$

$$CO(g) + \frac{1}{2}O_2(g) \longrightarrow CO_2(g); \quad \Delta H = -282 \text{ kJ} \ldots(ii)$$

$$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(g); \quad \Delta H = -242 \text{ kJ} \quad \ldots(iii)$$

$$C(g) + O_2(g) \longrightarrow CO_2(g); \Delta H = ? \ldots(iv)$$

On adding Eqs (i), (ii) and (iii), we get Eq (iv)

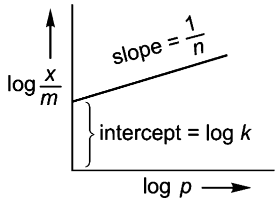
$$H_2O(g) + C(g) \longrightarrow CO(g) + H_2(g); \quad \Delta H = +131 \text{ kJ}$$

$$CO(g) + \frac{1}{2}O_2(g) \longrightarrow CO_2(g); \quad \Delta H = -282 \text{ kJ}$$

$$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(g); \quad \Delta H = -242 \text{ kJ}$$

$$C(g) + O_2(g) \longrightarrow CO_2(g)$$
$$\Delta H = (131 - 282 - 242) = -393 \text{ kJ}$$

**58.** When we plot a graph between log $(x/m)$ and log $p$, a straight line with positive slope will be obtained. This graph represents the Freundlich adsorption isotherm.




Graph of Freundlich

**59.** The reaction in which $H_2O_2$ is reduced while the other reactant is oxidised, represents the oxidising property of $H_2O_2$.

$$2\,K\overset{-1}{I} + H_2SO_4 + H_2\overset{-1}{O_2} \longrightarrow K_2SO_4 + \overset{0}{I_2} + 2H_2\overset{-2}{O}$$

reduction

(oxidising agent)

oxidation

**60.** (i) The alkali metal superoxides contain $O_2^-$ ion, which has an unpaired electron, hence they are paramagnetic in nature.

(ii) The basic character of alkali metal hydroxides increases on moving down the group.

(iii) The conductivity of alkali metal chlorides in their aqueous solution increases on moving down the group because in aqueous solution alkali metal chlorides ionise to give alkali metal ions. On moving down the group the size of alkali metal ion increases, thus degree of hydration decreases, due to this reason their conductivity in aqueous solution increases on moving down the group.

(iv) $M_2CO_3 \xrightleftharpoons[\text{aq solution}]{\text{Ionisation in}} 2M^+ + CO_3^{2-}$

$$CO_3^{2-} + 2H_2O \longrightarrow H_2CO_3 + 2OH^-$$

Thus, basic nature of carbonates in aqueous solution is due to anionic hydrolysis.

**61.** According to Lewis, the compound which can accept a lone pair of electron, are called acids. Boron halides, being electron deficient compounds, can accept a lone pair of electrons, so termed as Lewis acid.

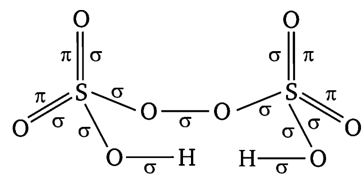
**62.** Orthosilicic acid ($H_4SiO_4$), on heating at high temperature, loses two water molecules and gives silica ($SiO_2$) which on reduction with carbon gives carborundum (SiC) and CO.

$$H_4SiO_4 \xrightarrow[-2H_2O]{1000°C} SiO_2 \xrightarrow[\Delta]{C} \underset{\text{carborundum}}{SiC} + CO$$

**63.** The reducing character of the hydrides of group V elements depends upon the stability of hydrides. With progressive decrease in stability, the reducing character of hydrides increases as we move down the group. Thus, ammonia, being stable, has least reducing ability. The order of reducing abilities of V group hydrides is

$$NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$$

**64.** The structure of peroxodisulphuric acid ($H_2S_2O_8$) is



Hence, it contains 11σ and 4π bonds.

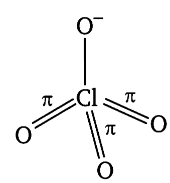
**65.** With progressive increase in atomic number, the reduction potential of halogens decreases, thus oxidising power also decreases. Hence, a halogen with lower atomic number will oxidise the halide ion of higher atomic number and therefore, will liberate them from their salt solution.

Hence, the reaction

$$Cl_2 + 2F^- \longrightarrow 2Cl^- + F_2$$

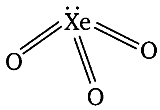
is not possible.

**66.** The structure of $ClO_4^-$ is



Thus, it contains 3 $d\pi$-$p\pi$ bonds.

$XeO_3$ also contains 3 $d\pi$-$p\pi$ bonds as



**67.** $[Co(NH_3)_5SO_4]Br \rightleftharpoons [Co(NH_3)_5SO_4]^+ + Br^-$

$[Co(NH_3)_5Br]SO_4 \rightleftharpoons [Co(NH_3)_5Br]^{2+} + SO_4^{2-}$

the molecular formula of both of the above compounds is same but on ionisation they give different ions in solution, so they are called ionisation isomers.

**68.** In stratosphere the following reactions takes place which are responsible for depletion of ozone layer.

$$NO + O_3 \longrightarrow NO_2 + O_2$$

$$CF_2Cl_2 \xrightarrow{h\nu} \dot{C}F_2Cl + \dot{C}l$$

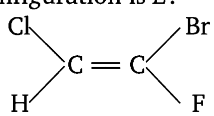
$$CFCl_3 \xrightarrow{h\nu} \dot{C}FCl_2 + \dot{C}l$$

$$\dot{C}l + O_3 \longrightarrow Cl\dot{O} + O_2$$

$$Cl\dot{O} + O \longrightarrow \dot{C}l + O_2$$

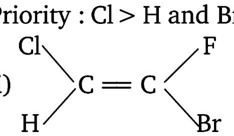
Hence, methane ($CH_4$) is not responsible for ozone layer depletion.

**69.** When the groups with higher priority (*ie*, with high atomic number) are present on same side of double bond, then the configuration is $Z$ but when present on opposite side of double bond, the configuration is $E$.

(i) 

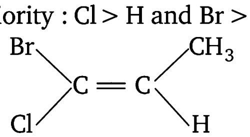
($Z$)

(Priority : Cl > H and Br > F)

(ii) 

($E$)

(Priority : Cl > H and Br > F)

(iii) 

($Z$)

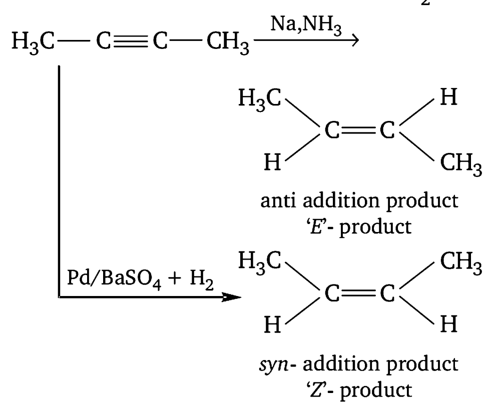
(Priority : Br > Cl and $CH_3$ > H)

Hence, compound (i) and (iii) have ($Z$) configuration.

**70.** According to Cahn-Ingold-Prelog sequence rules, the priority of groups is decided by the atomic number of their atoms. When the atom (which is directly attached to the asymmetric carbon atom) of a group has higher atomic number, then the group gets higher priority. Groups with atoms of comparable atomic number having double or triple bond, have high priority than those have single bond.

Hence, the order of priority of groups is

$$—OH > —COOH > —CHO > —CH_2OH$$

**71.** 


Hence, reagent $X$ and $Y$ are respectively Na, $NH_3$ and $Pd/BaSO_4 + H_2$.

**72.** In a reaction, the reagent, which is reduced or remove hydrogen from the other reactant (reagent), is termed as oxidising agent.

reduction

(i) $CH_3CH_2OH + \overset{0}{Cl_2} \longrightarrow CH_3CHO + H\overset{-1}{Cl}$

oxidising agent

removal of hydrogen *ie,* oxidation

reduction

(ii) $CH_3CHO + \overset{0}{Cl_2} \longrightarrow CCl_3{\cdot}CHO + H\overset{-1}{Cl}$

oxidising agent

oxidation (removal of H)

reduction

(iii) $CH_4 + \overset{0}{Cl_2} \xrightarrow{hn} CH_3Cl + H\overset{-1}{Cl}$

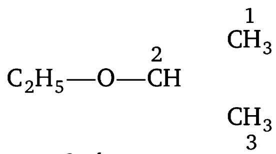
oxidising agent

oxidation

Hence, in all of the above reactions, chlorine acts as an oxidising agent.

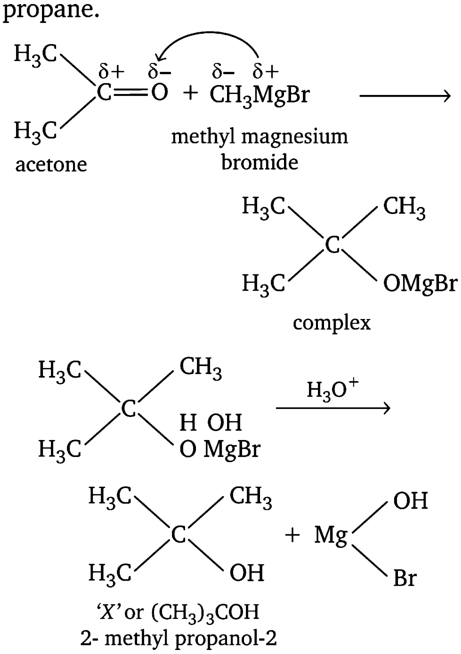
**73.** Among hydrogen halides, as the size of halide ion increases, its reactivity towards ethyl alcohol also increases. Thus, the order of reactivity of hydrogen halides is

$$HI > HBr > HCl > HF$$

**74.** 

2-ethoxy propane

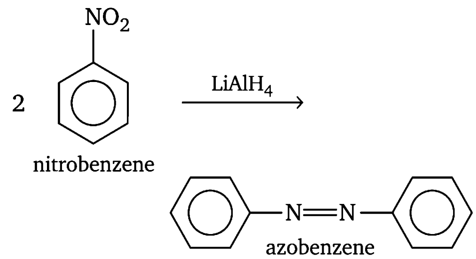
The above compound is an ether and its name is written as alkoxy alkane. Oxy is attached with the lower group. Hence, the IUPAC name of above compound is 2-ethoxy propane.

**75.** 

**76.** Acetic acid on reduction with lithium aluminiumhydride ($LiAlH_4$) gives ethyl alcohol while on reduction with HI and red P gives ethane.

$$CH_3COOH \xrightarrow{LiAlH_4} \underset{\text{ethyl alcohol}}{CH_3CH_2OH}$$

$$CH_3COOH \xrightarrow{\text{Red P + HI}} \underset{\text{ethane}}{CH_3—CH_3}$$



Hence, reagent *A* and *B* are respectively $LiAlH_4$ and HI/red P.

**77.** Nitrobenzene on reduction with lithium aluminium hydride ($LiAlH_4$) gives azobenzene.

**78.** The energy released in an atomic bomb explosion is mainly due to lesser mass of products than initial material, *ie*, mass destroyed is converted into energy.

**79.** The ratio of weight average molecular weight and the number average molecular weight is called poly dispersity index (PDI).

$$PDI = \frac{\overline{M}_w}{\overline{M}_n}$$

where,

$\overline{M}_w$ = weight average molecular weight

$\overline{M}_n$ = number average molecular weight

PDI is unity for natural monodispersed polymer but for synthetic polymers it is always greater than unity.

**80.** On hydrolysis with dilute aqueous sulphuric acid, sucrose gives a equimolar mixture of D-(+) glucose and D-(–)-fructose.

$$\underset{\text{sucrose}}{C_{12}H_{22}O_{11}} + H_2O \xrightarrow{H_2SO_4} \underset{\text{D-(+) glucose}}{C_6H_{12}O_6} + \underset{\text{D-(–) fructose}}{C_6H_{12}O_6}$$

$$1 : 1$$

Sucrose is dextrorotatory but after hydrolysis gives dextrorotatory glucose and laevorotatory fructose, laevorotatory fructose is more, so the mixture is laevorotatory.

**81.** Given that, $f(x)=e^{2ix}$ and $f: R \to C$.
Function $f(x)$ is not one-one, because after some values of $x$ (*ie*, $\pi$) it will give the same values.
Also, $f(x)$ is not onto, because it has minimum and maximum values $-1-i$ and $1+i$ respectively.

Hence, option (d) is correct.

**82.** Given that, $f(x)=|x|$ and $g(x)=[x-3]$

For $-\frac{8}{5}<x<\frac{8}{5}, 0 \le f(x)<\frac{8}{5}$

Now, for $0 \le f(x)<1$,

$g(f(x))=[f(x)-3]$

$=-3$ $[\because -3 \le f(x)-3<-2]$

Again, for $1 \le f(x)<1.6$

$g(f(x))=-2$

$[\because -2 \le f(x)-3<-1.4]$

Hence, required set is $\{-3,-2\}$.

**83.** Given, $f(x)=x^2-3$

Now, $f(-1)=(-1)^2-3=-2$

$\Rightarrow$ $fof(-1)=f(-2)=(-2)^2-3=1$

$\Rightarrow$ $fofof(-1)=f(1)=1^2-3=-2$

Now, $f(0)=0^2-3=-3$

$\Rightarrow$ $fof(0)=f(-3)=(-3)^2-3=6$

$\Rightarrow$ $fofof(0)=f(6)=6^2-3=33$

Again, $f(1)=1^2-3=-2$

$\Rightarrow$ $fof(1)=f(-2)=(-2)^2-3=1$

$\Rightarrow$ $fofof(1)=(1)^2-3=-2$

$\therefore fofof(-1)+fofof(0)+fofof(1)$

$=-2+33-2=29$

Now, $f(4\sqrt{2})=(4\sqrt{2})^2-3=32-3=29$

Hence, option (a) is correct.

**84.** Given that, $\left(a+\frac{b}{10}\right)^x=\left(\frac{a}{10}+\frac{b}{100}\right)^y=1000$

Let $a=0$ and $b=1$

$\therefore$ $\left(\frac{1}{10}\right)^x=\left(\frac{1}{100}\right)^y=1000$

$\Rightarrow$ $10^{-x}=10^{-2y}=10^3$

$\Rightarrow$ $x=-3, y=-\frac{3}{2}$

Now, $\frac{1}{x}-\frac{1}{y}=-\frac{1}{3}+\frac{2}{3}=\frac{1}{3}$

**85.** Now, $\sum_{k=1}^{n} k(k+2)$

$$=\sum_{k=1}^{n}(k^2+2k)$$

$$=\sum_{k=1}^{n} k^2+2\sum_{k=1}^{n} k$$

$$=\frac{n(n+1)(2n+1)}{6}+\frac{2\cdot n(n+1)}{2}$$

$$=n(n+1)\left(\frac{2n+1}{6}+1\right)$$

$$=\frac{n(n+1)(2n+7)}{6}$$

**86.** Required number of arrangements

$={}^6P_5 \times 4!$

$=720 \times 24=17280$

**87.** Given that, ${}^nP_r=30240$ and ${}^nC_r=252$

$\Rightarrow$ $\frac{n!}{(n-r)!}=30240$ and $\frac{n!}{(n-r)!\,r!}=252$

$\Rightarrow$ $r!=\frac{30240}{252}=120$

$\Rightarrow$ $r=5$

$\therefore$ $\frac{n!}{(n-5)!}=30240$

$\Rightarrow$ $n(n-1)(n-2)(n-3)(n-4)=30240$

$\Rightarrow$ $n(n-1)(n-2)(n-3)(n-4)$

$=10(10-1)(10-2)(10-3)(10-4)$

$\Rightarrow$ $n=10$

Hence, required ordered pair is (10, 5).

**88.** Given, $(1+x+x^2+x^3)^5=\sum_{k=0}^{15} a_k x^k$

$\Rightarrow$ $[(1+x)+x(1+x)]^5=\sum_{k=0}^{15} a_k x^k$

$\Rightarrow (1+x)^{10}=a_0x^0+a_1x+a_2x^2+\ldots+a_{15}x^{15}$

$\Rightarrow$ ${}^{10}C_0+{}^{10}C_1x+{}^{10}C_2x^2+\ldots+{}^{10}C_{10}x^{10}$

$=a_0+a_1x+a_2x^2+a_3x^3+\ldots+a_{15}x^{15}$

On equating the coefficient of constant and even powers of $x$, we get

$$a_0 = {}^{10}C_0,\ a_2 = {}^{10}C_2,$$
$$a_4 = {}^{10}C_4, \ldots, a_{10} = {}^{10}C_{10},$$
$$a_{12} = a_{14} = 0$$

$$\therefore \quad \sum_{k=0}^{7} a_2k = {}^{10}C_0 + {}^{10}C_2 + {}^{10}C_4 + {}^{10}C_6 + {}^{10}C_8 + {}^{10}C_{10} + 0 + 0$$
$$= 2^{10-1} = 2^9$$
$$= 512$$

**89.** Given that, $\alpha = \dfrac{5}{2!3} + \dfrac{5\cdot 7}{3!3^2} + \dfrac{5\cdot 7\cdot 9}{4!3^3} + \ldots$ ...(i)

We know that,

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots \quad \text{...(ii)}$$

On comparing Eqs. (i) and (ii), with respect to factorial

$$n(n-1)x^2 = \frac{5}{3} \quad \text{...(iii)}$$
$$n(n-1)(n-2)x^3 = \frac{5\cdot 7}{3^2} \quad \text{...(iv)}$$

and

$$n(n-1)(n-2)(n-3)x^4 = \frac{5\cdot 7\cdot 9}{3^3} \quad \text{...(v)}$$

On dividing Eq. (iv) by (iii) and Eq. (v) by (iv), we get

$$(n-2)x = \frac{7}{3} \quad \text{...(vi)}$$

and $\quad (n-3)x = 3 \quad$ ...(vii)

Again, dividing Eq. (vi) by (vii), we get

$$\frac{n-2}{n-3} = \frac{7}{9}$$
$$\Rightarrow \quad 9n - 18 = 7n - 21$$
$$\Rightarrow \quad 2n = -3$$
$$\Rightarrow \quad n = -\frac{3}{2}$$

On putting the value of $n$ in Eq. (vi), we get

$$\left(-\frac{3}{2} - 2\right)x = \frac{7}{3} \Rightarrow x = -\frac{2}{3}$$

$\therefore$ From Eq. (ii),

$$\left(1 - \frac{2}{3}\right)^{-3/2} = 1 + 1 + \frac{5}{2!3} + \frac{5\cdot 7}{3!3^2} + \ldots$$
$$\Rightarrow \quad 3^{3/2} - 2 = \frac{5}{2!3} + \frac{5\cdot 7}{3!3^2} + \ldots$$
$$\Rightarrow \quad \alpha = 3^{3/2} - 2 \quad \text{[from Eq. (i)]}$$

Now, $\alpha^2 + 4\alpha = (3^{3/2} - 2)^2 + 4(3^{3/2} - 2)$

$$= 27 + 4 - 4\cdot 3^{3/2} + 4\cdot 3^{3/2} - 8$$
$$= 23$$

**90.** $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k!}\left(\sum_{n=1}^{k} 2^{n-1}\right)$

$$= \sum_{k=1}^{\infty} \frac{1}{k!}[1(2^k - 1)] = \sum_{k=1}^{\infty} \frac{2^k - 1}{k!}$$
$$= \sum_{k=1}^{\infty} \frac{2^k}{k!} - \sum_{k=1}^{\infty} \frac{1}{k!} = e^2 - 1 - (e-1) = e^2 - e$$

**91.** Let $\quad S = \dfrac{1}{1\cdot 3} + \dfrac{1}{2\cdot 5} + \dfrac{1}{3\cdot 7} + \dfrac{1}{4\cdot 9} + \ldots$

$$\therefore \quad T_n = \frac{1}{n(2n+1)} = \frac{1}{n} - \frac{2}{(2n+1)}$$
$$\Rightarrow \quad S = \sum_{n=1}^{\infty} T_n = \sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{2}{2n+1}\right)$$
$$= \frac{1}{1} - \frac{2}{3} + \frac{1}{2} - \frac{2}{5} + \frac{1}{3} - \frac{2}{7} + \frac{1}{4} - \frac{2}{9} + \frac{1}{5} - \ldots$$
$$= 1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \ldots$$
$$= 1 - \left(-\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots\right)$$
$$= 1 - (-1 + \log_e 2) = 2 - \log_e 2$$

**92.** Given that, $\alpha + \beta = -2$ and $\alpha^3 + \beta^3 = -56$

$$\Rightarrow \quad (\alpha+\beta)(\alpha^2 + \beta^2 - \alpha\beta) = -56$$
$$\Rightarrow \quad \alpha^2 + \beta^2 - \alpha\beta = 28$$

Now, $\quad (\alpha+\beta)^2 = (-2)^2$

$$\Rightarrow \quad \alpha^2 + \beta^2 + 2\alpha\beta = 4$$
$$\Rightarrow \quad 28 + 3\alpha\beta = 4$$
$$\Rightarrow \quad \alpha\beta = -8$$

$\therefore$ Required equation is

$$x^2 - (-2)x + (-8) = 0$$
$$\Rightarrow \quad x^2 + 2x - 8 = 0$$

**93.** Given equation is

$$x^3 + 2x^2 - 4x + 1 = 0$$

Let $\alpha, \beta$ and $\gamma$ be the roots of the given equation

$\therefore \quad \alpha + \beta + \gamma = -2, \alpha\beta + \beta\gamma + \gamma\alpha = -4$

and $\quad \alpha\beta\gamma = -1$

Let the required cubic equation has the roots $3\alpha$, $3\beta$ and $3\gamma$.

$\Rightarrow \quad 3\alpha + 3\beta + 3\gamma = -6,$

$3\alpha \cdot 3\beta + 3\beta \cdot 3\gamma + 3\gamma \cdot 3\alpha = -36$

and $\quad 3\alpha \cdot 3\beta \cdot 3\gamma = -27$

$\therefore$ Required equation is

$$x^3 - (-6)x^2 + (-36)x - (-27) = 0$$

$$\Rightarrow \quad x^3 + 6x^2 - 36x + 27 = 0$$

**94.** Given that,

$$A = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} \text{ and } f(t) = t^2 - 3t + 7$$

Now, $A^2 = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} -7 & -12 \\ 24 & 17 \end{bmatrix}$

Now, $f(A) = A^2 - 3A + 7$

$$= \begin{bmatrix} -7 & -12 \\ 24 & 17 \end{bmatrix} - 3\begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -6 \\ 12 & 9 \end{bmatrix}$$

$$\therefore f(A) + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix} = \begin{bmatrix} -3 & -6 \\ 12 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

**95.** Let $\quad A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

Now, $\quad |A| = 7(1-0) + 3(-1-0) - 3(0+1)$
$= 1$

Cofactors of matrix $A$ are

$$C_{11} = 1, C_{12} = 1, C_{13} = 1$$
$$C_{21} = 3, C_{22} = 4, C_{23} = 3$$
$$C_{31} = 3, C_{32} = 3, C_{33} = 4$$

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 3 \\ 3 & 3 & 4 \end{bmatrix}^T = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{\text{adj}(A)}{|A|} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

**96.** Let $\Delta = \begin{bmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{bmatrix}$

Applying $R_1 \to R_1 + R_2 + R_3$ and taking common $(a+b+c)$ from $R_1$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$,

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & 0 \\ 2c & 0 & -a-b-c \end{vmatrix}$$

$$= (a+b+c)[(-b-c-a)(-a-b-c)]$$

$$= (a+b+c)^3$$

**97.** Given that, $\arg\left(\frac{z-2}{z-6i}\right) = \frac{\pi}{2}$

$$\therefore \quad \arg(z-2) - \arg(z-6i) = \frac{\pi}{2}$$

Let $z = x + iy$

$$\Rightarrow \arg[(x-2) + iy] - \arg[x + i(y-6)] = \frac{\pi}{2}$$

$$\Rightarrow \quad \tan^{-1}\frac{y}{x-2} - \tan^{-1}\frac{y-6}{x} = \frac{\pi}{2}$$

$$\Rightarrow \quad \left(\frac{\frac{y}{x-2} - \frac{y-6}{x}}{1 + \frac{y}{x-2} \cdot \frac{y-6}{x}}\right) = \tan\frac{\pi}{2}$$

$$\Rightarrow \quad 1 + \frac{y}{x-2} \cdot \frac{y-6}{x} = 0$$

$$\Rightarrow \quad x(x-2) + y(y-6) = 0$$

This is an equation of circle in diametric form.

**98.** Since, $\omega$ is a cube root of unity.

$$\therefore \quad \sin\left\{(\omega^{10}+\omega^{23})\pi-\frac{\pi}{4}\right\}$$
$$=\sin\left\{(\omega+\omega^2)\pi-\frac{\pi}{4}\right\}$$
$$=\sin\left(-\pi-\frac{\pi}{4}\right) \qquad (\because 1+\omega+\omega^2=0)$$
$$=-\sin\left(\pi+\frac{\pi}{4}\right)=\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$$

**99.** Let $z_1=1+4i$, $z_2=3+i$, $z_3=1-i$ and $z_4=2-3i$

$\therefore \quad m_1=|z_1|, m_2=|z_2|, m_3=|z_3|$

and $\quad m_4=|z_4|$

$\Rightarrow \quad m_1=\sqrt{1+4^2}, m_2=\sqrt{3^2+1^2},$

$m_3=\sqrt{1^2+1^2}$ and $m_4=\sqrt{2^2+3^2}$

$\Rightarrow \quad m_1=\sqrt{17}, m_2=\sqrt{10}, m_3=\sqrt{2}$

and $\quad m_4=\sqrt{13}$

$\Rightarrow \quad m_3<m_2<m_4<m_1$

**100.** $\sqrt{3}$ cosec $20°$ – sec $20°$

$$=\frac{\tan 60°}{\sin 20°}-\frac{1}{\cos 20°}$$
$$=\frac{\sin 60°\cos 20°-\sin 20°\cos 60°}{\cos 60°\sin 20°\cos 20°}$$
$$=\frac{\sin 40°}{\cos 60°\sin 20°\cos 20°}$$
$$=\frac{2\sin 20°\cos 20°}{\frac{1}{2}(\sin 20°\cos 20°)}=4$$

**101.** Given that, $A=35°$, $B=15°$ and $C=40°$

$\therefore \tan(A+B+C)$

$$=\frac{\begin{bmatrix}\tan A+\tan B+\tan C\\ -\tan A\tan B\tan C\end{bmatrix}}{\begin{bmatrix}1-\tan A\tan B-\tan B\tan C\\ -\tan C\tan A\end{bmatrix}}$$

$\Rightarrow \tan(90°)$

$$=\frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A}$$

$$\Rightarrow \quad \tan A\tan B+\tan B\tan C+\tan C\tan A=1$$

**102.** Given,

$$\tan\theta+\tan\left(\theta+\frac{\pi}{3}\right)+\tan\left(\theta+\frac{2\pi}{3}\right)=3$$
$$\Rightarrow \quad \tan\theta+\frac{\tan\theta+\sqrt{3}}{1-\sqrt{3}\tan\theta}+\frac{\tan\theta-\sqrt{3}}{1+\sqrt{3}\tan\theta}=3$$
$$\Rightarrow \quad \tan\theta+\frac{8\tan\theta}{1-3\tan^2\theta}=3$$
$$\Rightarrow \quad \frac{9\tan\theta-3\tan^3\theta}{1-3\tan^2\theta}=3$$
$$\Rightarrow \quad 3\tan 3\theta=3$$
$$\Rightarrow \quad \tan 3\theta=1$$

Hence, option (b) is correct.

**103.** Now,

$$\cos(\theta-\alpha)+\cos(\theta-\beta)+\cos\theta+\cos(\theta-\gamma)$$
$$=2\cos\left(\theta-\left(\frac{\alpha+\beta}{2}\right)\right)\cos\left(\frac{\beta-\alpha}{2}\right)+2\cos\left(\frac{\gamma}{2}\right)\cos\left(\theta-\frac{\gamma}{2}\right)$$
$$=2\cos\left(\frac{\gamma}{2}\right)\cos\left(\frac{\beta-\alpha}{2}\right)+2\cos\left(\frac{\gamma}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$
$$=2\cos\left(\frac{\gamma}{2}\right)\left[\cos\left(\frac{\beta-\alpha}{2}\right)+\cos\left(\frac{\alpha+\beta}{2}\right)\right]$$
$$=2\cos\left(\frac{\gamma}{2}\right)2\cos\frac{\alpha}{2}\cos\frac{\beta}{2}$$
$$=4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$$

**104.** Given equation is

$$\cos 2x+2\cos^2 x=2$$
$$\Rightarrow \quad 2\cos^2 x-1+2\cos^2 x=2$$
$$\Rightarrow \quad 4\cos^2 x=3$$
$$\Rightarrow \quad \cos^2 x=\frac{3}{4}$$
$$\Rightarrow \quad \cos x=\pm\frac{\sqrt{3}}{2}$$
$$\therefore \quad x=n\pi\pm\frac{\pi}{6}: n\in Z$$

**105.** Given that,

$$\sin^{-1}\left(\frac{3}{x}\right)+\sin^{-1}\left(\frac{4}{x}\right)=\frac{\pi}{2}$$

$$\therefore \quad \sin^{-1}\left(\frac{3}{x}\right)=\frac{\pi}{2}-\sin^{-1}\left(\frac{4}{x}\right)$$

$$\Rightarrow \quad \sin^{-1}\left(\frac{3}{x}\right)=\cos^{-1}\left(\frac{4}{x}\right)$$

$$\Rightarrow \quad \sin^{-1}\left(\frac{3}{x}\right)=\sin^{-1}\left(\frac{\sqrt{x^2-16}}{x}\right)$$

$$\Rightarrow \quad \frac{3}{x}=\frac{\sqrt{x^2-16}}{x}$$

$$\Rightarrow \quad 9=x^2-16 \quad \Rightarrow \quad x^2=25$$

$$\Rightarrow \quad x=\pm 5 \quad \Rightarrow \quad x=5$$

($\because$ $-5$ is not satisfied the given equation)

**106.** Given that,

$$\frac{1}{b+c}+\frac{1}{c+a}=\frac{3}{a+b+c}$$

$$\Rightarrow \quad 1+\frac{b}{a+c}+1+\frac{a}{b+c}=3$$

$$\Rightarrow \quad b(b+c)+a(a+c)=(a+c)(b+c)$$

$$\Rightarrow \quad b^2+bc+a^2+ac=ab+ac+bc+c^2$$

$$\Rightarrow \quad a^2+b^2-c^2=ab$$

We know that, $\cos C=\dfrac{a^2+b^2-c^2}{2ab}=\dfrac{ab}{2ab}=\dfrac{1}{2}$

$$\Rightarrow \quad C=60^\circ$$

**107.** Given that, $r_1=2r_2=3r_3$

$$\therefore \quad \frac{\Delta}{s-a}=\frac{2\Delta}{s-b}=\frac{3\Delta}{s-c}=\frac{\Delta}{k} \qquad \text{(say)}$$

Then, $\quad s-a=k, s-b=2k, s-c=3k$

$$\Rightarrow 3s-(a+b+c)=6k \quad \Rightarrow \quad s=6k$$

$$\therefore \quad \frac{a}{5}=\frac{b}{4}=\frac{c}{3}=k$$

Now, $\quad \dfrac{a}{b}+\dfrac{b}{c}+\dfrac{c}{a}=\dfrac{5}{4}+\dfrac{4}{3}+\dfrac{3}{5}$

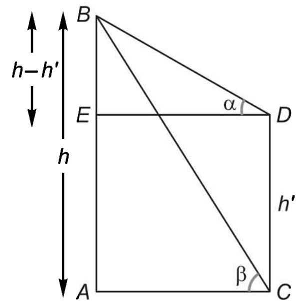
$$=\frac{75+80+36}{60}=\frac{191}{60}$$

**108.** Let $AB$ be a hill whose height is $h$ metres and $CD$ be a pillar of height $h'$ metres.

In $\Delta EDB$,

$$\tan\alpha=\frac{h-h'}{ED} \qquad \ldots\text{(i)}$$

and in $\Delta ACB$,




$$\tan\beta=\frac{h}{AC}=\frac{h}{ED} \qquad \ldots\text{(ii)}$$

Eliminate $ED$ from Eqs. (i) and (ii), we get

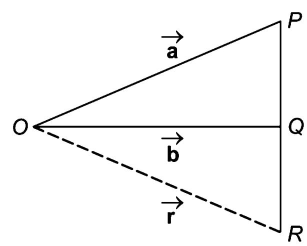
$$\tan\alpha=\frac{h-h'}{\dfrac{h}{\tan\beta}}$$

$$\Rightarrow \quad h\frac{\tan\alpha}{\tan\beta}=h-h'$$

$$\Rightarrow \quad h'=\frac{h(\tan\beta-\tan\alpha)}{\tan\beta}$$

**109.** Given that,

$$\vec{\mathbf{PR}}=5\,\vec{\mathbf{PQ}}$$




It mean $R$ divides $PQ$ externally in the ratio $5:4$.

$\therefore$Position vector of $R=\dfrac{5\,\vec{\mathbf{b}}-4\,\vec{\mathbf{a}}}{5-4}$

$$=5\,\vec{\mathbf{b}}-4\,\vec{\mathbf{a}}$$

**110.** Since, the position vectors $60\hat{\mathbf{i}}+3\hat{\mathbf{j}}$, $40\hat{\mathbf{i}}-8\hat{\mathbf{j}}$, and $a\hat{\mathbf{i}}-52\hat{\mathbf{j}}$ are collinear.

$$\therefore \quad \begin{vmatrix} 60 & 3 & 1 \\ 40 & -8 & 1 \\ a & -52 & 1 \end{vmatrix}=0$$

$\Rightarrow \ 60(-8+52)-3(40-a)$
$$+1(-2080+8a)=0$$
$\Rightarrow 2640-120+3a-2080+8a=0$

$\Rightarrow \quad 440+11a=0 \ \Rightarrow \ a=-40$

**111.** Let $\overrightarrow{\mathbf{OA}}=2\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$, $\overrightarrow{\mathbf{OB}}=\hat{\mathbf{i}}-3\hat{\mathbf{j}}-5\hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{OC}}=3\hat{\mathbf{i}}-4\hat{\mathbf{j}}-4\hat{\mathbf{k}}$

$\therefore \quad a=|\overrightarrow{\mathbf{OA}}|=\sqrt{6}, b=|\overrightarrow{\mathbf{OB}}|=\sqrt{35}$

and $\quad c=|\overrightarrow{\mathbf{OC}}|=\sqrt{41}$

$$\therefore \quad \cos A=\frac{b^2+c^2-a^2}{2bc}$$
$$=\frac{(\sqrt{35})^2+(\sqrt{41})^2-(\sqrt{6})^2}{2\sqrt{35}\sqrt{41}}$$
$$\Rightarrow \quad \cos A=\frac{70}{2\sqrt{35}\sqrt{41}}=\sqrt{\frac{35}{41}}$$
$$\Rightarrow \ \cos^2 A=\frac{35}{41}$$

**112.** Given that,

$$\vec{\mathbf{b}}=2\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}} \quad \text{and} \quad \vec{\mathbf{c}}=\hat{\mathbf{i}}+3\hat{\mathbf{k}}$$

Now, $\vec{\mathbf{b}}\times\vec{\mathbf{c}}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 2 & 1 & -1\\ 1 & 0 & 3\end{vmatrix}$

$$=\hat{\mathbf{i}}(3-0)-\hat{\mathbf{j}}(6+1)+\hat{\mathbf{k}}(0-1)$$
$$=3\hat{\mathbf{i}}-7\hat{\mathbf{j}}-\hat{\mathbf{k}}$$

Now, $[\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}]=\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})$

$$=|\vec{\mathbf{a}}||\vec{\mathbf{b}}\times\vec{\mathbf{c}}|\cos\theta$$
$$=1(\sqrt{3^2+7^2+1^2})\cos\theta$$
$$=\sqrt{59}\cos\theta$$

$\Rightarrow [\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}]_{\max}=\sqrt{59}\cdot 1$

($\because$ maximum value of $\cos\theta$ is1)

Hence, maximum value is $\sqrt{59}$.

**113.** Given that, $P(A\cap B)=\frac{1}{6}$ and $P(\bar{A}\cap\bar{B})=\frac{1}{3}$

Since, $A$ and $B$ are independent.

$\therefore \ P(A)P(B)=\frac{1}{6}$ and $P(\bar{A})P(\bar{B})=\frac{1}{3}$

$$\Rightarrow \quad [1-P(A)][1-P(B)]=\frac{1}{3}$$
$$\Rightarrow 1-[P(A)+P(B)]+P(A)P(B)=\frac{1}{3}$$
$$\Rightarrow \quad 1+\frac{1}{6}-\frac{1}{3}=P(A)+P(B)$$
$$\Rightarrow \quad P(A)+P(B)=\frac{5}{6}$$
$$\Rightarrow \quad P(A)=\frac{1}{2}, P(B)=\frac{1}{3}$$

and $\quad P(A)=\frac{1}{3}, P(B)=\frac{1}{2}$

Hence, options (b) and (c) are correct.

**114.** Given that, $E_k=\{(a,b)\in S : ab=k\}$ for $k\geq 1$ and $p_k=P(E_k)$

Now, $\quad E_1=\{(1,1)\}$

$$\Rightarrow \quad p_1=P(E_1)$$
$$\Rightarrow \quad p_1=\frac{1}{36}$$

$E_2=\{(1,2),(2,1)\} \ \Rightarrow \ p_2=P(E_2)$

$$\Rightarrow \quad p_2=\frac{2}{36}$$

$E_4=\{(1,4),(4,1),(2,2)\} \ \Rightarrow \ p_4=P(E_4)$

$$\Rightarrow \quad p_4=\frac{3}{36}$$

$E_6=\{(1,6),(6,1),(2,3),(3,2)\}$

$$\Rightarrow \quad p_6=P(E_6) \ \Rightarrow \ p_6=\frac{4}{36}$$

and $E_{30}=\{(5,6),(6,5)\} \ \Rightarrow \ p_{30}=P(E_{30})$

$$\Rightarrow \quad p_{30}=\frac{2}{36}$$

$\therefore$From the above results, we get

$$p_1<p_{30}<p_4<p_6$$

Hence, option (a) is correct.

**115.** In a box,

$$B_1 = 1R,\ 2W$$
$$B_2 = 2R,\ 3W$$
and $$B_3 = 3R,\ 4W$$

Also, given that,

$P(B_1) = \frac{1}{2}, P(B_2) = \frac{1}{3}$ and $P(B_3) = \frac{1}{6}$

$$\therefore P\left(\frac{B_2}{R}\right) = \frac{P(B_2)\,P\left(\frac{R}{B_2}\right)}{P(B_1)\,P\left(\frac{R}{B_1}\right) + P(B_2)\,P\left(\frac{R}{B_2}\right) + P(B_3)\,P\left(\frac{R}{B_3}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{2}{5}}{\frac{1}{2}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{5}+\frac{1}{6}\times\frac{3}{7}}$$

$$= \frac{\frac{2}{15}}{\frac{1}{6}+\frac{2}{15}+\frac{1}{14}} = \frac{\frac{2}{15}}{\frac{35+28+15}{210}}$$

$$= \frac{2}{15}\times\frac{210}{78} = \frac{14}{39}$$

**116.** Given, probability distribution is

| $X = x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|
| $P(X = x)$ | $\frac{1}{10}$ | $k$ | $\frac{1}{5}$ | $2k$ | $\frac{3}{10}$ | $k$ |

As we know, total probability distribution is 1.

$$\therefore \quad \frac{1}{10} + k + \frac{1}{5} + 2k + \frac{3}{10} + k = 1$$

$$\Rightarrow \quad \frac{6}{10} + 4k = 1$$

$$\Rightarrow \quad 4k = \frac{4}{10} \Rightarrow k = \frac{1}{10}$$

**117.** The sum of the distance of a point $P$ from two perpendicular lines in a plane is 1, then the locus of $P$ is a rhombus.

**118.** Since, the axes are rotated through an angle 45°, then we replace $(x, y)$ by

$$(x \cos 45° - y \sin 45°,\ x \sin 45° + y \cos 45°)$$

*ie,* $$\left(\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}, \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}\right)$$

in the given equation $3x^2 + 3y^2 + 2xy = 2$

$$\therefore \quad 3\left(\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}\right)^2 + 3\left(\frac{x+y}{\sqrt{2}}\right)^2 + 2\left(\frac{x-y}{\sqrt{2}}\right)\left(\frac{x+y}{\sqrt{2}}\right) = 2$$

$$\Rightarrow \frac{3}{2}(x^2 + y^2 + 2xy) + \frac{3}{2}(x^2 + y^2 - 2xy) + \frac{2}{2}(x^2 - y^2) = 2$$

$$\Rightarrow \quad 4x^2 + 2y^2 = 2$$

$$\Rightarrow \quad 2x^2 + y^2 = 1$$

**119.** Since, $l, m, n$ are in AP.

$$\therefore \quad 2m = l + n$$

Given equation of line is

$$lx + my + n = 0$$

Now, assume that the point $(1, -2)$ satisfy the given equation.

$$\therefore \quad l - 2m + n = 0$$

$$\Rightarrow \quad 2m = l + n$$

$\Rightarrow$ $l, m, n$ are in AP.

Hence, option (b) is correct.

**120.** Since, the lines

$2x - 3y + k = 0,\ 3x - 4y - 13 = 0$

and $8x - 11y - 33 = 0$

are concurrent

$$\therefore \quad \begin{vmatrix} 2 & -3 & k \\ 3 & -4 & -13 \\ 8 & -11 & -33 \end{vmatrix} = 0$$

$$\Rightarrow \quad 2(132 - 143) + 3(-99 + 104) + k(-33 + 32) = 0$$

$$\Rightarrow \quad -22 + 15 - k = 0$$

$$\Rightarrow \quad k = -7$$

**121.** Given pair of lines is

$$\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$$

On comparing with

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

we get,

$$a = \lambda, h = -5, b = 12, g = \frac{5}{2}, f = -8, c = -3$$

Condition for represents a pair of lines is

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\lambda \times 12 \times (-3) + 2(-8)\left(\frac{5}{2}\right)(-5) - \lambda(-8)^2 - 12\left(\frac{5}{2}\right)^2 + 3(-5)^2 = 0$$

$$\Rightarrow \quad -36\lambda + 200 - 64\lambda - 75 + 75 = 0$$

$$\Rightarrow \quad 100\lambda = 200 \quad \Rightarrow \quad \lambda = 2$$

**122.** To make the given curves $x^2 + y^2 = 4$ and $x + y = a$ homogeneous.

$$\therefore \quad x^2 + y^2 - 4\left(\frac{x+y}{a}\right)^2 = 0$$

$$\Rightarrow a^2(x^2 + y^2) - 4(x^2 + y^2 + 2xy) = 0$$

$$\Rightarrow \quad x^2(a^2 - 4) + y^2(a^2 - 4) - 8xy = 0$$

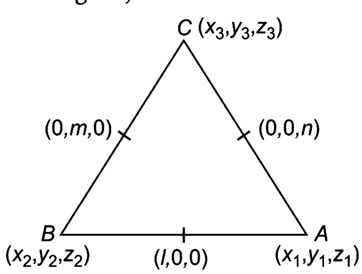
Since, this is a perpendicular pair of straight lines.

$$\therefore \quad a^2 - 4 + a^2 - 4 = 0$$

$$\Rightarrow \quad a^2 = 4 \quad \Rightarrow \quad a = \pm 2$$

Hence, required set of $a$ is $\{-2, 2\}$.

**123.** From the figure,




$$x_1 + x_2 = 2l, y_1 + y_2 = 0, z_1 + z_2 = 0,$$

$$x_2 + x_3 = 0, y_2 + y_3 = 2m, z_2 + z_3 = 0$$

and $\quad x_1 + x_3 = 0, y_1 + y_3 = 0, z_1 + z_3 = 2n$

On solving, we get

$$x_1 = l, x_2 = l, x_3 = -l,$$

$$y_1 = -m, y_2 = m, y_3 = m$$

and $\quad z_1 = n, z_2 = -n, z_3 = n$

$\therefore$ Coordinates are $A(l, -m, n)$, $B(l, m, -n)$ and $C(-l, m, n)$

$$\therefore \quad \frac{AB^2 + BC^2 + CA^2}{l^2 + m^2 + n^2}$$

$$= \frac{(4m^2 + 4n^2) + (4l^2 + 4n^2) + (4l^2 + 4m^2)}{l^2 + m^2 + n^2}$$

$$= 8$$

**124.** Given that,

$$l_1 = \frac{\sqrt{3}}{4}, m_1 = \frac{1}{4} \quad \text{and} \quad n_1 = \frac{\sqrt{3}}{2}$$

and $l_2 = \frac{\sqrt{3}}{4}, m_2 = \frac{1}{4} \quad \text{and} \quad n_2 = \frac{-\sqrt{3}}{2}$

$$\therefore \cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$$

$$= \left|\frac{\sqrt{3}}{4} \times \frac{\sqrt{3}}{4} + \frac{1}{4} \times \frac{1}{4} + \frac{\sqrt{3}}{2} \times \left(\frac{-\sqrt{3}}{2}\right)\right|$$

$$= \left|\frac{3}{16} + \frac{1}{16} - \frac{3}{4}\right| = \left|-\frac{2}{4}\right| = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{3}$$

**125.** Since, the lines $2x - 3y = 5$ and $3x - 4y = 7$ are the diameters of a circle. Therefore, the point of intersection is the centre of the circle. On solving the given equations, we get $x = 1$ and $y = -1$ *ie*, the centre of the circle.

$\therefore$Required equation of circle is

$$(x - 1)^2 + (y + 1)^2 = 7^2$$

$$\Rightarrow \quad x^2 + y^2 - 2x + 2y + 2 = 49$$

$$\Rightarrow \quad x^2 + y^2 - 2x + 2y - 47 = 0$$

**126.** The equation of pole w.r.t. the point (1, 2) to the circle $x^2 + y^2 - 4x - 6y + 9 = 0$ is

$$x + 2y - 2(x + 1) - 3(y + 2) + 9 = 0$$

$$\Rightarrow \quad x + y - 1 = 0$$

Since, the inverse of the point (1, 2) is the foot $(\alpha, \beta)$ of the perpendicular from the point (1, 2) to the line $x + y - 1$.

$$\therefore \quad \frac{\alpha - 1}{1} = \frac{\beta - 2}{1} = -\frac{(1 \cdot 1 + 1 \cdot 2 - 1)}{1^2 + 1^2}$$

$$\Rightarrow \quad \alpha - 1 = \beta - 2 = -1$$

$$\Rightarrow \quad \alpha = 0, \beta = 1$$

Hence, required point is (0, 1).

**127.** We know that, the angle between the two tangents from $(\alpha, \beta)$ to the circle $x^2 + y^2 = r^2$ is

$$2\tan^{-1}\frac{r}{\sqrt{S_1}}$$

Given equation of circle is $x^2 + y^2 - 5x + 4y - 2 = 0$.

Now, radius, $r = \sqrt{\left(-\frac{5}{2}\right)^2 + (2)^2 + 2} = \sqrt{\frac{49}{4}}$

$$= \frac{7}{2}$$

At point $(-1, 0)$

$$S_1 = (-1)^2 + (0)^2 - 5(-1) + 4(0) - 2$$

$$= 1 + 5 - 2 = 4$$

$\therefore$Required angle, $\theta = 2\tan^{-1}\dfrac{\frac{7}{2}}{\sqrt{4}} = 2\tan^{-1}\left(\dfrac{7}{4}\right)$

**128.** Using the condition that if two lines $l_1x + m_1y + n_1 = 0$ and $l_2x + m_2y + n_2 = 0$ are conjugate w.r.t. parabola $y^2 = 4ax$, then

$$l_1n_2 + l_2n_1 = 2am_1m_2 \quad \ldots(i)$$

Given conjugate lines are $2x + 3y + 12 = 0$ and $x - y + 4\lambda = 0$ and equation of parabola is $y^2 = 8x$.

Here, $l_1 = 2, m_1 = 3, n_1 = 12; l_2 = 1, m_2 = -1,$
$n_2 = 4\lambda$ and $a = 2$

$\therefore$From Eq. (i),

$$2 \times 4\lambda + 1 \times 12 = 2 \times 2 \times 3 \times (-1)$$

$$8\lambda = -12 - 12 \Rightarrow \lambda = -3$$

**129.** Given that, $e = \dfrac{1}{2}$ and $\dfrac{a}{e} = 4$

$$\Rightarrow \frac{a}{1/2} = 4 \Rightarrow a = 2$$

Using $\quad b^2 = a^2(1 - e^2)$

$$\Rightarrow b^2 = 4\left(1 - \frac{1}{4}\right) = 3$$

$\therefore$Equation of ellipse is

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

$$\Rightarrow 3x^2 + 4y^2 = 12$$

**130.** Given, equation of hyperbola is

$$x^2 - 3y^2 - 4x - 6y - 11 = 0$$

$$\Rightarrow (x^2 - 4x + 4) - 3(y^2 + 2y + 1) - 11 = 4 - 3$$

$$\Rightarrow (x-2)^2 - 3(y+1)^2 = 12$$

$$\Rightarrow \frac{(x-2)^2}{12} - \frac{(y+1)^2}{4} = 1$$

Now, $\quad e = \sqrt{1 + \dfrac{4}{12}} = \dfrac{2}{\sqrt{3}}$

$\therefore$Distance between foci

$$= 2ae = 2 \times \sqrt{12} \times \frac{2}{\sqrt{3}} = 8$$

**131.** Given polar equation of circle is

$$r^2 - 8r(\sqrt{3}\cos\theta + \sin\theta) + 15 = 0$$

or $r^2 - 8(\sqrt{3}\, r\cos\theta + r\sin\theta) + 15 = 0$

where $r\cos\theta = x$ and $y = r\sin\theta$.

It can be rewritten in cartesian form

$$x^2 + y^2 - 8(\sqrt{3}x + y) + 15 = 0$$

$$\Rightarrow x^2 + y^2 - 8\sqrt{3}x - 8y + 15 = 0$$

Now, $\quad$ radius $= \sqrt{(4\sqrt{3})^2 + (4)^2 - 15}$

$$= \sqrt{48 + 16 - 15} = 7$$

**132.** $\displaystyle\lim_{x \to 0} \frac{(1 - e^x)\sin x}{(x + x^2)\,x}$

$$= \lim_{x \to 0} \frac{\left(-x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots\right)}{x(1+x)} \times \lim_{x \to 0}\frac{\sin x}{x}$$

$$= -1 \times 1 = -1$$

**133.** Given that,

$$f(x) = [x - 3] + |x - 4|$$

$$\therefore \lim_{x \to 3^-} f(x) = \lim_{x \to 3^-}([x - 3] + |x - 4|)$$

$$= \lim_{h \to 0}([3 - h - 3] + |3 - h - 4|)$$

$$= \lim_{h \to 0}([-h] + 1 + h)$$

$$= -1 + 1 + 0 = 0$$

**134.** Given that,

$$f(x)=\begin{cases}\dfrac{\cos 3x-\cos x}{x^2}, & \text{for } x\neq 0\\ \lambda, & \text{for } x=0\end{cases}$$

Now, LHL $=\lim\limits_{x\to 0^-} f(x)$

$$=\lim_{x\to 0^-}\frac{\cos 3x-\cos x}{x^2}$$

$$=\lim_{h\to 0}\frac{\cos 3(0-h)-\cos(0-h)}{(0-h)^2}$$

$$=\lim_{h\to 0}\frac{\cos 3h-\cos h}{h^2}=\lim_{h\to 0}\frac{-3\sin 3h+\sin h}{2h}$$

(using L' Hospital's rule)

$$=\lim_{h\to 0}\frac{-9\cos 3h+\cos h}{2}$$

(using L' Hospital's rule)

$$=\frac{-9+1}{2}=-4$$

Since, $f(x)$ is continuous at $x=0$

$\therefore \quad \lim\limits_{x\to 0^-} f(x)=f(0)$

$\Rightarrow \quad -4=\lambda \;\Rightarrow\; \lambda=-4$

**135.** Given that, $f(2)=4$ and $f'(2)=1$

$$\therefore \quad \lim_{x\to 2}\frac{xf(2)-2f(x)}{x-2}$$

$$=\lim_{x\to 2}\frac{xf(2)-2f(2)+2f(2)-2f(x)}{x-2}$$

$$=\lim_{x\to 2}f(2)-2\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}$$

$$=f(2)-2f'(2)$$

$$=4-2(1)$$

$$=2$$

**136.** Given that,

$$x=a\left(\cos\theta+\log\tan\left(\frac{\theta}{2}\right)\right) \text{ and } y=a\sin\theta$$

On differentiating w.r.t. $\theta$ respectively, we get

$$\frac{dx}{d\theta}=a\left(-\sin\theta+\frac{1}{\tan\left(\frac{\theta}{2}\right)}\cdot\sec^2\frac{\theta}{2}\cdot\frac{1}{2}\right)$$

$$=a\left(-\sin\theta+\frac{1}{\sin\theta}\right)=\frac{a\cos^2\theta}{\sin\theta}$$

and $\dfrac{dy}{d\theta}=a\cos\theta$

$$\therefore \quad \frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{a\cos\theta}{a\cos^2\theta/\sin\theta}=\tan\theta$$

**137.** Given that,

$$y=\sin(\log_e x) \qquad \ldots(\text{i})$$

On differentiating w.r.t. $x$, we get

$$\frac{dy}{dx}=\cos(\log_e x)\cdot\frac{1}{x}$$

Again differentiating, we get

$$\frac{d^2y}{dx^2}=\frac{-x\cdot\sin(\log_e x)\cdot\frac{1}{x}-\cos(\log_e x)\cdot 1}{x^2}$$

$$=\frac{-\sin(\log_e x)-\cos(\log_e x)}{x^2}$$

Now, $\quad x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}$

$$=\frac{x^2[-\sin(\log_e x)-\cos(\log_e x)]}{x^2}+\frac{x\cdot\cos(\log_e x)}{x}$$

$$=-\sin(\log_e x)$$

$$=-y \qquad \text{[from Eq. (i)]}$$

**138.** Given curves are

$$y^2=4x+4 \text{ and } y^2=36(9-x) \qquad \ldots(\text{i})$$

On solving, we get the points (8, 6) and (8, −6).

On differentiating Eq. (i), we get

$$2y\frac{dy}{dx}=4 \quad\text{and}\quad 2y\frac{dy}{dx}=-36$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{2}{y} \quad\text{and}\quad \frac{dy}{dx}=\frac{-18}{y}$$

At point (8, 6),

$$m_1 = \frac{dy}{dx} = \frac{2}{6} = \frac{1}{3} \text{ and } m_2 = \frac{dy}{dx} = \frac{-18}{6} = -3$$

$$\therefore \quad \tan\theta = \frac{\frac{1}{3} + 3}{1 + \frac{1}{3} \times (-3)} = \frac{10/3}{0} = \infty$$

$$\Rightarrow \quad \theta = \frac{\pi}{2}$$

And at point (8, –6),

$$m_1 = \frac{dy}{dx} = \frac{2}{-6} = -\frac{1}{3} \text{ and } m_2 = \frac{dy}{dx} = \frac{-18}{-6} = 3$$

$$\therefore \quad \tan\theta = \frac{\left|-\frac{1}{3} - 3\right|}{1 + \left(-\frac{1}{3}\right)\cdot 3} = \frac{10/3}{0} = \infty$$

$$\Rightarrow \quad \theta = \frac{\pi}{2}$$

**139.** Given that,

$$f(x) = (x-1)^2 + 3, \quad x \in [-3, 1]$$

On differentiating w.r.t. $x$, we get

$$f'(x) = 2(x-1)$$

For maxima and minima, put $f'(x) = 0$

$$\Rightarrow \quad 2(x-1) = 0 \quad \Rightarrow \quad x = 1$$

Now, $f''(x) = 2$, minima $\forall\ x \in R$

At $\quad x = 1,$

$$f(1) = (1-1)^2 + 3 = 3$$

At $\quad x = -3,$

$$f(-3) = (-3-1)^2 + 3 = 19$$

Here, $m = 3$ and $M = 19$

Hence, required ordered pair is (3, 19).

**140.** Given that,

$$2y^4 = x^5$$

On differentiating w.r.t. $x$, we get

$$8y^3 \frac{dy}{dx} = 5x^4$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(2,\,2)} = \frac{5(2)^4}{8(2)^3} = \frac{5}{4}$$

$$\therefore \text{ Length of subtangent} = \frac{y}{dy/dx} = \frac{2}{5/4} = \frac{8}{5}$$

**141.** $\int e^x \left(\frac{1-\sin x}{1-\cos x}\right) dx$

$$= \int e^x \left(\frac{1 - 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right) dx$$

$$= \frac{1}{2}\int \underset{\text{I}}{e^x} \left(\underset{\text{II}}{\operatorname{cosec}^2 \frac{x}{2}}\right) dx - \int e^x \cot\frac{x}{2}\, dx$$

$$= \frac{1}{2}\left[-e^x \cot\frac{x}{2}\cdot 2 + \int e^x \cot\frac{x}{2}\, 2dx\right] - \int e^x \cot\frac{x}{2}\, dx + c$$

$$= -e^x \cot\frac{x}{2} + c$$

**142.** Given that,

$$I_n = \int x^n \cdot e^{cx}\, dx$$

$$\Rightarrow \quad I_n = \frac{e^{cx}}{c}\cdot x^n - \int \frac{e^{cx}}{c}\cdot nx^{n-1}\, dx$$

$$\Rightarrow \quad I_n = \frac{e^{cx}\cdot x^n}{c} - \frac{n}{c} I_{n-1}$$

$$\Rightarrow \quad cI_n + nI_{n-1} = e^{cx}\cdot x^n$$

**143.** Given that,

$$\int e^x (1+x)\cdot \sec^2(xe^x)\, dx = f(x) + \text{constant}$$

Put $\quad xe^x = t$ in LHS

$$\Rightarrow \quad (e^x + xe^x)\, dx = dt$$

$$\therefore \text{ LHS} = \int \sec^2 t\, dt = \tan t + \text{constant}$$

$$\Rightarrow \quad \tan(xe^x) + \text{constant} = f(x) + \text{constant}$$

$$\Rightarrow \quad f(x) = \tan(xe^x)$$

**144.** Let $I = \int_0^1 x^{3/2}\sqrt{1-x}\, dx$

Put $x = \sin^2\theta \Rightarrow dx = 2\sin\theta\cos\theta\, d\theta$

$$\therefore I = \int_0^{\pi/2} \sin^3\theta \cdot \sqrt{1-\sin^2\theta}\; 2\sin\theta\cos\theta\, d\theta$$

$$= 2\int_0^{\pi/2} \sin^4\theta\cos^2\theta\, d\theta$$

$$= 2\left[\frac{3\cdot 1\cdot 1}{6\cdot 4\cdot 2}\cdot\frac{\pi}{2}\right] \text{ (using Wallis formula)}$$

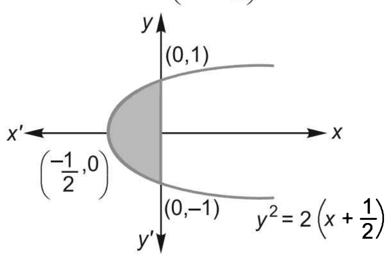
$$= \frac{\pi}{16}$$

**145.** Let $I = \int_{-\pi/2}^{\pi/2} \sin|x|\,dx$

$$= 2\int_0^{\pi/2} \sin x\,dx = 2[-\cos x]_0^{\pi/2} = 2$$

**146.** Given curve can be rewritten as

$$y^2 = 2\left(x + \frac{1}{2}\right)$$




$\therefore$Required area $= \int_{-1}^{1} x\,dy = 2\int_0^1 \frac{y^2-1}{2}\,dy$

$$= \left|\left[\frac{y^3}{3} - y\right]_0^1\right| = \left|\left[\frac{1}{3} - 1\right]\right| = \frac{2}{3} \text{ sq unit}$$

**147.** Given differential equation is

$$\frac{dy}{dx} = \frac{xy+y}{xy+x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y(1+x)}{x(1+y)}$$

$$\Rightarrow \quad \frac{(1+y)}{y}\,dy = \frac{(1+x)}{x}\,dx$$

$$\Rightarrow \quad \int\left(\frac{1}{y}+1\right)dy = \int\left(\frac{1}{x}+1\right)dx$$

$$\Rightarrow \quad \log y + y = \log x + x + \log c$$

$$\Rightarrow \quad y - x = \log\left(\frac{cx}{y}\right)$$

**148.** Given that,

$$\frac{dy}{dx} = \frac{x-2y+1}{2x-4y}$$

Put $x - 2y = z \Rightarrow 1 - 2\frac{dy}{dx} = \frac{dz}{dx}$

$$\therefore \quad \frac{1}{2}\left[-\frac{dz}{dx}+1\right] = \frac{z+1}{2z}$$

$$\Rightarrow \quad -\frac{dz}{dx} + 1 = \frac{z+1}{z}$$

$$\Rightarrow \quad \frac{dz}{dx} = -\frac{1}{z}$$

$$\Rightarrow \quad z\,dz = -dx$$

$$\Rightarrow \quad \frac{z^2}{2} = -x + c_1$$

$$\Rightarrow \quad (x-2y)^2 + 2x = c$$

**149.** Given linear differential equation is

$$\frac{dy}{dx} - y\tan x = e^x \sec x$$

$$\therefore \quad \text{IF} = e^{\int -\tan x\,dx} = e^{-\log\sec x} = \frac{1}{\sec x}$$

$\therefore$ Complete solution is

$$y \cdot \frac{1}{\sec x} = \int e^x \sec x \cdot \frac{1}{\sec x}\,dx$$

$$\Rightarrow \quad \frac{y}{\sec x} = e^x + c$$

$$\Rightarrow \quad y\cos x = e^x + c$$

**150.** Given differential equation can be rewritten as

$$\frac{dy}{dx} = \frac{x^3+y^3}{xy^2}$$

It is a homogeneous differential equation.

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore \quad x\frac{dv}{dx} + v = \frac{x^3+v^3x^3}{x^3v^2}$$

$$\Rightarrow \quad x\frac{dv}{dx} + v = \frac{1+v^3}{v^2}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1}{v^2}$$

$$\Rightarrow \quad v^2\,dv = \frac{dx}{x}$$

On integrating both sides, we get

$$\frac{v^3}{3} = \log x + \log c$$

$$\Rightarrow \quad \frac{1}{3}\left(\frac{y}{x}\right)^3 = \log x + \log c$$

$$\Rightarrow \quad y^3 = 3x^3 \log cx$$