Solved Paper 2007

AMU

Engineering Entrance Exam

Physics

1. A particle is projected with velocity $2\sqrt{gh}$, such that it just crosses two walls of height h. Find the angle of projection.
   (a) 15° (b) 75°
   (c) 60° (d) 30°

2. For a projectile, $(\text{range})^2$ is 48 times of $(\text{maximum height})^2$ obtained. Find angle of projection.
   (a) 60° (b) 30°
   (c) 45° (d) 75°

3. Which of the following cannot be explained on the basis of wave nature of light ?
   (i) Polarization
   (ii) Optical activity
   (iii) Photoelectric effect
   (iv) Compton effect
   (a) (iii) and (iv)
   (b) (ii) and (iii)
   (c) (i) and (iii)
   (d) (ii) and (iv)

4. An ice cube is sliding down on an inclined plane of angle 30°. Coefficient of kinetic friction between block and inclined plane is $\frac{1}{\sqrt{3}}$. What is acceleration of block ?
   (a) Zero (b) 2 m/s^2
   (c) 1.5 m/s^2 (d) 5 m/s^2

5. A round disc of moment of inertia I_2 about its axis perpendicular to its plane and passing through its centre is placed over another disc of moment of inertia I_1 rotating with an angular velocity ω about the same axis. The final angular velocity of the combination of discs is
   (a) $\frac{I_2\omega}{I_1 + I_2}$ (b) ω
   (c) $\frac{I_1\omega}{I_1 + I_2}$ (d) $\frac{(I_1 + I_2)\omega}{I_1}$

6. A particle is executing SHM at mid-point of mean position and extremity. What is the potential energy in terms of total energy (E) ?
   (a) $\frac{E}{4}$ (b) $\frac{E}{16}$
   (c) $\frac{E}{2}$ (d) $\frac{E}{8}$

7. A train is approaching with velocity 25 m/s towards a pedestrian standing on track, frequency of horn of train is 1 kHz. Frequency heard by the pedestrian is $(v = 350\text{ m/s})$
   (a) 1077 Hz (b) 1167 Hz
   (c) 985 Hz (d) 945 Hz

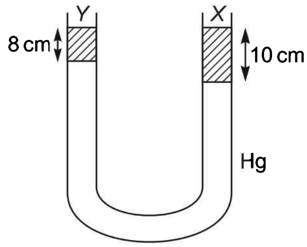
8. A force of 200 N acts tangentially on the rim of a wheel 25 cm in radius. Find the torque.
   (a) 50 N-m (b) 150 N-m
   (c) 75 N-m (d) 30 N-m

9. Focal length of objective and eyepiece of telescope are 200 cm and 4 cm respectively. What is length of telescope for normal adjustment ?
   (a) 196 cm (b) 204 cm
   (c) 250 cm (d) 225 cm

10. Two lenses of power 3D and – 1D are kept in contact. What is focal length and nature of combined lens ?
    (a) 50 cm, convex
    (b) 200 cm, convex
    (c) 50 cm, concave
    (d) 200 cm, concave

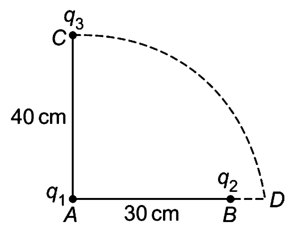
11. Intensity of wave A is $9I$, while that of wave B is I. What is maximum and minimum intensity in YDSE?
    (a) $82I$, $80I$ (b) $8I$, $10I$
    (c) $16I$, $4I$ (d) $4I$, I

12. A liquid X of density 3.36 g/cm^3 is poured in a U-tube, which contains Hg. Another liquid Y is poured in left arm with height 8 cm, upper levels of X and Y are same. What is density of Y ?



(a) 0.8 g/cc (b) 1.2 g/cc
(c) 1.4 g/cc (d) 1.6 g/cc

13. Two charges q_1 and q_2 are placed 30 cm apart, as shown in the figure. A third charge q_3 is moved along the arc of a circle of radius 40 cm from C to D. The change in the potential energy of the system is $\frac{q_3}{4\pi\varepsilon_0}k$, where k is



(a) $8q_2$ (b) $8q_1$
(c) $6q_2$ (d) $6q_1$

14. What is order of energy of X-rays (E_X), radio waves (E_R) and microwaves (E_M) ?
(a) $E_X < E_R < E_M$ (b) $E_X > E_M > E_R$
(c) $E_M > E_X > E_R$ (d) $E_M < E_R < E_X$

15. A ray of light is incident on a plane mirror, along the direction given by vector $A = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$. Find the unit vector along reflected ray. Take normal to mirror along the direction of vector $B = 3\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$.

(a) $\frac{-94\hat{\mathbf{i}} + 237\hat{\mathbf{j}} + 68\hat{\mathbf{k}}}{49\sqrt{29}}$

(b) $\frac{-94\hat{\mathbf{i}} + 68\hat{\mathbf{j}} - 237\hat{\mathbf{k}}}{49\sqrt{29}}$

(c) $\frac{3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}}{7}$

(d) None of the above

16. Motion of two particles is given by

$$y_1 = 0.25 \sin(310t)$$
$$y_2 = 0.25 \sin(316t).$$

Find beat frequency.

(a) 3 (b) $\frac{3}{\pi}$
(c) $\frac{6}{\pi}$ (d) 6

17. A coin is of mass 4.8 kg and radius 1 m rolling on a horizontal surface without sliding with angular velocity 600 rotation/min. What is total kinetic energy of the coin ?
(a) 360 J (b) $1440\pi^2$J
(c) $4000\pi^2$J (d) $600\pi^2$J

18. In BJT, maximum current flows in which of the following ?
(a) Emitter region
(b) Base region
(c) Collector region
(d) Equal in all the regions

19. In semiconductors at a room temperature
(a) the valence band is partially empty and the conduction band is partially filled
(b) the valence band is completely filled and the conduction band is partially filled
(c) the valence band is completely filled
(d) the conduction band is completely empty

20. In a circuit L, C and R are connected in series with an alternating voltage source of frequency f. The current leads the voltage by 45°. The value of C is

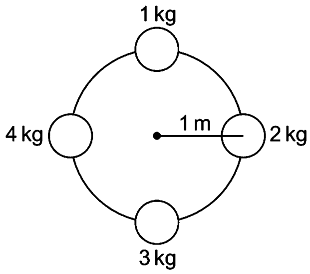
(a) $\frac{1}{2\pi f\,(2\pi fL + R)}$ (b) $\frac{1}{\pi f\,(2\pi fL + R)}$

(c) $\frac{1}{2\pi f\,(2\pi fL - R)}$ (d) $\frac{1}{\pi f\,(2\pi fL - R)}$

21. What is the angle between electric field and equipotential surface ?
(a) 90° always (b) 0° always
(c) 0° to 90° (d) 0° to 180°

22. A satellite moves in elliptical orbit about a planet. The maximum and minimum velocities of satellites are 3×10^4 m/s and 1×10^3m/s respectively. What is the minimum distance of satellite from planet, if maximum distance is 4×10^4 km ?
(a) 4×10^3 km (b) 3×10^3 km
(c) $4/3 \times 10^3$ km (d) 1×10^3 km

23. Four balls each of radius 10 cm and mass 1 kg, 2 kg, 3 kg and 4 kg are attached to the periphery of massless plate of radius 1 m. What is moment of inertia of the system about the centre of plate ?



(a) $12.04\ kg\text{-}m^2$ (b) $10.04\ kg\text{-}m^2$
(c) $11.50\ kg\text{-}m^2$ (d) $5.04\ kg\text{-}m^2$

24. A Carnot engine has efficiency 1/5. Efficiency becomes 1/3 when temperature of sink is decreased by 50 K. What is the temperature of sink ?
(a) 325 K (b) 375 K
(c) 300 K (d) 350 K

25. A cyclist moves in such a way that he takes 60° turn after 100 m. What is the displacement when he takes 7th turn ?
(a) 100 m (b) 200 m
(c) $100\sqrt{3}$ m (d) $100/\sqrt{3}$ m

26. A spring of spring constant k is cut into two equal parts. A block of mass m is attached with one part of spring. What is the frequency of the system if ν is frequency of block with original spring ?
(a) $\sqrt{2}\nu$ (b) $\nu/2$
(c) 2ν (d) ν

27. Why is there sudden increase in current in zener diode ?
(a) Due to rupture of bonds
(b) Resistance of depletion layer becomes less
(c) Due to high doping
(d) None of the above

28. Coefficient of coupling between two coils of self-inductances L_1 and L_2 is unity. It means
(a) 50% flux of L_1 is linked with L_2
(b) 100% flux of L_1 is linked with L_2
(c) $\sqrt{L_1}$ time of flux of L_1 is linked with L_2
(d) None of the above

29. One mole of an ideal gas at an initial temperature of T kelvin does $6R$ joule of work adiabatically. If the ratio of specific heats of this gas at constant pressure and at constant volume is 5/3, the final temperature of gas will be
(a) $(T + 2.4)$ K (b) $(T - 2.4)$ K
(c) $(T + 4)$ K (d) $(T - 4)$ K

30. A drum of radius R and mass M, rolls down without slipping along an inclined plane of angle θ. The frictional force
(a) converts translational energy to rotational energy
(b) dissipates energy as heat
(c) decreases the rotational motion
(d) decreases the rotational and translational motion

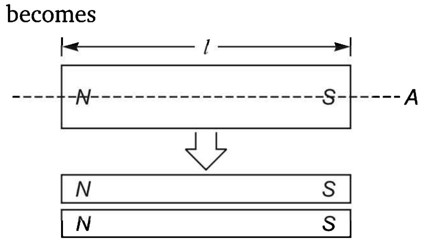
31. If in a nuclear fusion process, the masses of the fusing nuclei be m_1 and m_2 and the mass of the resultant nucleus be m_3, then
(a) $m_3 = m_1 + m_2$ (b) $m_3 = |m_1 - m_2|$
(c) $m_3 < (m_1 + m_2)$ (d) $m_3 > (m_1 + m_2)$

32. If the angle between the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is θ, the value of the product $(\vec{\mathbf{B}} \times \vec{\mathbf{A}}) \cdot \vec{\mathbf{A}}$ is equal to
(a) $BA^2 \cos\theta$ (b) $BA^2 \sin\theta$
(c) $BA^2 \sin\theta\cos\theta$ (d) zero

33. A beam of light composed of red and green rays is incident obliquely at a point on the face of a rectangular glass slab. When coming out on the opposite parallel face, the red and green rays emerge from
(a) two points propagating in two different non-parallel directions
(b) two points propagating in two different parallel directions
(c) one point propagating in two different directions
(d) one point propagating in the same direction

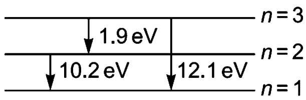
34. A car runs at a constant speed on a circular track of radius 100 m, taking 62.8 s for every circular lap. The average velocity and average speed for each circular lap respectively is
(a) 0, 0 (b) 0, 10 m/s
(c) 10 m/s, 10 m/s (d) 10 m/s, 0

35. If a bar magnet of length l and cross-sectional area A is cut into two equal parts as shown in figure, then the pole strength of each pole becomes



(a) half (b) double
(c) one-fourth (d) four times

36. Three photons coming from excited atomic hydrogen sample are observed, their energies are 12.1 eV, 10.2 eV and 1.9 eV. These photons must come from



(a) single atom
(b) two atoms
(c) three atoms
(d) either two or three atoms

37. A police car is travelling in a straight line with a constant speed v. A truck travelling in the same direction with constant velocity $3v/2$ passes, the police car at $t = 0$. The police car starts accelerating 10 s after passing the truck, at a constant rate of $3\,m/s^2$, while truck continues to move at constant speed. If the police car takes 10 s further to catch the truck, find the value of v.

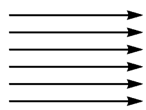
(a) 10 m/s (b) 15 m/s
(c) 20 m/s (d) 30 m/s

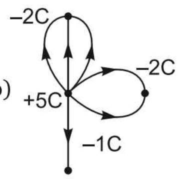
38. Consider the statements.

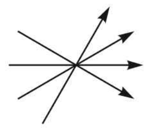
(I) If magnetic field, $\vec{\mathbf{B}} = 0$, then magnetic flux is also zero.
(II) If magnetic flux, $\phi = 0$, then magnetic field is also zero.

(a) (I) is true, (II) may be true
(b) Both (I) and (II) are true
(c) (I) may be true, (II) is true
(d) (I) and (II) both are false

39. Which of the following configurations of electric lines of force is not possible ?

(a) 

(b) 

(c) 

(d) Both (b) and (c)

40. Three guns are aimed at the centre of a circle. They are mounted on the circle, 120° apart. They fire in a timed sequence, such that the three bullets collide at the centre and mash into a stationary lump. Two of the bullets have identical masses of 4.5 g each and speeds of v_1 and v_2. The third bullet has a mass of 2.50 g and a speed of 575 m/s. Find the unknown speeds.

(a) 200 m/s each
(b) 145 m/s and 256 m/s
(c) 536 m/s and 320 m/s
(d) None of the above

Chemistry

41. $Sn^{4+} + 2e^- \longrightarrow Sn^{2+}$ $E° = 0.13\,V$

$Br_2 + 2e^- \longrightarrow 2Br^-$ $E° = 1.08\,V$

Calculate K_{eq} for the cell reaction for the cell formed by two electrodes.

(a) 10^{41} (b) 10^{32}
(c) 10^{-32} (d) 10^{-42}

42. Consider the reaction,

$$NO_2 \rightleftharpoons \frac{1}{2}N_2 + O_2;\ K_1$$

$$N_2O_4 \rightleftharpoons 2NO_2;\quad K_2$$

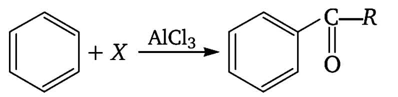
Give the equilibrium constant for the formation of N_2O_4 from N_2 and O_2.

(a) $\frac{1}{K_1^2} + \frac{1}{K_2}$ (b) $\frac{1}{K_1^2 K_2}$
(c) $\sqrt{\frac{1}{K_1 K_2}}$ (d) $\frac{K_2}{K_1}$

43. Half-life of radioactive element is 16 h. What time it will take for 75% disintegration ?

(a) 32 days (b) 32 h
(c) 48 h (d) 16 h

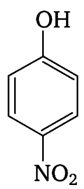
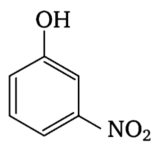
44. Friedel-Craf't acylation can be given by

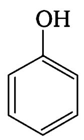
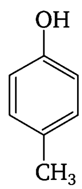


X is

(a) $R—\overset{\overset{\large O}{\|}}{C}—Cl$ (b) $R—\overset{\overset{\large O}{\|}}{C}—R$
(c) $R—\overset{\overset{\large O}{\|}}{C}—H$ (d) $R—O—R$

45. Which of the following is having maximum acidic strength ?

(a) 
(b) 

(c) 
(d) 

46. For the reaction,

$Na_2CO_3 + 2HCl \longrightarrow NaCl + H_2O + CO_2$

equivalent weight of Na_2CO_3 is

(a) $M/2$ (b) M
(c) $2M$ (d) $M/4$

47. Excited state configuration of Mn^{2+} is

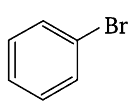
(a) t_{2g}^{4} (b) $t_{2g}^{3}e_g^{2}$
(c) $t_{2g}^{4}e_g^{2}$ (d) $t_{2g}^{5}e_g^{0}$

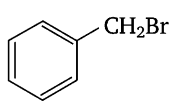
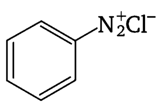
48. Which of the following is thermosetting polymer ?

(a) Nylon-6 (b) Bakelite
(c) Nylon-66 (d) SBR

49. $X \xrightarrow[HNO_3]{AgNO_3}$ Yellow or white ppt.

Which of the following cannot be X ?

(a) 
(b) $(CH_3)_2CHCl$

(c) 
(d) 

50. Rate of a reaction can be expressed by following rate expression

Rate $= k[A]^2[B]$, if concentration of A is increased by 3 times and concentration of B is increased by 2 times, how many times rate of reaction increases ?

(a) 9 times (b) 27 times
(c) 18 times (d) 8 times

51. What is the reason for unusual high b.p. of water ?

(a) Due to presence of H^+ and OH^- ions in water
(b) Due to dipole-dipole interactions
(c) Due to London forces
(d) Strong London forces

52. Shine at freshly cut sodium is

(a) due to oscillation of free electrons
(b) due to weak metallic bonding
(c) due to absorption of light in crystal lattice
(d) due to presence of free valency at the surface

53. Most acidic oxide among the following is

(a) Cl_2O_5 (b) Cl_2O
(c) Cl_2O_3 (d) Cl_2O_7

54. $(H_3C)_2CH-OH \xrightarrow{P + Br_2} \xrightarrow{Na} X$, X is

(a) $CH_3-CH(CH_3)-CH_2-CH_3$
(b) $H_3C-C(CH_3)_2-CH_3$
(c) $(H_3C)_2CH-CH(CH_3)_2$
(d) $CH_3-CH(CH_3)-CH_2CH_2CH_3$

55. Which of the following statements is wrong ?

(a) Metals are more than non-metals
(b) There are only few metalloids
(c) Hydrogen can be placed with alkali metals as well as with halogen in periodic table
(d) Non-metals are more than metals

56. What volume of M/10 NaOH is added in 50 mL M/10 acetic acid solution to get a buffer solution having highest buffer capacity ?

(a) 50 mL (b) 25 mL
(c) 10 mL (d) 40 mL

57. Monomer of nucleic acid is

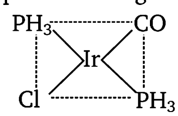
(a) nucleotide
(b) nucleoside
(c) amino acid
(d) carboxylic acid

58. If volume containing gas is compressed to half, how many moles of gas remained in the vessel ?
(a) just double
(b) just half
(c) same
(d) more than double

59. At same temperature, calculate the ratio of average velocity of SO_2 to CH_4.
(a) 2 : 3 (b) 3 : 4
(c) 1 : 2 (d) 1 : 6

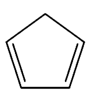
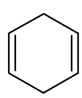
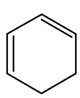
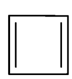
60. If temperature of 1 mole of gas is increased by 50°C, calculate the change in kinetic energy of the system.
(a) 623.25 J (b) 6.235 J
(c) 623.5 J (d) 6235.0 J

61. Give name of the complex, name should specify the position of ligands.



(a) bis*trans*phosphinecarbonylchloroiridium [II]
(b) carbonylchlorobis*trans*phosphineirdium [III]
(c) carbonylchlorobis*trans*phosphineiridium [I]
(d) chlorocarbonylbis*trans*phosphineiridium [I]

62. Ozonolysis products of an olefin are
CHO
|
CHO
and
CH_2CHO
|
CH_2CHO
· Olefin is

(a)  (b) 
(c)  (d) 

63. A bubble of volume V_1 is in the bottom of a pond at 15°C and 1.5 atm pressure. When it comes at the surface it observes a pressure of 1 atm at 25°C and have volume V_2, give $\frac{V_2}{V_1}$.
(a) 15.5 (b) 0.155
(c) 155.0 (d) 1.55

64. Volume of 0.6 M NaOH required to neutralize 30 cm^3 of 0.4 M HCl is
(a) 20 cm^3 (b) 40 cm^3
(c) 45 cm^3 (d) 30 cm^3

65. The orbital angular momentum of an electron in 3*s* orbital is
(a) $\frac{1}{2}\cdot\frac{h}{2\pi}$ (b) $\frac{h}{2\pi}$
(c) $\frac{1}{3}\cdot\frac{h}{2\pi}$ (d) zero

66. Decomposition of H_2O_2 is prevented by
(a) KOH (b) MnO_2
(c) acetanilide (d) oxalic acid

67. The bad smelling substance formed by the action of alcoholic caustic potash on chloroform and aniline is
(a) nitrobenzene
(b) phenyl isocyanide
(c) phenyl cyanide
(d) phenyl isocynate

68. What is the EAN of nickel in $Ni(CO)_4$?
(a) 38 (b) 30
(c) 36 (d) 32

69. $P_4 + 3NaOH + 3H_2O \longrightarrow A + 3NaH_2PO_2$
here *A* is
(a) NH_3 (b) PH_3
(c) H_3PO_4 (d) H_3PO_3

70. If solubility of calcium hydroxide is $\sqrt{3}$, then its solubility product will be
(a) 27 (b) 3
(c) 9 (d) $12\sqrt{3}$

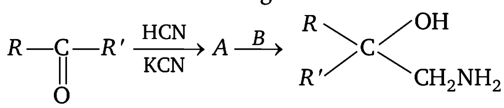
71. Which of the following compounds is optically active ?
(a) $(CH_3)_2CHCH_2OH$
(b) CH_3CH_2OH
(c) CCl_2F_2
(d) $CH_3CHOHC_2H_5$

72. Which compound is soluble in water ?
(a) CS_2 (b) C_2H_5OH
(c) CCl_4 (d) $CHCl_3$

73. In a cubic structure of diamond which is made from *X* and *Y*, *X* atoms are at the corners of the cube and *Y* at the face centres of the cube. The molecular formula of the compound is
(a) X_2Y (b) X_3Y
(c) XY_2 (d) XY_3

74. Bithional is an example of
(a) disinfectant (b) antiseptic
(c) antibiotic (d) analgesic

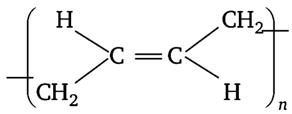
75. A and B in the following reactions are

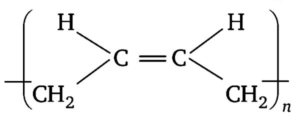


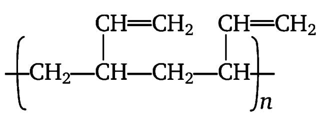
(a) $A = RR'C\begin{smallmatrix} CN \\ OH \end{smallmatrix}$, $B = LiAlH_4$
(b) $A = RR'C\begin{smallmatrix} OH \\ COOH \end{smallmatrix}$, $B = NH_3$
(c) $A = RR'C\begin{smallmatrix} OH \\ CN \end{smallmatrix}$, $B = H_3O^{\oplus}$
(d) $A = RR'CH_2CN$, $B = NaOH$

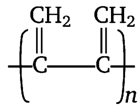
76. Structure of XeF_5^+ ion is
(a) trigonal bipyramidal
(b) square pyramidal
(c) octahedral
(d) pentagonal

77. Mark out the most unlike form of polymerisation of $CH_2{=}CH{-}CH{=}CH_2$.

(a) 

(b) 

(c) 

(d) 

78. $R_3\overset{\oplus}{N}{-}CH{=}CH_2 \xrightarrow{HBr}$ product
Predominant product is
(a) $R_3\overset{\oplus}{N}{-}CH(Br){-}CH_3$
(b) $R_3\overset{\oplus}{N}{-}CH_2{-}CH_2{-}Br$
(c) $CH_2{=}CH{-}\overset{\oplus}{N}R_3\ \overset{\ominus}{Br}$
(d) No reaction

79. Mark out the enthalpy of formation of carbon monoxide (CO).
Given, $C(s) + O_2(g) \longrightarrow CO_2(g)$; $\Delta H = -393.3$ kJ/mol
$CO(g) + \frac{1}{2}O_2(g) \longrightarrow CO_2(g)$; $\Delta H = -282.8$ kJ/mol
(a) -110.5 kJ/mol
(b) -676.1 kJ/mol
(c) -282.8 kJ/mol
(d) -300.0 kJ/mol

80. The magnetic moment of a transition metal ion is 3.87 BM. The number of unpaired electrons present in it, is
(a) 2 (b) 3
(c) 4 (d) 5

Mathematics

81. The function $f : R \to R$ defined by
$f(x) = (x-1)(x-2)(x-3)$ is
(a) one-one but not onto
(b) onto but not one-one
(c) both one-one and onto
(d) neither one-one nor onto

82. If R is an equivalence relation on a set A, then R^{-1} is
(a) reflexive only
(b) symmetric but not transitive
(c) equivalence
(d) None of the above

83. If the complex numbers z_1, z_2, z_3 are in AP, then they lie on a
(a) a circle (b) a parabola
(c) line (d) ellipse

84. Let a, b, c be in AP and $|a| < 1, |b| < 1, |c| < 1$.
If $x = 1 + a + a^2 + \ldots$ to ∞,
$y = 1 + b + b^2 + \ldots$ to ∞,
$z = 1 + c + c^2 + \ldots$ to ∞, then, x, y, z are in
(a) AP (b) GP
(c) HP (d) None of these

85. If $\log_e\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log_e a + \log_e b)$, then

(a) $a = b$ (b) $a = \frac{b}{2}$

(c) $2a = b$ (d) $a = \frac{b}{3}$

86. The number of real solutions of the equation $\left(\frac{9}{10}\right) = -3 + x - x^2$ is

(a) 0 (b) 1

(c) 2 (d) None of these

87. If $f(x) = ax + b$ and $g(x) = cx + d$, then $f\{g(x)\} = g\{f(x)\}$ is equivalent to

(a) $f(a) = g(c)$ (b) $f(b) = g(b)$

(c) $f(d) = g(b)$ (d) $f(c) = g(a)$

88. $(1 + i)^8 + (1 - i)^8$ equals

(a) 2^8 (b) 2^5

(c) $2^4 \cos\frac{\pi}{4}$ (d) $2^8 \cos\frac{\pi}{8}$

89. The value of $\sqrt{3}$ cosec 20°– sec 20° is

(a) 2 (b) 4

(c) – 4 (d) None of these

90. If x, y, z are in HP, then $\log(x + z) + \log(x - 2y + z)$ is equal to

(a) $\log(x - z)$ (b) $2\log(x - z)$

(c) $3\log(x - z)$ (d) $4\log(x - z)$

91. The lines $2x - 3y - 5 = 0$ and $3x - 4y = 7$ are diameters of a circle of area 154 sq unit, then the equation of the circle is

(a) $x^2 + y^2 + 2x - 2y - 62 = 0$

(b) $x^2 + y^2 + 2x - 2y - 47 = 0$

(c) $x^2 + y^2 - 2x + 2y - 47 = 0$

(d) $x^2 + y^2 - 2x + 2y - 62 = 0$

92. Which of the following is a point on the common chord of the circle $x^2 + y^2 + 2x - 3y + 6 = 0$ and $x^2 + y^2 + x - 8y - 13 = 0$?

(a) (1, – 2) (b) (1, 4)

(c) (1, 2) (d) (1, – 4)

93. The angle of depressions of the top and the foot of a chimney as seen from the top of a second chimney, which is 150 m high and standing on the same level as the first are θ and ϕ respectively, then the distance between their tops when $\tan\theta = \frac{4}{3}$ and $\tan\phi = \frac{5}{2}$ is

(a) $\frac{150}{\sqrt{3}}$ m (b) $100\sqrt{3}$ m

(c) 150 m (d) 100 m

94. If one root is square of the other root of the equation $x^2 + px + q = 0$, then the relation between p and q is

(a) $p^3 - (3p - 1)q + q^2 = 0$

(b) $p^3 - q(3p + 1) + q^2 = 0$

(c) $p^3 + q(3p - 1) + q^2 = 0$

(d) $p^3 + q(3p + 1) + q^2 = 0$

95. The coefficient of x^{53} in the following expansions

$\sum_{m=0}^{100} {}^{100}C_m (x - 3)^{100 - m} \cdot 2^m$ is

(a) ${}^{100}C_{47}$ (b) ${}^{100}C_{53}$

(c) $-{}^{100}C_{53}$ (d) $-{}^{100}C_{100}$

96. If (– 3,2) lies on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ which is concentric with the circle $x^2 + y^2 + 6x + 8y - 5 = 0$, then c is equal to

(a) 11 (b) –11

(c) 24 (d) 100

97. If $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\vec{\mathbf{b}} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = 7\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$, then the area of parallelogram having diagonals $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ and $\vec{\mathbf{b}} + \vec{\mathbf{c}}$ is

(a) $4\sqrt{6}$ sq unit (b) $\frac{1}{2}\sqrt{21}$ sq unit

(c) $\frac{\sqrt{6}}{2}$ sq unit (d) $\sqrt{6}$ sq unit

98. The centre of the circle given by $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 15$ and $|\vec{\mathbf{r}} - (\hat{\mathbf{j}} + 2\hat{\mathbf{k}})| = 4$ is

(a) (0, 1, 2) (b) (1, 3, 4)

(c) (–1, 3, 4) (d) None of these

99. If $A = \begin{bmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{bmatrix}$, then trace of matrix A is

(a) 17 (b) 25

(c) 3 (d) 12

100. The value of the determinant

$$\begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ \cos(\alpha + \beta) & -\sin(\alpha + \beta) & 1 \end{vmatrix}$$ is

(a) independent of α
(b) independent of β
(c) independent of α and β
(d) None of the above

101. A committee of five is to be chosen from a group of 9 people. The probability that a certain married couple will either serve together or not at all, is
(a) $\frac{1}{2}$ (b) $\frac{5}{9}$
(c) $\frac{4}{9}$ (d) $\frac{2}{3}$

102. The maximum value of $4\sin^2 x - 12\sin x + 7$ is
(a) 25 (b) 4
(c) does not exist (d) None of these

103. If a point $P(4, 3)$ is shifted by a distance $\sqrt{2}$ unit parallel to the line $y = x$, then coordinates of P in new position are
(a) (5, 4)
(b) $(5+\sqrt{2}, 4+\sqrt{2})$
(c) $(5-\sqrt{2}, 4-\sqrt{2})$
(d) None of the above

104. A straight line through the point $A(3, 4)$ is such that its intercept between the axes is bisected at A. Its equation is
(a) $3x - 4y + 7 = 0$ (b) $4x + 3y = 24$
(c) $3x + 4y = 25$ (d) $x + y = 7$

105. If $(-4, 5)$ is one vertex and $7x - y + 8 = 0$ is one diagonal of a square, then the equation of second diagonal is
(a) $x + 3y = 21$ (b) $2x - 3y = 7$
(c) $x + 7y = 31$ (d) $2x + 3y = 21$

106. The equation $2x^2 - 24xy + 22y^2 = 0$ represents
(a) two parallel lines
(b) two perpendicular lines
(c) two lines passing through the origin
(d) a circle

107. The tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle
$$x^2 + y^2 + 16x + 12y + c = 0 \text{ at}$$
(a) (6, 7) (b) (– 6, 7)
(c) (6, – 7) (d) (–6, – 7)

108. The equation of straight line through the intersection of the lines $x - 2y = 1$ and $x + 3y = 2$ and parallel to $3x + 4y = 0$ is
(a) $3x + 4y + 5 = 0$
(b) $3x + 4y - 10 = 0$
(c) $3x + 4y - 5 = 0$
(d) $3x + 4y + 6 = 0$

109. $\int \frac{dx}{\sin x - \cos x + \sqrt{2}}$ equals
(a) $-\frac{1}{\sqrt{2}} \tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$
(b) $\frac{1}{2} \tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$
(c) $\frac{1}{\sqrt{2}} \cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$
(d) $-\frac{1}{\sqrt{2}} \cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$

110. If $\int \frac{2x^2 + 3}{(x^2 - 1)(x^2 + 4)} dx = a \log\left(\frac{x-1}{x+1}\right) + b \tan^{-1}\left(\frac{x}{2}\right) + c$, then value of a and b are
(a) (1, – 1) (b) (– 1, 1)
(c) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, \frac{1}{2}\right)$

111. $\int \operatorname{cosec}^4 x\, dx$ is equal to
(a) $\cot x + \frac{\cot^3 x}{3} + c$
(b) $\tan x + \frac{\tan^3 x}{3} + c$
(c) $-\cot x - \frac{\cot^3 x}{3} + c$
(d) $-\tan x - \frac{\tan^3 x}{3} + c$

112. The value of integral $\int_0^1 \sqrt{\frac{1-x}{1+x}}\, dx$ is
(a) $\frac{\pi}{2} + 1$ (b) $\frac{\pi}{2} - 1$
(c) – 1 (d) 1

113. The value of $I = \int_0^1 x \left| x - \frac{1}{2} \right| dx$ is
(a) $\frac{1}{3}$ (b) $\frac{1}{4}$
(c) $\frac{1}{8}$ (d) None of these

114. The slope of tangents drawn from a point (4,10) to the parabola $y^2 = 9x$ are
(a) $\frac{1}{4}, \frac{3}{4}$ (b) $\frac{1}{4}, \frac{9}{4}$
(c) $\frac{1}{4}, \frac{1}{3}$ (d) None of these

115. The line $x = at^2$ meets the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the real points, iff

(a) $|t| < 2$ (b) $|t| \leq 1$
(c) $|t| > 1$ (d) None of these

116. The eccentricity of the ellipse which meets the straight line $\frac{x}{7} + \frac{y}{2} = 1$ on the axis of x and the straight line $\frac{x}{3} - \frac{y}{5} = 1$ on the axis of y and whose axes lie along the axes of coordinates, is

(a) $\frac{3\sqrt{2}}{7}$ (b) $\frac{2\sqrt{6}}{7}$
(c) $\frac{\sqrt{3}}{7}$ (d) None of these

117. If $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ and $x^2 - y^2 = c^2$ cut at right angles, then

(a) $a^2 + b^2 = 2c^2$ (b) $b^2 - a^2 = 2c^2$
(c) $a^2 - b^2 = 2c^2$ (d) $a^2b^2 = 2c^2$

118. The equation of the conic with focus at (1, – 1) directrix along $x - y + 1 = 0$ and with eccentricity $\sqrt{2}$, is

(a) $x^2 - y^2 = 1$
(b) $xy = 1$
(c) $2xy - 4x + 4y + 1 = 0$
(d) $2xy + 4x - 4y - 1 = 0$

119. The sum of all five digits numbers that can be formed using the digits 1, 2, 3, 4, 5 when repetition of digits is not allowed, is

(a) 366000 (b) 660000
(c) 360000 (d) 3999960

120. There are 5 letters and 5 different envelopes. The number of ways in which all the letters can be put in wrong envelope, is

(a) 119 (b) 44
(c) 59 (d) 40

121. The sum of the series

$$1 + \frac{1^2 + 2^2}{2!} + \frac{1^2 + 2^2 + 3^2}{3!} + \frac{1^2 + 2^2 + 3^2 + 4^2}{4!} + \ldots \text{ is}$$

(a) $3e$ (b) $\frac{17}{6}e$
(c) $\frac{13}{6}e$ (d) $\frac{19}{6}e$

122. The coefficient of x^n in the expansion of $\log_a(1 + x)$ is

(a) $\frac{(-1)^{n-1}}{n}$ (b) $\frac{(-1)^{n-1}}{n}\log_a e$
(c) $\frac{(-1)^{n-1}}{n}\log_e a$ (d) $\frac{(-1)^n}{n}\log_a e$

123. If the mean of n observations $1^2, 2^2, 3^2 \ldots, n^2$ is $\frac{46n}{11}$, then n is equal to

(a) 11 (b) 12
(c) 23 (d) 22

124. If a plane meets the coordinate axes at A, B and C in such a way that the centroid of $\Delta\, ABC$ is at the point (1, 2, 3), then equation of the plane is

(a) $\frac{x}{1} + \frac{y}{2} + \frac{z}{3} = 1$ (b) $\frac{x}{3} + \frac{y}{6} + \frac{z}{9} = 1$
(c) $\frac{x}{1} + \frac{y}{2} + \frac{z}{3} = \frac{1}{3}$ (d) None of these

125. The projections of a directed line segment on the coordinate axes are 12, 4, 3. The DC's of the line are

(a) $\frac{12}{13}, -\frac{4}{13}, \frac{3}{13}$ (b) $-\frac{12}{13}, -\frac{4}{13}, \frac{3}{13}$
(c) $\frac{12}{13}, \frac{4}{13}, \frac{3}{13}$ (d) None of these

126. The value of $\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} + \vec{\mathbf{c}}) \times (\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}})$ is

(a) $2[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$ (b) $[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$
(c) 0 (d) None of these

127. Let $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\vec{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and a unit vector $\vec{\mathbf{c}}$ be coplanar. If $\vec{\mathbf{c}}$ is perpendicular to $\vec{\mathbf{a}}$, then $\vec{\mathbf{c}}$ is equal to

(a) $\frac{1}{\sqrt{2}}(-\hat{\mathbf{j}} + \hat{\mathbf{k}})$ (b) $\frac{1}{\sqrt{3}}(-\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$
(c) $\frac{1}{\sqrt{5}}(\hat{\mathbf{i}} - 2\hat{\mathbf{j}})$ (d) $\frac{1}{\sqrt{3}}(\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$

128. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are the position vectors of the vertices of an equilateral triangle whose orthocentre is at the origin, then

(a) $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$ (b) $\vec{\mathbf{a}}^2 = \vec{\mathbf{b}}^2 + \vec{\mathbf{c}}^2$
(c) $\vec{\mathbf{a}} + \vec{\mathbf{b}} = \vec{\mathbf{c}}$ (d) None of these

129. The points with position vectors $60\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$, $40\hat{\mathbf{i}} - 8\hat{\mathbf{j}}$, $a\hat{\mathbf{i}} - 52\hat{\mathbf{j}}$ are collinear, if

(a) $a = -40$ (b) $a = 40$
(c) $a = 20$ (d) None of these

130. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$, the line $x = \sqrt{3}y$ and x-axis is

(a) π sq unit (b) $\frac{\pi}{2}$ sq unit

(c) $\frac{\pi}{3}$ sq unit (d) None of these

131. The value of $\lim_{x \to \infty} \left(\frac{\pi}{2} - \tan^{-1} x \right)^{1/x}$ is

(a) 0 (b) 1

(c) –1 (d) e

132. If $f(x) = \begin{cases} mx + 1, & x \le \frac{\pi}{2} \\ \sin x + n, & x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then

(a) $m = 1, n = 0$ (b) $m = \frac{n\pi}{2} + 1$

(c) $n = m\frac{\pi}{2}$ (d) $m = n = \frac{\pi}{2}$

133. The domain of the function $f(x) = \frac{\sqrt{4 - x^2}}{\sin^{-1}(2 - x)}$ is

(a) [0, 2] (b) [0, 2)

(c) [1, 2) (d) [1, 2]

134. The general solution of the differential equation $(1 + y^2)dx + (1 + x^2)\,dy = 0$ is

(a) $x - y = c(1 - xy)$

(b) $x - y = c(1 + xy)$

(c) $x + y = c(1 - xy)$

(d) $x + y = c(1 + xy)$

135. The order and degree of the differential equation $\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ are respectively

(a) 2, 2 (b) 2, 3

(c) 2, 1 (d) None of these

136. The matrix A satisfying the equation $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 4 \\ -1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -4 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$ (d) None of these

137. The relation R defined on the set of natural numbers as $\{(a, b) : a$ differs from b by $3\}$ is given

(a) {(1, 4), (2, 5), (3, 6), ...}

(b) {(4, 1), (5, 2), (6, 3), ...}

(c) {(1, 3), (2, 6), (3, 9), ...}

(d) None of the above

138. The solution of $\frac{dy}{dx} = \frac{ax + h}{by + k}$ represents a parabola when

(a) $a = 0, b = 0$ (b) $a = 1, b = 2$

(c) $a = 0, b \ne 0$ (d) $a = 2, b = 1$

139. The solution of the differential equation $\frac{dy}{dx} + \frac{2yx}{1 + x^2} = \frac{1}{(1 + x^2)^2}$ is

(a) $y(1 + x^2) = c + \tan^{-1} x$

(b) $\frac{y}{1 + x^2} = c + \tan^{-1} x$

(c) $y \log(1 + x^2) = c + \tan^{-1} x$

(d) $y(1 + x^2) = c + \sin^{-1} x$

140. If x, y, z are all distinct and $\begin{vmatrix} x & x^2 & 1 + x^3 \\ y & y^2 & 1 + y^3 \\ z & z^2 & 1 + z^3 \end{vmatrix} = 0$, then the value of xyz is

(a) –2 (b) –1

(c) –3 (d) None of these

141. The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\bar{A}) + P(\bar{B})$ is

(a) 0.4 (b) 0.8

(c) 1.2 (d) 1.4

142. If A and B are two events such that $P(A) > 0$ and $P(B) \ne 1$, then $P(\bar{A}/\bar{B})$ is equal to

(a) $1 - P(A/\bar{B})$ (b) $1 - P(\bar{A}/B)$

(c) $\frac{1 - P(A \cap B)}{P(\bar{B})}$ (d) $\frac{P(\bar{A})}{P(\bar{B})}$

143. A letter is taken out at random from 'ASSISTANT' and another is taken out from 'STATISTICS'. The probability that they are the same letters, is

(a) $\frac{1}{45}$ (b) $\frac{13}{90}$

(c) $\frac{19}{90}$ (d) None of these

144. If $3p$ and $4p$ are resultant of a force $5p$, then angle between $3p$ and $5p$ is

(a) $\sin^{-1}\left(\frac{3}{5}\right)$ (b) $\sin^{-1}\left(\frac{4}{5}\right)$

(c) 90° (d) None of these

145. Resultant velocity of two velocities 30 km/h and 60 km/h making an anlge 60° with each other is

(a) 90 km/h (b) 30 km/h

(c) $30\sqrt{7}$ km/h (d) None of these

146. A ball falls from rest from top of a tower. If the ball reaches the foot of the tower is 3s, then height of tower is (take $g = 10$ m/s^2)

(a) 45 m (b) 50 m

(c) 40 m (d) None of these

147. Two trains A and B 100 km apart are travelling towards each other with starting speeds of 50 km/h. The train A is accelerating at 18 km/h^2 and B deaccelerating at 18 km/h^2. The distance where the engines cross each other from the initial position of A is

(a) 50 km (b) 68 km

(c) 32 km (d) 59 km

148. If $2\tan^{-1}(\cos x) = \tan^{-1}(2\,\text{cosec}\, x)$, then the value of x is

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) None of these

149. Let a be any element in a boolean algebra B. If $a + x = 1$ and $ax = 0$, then

(a) $x = 1$ (b) $x = 0$

(c) $x = a$ (d) $x = a'$

150. Dual of $(x + y)\cdot(x + 1) = x + x\cdot y + y$ is

(a) $(x\cdot y) + (x\cdot 0) = x\cdot(x + y)\cdot y$

(b) $(x + y) + (x\cdot 1) = x\cdot(x + y)\cdot y$

(c) $(x\cdot y)(x\cdot 0) = x\cdot(x + y)\cdot y$

(d) None of the above

Answers

PHYSICS

1. (d)	**2.** (b)	**3.** (a)	**4.** (a)	**5.** (c)	**6.** (a)	**7.** (a)	**8.** (a)	**9.** (b)	**10.** (a)
11. (c)	**12.** (a)	**13.** (a)	**14.** (b)	**15.** (a)	**16.** (b)	**17.** (b)	**18.** (a)	**19.** (a)	**20.** (c)
21. (a)	**22.** (c)	**23.** (b)	**24.** (c)	**25.** (a)	**26.** (a)	**27.** (a)	**28.** (b)	**29.** (d)	**30.** (a)
31. (c)	**32.** (d)	**33.** (b)	**34.** (b)	**35.** (a)	**36.** (c)	**37.** (b)	**38.** (a)	**39.** (d)	**40.** (d)

CHEMISTRY

41. (b)	**42.** (b)	**43.** (b)	**44.** (a)	**45.** (a)	**46.** (a)	**47.** (b)	**48.** (b)	**49.** (a)	**50.** (c)
51. (b)	**52.** (a)	**53.** (d)	**54.** (c)	**55.** (d)	**56.** (b)	**57.** (a)	**58.** (c)	**59.** (c)	**60.** (a)
61. (c)	**62.** (c)	**63.** (d)	**64.** (a)	**65.** (d)	**66.** (c)	**67.** (b)	**68.** (c)	**69.** (b)	**70.** (d)
71. (d)	**72.** (b)	**73.** (d)	**74.** (b)	**75.** (a)	**76.** (b)	**77.** (d)	**78.** (a)	**79.** (a)	**80.** (b)

MATHEMATICS

81. (b)	**82.** (c)	**83.** (c)	**84.** (c)	**85.** (a)	**86.** (a)	**87.** (c)	**88.** (b)	**89.** (b)	**90.** (b)
91. (c)	**92.** (d)	**93.** (d)	**94.** (a)	**95.** (c)	**96.** (b)	**97.** (a)	**98.** (b)	**99.** (a)	**100.** (a)
101. (c)	**102.** (d)	**103.** (a)	**104.** (b)	**105.** (c)	**106.** (c)	**107.** (d)	**108.** (c)	**109.** (c)	**110.** (d)
111. (c)	**112.** (b)	**113.** (c)	**114.** (b)	**115.** (b)	**116.** (b)	**117.** (c)	**118.** (c)	**119.** (d)	**120.** (b)
121. (b)	**122.** (b)	**123.** (a)	**124.** (b)	**125.** (c)	**126.** (c)	**127.** (a)	**128.** (a)	**129.** (a)	**130.** (c)
131. (b)	**132.** (c)	**133.** (c)	**134.** (c)	**135.** (a)	**136.** (c)	**137.** (b)	**138.** (c)	**139.** (a)	**140.** (b)
141. (c)	**142.** (a)	**143.** (c)	**144.** (b)	**145.** (c)	**146.** (a)	**147.** (d)	**148.** (b)	**149.** (d)	**150.** (a)

Hints & Explanations

Physics

1. For vertically upward motion of a projectile,

$$y = (u \sin \alpha)t - \frac{1}{2}gt^2$$

or $$h = (u \sin \alpha)t - \frac{1}{2}gt^2$$

or $$gt^2 - (2u \sin \alpha)t + 2h = 0 \quad ...(i)$$

$$\therefore\ t = \frac{2u \sin \alpha \pm \sqrt{(4u^2 \sin^2 \alpha) - 8gh}}{2g}$$

If two roots of quadratic Eq. (i) are t_1, t_2 then

$$t_1 = \frac{2u \sin \alpha + \sqrt{4u^2 \sin^2 \alpha - 8gh}}{2g}$$

and $$t_2 = \frac{2u \sin \alpha - \sqrt{4u^2 \sin^2 \alpha - 8gh}}{2g}$$

If particle crosses the walls at times t_1 and t_2 respectively, then time of flight t is

$$t = \sqrt{t_1 t_2}$$

or $$t^2 = t_1 t_2$$

$$\therefore \left(\frac{2u \sin \alpha}{g}\right)^2 = \frac{(2u \sin \alpha)^2 - (4u^2 \sin^2 \alpha - 8gh)}{4g^2}$$

or $$\frac{4u^2 \sin^2 \alpha}{g^2} = \frac{8gh}{4g^2}$$

or $$2u^2 \sin^2 \alpha = gh$$

Given, $$u = \sqrt{2gh}$$

$$\therefore\ 2(2gh) \sin^2 \alpha = gh$$

or $$\sin^2 \alpha = \frac{1}{4}$$

or $$\sin \alpha = \frac{1}{2}$$

$$\therefore\ \alpha = 30°$$

2. $(\text{Range})^2 = 48\ (\text{maximum height})^2$

$$\therefore \left(\frac{u^2 \sin 2\alpha}{g}\right)^2 = 48\left(\frac{u^2 \sin^2 \alpha}{2g}\right)^2$$

or $$\frac{u^2 \sin 2\alpha}{g} = 4\sqrt{3}\left(\frac{u^2 \sin^2 \alpha}{2g}\right)$$

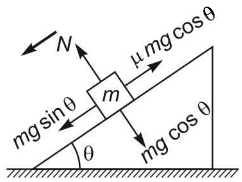
or $$\frac{2 \sin \alpha \cos \alpha}{4\sqrt{3}} = \frac{\sin^2 \alpha}{2}$$

or $$\tan \alpha = \frac{4}{4\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\therefore\ \alpha = 30°$$

3. Photoelectric effect and Compton effect cannot be explained on the basis of wave nature of light while polarization and optical activity can be explained.

4. We can realise the situation as shown.



Maximum retardation

$$a = \frac{f_k - mg \sin \theta}{m}$$

$$= \frac{-\mu_k mg \cos \theta + mg \sin \theta}{m}$$

$$= \frac{-\frac{1}{\sqrt{3}} mg \cos 30° + mg \sin 30°}{m} \quad (\because \theta = 30°)$$

$$= \frac{-\frac{1}{\sqrt{3}} g \times \frac{\sqrt{3}}{2} + g \times \frac{1}{2}}{1} = 0$$

Hence, under the effect of kinetic friction between block and inclined plane, acceleration of block is zero.

5. **Key Idea** *In the absence of any external torque angular momentum remains conserved.*

The angular momentum of a disc of moment of inertia I_1 and rotating about its axis with angular velocity ω is

$$L_1 = I_1 \omega$$

When a round disc of moment of inertia I_2 is placed on first disc, then angular momentum of the combination is

$$L_2 = (I_1 + I_2)\,\omega'$$

As $\tau_{ext} = 0$

$$L_1 = L_2$$

$$I_1 \omega = (I_1 + I_2)\,\omega'$$

$$\Rightarrow\ \omega' = \frac{I_1 \omega}{I_1 + I_2}$$

6. If a particle executes SHM, its kinetic energy is given by

$$KE = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

or $$KE = \frac{1}{2} k (A^2 - x^2)$$

where $k = m\omega^2$ = constant

Its potential energy is given by

$$PE = \frac{1}{2} m\omega^2 x^2 = \frac{1}{2} k x^2$$

Thus, total energy of particle

$$E = KE + PE$$
$$= \frac{1}{2} k (A^2 - x^2) + \frac{1}{2} k x^2 = \frac{1}{2} k A^2$$

Hence, $$PE = \frac{1}{2} k x^2 = \frac{1}{2} k \left(\frac{A}{2}\right)^2 \quad \left(\because x = \frac{A}{2}\right)$$
$$= \frac{1}{4}\left(\frac{1}{2} k A^2\right)$$
$$= \frac{1}{4} E$$

Hence, potential energy is one-fourth of total energy.

7. **Key Idea** *Apply Doppler's effect.*

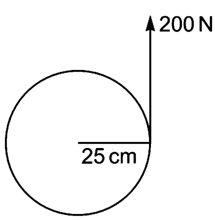
Since, train (source) is moving towards pedestrian (observer), the perceived frequency will be higher than the original.

$$f' = f\left(\frac{v + v_o}{v - v_s}\right)$$

Here, $v_o = 0$ (as observer is stationary)
$v_s = 25$ m/s (velocity of source)
$v = 350$ m/s (velocity of sound)
and $f = 1$ kHz (original frequency)

Hence, $$f' = 1000\left(\frac{350 + 0}{350 - 25}\right)$$
$$= 1000 \times \frac{350}{325} = 1077 \text{ Hz}$$

8. Clearly, the question refers to the torque about an axis through the centre of wheel. Then, since the radius to the point of application of the force is the lever or moment arm,



we have

$$\tau = 0.25 \times 200 = 50 \text{ N-m}$$

9. Length of telescope tube

$$L = f_o + f_e$$

Here, $f_o = 200$ cm, $f_e = 4$ cm

$\therefore$ $$L = 200 + 4$$
$$= 204 \text{ cm}$$

10. **Key Idea** *If two lenses are kept in contact, their powers should be added to get resultant power of the combination.*

$$P = P_1 + P_2$$
$$= 3D - 1D = 2D$$

As power is positive in sign, so it behaves as converging or convex lens.

Now, focal length of the combined lens is given by

$$f = \frac{1}{P \text{ (in D)}} = \frac{1}{2} \text{ m} = 50 \text{ cm}$$

11. For two coherent sources, the resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$$

For maximum intensity, $\cos\phi = +1$

$\therefore$ $$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$
$$= (\sqrt{I_1} + \sqrt{I_2})^2$$

For minimum intensity, $\cos\phi = -1$

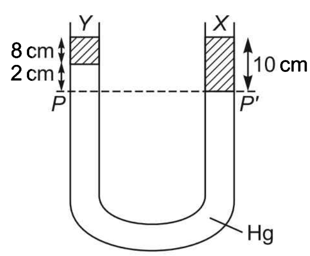
$\therefore$ $$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$
$$= (\sqrt{I_1} - \sqrt{I_2})^2$$

Hence, $$I_{max} = (\sqrt{9I} + \sqrt{I})^2$$
$$= (3\sqrt{I} + \sqrt{I})^2 = 16I$$

and $$I_{min} = (\sqrt{9I} - \sqrt{I})^2$$
$$= (3\sqrt{I} - \sqrt{I})^2 = 4I$$

12. **Key Idea** *In a liquid at same level, the pressure will be same at all the points.*

As shown in figure, in the two arms of a tube pressure remains same on surface PP'.



Hence, $8 \times \rho_y \times g + 2 \times \rho_{Hg} \times g = 10 \times \rho_x \times g$

$\therefore \quad 8\rho_y + 2 \times 13.6 = 10 \times 3.36$

or $\quad \rho_y = \dfrac{33.6 - 27.2}{8}$

$= 0.8 \text{ g/cc}$

13. Key Idea *Change in potential energy is the difference of between final and initial potential energies.*

When charge q_3 is at C, then its potential energy is

$$U_C = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_3}{0.4} + \frac{q_2q_3}{0.5}\right)$$

When charge q_3 is at D, then

$$U_D = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_3}{0.4} + \frac{q_2q_3}{0.1}\right)$$

Hence, change in potential energy

$$\Delta U = U_D - U_C = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_2q_3}{0.1} - \frac{q_2q_3}{0.5}\right)$$

but $\quad \Delta U = \dfrac{q_3}{4\pi\varepsilon_0} k$

$$\therefore \quad \frac{q_3}{4\pi\varepsilon_0} k = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_2q_3}{0.1} - \frac{q_2q_3}{0.5}\right)$$

$$\Rightarrow \quad k = q_2 (10 - 2) = 8q_2$$

14. Key Idea *The waves with less wavelength will have more energy.*

X-rays, radiowaves and microwaves are electromagnetic waves. They travel with the speed of light in air or vacuum.

The wavelength of X-rays is of the order of 1 Å to 100 Å. The wavelength of radiowaves is of the order of 10^9 Å to $10^{1.4}$ Å. The wavelength of microwaves is of the order of 10^7 Å to 10^9 Å.

Thus, $\quad \lambda_X < \lambda_M < \lambda_R$

Hence, $\quad E_X > E_M > E_R$

15. Unit vector along incident ray

$$\hat{\mathbf{I}} = \frac{(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})}{\sqrt{29}}$$

Unit vector along normal

$$\hat{\mathbf{N}} = \frac{(3\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}})}{7}$$

Unit vector along reflected ray

$$\hat{\mathbf{R}} = \hat{\mathbf{I}} - 2(\hat{\mathbf{I}} \cdot \hat{\mathbf{N}})\hat{\mathbf{N}}$$

$$\Rightarrow \quad \hat{\mathbf{R}} = \frac{(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})}{\sqrt{29}} - 2\left(\frac{32}{7\sqrt{29}}\right) \times \left[\frac{(3\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}})}{7}\right]$$

$$\Rightarrow \quad \hat{\mathbf{R}} = \frac{(-94\hat{\mathbf{i}} + 237\hat{\mathbf{j}} + 68\hat{\mathbf{k}})}{49\sqrt{29}}$$

16. Key Idea *Compare the given equations with standard equation to obtain the frequencies.*

The given equations of waves can be written as

$$y_1 = 0.25 \sin (310\,t) \quad \text{...(i)}$$

and $$y_2 = 0.25 \sin (316\,t) \quad \text{...(ii)}$$

Comparing Eqs. (i) and (ii) with the standard wave equation, written as

$$y = a \sin (\omega t) \quad \text{...(iii)}$$

We have,

$$\omega_1 = 310 \Rightarrow f_1 = \frac{310}{2\pi} \text{ unit}$$

and $$\omega_2 = 316 \Rightarrow f_2 = \frac{316}{2\pi} \text{ unit}$$

Hence, beat frequency $= f_2 - f_1$

$$= \frac{316}{2\pi} - \frac{310}{2\pi} = \frac{3}{\pi} \text{ unit}$$

17. Angular velocity

$$\omega = 600 \text{ rotation/min} = \frac{600 \times 2\pi}{60} \text{ rad/s} = 20\,\pi \text{ rad/s}$$

Kinetic energy of coin which is due to rotation and translation is

$$K = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$$

$$= \frac{1}{2} \times \frac{1}{2} mr^2\omega^2 + \frac{1}{2} m(\omega r)^2$$

$$= \frac{1}{4} \times 4.8 \times (1)^2 (20\pi)^2 + \frac{1}{2} \times 4.8 \times (20\pi \times 1)^2$$

$$= 480\,\pi^2 + 960\,\pi^2$$

$$= 1440\,\pi^2 \text{ J}$$

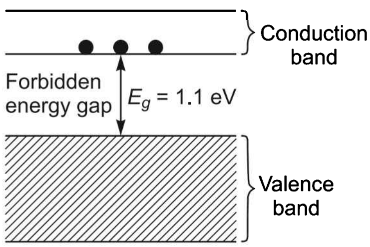
18. In a junction transistor, both the electrons and holes play important role, hence they are called the bi-polar devices or the bi-polar transistors and they are abbreviated as BJT in short form.

There are three parts in a transistor; namely emitter, base and collector. In the emitter part of the transistor the doping is more and it is less in collector part. The doping is very less in the base part. So, emitter current is actually the sum of base and collector current.

ie, $$I_E = I_B + I_C$$

Hence, maximum current flows in emitter region.

19. The energy band scheme of semiconductor is shown here.



In semiconductors, valence band and conduction band are separated by an energy gap called the forbidden energy gap. It is very small. At room temperature some electrons in valence band acquire thermal energy. This energy is more than forbidden energy gap E_g, thus they jump into the conduction band and leaves their vacancy in the valence band which act as holes. Hence, at room temperature valence band is partially empty and conduction band is partially filled.

20. $$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

ϕ being the angle by which the current leads the voltage.

Given, $\phi = 45°$

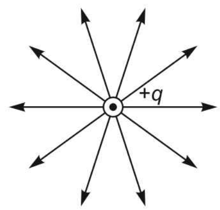
$$\therefore \quad \tan 45° = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$\Rightarrow \quad 1 = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$\Rightarrow \quad C = \frac{1}{\omega(\omega L - R)}$$

$$= \frac{1}{2\pi f\,(2\pi f L - R)}$$

21. The imaginary surface joining the points of same potential in an electric field is called the equipotential surface, *ie,* the potential difference between any two points on an equipotential surface is zero. Hence, if a charge is moved on an equipotential surface from one point to the other, no work is needed to be done. But this is possible only when the charge is moved perpendicular to the electric field (*ie,* perpendicular to the lines of force). It means that the electric lines of force at each point of an equipotential surface are normal to the surface.



Hence, the angle between electric field and equipotential surface is 90°.

Alternative Potential gradient along equipotential surface is zero.

ie, $$E\cos\theta = -\frac{dV}{dr} = 0$$

$$\therefore \quad \theta = 90°$$

22. Key Idea *Torque is rate change of angular momentum.*

If no external torque acts on a system, then angular momentum of the system does not change.

ie, If $$\tau = 0$$

$$\Rightarrow \quad \frac{dL}{dt} = 0$$

$$\therefore \quad L = \text{constant}$$

Hence, $$mv_{\max}\, r_{\min} = mv_{\min}\, r_{\max}$$

$$\Rightarrow \quad r_{\min} = \frac{v_{\min} \times r_{\max}}{v_{\max}}$$

$$= \frac{1 \times 10^3 \times 4 \times 10^4}{3 \times 10^4}$$

$$= \frac{4}{3} \times 10^3 \text{ km}$$

23. Moment of inertia of the system about the centre of plate is given by

$$I = \left[\frac{2}{5} \times 1 \times (0.1)^2 + 1 \times (1)^2\right] + \left[\frac{2}{5} \times 2 \times (0.1)^2 + 2 \times (1)^2\right] + \left[\frac{2}{5} \times 3 \times (0.1)^2 + 3 \times (1)^2\right] + \left[\frac{2}{5} \times 4 \times (0.1)^2 + 4 \times (1)^2\right]$$

$= 1.004 + 2.008 + 3.012 + 4.016$

$= 10.04 \text{ kg-m}^2$

24. $\eta = 1 - \dfrac{T_L}{T_H}$

where T_L is temperature of sink and T_H is temperature of hot reservoir.

According to question

$$\frac{1}{5} = 1 - \frac{T_L}{T_H} \quad \text{...(i)}$$

and $$\frac{1}{3} = 1 - \frac{T_L - 50}{T_H} \quad \text{...(ii)}$$

From Eq. (i)

$$\frac{T_L}{T_H} = \frac{4}{5} \Rightarrow T_H = \frac{5}{4} T_L$$

Substituting value of T_H in Eq. (ii), we get

$$\frac{1}{3} = 1 - \frac{T_L - 50}{\frac{5}{4} T_L}$$

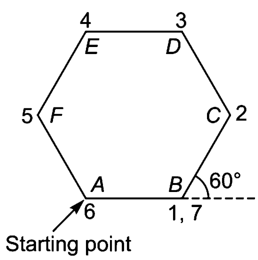
or $$\frac{4(T_L - 50)}{5T_L} = \frac{2}{3}$$

or $$T_L - 50 = \frac{2}{3} \times \frac{5}{4} T_L$$

or $$T_L - \frac{5}{6} T_L = 50$$

$\therefore \quad T_L = 50 \times 6 = 300 \text{ K}$

25. In 6 turns each of 60°, the cyclist traversed a regular hexagon path having each side 100 m. So, at 7th turn, he will be again at



Point B (as shown) which is at a distance 100 m from starting point A. Hence, net displacement of cyclist is 100 m.

26. **Key Idea** *When the spring is cut in two equal parts, the spring constant of each part is twice the spring constant of the original spring.*

Original frequency of system

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

where m is the mass of the block.

New frequency of system

$$\nu' = \frac{1}{2\pi}\sqrt{\frac{k'}{m}}$$

or $$\nu' = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$$

$\therefore \quad \nu' = \sqrt{2}\,\nu$

27. The reverse bias potential that results in this sudden change in characteristics is called the zener potential and is given by the symbol V_Z. When the voltage across diode is increased in the reverse bias region, the minority carriers gain velocity and associated kinetic energy. These minority carriers are responsible for the reverse saturation current. The collisions of these minority carriers with atomic structure will result in an ionisation process and a very high current is established. This current is called avalanche current and the region in which this current is established is called avalanche breakdown region. The magnitude of zener potential may be decreased by increasing doping levels in the p and n-type materials.

When the V_Z decreases to a very low level, there is a strong electric field in the region of the junction that can break the bonds within the atom and generate charge carriers. This mechanism is called zener breakdown.

28. **Key Idea** *If L_1 and L_2 be the self-inductances of the coils and M be their mutual inductances, then*

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

When 100% flux produced by one coil links with the other, then mutual inductance between the two is maximum and is given by

$$M = \sqrt{L_1 L_2}$$

In that case, $\quad k = 1$ (unity)

29. Key Idea *In an adiabatic process*

$$Q = 0$$

So, from 1st law of thermodynamics

$$W = -\Delta U$$
$$= -nC_V\Delta T$$
$$= -n\left(\frac{R}{\gamma - 1}\right)(T_f - T_i)$$
$$= \frac{nR}{\gamma - 1}(T_i - T_f) \quad \text{...(i)}$$

Here, $W = 6R$ Joule, $n = 1$ mol

$$R = 8.31 \text{ J/mol-K}, \gamma = \frac{5}{3}, T_i = T \text{ K}$$

Substituting given values in Eq. (i), we get

$$\therefore \quad 6R = \frac{R}{(5/3 - 1)}(T - T_f)$$
$$\Rightarrow \quad 6R = \frac{3R}{2}(T - T_f)$$
$$\Rightarrow \quad T - T_f = 4$$
$$\therefore \quad T_f = (T - 4) \text{ K}$$

30. When a body rolls down without slipping along an inclined plane of inclination θ, it rotates about a horizontal axis through its centre of mass and also its centre of mass moves. Therefore, rolling motion may be regarded as a rotational motion about an axis through its centre of mass plus a translational motion of the centre of mass. As it rolls down, it suffers loss in gravitational potential energy provided translational energy due to frictional force is converted into rotational energy.

31. Key Idea *In a nuclear fusion, when two light nuclei of different masses are combined to form a stable nucleus, then some mass is lost and appears in the form of energy, called the mass defect.*

The mass of resultant nucleus is always less than the sum of masses of fusing nuclei, *ie,*

$$m_3 < (m_1 + m_2)$$

32. $(\vec{\mathbf{B}} \times \vec{\mathbf{A}}) \cdot \vec{\mathbf{A}}$

Interchange the cross and dot, we have

$$(\vec{\mathbf{B}} \times \vec{\mathbf{A}}) \cdot \vec{\mathbf{A}} = \vec{\mathbf{B}} \cdot (\vec{\mathbf{A}} \times \vec{\mathbf{A}}) = 0 \qquad (\because \vec{\mathbf{A}} \times \vec{\mathbf{A}} = 0)$$

33. Key Idea *In any medium other than air or vacuum, the velocities of different colours are different.*

Both red and green colours are refracted at different angles of refraction. Hence, after emerging from glass slab through opposite parallel face, they appear at two different points and move in the two different parallel directions.

34. Key Idea *Average velocity is defined as the ratio of displacement to time taken while the average speed of a particle in a given interval of time is defined as the ratio of distance travelled to the time taken.*

On a circular path in completing one turn, the distance travelled is $2\pi r$ while displacement is zero.

$$\text{Hence, average velocity} = \frac{\text{displacement}}{\text{time interval}}$$
$$= \frac{0}{t} = 0$$
$$\text{Average speed} = \frac{\text{distance}}{\text{time interval}}$$
$$= \frac{2\pi r}{t} = \frac{2 \times 3.14 \times 100}{62.8}$$
$$= 10 \text{ m/s}$$

35. Pole strength depends on material of magnet, state of magnetization and cross-sectional area. As $m \propto A$, so if A becomes half, pole strength gets half.

36. These photons will be emitted when electron makes transitions in the shown way.

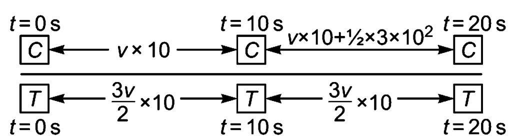
So, these transitions are possible from two or three atoms.

From three atoms separately.

or

From two atoms, one atom emit 12.1 eV and other atom emits 1.9 and 10.2 eV.

37.



The diagram is showing the position of car and truck at various instants.

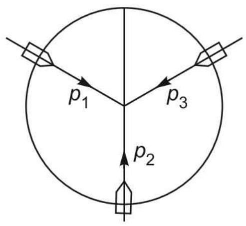
$$v \times 20 + \frac{1}{2} \times 3 \times 10^2 = \frac{3v}{2} \times 20$$
$$\frac{3}{2} \times 100 = \frac{v}{2} \times 20$$
$$v = 15 \text{ m/s}$$

38. If $\vec{\mathbf{B}} = 0$ then $\phi = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = 0$. If $\phi = 0$, then $\phi = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = 0$, $\vec{\mathbf{B}}$ may or may not be zero because angle between $\vec{\mathbf{B}}$ and $\vec{\mathbf{A}}$ may be 90°.

For same part ϕ may be positive and for remaining part, it may be negative so that the resultant ϕ becomes zero but $\vec{\mathbf{B}}$ is non-zero.

39. Option (b) is not possible because it is not obeying the fact that number of lines of force has to be proportional to magnitude of charge. Option (c) is not possible because it is violating the fact that electric lines of force can never intersect.

40. Three guns are fired towards the centre of circle as shown in figure.



Since, total final momentum is zero, and no external force is acting on the system, so total initial momentum should be also zero.

So, $$\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 + \vec{\mathbf{p}}_3 = 0$$

Three vectors, which are at an angle of 120° leads to zero resultant if and only if they have same magnitude.

So, $4.5 v_1 = 2.5 \times 575 = 4.5 v_2$

After solving, we will get v_1 and v_2 come out to be 320 m/s.

Chemistry

41. Key Idea

(i) Find the E°_{cell} by adding oxidation half-cell and reduction half-cell.

(ii) To calculate K_{eq} use the following formula.

$$E^\circ_{cell} = \frac{0.059}{n} \log K_{eq}$$

where, n = number of electrons taking part in reaction.

$Br_2 + 2e^- \longrightarrow 2Br^-$; $E^\circ = 1.08$ V

(reduction half-cell) ...(i)

$Sn^{2+} \longrightarrow Sn^{4+} + 2e^-$; $E^\circ = -0.13$ V

(oxidation half-cell) ...(ii)

On adding Eqs. (i) and (ii), the cell reaction will be

$Br_2 + Sn^{2+} \longrightarrow 2Br^- + Sn^{4+}$; $E^\circ = 0.95$ V

$$E^\circ_{cell} = \frac{0.059}{2} \log K_{eq}$$

$$0.95 = \frac{0.059}{2} \log K_{eq}$$

$$\frac{0.95 \times 2}{0.059} = \log K_{eq}$$

$$K_{eq} \approx 10^{32}$$

42. Key Idea *On adding, equilibrium constants are multiplied and on subtracting they are divided.*

Given, $NO_2 \rightleftharpoons \frac{1}{2} N_2 + O_2$; K_1 ...(i)

$N_2O_4 \rightleftharpoons 2NO_2$; K_2 ...(ii)

Required reaction

$N_2 + 2O_2 \rightleftharpoons N_2O_4$; K ...(iii)

By squaring Eq. (i) we get,

$2NO_2 \rightleftharpoons N_2 + 2O_2$; K_1^2 ...(iv)

Now, Eq. (iii) is obtained by first inverting Eq. (ii) and Eq. (iv) and then, by adding them as,

$$N_2 + 2O_2 \rightleftharpoons 2NO_2; \frac{1}{K_1^2}$$

$$2NO_2 \rightleftharpoons N_2O_4; \frac{1}{K_2}$$

$$N_2 + 2O_2 \rightleftharpoons N_2O_4; \frac{1}{K_1^2} \times \frac{1}{K_2}$$

43. $$N = N_0 \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

$$\frac{25}{100} = \left(\frac{1}{2}\right)^n$$

or $$\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^n$$

$\therefore$ $$n = 2$$

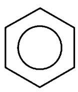
$\Rightarrow \quad n = \dfrac{\text{total time}}{\text{half-life}}$

$\Rightarrow$ Total time $= n \times$ Half-life

or $\quad = 2 \times 16 = 32\,\text{h}$

44. Key Idea *Reactive acyl group containing compounds (like acetyl chloride and acetic anhydride) Give Friedel-Craft's acylation reaction.*

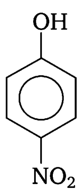
In this reaction benzene reacts with acetyl chloride or acetic anhydride in presence of anhy. $AlCl_3$.

 $+ CH_3COCl \xrightarrow{AlCl_3}$ product
or
$(CH_3CO)_2O$

Thus, X is $R—\overset{\overset{\large O}{||}}{C}—Cl$.

45. Key Idea *Electron withdrawing group (like* $—NO_2$, $—OH$*) increases the acidic strength while electron releasing group (like* $—CH_3$*) decreases the acidic strength.*

$—OH$ and $—NO_2$ both show $-I$ effect. When $—NO_2$ group is present at *para*-position to phenolic group, $-M$ effect takes place. That's why



has maximum acidic strength.

46. $Na_2CO_3 + 2HCl \longrightarrow 2NaCl + H_2O + CO_2$

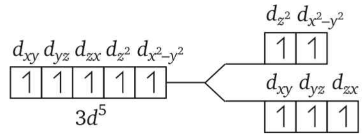
In the above reaction equivalent weight of Na_2CO_3 is $\dfrac{M}{2}$ because 2 moles of Na^+ being transferred per mole of Na_2CO_3.

47. Key Idea *In excited state, d-orbitals split into two sets of orbitals*

(i) One of higher energy, having two orbitals, called e_g *set.*

(ii) Other of lower energy, having three orbitals, called t_{2g} *set.*

Configuration of Mn^{2+} is [Ar] $3d^5$.



According to CFSE (crystal field stabilisation energy), in excited state of Mn^{2+} ion, 3 electrons go in t_{2g} level (d_{xy}, d_{yz} and d_{zx}) and 2 electrons go in e_g level (d_{z^2} and $d_{x^2-y^2}$).

48. Key Idea *A thermosetting polymer is one which becomes hard on heating and have three dimensional network structure. It cannot be softened by heating.*

$\because$ Bakelite is a thermosetting polymer because it hardens on heating and has three dimensional network. It is formed by reaction between phenol and formaldehyde.

n Phenol + n HCHO $\longrightarrow$ bakelite.

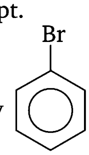
49. Key Idea

(i) More reactive halogen containing compounds give yellow or white ppt with $AgNO_3/HNO_3$.

(ii) Alkyl halide are more reactive than aryl halide towards substitution.

$$X \xrightarrow[HNO_3]{AgNO_3} \text{yellow or white ppt.}$$

The above reaction is not given by



because in bromobenzene, halogen is directly attached with the benzene ring.

50. Given, $\quad R_1 = k[A]^2[B]$

According to question $R_2 = k[3A]^2[2B]$

$= k \times 9[A]^2\, 2[B]$

$= 18 \times k[A]^2[B]$

$= 18\, R_1$

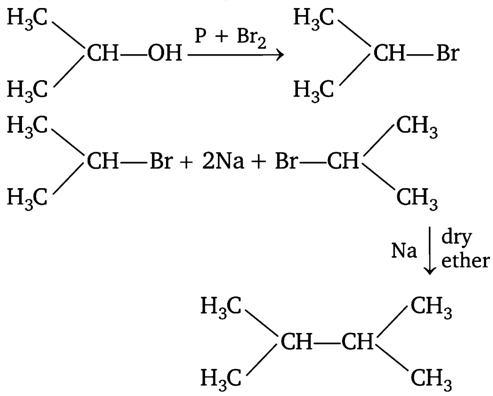
51. High b.p. of water is due to dipole-dipole interaction.

52. Shining at freshly cut sodium is due to oscillation of free electrons.

53. Key Idea *As the oxidation number of central atom increases, acidic nature of oxide increases.*

In case of Cl_2O_7, Cl has +7 oxidation state (maximum oxidation state) and also have highest oxygen content. So, it is most acidic oxide.

54. Key Idea *In presence of P* + Br_2, —OH *group of alcohol is replaced by* —Br *group.*
Alkyl halide with Na *gives Wurtz reaction.*



2, 3-dimethyl butane
(*X*)

55. Key Idea *All s-, d- and f-block elements are metals. p-block contains metals, metalloids and non-metals all.*

∴ Non-metals are less than metals.

56. Key Idea *Buffer capacity of a buffer solution is maximum when the concentration of either weak acid and its salt or weak base and its salt are equal.*

∴ For highest buffer capacity $pH = pK_a$

For this [Salt] = [Acid]

Thus, 25 mL.

57. Monomer of nucleic acid (DNA or RNA) is nucleotide.

58. The gas is not escaped or injected, so number of moles remains the same. When volume of gas is compressed to half, no change will occur in the vessel.

59. Key Idea *At constant temperature,*

$$U_{av} \propto \frac{1}{\sqrt{M}}$$

$$\frac{U_{av(SO_2)}}{U_{av(CH_4)}} = \sqrt{\frac{M_{CH_4}}{M_{SO_2}}}$$

$$M_{SO_2} = 32 + 2 \times 16 = 64$$

$$M_{CH_4} = 12 + 4 \times 1 = 16$$

$$\frac{U_{av(SO_2)}}{U_{av(CH_4)}} = \sqrt{\frac{16}{64}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$U_{SO_2} : U_{CH_4} = 1 : 2$$

60. Key Idea *Use the following formula*

$$KE = \frac{3}{2} nRT$$

where, n = number of moles.

$\therefore$ KE $= \frac{3}{2} RT$ for 1 mole of a gas

$$\Delta KE = \frac{3}{2} \times 8.315 \times (50 - 0)$$

$$= \frac{3}{2} \times 8.315 \times 50 = 623.25 \text{ J}$$

61. Key Idea *First write the name of the ligands in alphabetical order and then write the name of the metal with its oxidation state in bracket.*

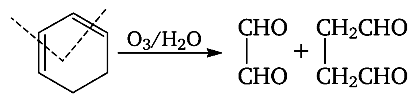
$\because$ Oxidation state of metal in $[IrClCO(PH_3)_2]$ is

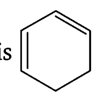
$$x + 0 + (-1) + 0 = 0$$

$$x = +1$$

∴ The name of the complex is carbonylchlorobis*trans*phosphineiridium(I).

62.



So, the olefin is .

63.

$$pV = nRT \quad \text{(Ideal gas equation)}$$

or $$V = \frac{nRT}{p}$$

or $$\frac{V_1}{V_2} = \frac{T_1}{T_2} \times \frac{p_2}{p_1}$$

$$\frac{V_1}{V_2} = \frac{273 + 15}{273 + 25} \times \frac{1}{1.5}$$

$$\frac{V_1}{V_2} = \frac{288}{298} \times \frac{1}{1.5}$$

or $$\frac{V_1}{V_2} = \frac{1}{1.55}$$

∴ $$\frac{V_2}{V_1} = 1.55$$

64. Key Idea *For neutralisation,*
Milliequivalents of acid ≡ Milliequivalents of base

According to molarity equation

$$NaOH \equiv HCl$$

∴ $$M_1V_1 = M_2V_2$$

$$0.6 \times V_1 = 0.4 \times 30$$

$$V_1 = \frac{0.4 \times 30}{0.6} = 20 \text{ cm}^3$$

65. Key Idea *The orbital angular momentum*

$$= \frac{h}{2\pi}\sqrt{l(l+1)}$$

For $3s$ electron, $l = 0$

$$\therefore \text{Orbital angular momentum} = \frac{h}{2\pi}\sqrt{0\,(0+1)}$$

$$= 0 \text{ (zero)}$$

66. Pure hydrogen peroxide is an unstable liquid and decomposes into water and oxygen either upon standing or on heating.

$$2H_2O_2 \longrightarrow 2H_2O + O_2\,; \qquad \Delta H = -196.0 \text{ kJ}$$

To prevent decomposition of H_2O_2, phosphoric acid, acetanilide or glycerol are added. These acts as negative catalyst.

67. Action of alcoholic caustic potash on chloroform and aniline forms a bad smelling compound phenyl isocyanide.

$$\underset{\text{aniline}}{C_6H_5NH_2} + \underset{\text{chloroform}}{CHCl_3} + 3KOH(alc.) \longrightarrow \underset{\text{phenyl isocyanide}}{C_6H_5NC} + 3KCl + 3H_2O$$

This reaction is called carbylamine reaction and it is actually the test of primary amines.

68. Effective atomic number (EAN)

$$= \text{Atomic no.} - \text{O}\cdot\text{S} + 2 \times \text{C}\cdot\text{N}$$

$$= 28 - 0 + 2 \times 4$$

$$= 28 + 8 = 36$$

69. This is the laboratory method of preparing phosphine gas.

$$P_4 + 3NaOH + 3H_2O \longrightarrow \underset{\text{phosphine}}{PH_3} + 3NaH_2PO_2$$

70. $Ca(OH)_2 \rightleftharpoons \underset{s}{Ca^{2+}} + \underset{2s}{2OH^-}$

$$K_{sp} = [Ca^{2+}][OH^-]^2$$

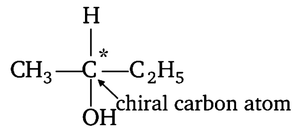
$$K_{sp} = (s)(2s)^2 \qquad \text{(where } s = \text{solubility)}$$

$$K_{sp} = 4s^3$$

$$K_{sp} = 4\cdot(\sqrt{3})^3 = 12\sqrt{3}$$

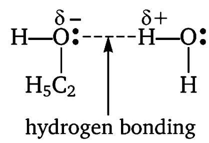
71. Key Idea *Compound containing chiral carbon atom is optically active. The carbon whose four valencies are satified by four different groups is chiral.*

$\therefore$ $CH_3CHOHC_2H_5$ is optically active because it has chiral C^*-atom.



72. Key Idea *The compound which is able to form H-bond, is soluble in water.*

Ethyl alcohol (C_2H_5OH) is soluble in water due to intermolecular H-bonding between water and ethyl alcohol.



73. Key Idea *In a cubic structure one corner atom is shared by eight unit cells and one face centre atom is shared by two unit cells.*

Thus, number of X atoms at corner per unit cell

$$= 8 \times \frac{1}{8} = 1$$

Number of Y atoms at face centres per unit cel

$$= 6 \times \frac{1}{2} = 3$$

$\therefore$ The formula of the compound is XY_3.

74. (a) **Disinfectant** The substance which kill micro-organisms but are not safe for living tissue.

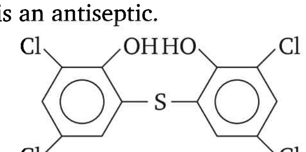
(b) **Antiseptic** The substance which kill microorganisms and also are safe for living tissue.

(c) **Antibiotic** The substance which are produced by micro-organisms and inhibits the growth of other micro-organisms.

(d) **Analgesic** The substance used to relieve pains.

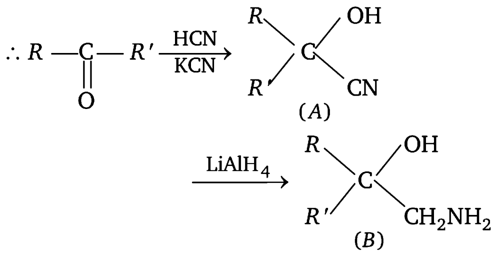
$\because$ Bithional is generally added to bathing soaps to reduce the odour produced by bacterial decomposition of organic matter on the skin.

$\therefore$ It is an antiseptic.



bithional

75. Key Idea *Ketones gives nucleophilic addition with* HCN *and cyanides on reduction form primary amines.*



Hence, A and B are $RR'C(OH)CN$ and $LiAlH_4$.

76. Key Idea

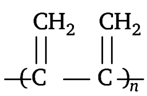
(i) Xe *is a noble metal, have complete octet, ie,* 8 *electrons in outer shell.*

(ii) +ve *sign indicates the loss of electrons from central atom.*

(iii) Find the number of lp + bp and then find the geometry.

In XeF_5^+, Xe atom has only seven electrons, *ie,* $5s^2 5p^5$. Here two $5p$ electrons are promoted to $5d$ sub-level. Thus, $5s$, three $5p$ and two $5d$ orbitals hybridize to give six sp^3d^2 hybrid orbitals in an octahedral geometry. Out of these, five orbitals are singly occupied which form sigma bonds with five F atoms. The sixth hybrid orbital is occupied by a lone pair in *trans* position giving a square pyramid structure.

77. Key Idea *Vinylic* H-*atom are more stable due to resonance.*

 , it suggests polymerisation on the lost of vinylic hydrogen atom, which is not possible.

78. Key Idea *Due to* $R_3\overset{\oplus}{N}$—(e^-*withdrawing tendency), carbocation will appear nearer to that terminal.*

$$R_3\overset{\oplus}{N}\text{—}\leftarrow CH{=}CH_2 \xrightarrow{H^+}$$

$$R_3\overset{\oplus}{N}\leftarrow\overset{\oplus}{C}H_2\text{—}CH_3 \xrightarrow{Br^-} R_3\overset{\oplus}{N}\text{—}\underset{}{\overset{Br}{\overset{|}{C}}}H_2\text{—}CH_3$$

Hence, product is $R_3\overset{\oplus}{N}$—$CH_2(Br)$—CH_3.

79. $C(s) + \frac{1}{2}O_2(g) \longrightarrow CO(g)$

This equation can be obtained by subtraction of $[CO(g) + \frac{1}{2}O_2(g) \longrightarrow CO_2(g)]$

from $[C(s) + O_2(g) \longrightarrow CO_2(g)]$.

Hence, $\Delta H_f(CO) = [-393.3 - (-282.8)]$ kJ

$= -110.5$ kJ/mol

80. Key Idea *Use the following formula,*

Magnetic moment $= \sqrt{n(n+2)}$

where n = number of unpaired electrons.

$$\therefore \quad 3.87 = \sqrt{n(n+2)}$$

$$(3.87)^2 = n(n+2)$$

$$15 = n^2 + 2n$$

$$n^2 + 2n - 15 = 0$$

$$\therefore \quad n = 3$$

Mathematics

81. Given, $f(x) = (x-1)(x-2)(x-3)$

$\Rightarrow \quad f(1) = f(2) = f(3) = 0$

$\Rightarrow f(x)$ is not one-one

For each $y \in R$, there exists $x \in R$ such that $f(x) = y$.

Therefore, f is onto.

NOTE If a continuous function has more than one roots, then a function is always a many-one.

82. If R is an equivalence relation, then R^{-1} is also an equivalence relation.

83. Let z_1, z_2, z_3 be affixes of points A, B, C respectively. Since, z_1, z_2, z_3 are in AP, therefore

$$2z_2 = z_1 + z_3$$

$$\Rightarrow \quad z_2 = \frac{z_1 + z_3}{2}$$

$\Rightarrow$ B is the mid point of the line AC

$\Rightarrow$ A, B, C are collinear

$\Rightarrow$ z_1, z_2, z_3 lie on a line.

84. Given, $x = 1 + a + a^2 + \dots \infty = \frac{1}{1-a}$,

$$y = 1 + b + b^2 + \dots \infty = \frac{1}{1-b}$$

and $\quad z = 1 + c + c^2 + \dots \infty = \frac{1}{1-c}$

Since, a, b, c are in AP

$\Rightarrow 1-a, 1-b, 1-c$, are in AP

$\Rightarrow \frac{1}{1-a}, \frac{1}{1-b}, \frac{1}{1-c}$ are in HP

$\Rightarrow x, y, z$ are in HP.

NOTE If the common ratio of a GP is not less than 1, then we do not determined the sum of an infinite GP series.

85. Given, $\log_e\left(\frac{a+b}{2}\right)=\frac{1}{2}(\log_e a+\log_e b)$

$$\Rightarrow \frac{a+b}{2}=\sqrt{ab}$$

$$\Rightarrow a+b-2\sqrt{ab}=0$$

$$\Rightarrow \sqrt{a}=\sqrt{b} \Rightarrow a=b$$

86. Let $f(x)=-3+x-x^2$

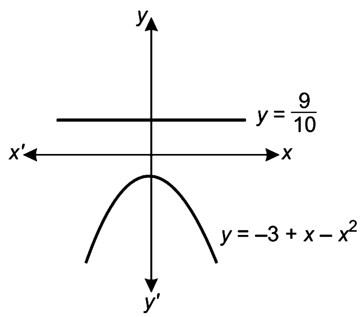
Then, $f(x)<0$ for all x, because coefficient of $x^2<0$ and disc <0. Thus, LHS of the given equation is always positive whereas the RHS is always less than zero.

Hence, the given equation has no solution.

Alternate

Given equation is

$$\frac{9}{10}=-3+x-x^2$$



Let $y=\frac{9}{10}$, therefore,

$$y=-3+x-x^2$$

$$y=-\left[x^2-x+\frac{1}{4}\right]-3+\frac{1}{4}$$

$$\Rightarrow y+\frac{11}{4}=-\left(x-\frac{1}{2}\right)^2$$

It is clear from the graph that two curves do not intersect. Hence, no solution exist.

87. Given, $f(x)=ax+b, g(x)=cx+d$

and $f\{g(x)\}=g\{f(x)\}$

$$\Rightarrow f(cx+d)=g(ax+b)$$

$$\Rightarrow a(cx+d)+b=c(ax+b)+d$$

$$\Rightarrow acx+ad+b=cax+bc+d$$

$$\Rightarrow ad+b=bc+d$$

$$\Rightarrow f(d)=g(b)$$

88. We know that, $1+i=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

and $1-i=\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)$

$$\therefore (1+i)^8+(1-i)^8=2^4\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)^8 + 2^4\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)^8$$

$$=2^4(\cos 2\pi+i\sin 2\pi)+2^4(\cos 2\pi-i\sin 2\pi)$$

$$=2^4(2\cos 2\pi)=2^5$$

89. Given, $\sqrt{3}$ cosec 20°– sec 20°

$$=\tan 60°\ \text{cosec}\ 20°-\sec 20°$$

$$=\frac{\sin 60°}{\cos 60°}\cdot\frac{1}{\sin 20°}-\frac{1}{\cos 20°}$$

$$=\frac{\sin 60°\cos 20°-\cos 60°\sin 20°}{\cos 60°\sin 20°\cos 20°}$$

$$=\frac{\sin 40°}{\cos 60°\sin 20°\cos 20°}$$

$$=\frac{2\sin 20°\cos 20°}{\frac{1}{2}\sin 20°\cos 20°}=4$$

90. Since, x, y, z are in HP.

Therefore, $y=\frac{2xz}{x+z}$

Now, $x-2y+z=x+z-2\left(\frac{2xz}{x+z}\right)$

$$=x+z-\frac{4xz}{x+z}$$

$$=\frac{(x-z)^2}{x+z}$$

$$\Rightarrow \log(x-2y+z)=\log(x-z)^2-\log(x+z)$$

$$\Rightarrow \log(x-2y+z)+\log(x+z)=2\log(x-z)$$

NOTE We do not determine the sum of HP series.

91. **Key Idea** *The intersection of two diameter lines of a circle is the centre of the circle.*

The centre of the required circle lies at the intersection of $2x-3y-5=0$ and $3x-4y-7=0$. Thus, the coordinates of the centre are $(1,-1)$.

Let r be the radius of the circle.

Then, $\pi r^2 = 154$

$\Rightarrow \frac{22}{7} r^2 = 154$

$\Rightarrow r = 7$

Hence, the equation of required circle is

$$(x-1)^2 + (y+1)^2 = 7^2$$

$\Rightarrow x^2 + y^2 - 2x + 2y - 47 = 0$

92. Key Idea *The equation of common chord of two circles S_1 and S_2 is $S_1 - S_2 = 0$.*

Let the equation of circles be

$$S_1 \equiv x^2 + y^2 + 2x - 3y + 6 = 0 \quad \text{...(i)}$$

and $S_2 \equiv x^2 + y^2 + x - 8y - 13 = 0 \quad \text{...(ii)}$

$\therefore$ Equation of common chord is

$$S_1 - S_2 = 0$$

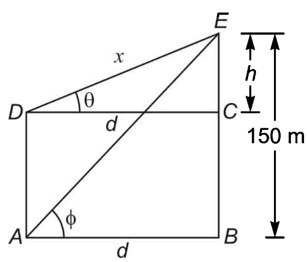
$\Rightarrow (x^2 + y^2 + 2x - 3y + 6) - (x^2 + y^2 + x - 8y - 13) = 0$

$\Rightarrow x + 5y + 19 = 0$

In the given options only the point (1, – 4) satisfies this equation.

93. Given that, $\tan\theta = \frac{4}{3}$ and $\tan\phi = \frac{5}{2}$

In Δ *ABE*,



$$\tan\phi = \frac{150}{d}$$

$\Rightarrow d = 150 \cot\phi$

$= 150 \times \frac{2}{5}$

$= 60$ m

In Δ *DCE*, $\tan\theta = \frac{h}{d}$

$\Rightarrow \frac{4}{3} = \frac{h}{d}$

$\Rightarrow h = \frac{4}{3} \times 60$

$\Rightarrow h = 80$ m

Now, In Δ *DCE*, $DE^2 = DC^2 + CE^2$

$\Rightarrow x^2 = 60^2 + 80^2 = 10000$

$\Rightarrow x = 100$ m

94. Given equation $x^2 + px + q = 0$ has roots α and α^2.

$\Rightarrow \alpha + \alpha^2 = -p$ and $\alpha^3 = q$

$\Rightarrow \alpha(\alpha + 1) = -p$

$\Rightarrow \alpha^3[\alpha^3 + 1 + 3\alpha(\alpha + 1)] = -p^3$

$\Rightarrow q(q + 1 - 3p) = -p^3$

$\Rightarrow p^3 - (3p - 1)q + q^2 = 0$

95. The given sigma expansion

$\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} \cdot 2^m$ can be rewritten as

$[(x-3)+2]^{100} = (x-1)^{100} = (1-x)^{100}$

$\therefore x^{53}$ will occur in T_{54}.

$\Rightarrow T_{54} = {}^{100}C_{53} (-x)^{53}$

$\therefore$ Required coefficient is $-{}^{100}C_{53}$.

96. Key Idea *Two circles are said to be concentric, if they same centre.*

Equation of family of concentric circles to the circle $x^2 + y^2 + 6x + 8y - 5 = 0$ is

$$x^2 + y^2 + 6x + 8y + \lambda = 0$$

which is similar to

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Thus, the point (– 3, 2) lies on the circle

$$x^2 + y^2 + 6x + 8y + c = 0$$

$\therefore (-3)^2 + (2)^2 + 6(-3) + 8(2) + c = 0$

$\Rightarrow 9 + 4 - 18 + 16 + c = 0$

$\Rightarrow c = -11$

97. Key Idea *If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are diagonals of a parallelogram, then area of parallelogram is $\frac{1}{2}|\vec{\mathbf{a}} \times \vec{\mathbf{b}}|$.*

Given, $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\vec{\mathbf{b}} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

and $\vec{\mathbf{c}} = 7\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$

Let $\vec{\mathbf{A}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$

$= (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + (\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$

$= 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$

and $\vec{\mathbf{B}} = \vec{\mathbf{b}} + \vec{\mathbf{c}}$

$= (\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) + (7\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 11\hat{\mathbf{k}})$

$= 8\hat{\mathbf{i}} + 12\hat{\mathbf{j}} + 16\hat{\mathbf{k}}$

Since $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are diagonals.

$\therefore$ Area of parallelogram

$$= \frac{1}{2}|\vec{\mathbf{A}} \times \vec{\mathbf{B}}|$$

$$= \frac{1}{2}\left\|\begin{matrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 4 & 6 \\ 8 & 12 & 16 \end{matrix}\right\|$$

$$= \frac{1}{2}|\hat{\mathbf{i}}(64 - 72) - \hat{\mathbf{j}}(32 - 48) + \hat{\mathbf{k}}(24 - 32)|$$

$$= \frac{1}{2}|- 8\hat{\mathbf{i}} + 16\hat{\mathbf{j}} - 8\hat{\mathbf{k}}|$$

$$= \sqrt{(-4)^2 + (8)^2 + (-4)^2}$$

$$= \sqrt{96}$$

$$= 4\sqrt{6} \text{ sq unit}$$

98. The equation of a line through the centre $\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and normal to the given plane is

$$\vec{\mathbf{r}} = \hat{\mathbf{j}} + 2\hat{\mathbf{k}} + \lambda(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \quad \text{...(i)}$$

This meets the plane at a point for which we must have

$$[(\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) + \lambda(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})] \cdot (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 15$$

$$\Rightarrow \quad [\lambda\hat{\mathbf{i}} + (2\lambda + 1)\hat{\mathbf{j}} + (2 + 2\lambda)\hat{\mathbf{k}}] \cdot (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 15$$

$$\Rightarrow \quad 6 + 9\lambda = 15$$

$$\Rightarrow \quad \lambda = 1$$

On putting this value in Eq. (i), we get

$$\vec{\mathbf{r}} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

$\therefore$ Centre of the circle is (1,3,4).

99. Key Idea *The tracing of a matrix is the sum of the diagonal elements.*

We know that $\text{tr}(A) = \sum_{i=1}^{n} a_{ii}$

$$\therefore \text{ If } \quad A = \begin{bmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{bmatrix}, \text{ then}$$

$$\text{tr}(A) = 1 + 7 + 9 = 17$$

100. Given, $\begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ \cos(\alpha + \beta) & -\sin(\alpha + \beta) & 1 \end{vmatrix}$

Applying $R_3 \to R_3 - R_1(\cos\beta) + R_2(\sin\beta)$

$$= \begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ 0 & 0 & 1 + \sin\beta - \cos\beta \end{vmatrix}$$

$$= (1 + \sin\beta - \cos\beta)(\cos^2\alpha + \sin^2\alpha)$$

$= 1 + \sin\beta - \cos\beta$, which is independent of α

101. The total number of ways in which 5 persons can be chosen out of 9 persons is ${}^9C_5 = 126$

The couple serves the committee in ${}^7C_3 \times {}^2C_2 = 35$ ways

The couple does not serve the committee in ${}^7C_5 = 21$ ways.

Since, the couple will serve either together or not at all.

So, favourable number of cases = 35 + 21 = 56

Thus, the required probability $= \frac{56}{126} = \frac{4}{9}$

102. Given, $4\sin^2 x - 12\sin x + 7$

$$= 4(\sin^2 x - 3\sin x) + 7$$

$$= 4\left[\left(\sin x - \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 7$$

$$= 4\left(\sin x - \frac{3}{2}\right)^2 - 9 + 7$$

$$= 4\left(\sin x - \frac{3}{2}\right)^2 - 2$$

Since, $-1 \le \sin x \le 1$

$$\Rightarrow \quad -\frac{5}{2} \le \sin x - \frac{3}{2} \le -\frac{1}{2}$$

$$\Rightarrow \quad \frac{1}{4} \le \left(\sin x - \frac{3}{2}\right)^2 \le \frac{25}{4}$$

$$\Rightarrow \quad 1 \le 4\left(\sin x - \frac{3}{2}\right)^2 \le 25$$

$$\Rightarrow \quad -1 \le 4\left(\sin x - \frac{3}{2}\right)^2 - 2 \le 23$$

103. Let $\quad x = 4 + r\cos\theta$

and $\quad y = 3 + r\sin\theta$

Given, distance $r = \sqrt{2}$

and $\quad \theta = 45°$

$\therefore \quad x = 4 + \sqrt{2}\cos 45° = 4 + \sqrt{2}\cdot\frac{1}{\sqrt{2}} = 5$

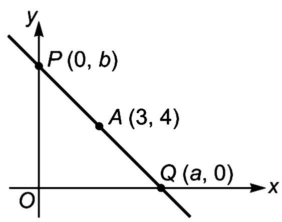
and $\quad y = 3 + \sqrt{2}\sin 45° = 3 + \sqrt{2}\cdot\frac{1}{\sqrt{2}} = 4$

104. Since, A is mid point of line PQ.

$\therefore \quad 3 = \frac{a+0}{2}$

$\Rightarrow \quad a = 6$

and $\quad 4 = \frac{0+b}{2}$



$\Rightarrow \quad b = 8$

Thus, equation of line is

$$\frac{x}{6} + \frac{y}{8} = 1$$

$\Rightarrow \quad 4x + 3y = 24$

105. Key Idea *Diagonals of a square are perpendicular to each other.*

Equation of perpendicular line to $7x - y + 8 = 0$ is $x + 7y = \lambda$, which passes through $(-4, 5)$.

$\therefore \quad -4 + 7 \times 5 = \lambda$

$\Rightarrow \quad \lambda = 31$

So, equation of another diagonal is $x + 7y = 31$

106. Given equation can be rewritten as

$$2x^2 - 22xy - 2xy + 22y^2 = 0$$

$\Rightarrow \quad 2x(x - 11y) - 2y(x - 11y) = 0$

$\Rightarrow \quad (2x - 2y)(x - 11y) = 0$

$\Rightarrow \quad 2x - 2y = 0$ or $x - 11y = 0$

$\Rightarrow \quad y = x$ or $y = \frac{x}{11}$

Since, both the lines passing through the origin.

Alternate

Given equation is

$$2x^2 - 24xy + 22y^2 = 0$$

Here, $a = 2, \quad h = -12, \quad b = 22$

Now, $\quad \tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$

$= \frac{2\sqrt{144 - 44}}{2 + 22}$

$= \frac{2 \times 10}{24} = 0.83$

It means that the two lines are neither parallel nor perpendicular.

Also, it is passing through origin.

Hence, option (c) is correct.

107. The tangent at (1, 7) to the curve $x^2 = y - 6$ is

$$x = \frac{1}{2}(y + 7) - 6$$

$\Rightarrow \quad 2x = y + 7 - 12$

$\Rightarrow \quad y = 2x + 5$

Which is also tangent to the circle

$$x^2 + y^2 + 16x + 12y + c = 0$$

ie, $x^2 + (2x + 5)^2 + 16x + 12(2x + 5) + c = 0$

$\Rightarrow 5x^2 + 60x + 85 + c = 0$, which must have equal roots. Let α and β are the roots of the equation.

Then $\alpha + \beta = -12 \Rightarrow \alpha = -6 \quad [\because \alpha = \beta]$

$\therefore \quad x = -6, y = 2x + 5 = -7$

Hence, point of contact is $(-6, -7)$.

108. Key Idea *The intersection of two lines L_1 and L_2 is $L_1 + \lambda L_2 = 0$.*

The intersection point of lines $x - 2y = 1$ and $x + 3y = 2$ is $\left(\frac{7}{5}, \frac{1}{5}\right)$.

Since, required line is parallel to $3x + 4y = 0$.

Therefore, the slope of required line is $-\frac{3}{4}$.

$\therefore$ Equation of required line which passes through $\left(\frac{7}{5}, \frac{1}{5}\right)$ and slope $-\frac{3}{4}$, is

$$y - \frac{1}{5} = -\frac{3}{4}\left(x - \frac{7}{5}\right)$$

$\Rightarrow \quad \frac{3x}{4} + y = \frac{21}{20} + \frac{1}{5}$

$\Rightarrow \quad \frac{3x + 4y}{4} = \frac{21 + 4}{20}$

$\Rightarrow \quad 3x + 4y - 5 = 0$

109. Let $I = \int \frac{dx}{\sin x - \cos x + \sqrt{2}}$

$$= \int \frac{dx}{\sin x \frac{\sqrt{2}}{\sqrt{2}} - \cos x \frac{\sqrt{2}}{\sqrt{2}} + \sqrt{2}}$$

$$= \int \frac{dx}{\sqrt{2}\left(\sin x \sin \frac{\pi}{4} - \cos x \cos \frac{\pi}{4} + 1\right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{1 - \cos\left(x + \frac{\pi}{4}\right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{2\sin^2\left(\frac{x}{2} + \frac{\pi}{8}\right)}$$

$$= \frac{1}{2\sqrt{2}} \int \operatorname{cosec}^2\left(\frac{x}{2} + \frac{\pi}{8}\right) dx$$

$$= \frac{-1}{2\sqrt{2}} \cdot \frac{-\cot\left(\frac{x}{2} + \frac{\pi}{8}\right)}{\frac{1}{2}} + c$$

$$= \frac{1}{\sqrt{2}} \cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$$

110. Let $I = \int \frac{2x^2 + 3}{(x^2 - 1)(x^2 + 4)} dx$

Now, $\frac{2x^2 + 3}{(x^2 - 1)(x^2 + 4)} = \frac{1}{x^2 - 1} + \frac{1}{x^2 + 4}$

$\therefore \quad I = \int \frac{dx}{x^2 - 1} + \int \frac{dx}{x^2 + 4}$

$\Rightarrow \quad I = \frac{1}{2} \log\left(\frac{x - 1}{x + 1}\right) + \frac{1}{2} \tan^{-1} \frac{x}{2} + c$

But $\quad I = a \log\left(\frac{x - 1}{x + 1}\right) + b \tan^{-1}\left(\frac{x}{2}\right) + c$

$\therefore \quad a = \frac{1}{2}, b = \frac{1}{2}$

111. Let $I = \int \operatorname{cosec}^4 x \, dx$

$$= \int \operatorname{cosec}^2 x \cdot \operatorname{cosec}^2 x \, dx$$

$$= \int \operatorname{cosec}^2 x (1 + \cot^2 x) \, dx$$

$$= \int \operatorname{cosec}^2 x \, dx + \int \cot^2 x \operatorname{cosec}^2 x \, dx$$

$$= -\cot x - \frac{\cot^3 x}{3} + c$$

112. Let $I = \int_0^1 \sqrt{\frac{1 - x}{1 + x}} \, dx$

$$= \int_0^1 \frac{1 - x}{\sqrt{1 - x^2}} dx$$

$$= \int_0^1 \frac{1}{\sqrt{1 - x^2}} dx - \int_0^1 \frac{x}{\sqrt{1 - x^2}} dx$$

$$= [\sin^{-1} x]_0^1 - \int_0^1 \frac{x}{\sqrt{1 - x^2}} dx$$

Put $t^2 = 1 - x^2 \quad \Rightarrow \quad 2t \, dt = -2x \, dx$

$\Rightarrow \quad t \, dt = -x \, dx$

$\therefore \quad I = [\sin^{-1} 1 - \sin^{-1} 0] + \int_1^0 \frac{t}{t} dt$

$$= \frac{\pi}{2} + [t]_1^0$$

$$= \frac{\pi}{2} - 1$$

Alternate

Let $I = \int_0^1 \sqrt{\frac{1 - x}{1 + x}} \, dx$

Put $x = \cos 2\theta \quad \Rightarrow \quad dx = -2 \sin 2\theta \, d\theta$

$\therefore \quad I = -\int_{\pi/4}^0 \sqrt{\frac{1 - \cos 2\theta}{1 + \cos 2\theta}} \cdot 2 \sin 2\theta \, d\theta$

$$= -2\int_{\pi/4}^0 \frac{\sin \theta}{\cos \theta} \cdot 2 \sin \theta \cos \theta \, d\theta$$

$$= -2\int_{\pi/4}^0 (1 - \cos 2\theta) \, d\theta$$

$$= -2\left[\theta - \frac{\sin 2\theta}{2}\right]_{\pi/4}^0$$

$$= -2\left[0 - \left(\frac{\pi}{4} - \frac{1}{2}\right)\right]$$

$$= \frac{\pi}{2} - 1$$

113. **Key Idea** *If $a < c < b$, then*

$$\int_a^b |x - c| \, dx = -\int_a^c (x - c) \, dx + \int_c^b (x - c) \, dx$$

Let $I = \int_0^1 x \left| x - \frac{1}{2} \right| dx$

$$= -\int_0^{1/2} x\left(x - \frac{1}{2}\right) dx + \int_{1/2}^1 x\left(x - \frac{1}{2}\right) dx$$

$$= \int_0^{1/2} \left(\frac{x}{2} - x^2\right) dx + \int_{1/2}^1 \left(x^2 - \frac{x}{2}\right) dx$$

$$= \left[\frac{x^2}{4} - \frac{x^3}{3}\right]_0^{1/2} + \left[\frac{x^3}{3} - \frac{x^2}{4}\right]_{1/2}^1$$

$$= \left(\frac{1}{16} - \frac{1}{24}\right) + \left(\frac{1}{3} - \frac{1}{4} - \frac{1}{24} + \frac{1}{16}\right)$$

$$= \left(\frac{6-4}{96}\right) + \left(\frac{32 - 24 - 4 + 6}{96}\right)$$

$$= \frac{12}{96} = \frac{1}{8}$$

114. The equation of a tangent of slope m to the parabola $y^2 = 9x$ is

$$y = mx + \frac{9}{4m}$$

If it passes through (4, 10), then

$$10 = 4m + \frac{9}{4m}$$

$\Rightarrow \quad 16m^2 - 40m + 9 = 0$

$\Rightarrow \quad (4m - 1)(4m - 9) = 0$

$\Rightarrow \quad m = \frac{1}{4}, \frac{9}{4}$

115. Given, ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Put $x = at^2$ in this equation, we get

$$\frac{a^2t^4}{a^2} + \frac{y^2}{b^2} = 1$$

$\Rightarrow \quad y^2 = b^2(1 - t^4)$

$\Rightarrow \quad y^2 = b^2(1 - t^2)(1 + t^2)$

This will give real values of y, if

$$1 - t^2 \geq 0$$

$\Rightarrow \quad |t| \leq 1$

116. Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

It is given that it passes through (7, 0) and (0, – 5).

Therefore, $a^2 = 49$ and $b^2 = 25$.

The eccentricity of the ellipse is

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \sqrt{1 - \frac{25}{49}} = \sqrt{\frac{24}{49}}$$

$$= \frac{2\sqrt{6}}{7}$$

117. Given, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \ldots(i)$

On differentiating w.r.t. x, we get

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{xb^2}{a^2y}$ and $x^2 - y^2 = c^2$

On differentiating w.r.t. x, we get

$$2x - 2y\frac{dy}{dx} = 0$$

$\Rightarrow \quad \frac{dy}{dx} = \frac{x}{y}$

The two curves will cut at right angles, if

$$\left(\frac{dy}{dx}\right)_{C_1} \times \left(\frac{dy}{dx}\right)_{C_2} = -1$$

$\Rightarrow \quad -\frac{b^2x}{a^2y} \cdot \frac{x}{y} = -1$

$\Rightarrow \quad \frac{x^2}{a^2} = \frac{y^2}{b^2}$

$\Rightarrow \quad \frac{x^2}{a^2} = \frac{y^2}{b^2} = \frac{1}{2} \quad$ [using Eq. (i)]

On substituting these values in $x^2 - y^2 = c^2$, we get

$$\frac{a^2}{2} - \frac{b^2}{2} = c^2$$

$\Rightarrow \quad a^2 - b^2 = 2c^2$

118. **Key Idea** *If P is any point on the curve, S be a focus and M be a point on directrix and e be the eccentricity, then PS = ePM.*

Let $P(x, y)$ be any point on the conic. Then

$$\sqrt{(x-1)^2 + (y+1)^2} = \sqrt{2}\left(\frac{x - y + 1}{\sqrt{2}}\right)$$

$\Rightarrow \quad (x-1)^2 + (y+1)^2 = (x - y + 1)^2$

$\Rightarrow \quad 2xy - 4x + 4y + 1 = 0$

119. Required sum

$$= (\text{sum of the digits}) \times (n-1)!\left(\frac{10^n - 1}{10 - 1}\right)$$

$$= (1 + 2 + 3 + 4 + 5)(5 - 1)!\left(\frac{10^5 - 1}{10 - 1}\right)$$

$$= 360\left(\frac{100000 - 1}{9}\right)$$

$$= 40 \times 99999 = 3999960$$

120. Required numbers

$$= 5!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right]$$

$$= 44$$

Note If $r\ (0 \le r \le n)$ objects occupy the original places and none of the remaining $(n - r)$ objects occupies its original places, then the number of such arrangements

$$= {}^nC_r \cdot (n - r)!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^{n-r}\frac{1}{(n-r)!}\right)$$

121. We have, $T_n = \dfrac{1^2 + 2^2 + 3^2 + \dots + n^2}{n!}$

$$= \frac{\Sigma n^2}{n!}$$

$$= \frac{n(n+1)(2n+1)}{6\,n!}$$

$$= \frac{1}{6}\left(\frac{2n^3 + 3n^2 + n}{n!}\right)$$

$$= \frac{1}{6}\left(2\cdot\frac{n^3}{n!} + \frac{3n^2}{n!} + \frac{n}{n!}\right)$$

$\therefore$ Sum of the series

$$= \frac{1}{6}\left(2\sum_{n=1}^{\infty}\frac{n^3}{n!} + 3\sum_{n=1}^{\infty}\frac{n^2}{n!} + \sum_{n=1}^{\infty}\frac{n}{n!}\right)$$

$$= \frac{1}{6}(2 \times 5e + 3 \times 2e + e)$$

$$= \frac{1}{6}(10e + 6e + e) = \frac{17}{6}e$$

122. We have,

$$\log_a(1 + x) = \log_e(1 + x)\log_a e$$

$$= \log_a e\left[\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}\right]$$

So, the coefficient of x^n in $\log_a(1 + x)$ is $\dfrac{(-1)^{n-1}}{n}\log_a e$.

123. Mean of $1^2, 2^2, 3^2, \dots, n^2$ is

$$\frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n} = \frac{\Sigma n^2}{n}$$

$$\therefore \quad \frac{46n}{11} = \frac{n(n+1)(2n+1)}{6n}$$

$$\Rightarrow \quad 22n^2 + 33n + 11 - 276n = 0$$

$$\Rightarrow \quad 22n^2 - 243n + 11 = 0$$

$$\Rightarrow \quad (n - 11)(22n - 1) = 0$$

$$\Rightarrow \quad n = 11 \quad \text{and} \quad n \ne \frac{1}{22}$$

124. Let the equation of the required plane be

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

This meets the coordinate axes at A, B and C, the coordinates of the centroid of ΔABC are $\left(\dfrac{a}{3}, \dfrac{b}{3}, \dfrac{c}{3}\right)$.

$$\therefore \quad \frac{a}{3} = 1, \frac{b}{3} = 2, \frac{c}{3} = 3$$

$$\Rightarrow \quad a = 3, \quad b = 6, \quad c = 9$$

Hence, the equation of the plane is

$$\frac{x}{3} + \frac{y}{6} + \frac{z}{9} = 1$$

125. Let AB be the given line and the DC's of AB be l, m, n.

Then, projection on x-axis $= AB \times l = 12$

Projection on y-axis $= AB \times m = 4$

Projection on z-axis $= AB \times n = 3$

$$\therefore \quad (AB)^2(l^2 + m^2 + n^2) = 12^2 + 4^2 + 3^2$$

$$\Rightarrow \quad (AB)^2 = 169$$

$$\Rightarrow \quad AB = 13$$

Hence, DC's of AB are $\dfrac{12}{13}, \dfrac{4}{13}, \dfrac{3}{13}$.

126. Given, $\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} + \vec{\mathbf{c}}) \times (\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}})$

$$= \vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{a}} + \vec{\mathbf{b}} \times \vec{\mathbf{b}} + \vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}} + \vec{\mathbf{c}} \times \vec{\mathbf{b}} \times \vec{\mathbf{c}} \times \vec{\mathbf{c}})$$

$$= \vec{\mathbf{a}} \cdot (-\vec{\mathbf{a}} \times \vec{\mathbf{b}} + \vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}} + \vec{\mathbf{c}} \times \vec{\mathbf{b}}) \quad [\because \vec{\mathbf{b}} \times \vec{\mathbf{b}} = 0]$$

$$= \vec{\mathbf{a}} \cdot (-\vec{\mathbf{a}} \times \vec{\mathbf{b}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}}) \quad [\vec{\mathbf{c}} \times \vec{\mathbf{b}} = -\vec{\mathbf{b}} \times \vec{\mathbf{c}}]$$

$$= 0$$

127. Given, $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

and $\vec{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$

Required unit vector $\vec{\mathbf{c}} = \dfrac{\vec{\mathbf{a}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}})}{|\vec{\mathbf{a}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}})|}$

Now, $\vec{\mathbf{a}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) = (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\,\vec{\mathbf{a}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{a}})\,\vec{\mathbf{b}}$

$$= 3(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - 6(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})$$

$$= -9\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$$

$\therefore$ $$\vec{\mathbf{c}} = \frac{-9\hat{\mathbf{j}} + 9\hat{\mathbf{k}}}{\sqrt{9^2 + 9^2}} = \frac{1}{\sqrt{2}}(-\hat{\mathbf{j}} + \hat{\mathbf{k}})$$

128. Key Idea *In an equilateral triangle orthocentre and centroid are coincide.*

We know that the position vector of the centroid of the triangle is $\frac{\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}}{3}$. Since, the triangle is an equilateral, therefore the orthocentre coincides with the centroid and hence,

$$\frac{\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}}{3} = \vec{\mathbf{0}}$$

$\Rightarrow$ $$\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$$

129. Key Idea *If three position vectors* $\vec{\mathbf{A}}, \vec{\mathbf{B}}$ *and* $\vec{\mathbf{C}}$ *are collinear, then* $\overrightarrow{\mathbf{AB}} = \lambda\,\overrightarrow{\mathbf{BC}}$.

Let $P(60\hat{\mathbf{i}} + 3\hat{\mathbf{j}}), Q(40\hat{\mathbf{i}} - 8\hat{\mathbf{j}})$ and $R(a\hat{\mathbf{i}} - 52\hat{\mathbf{j}})$ be the collinear points. Then

$\overrightarrow{\mathbf{PQ}} = \lambda\,\overrightarrow{\mathbf{QR}}$ for some scalar λ

$\Rightarrow (-20\hat{\mathbf{i}} - 11\hat{\mathbf{j}}) = \lambda[(a-40)\hat{\mathbf{i}} - 44\hat{\mathbf{j}}]$

On comparing the coefficients of $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ on both sides, we get

$$\lambda(a-40) = -20, \; -44\lambda = -11$$

$\Rightarrow$ $$\lambda(a-40) = -20, \; \lambda = \frac{1}{4}$$

$\therefore$ $$a - 40 = -20 \times 4$$

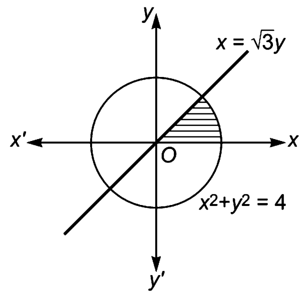
$\Rightarrow$ $$a = -40$$

130. Required area

$$= \int_0^1 (x_2 - x_1)\,dy$$

$$= \int_0^1 (\sqrt{4-y^2} - \sqrt{3}y)\,dy$$

$$= \left[\frac{1}{2}y\sqrt{4-y^2} + \frac{1}{2}(4)\sin^{-1}\frac{y}{2} - \frac{\sqrt{3}y^2}{2}\right]_0^1$$



$$= \frac{\sqrt{3}}{2} + 2\sin^{-1}\left(\frac{1}{2}\right) - \frac{\sqrt{3}}{2} - 2\sin^{-1} 0$$

$$= \frac{\sqrt{3}}{2} + 2\left(\frac{\pi}{6}\right) - \frac{\sqrt{3}}{2}$$

$$= \frac{\pi}{3} \text{ sq unit}$$

131. Let $y = \lim_{x\to\infty}\left(\frac{\pi}{2} - \tan^{-1}x\right)^{1/x}$

Taking log on both sides, we get

$$\log y = \lim_{x\to\infty}\frac{1}{x}\log\left(\frac{\pi}{2} - \tan^{-1}x\right) \qquad \left(\text{form } \frac{\infty}{\infty}\right)$$

$$= \lim_{x\to\infty}\frac{\left(-\frac{1}{1+x^2}\right)}{\frac{\pi}{2} - \tan^{-1}x} \qquad \text{(using L'Hospital's rule)}$$

$$= \lim_{x\to\infty}\frac{\frac{2x}{(1+x^2)^2}}{-\left(\frac{1}{1+x^2}\right)} \qquad \text{(using L' Hospital's rule)}$$

$$= \lim_{x\to\infty}\frac{-2x}{1+x^2} = 0$$

$\Rightarrow$ $$y = e^0 = 1$$

132. Since, $f(x)$ is continuous at $x = \frac{\pi}{2}$

So, $$\lim_{x\to\frac{\pi^-}{2}} f(x) = \lim_{x=\frac{\pi^+}{2}} f(x)$$

$\Rightarrow$ $$m\frac{\pi}{2} + 1 = \sin\frac{\pi}{2} + n$$

$\Rightarrow$ $$m\frac{\pi}{2} + 1 = 1 + n$$

$\Rightarrow$ $$\frac{m\pi}{2} = n$$

133. Given, $f(x) = \dfrac{\sqrt{4-x^2}}{\sin^{-1}(2-x)}$

$\sqrt{4-x^2}$ is defined for $4 - x^2 \geq 0$

$\Rightarrow \quad x^2 \leq 4$

$\Rightarrow \quad -2 \leq x \leq 2$

and $\sin^{-1}(2-x)$ is defined for $-1 \leq 2 - x \leq 1$

$\Rightarrow \quad -3 \leq -x \leq -1$

$\Rightarrow \quad 1 \leq x \leq 3$

Also, $\sin^{-1}(2-x) = 0$ for $x = 2$

$\therefore$ Domain of $f(x) = [-2, 2] \cap [1, 3] - \{2\}$

$= [1, 2)$

134. Given, $(1 + y^2)\,dx + (1 + x^2)\,dy = 0$

$$\Rightarrow \quad \frac{dx}{1+x^2} + \frac{dy}{1+y^2} = 0$$

On integrating, we get

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} c$$

$$\Rightarrow \quad \frac{x+y}{1-xy} = c$$

$$\Rightarrow \quad x + y = c\,(1 - xy)$$

135. Given, $$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$\Rightarrow \quad \rho\left(\frac{d^2y}{dx^2}\right) = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$$

On squaring both sides, we get

$$\rho^2\left(\frac{d^2y}{dx^2}\right)^2 = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^3$$

Clearly, it is a second order differential equation of degree 2.

NOTE If the higher order derivative is in the transcendental, then we do not determined the degree of that equation.

136. Given that, $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$

$$\Rightarrow \quad A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow \quad A = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow \quad A = \begin{bmatrix} 1 & 1+3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$$

137. Let $R = \{(a, b) : a, b \in N,\ a - b = 3\}$

$= \{((n + 3), n) : n \in N\}$

$= \{(4, 1), (5, 2), (6, 3), \ldots\}$

138. Given, $$\frac{dy}{dx} = \frac{ax + h}{by + k}$$

$$\Rightarrow \quad (by + k)\,dy = (ax + h)\,dx$$

On integrating, we get

$$\frac{b}{2}y^2 + ky = \frac{a}{2}x^2 + hx + c$$

For this to represent a parabola, one of the two terms x^2 or y^2 is zero.

Therefore, either $a = 0,\ b \neq 0$

or $\quad a \neq 0,\ b = 0$

139. Given, $$\frac{dy}{dx} + \frac{2yx}{1+x^2} = \frac{1}{(1+x^2)^2}$$

which is a linear differential equation.

$$\therefore \quad P = \frac{2x}{1+x^2}, \quad Q = \frac{1}{(1+x^2)^2}$$

Now, $\quad \text{IF} = e^{\int P\,dx}$

$$= e^{\int \frac{2x}{1+x^2}\,dx} = e^{\log(1+x^2)}$$

$$= (1 + x^2)$$

Solution of differential equation is

$$y\,(1 + x^2) = \int \frac{1}{(1+x^2)^2} \cdot (1 + x^2)\,dx + c$$

$$\Rightarrow \quad y(1 + x^2) = \int \frac{1}{1+x^2}\,dx + c$$

$$\Rightarrow \quad y(1 + x^2) = \tan^{-1} x + c$$

140. Given, $$\begin{vmatrix} x & x^2 & 1 + x^3 \\ y & y^2 & 1 + y^3 \\ z & z^2 & 1 + z^3 \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$\Rightarrow (1 + xyz) \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (1 + xyz)[x(y^2 - z^2) - y(x^2 - z^2) + z(x^2 - y^2)] = 0$$

$$\Rightarrow (1 + xyz)(x - y)(y - z)(z - x) = 0$$

$$\Rightarrow 1 + xyz = 0$$

$$\Rightarrow xyz = -1$$

141. Given, $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.2$

We know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow 0.6 = P(A) + P(B) - 0.2$$

$$\Rightarrow P(A) + P(B) = 0.8$$

$$\Rightarrow 1 - P(\bar{A}) + 1 - P(\bar{B}) = 0.8$$

$$\Rightarrow -[P(\bar{A}) + P(\bar{B})] = 0.8 - 2$$

$$\Rightarrow P(\bar{A}) + P(\bar{B}) = 1.2$$

142. We know that $P(A/\bar{B}) + P(\bar{A}/\bar{B}) = 1$

$$\Rightarrow P(\bar{A}/\bar{B}) = 1 - P(A/\bar{B})$$

143. ASSISTANT → AA I N SSS TT

STATISTICS → AC II SSS TTT

Here N and C are not common.

Same letters can be A, I, S, T

Probability of choosing A

$$= \frac{^2C_1}{^9C_1} \times \frac{^1C_1}{^{10}C_1}$$

$$= \frac{2}{9} \times \frac{1}{10} = \frac{1}{45}$$

Probability of choosing I

$$= \frac{1}{^9C_1} \times \frac{^2C_1}{^{10}C_1}$$

$$= \frac{1}{9} \times \frac{2}{10} = \frac{1}{45}$$

Probability of choosing S

$$= \frac{^3C_1}{^9C_1} \times \frac{^3C_1}{^{10}C_1}$$

$$= \frac{3}{9} \times \frac{3}{10} = \frac{1}{10}$$

Probability of choosing T

$$= \frac{^2C_1}{^9C_1} \times \frac{^3C_1}{^{10}C_1}$$

$$= \frac{2}{9} \times \frac{3}{10} = \frac{1}{15}$$

Hence, required probability

$$= \frac{1}{45} + \frac{1}{45} + \frac{1}{10} + \frac{1}{15}$$

$$= \frac{19}{90}$$

144. Since, $Q = \dfrac{R \sin \alpha}{\sin(\alpha + \beta)}$

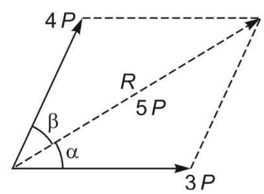
Also, $(5P)^2 = (4P)^2 + (3P)^2 + 2(4P)(3P)\cos(\alpha + \beta)$

$$\Rightarrow 25P^2 = 16P^2 + 9P^2 + 24P^2 \cos(\alpha + \beta)$$

$$\Rightarrow 24P^2 \cos(\alpha + \beta) = 0$$

$$\Rightarrow \cos(\alpha + \beta) = 0 = \cos 90°$$

$$\Rightarrow \alpha + \beta = 90°$$



Now, $4P = \dfrac{5P \sin \alpha}{\sin 90°}$

$$\Rightarrow \sin \alpha = \frac{4}{5}$$

$$\Rightarrow \alpha = \sin^{-1}\left(\frac{4}{5}\right)$$

145. Since, $R^2 = u^2 + v^2 + 2uv \cos \alpha$

$$\therefore R^2 = (30)^2 + (60)^2 + 2(30)(60)\cos 60°$$

$$\Rightarrow R^2 = 30(30 + 120 + 60)$$

$$\Rightarrow R^2 = 30 \times 30 \times 7$$

$$\Rightarrow R = 30\sqrt{7} \text{ km/h}$$

146. Height $h = \dfrac{1}{2}gt^2$

$$\Rightarrow h = \frac{1}{2}(10)(3)^2$$

$\Rightarrow \quad h = 5 \times 9$

$\Rightarrow \quad h = 45$ m

147. Let the two trains meet after time t hour at a distance of s kilometre from starting point of A, then for the train A

$$s = 50t + \frac{1}{2}(18t^2) \quad \text{...(i)}$$

and for the train B,

$$100 - s = 50t + \frac{1}{2}(-18t^2) \quad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$100 = 100t$$

$$\Rightarrow \quad t = 1$$

From Eq. (i),

$$s = 50 + \frac{1}{2}(18) = 59 \text{ km}$$

So, the two trains meet at a distance of 59 km from the starting position of A.

148. Given, $2\tan^{-1}(\cos x) = \tan^{-1}(2\,\text{cosec}\ x)$

$$\Rightarrow \quad \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\,\text{cosec}\ x)$$

$$\Rightarrow \quad \frac{2\cos x}{1-\cos^2 x} = 2\,\text{cosec}\ x$$

$$\Rightarrow \quad \frac{2\cos x}{\sin^2 x} = 2\,\text{cosec}\ x$$

$$\Rightarrow \quad \sin x = \cos x$$

$$\Rightarrow \quad x = \frac{\pi}{4}$$

149. Given, condition are $a + x = 1$ and $ax = 0$

These two condition will be true, if $x = a'$

150. Given $(x + y)\cdot(x + 1) = x + x\cdot y + y$

Replace '·' by '+', '+' by '·', '1' by '0', we get

$$(x\cdot y) + (x\cdot 0) = x\cdot(x + y)\cdot y$$