AMU

Engineering Entrance Exam

Solved Paper 2018

Physics

1. The nearest star to our solar system is 4.3 light years away. The distance of this star in Parsec is (Mean distance between the earth and the sun $= 1.5 \times 10^{11}$ m and one light year $= 9.46 \times 10^{15}$ m)

(a) 1.3 (b) 8.0 (c) 13.0 (d) 3.3×10^4

2. Consider the following statements about the four fundamental forces in nature.

(i) The strong nuclear force binds protons and neutrons in a nucleus.

(ii) The strong nuclear force is about 10^3 times the electromagnetic force in strength.

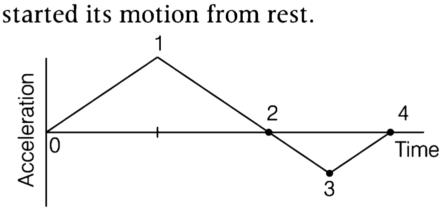
(iii) The weak nuclear force is the weakest of all the four fundamental forces.

(iv) The range of the weak nuclear is about 10^{-16} m.

The correct statement(s) is (are)

(a) (i) only (b) (ii) and (iii)
(c) (i) and (iv) (d) (i), (ii) and (iv)

3. Acceleration-time graph of a body moving in a straight line is as shown in figure. The body started its motion from rest.



At which point is the body moving with the largest speed?

(a) 1 (b) 2 (c) 3 (d) 4

4. The displacement of a body is given by $x = 4t + 5t^3$, where x is in metre and t is in second. The difference between the average velocity of the body in the time-interval $t = 1$ s to $t = 2$ s and its instantaneous velocity at $t = 1$ s is

(a) 20.0 m/s (b) 22.5 m/s
(c) 27.0 m/s (d) 39.0 m/s

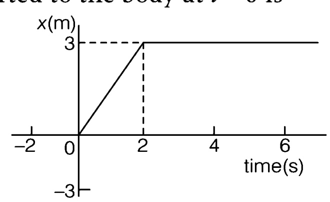
5. The relation between time t and distance x for a moving particle is $t = \alpha x^2 + \beta x$, where α and β are constants. If v is the velocity at distance x, then the retardation of the particle is

(a) $2\alpha v^3$ (b) $2\beta v^3$ (c) $2\alpha\beta v^3$ (d) $2\beta^2 v^2$

6. Two seconds after projection, a projectile is moving at 30° above the horizontal, after one more second, it is moving horizontally. The initial speed of the projectile is (Take $g = 10$ ms^{-2})

(a) 10 m/s (b) $10\sqrt{3}$ m/s
(c) 20 m/s (d) $20\sqrt{3}$ m/s

7. Figure represents the position-time graph of a body of mass 4 kg. Impulse (kg ms^{-1}) imparted to the body at $t = 0$ is



(a) 6 (b) 4
(c) 3 (d) 0

8. A particle of mass m is moving with a constant velocity along a line parallel to the positive direction of X-axis. The magnitude of its angular momentum with respect to the origin

(a) is zero.
(b) goes on increasing as x increases.
(c) goes on decreasing as x increases.
(d) remains constant for all positions of the particle.

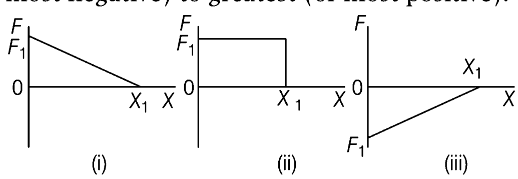
9. A man throws a ball of mass 3.0 kg with a speed of 5.0 m/s. His hand is in contact with the ball for 0.2 s. If he throws 4 balls in 2 seconds, the average force exerted by him in 1 second is

(a) 15 N (b) 30 N
(c) 150 N (d) 75 N

10. One end of massless spring of spring constant 100 N/m and natural length 0.49 m is fixed and other end is connected to a body of mass 0.5 kg lying on a frictionless horizontal table. The spring remains horizontal. If the body is made to rotate at an angular velocity of 2 rad/s, then the elongation of the spring will be

(a) 2 cm (b) 1 cm
(c) 0.5 cm (d) 0.25 cm

11. The graphs below show the magnitude of the force on a particle as it moves along the positive X-axis from the origin to $X = X_1$. The force is parallel to the X-axis and conservative. The maximum magnitude F_1 has the same value for all graphs. Rank the situations according to the change in the potential energy associated with the force, least (or most negative) to greatest (or most positive).



(a) (i), (ii), (iii) (b) (i), (iii), (ii)
(c) (iii), (ii), (i) (d) (ii), (i), (iii)

12. The only force acting on a 2 kg body that is moving in xy-plane has a magnitude of 5 N. The body initially has a velocity of 4 m/s in the positive x-direction. Some time later, the body has a velocity of 6 m/s in the positive y-direction. The work done on the body by the 5 N force during this time is

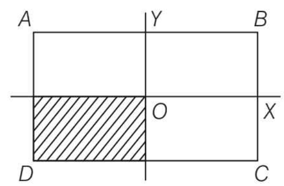
(a) 20 J (b) 40 J (c) 52 J (d) 72 J

13. The time-period of a physical pendulum is $2\pi\sqrt{I / mgd}$, where I is the moment of inertia of the pendulum about the axis of rotation and d is perpendicular distance between the axis of rotation and the centre of mass of the pendulum.

A circular ring hangs from a nail on a wall. The mass of the ring is 3 kg and its radius is 20 cm. If the ring is slightly displaced, the time of resulting oscillations will be

(a) 1.0 s (b) 1.3 s (c) 1.8 s (d) 2.1 s

14. Figure shows a rectangular copper plate with is centre of mass at the origin O and side $AB = 2BC = 2$ m. If a quarter part of the plate (shown as shaded) is removed, the centre of mass of the remaining plate would lie at

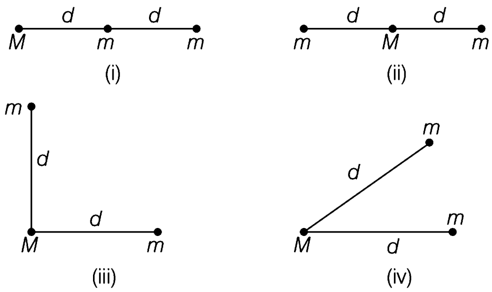


(a) $\frac{1}{12}$ m, $\frac{1}{6}$ m (b) $\frac{1}{6}$ m, $\frac{1}{12}$ m
(c) $\frac{1}{3}$ m, $\frac{1}{6}$ m (d) $\frac{1}{3}$ m, $\frac{1}{2}$ m

15. A block is released from rest on a 45° smooth incline and slide a distance d. The time taken to slide the same distance is n times as much to slide on a 45° rough incline than on the smooth incline. The coefficients of friction for the rough incline is

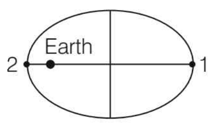
(a) $\sqrt{1 - \frac{1}{n^2}}$ (b) $1 - \frac{1}{n^2}$
(c) $\sqrt{1 - \frac{1}{2n^2}}$ (d) $1 - \frac{1}{2n^2}$

16. Three particles, two with masses m and one with mass M, might be arranged in any of the four configurations shown below. Rank the configurations according to the magnitude of the gravitational force on M, least to greatest



(a) (i), (ii), (iii), (iv)
(b) (ii), (i), (iii), (iv)
(c) (ii), (i), (iv), (iii)
(d) (ii), (iii), (iv), (i)

17. A small satellite is in elliptical orbit around the earth as shown in figure. L denotes the magnitude of its angular momentum and K denotes its kinetic energy. If 1 and 2 denote two positions of the satellite, then

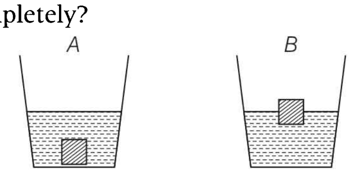


(a) $L_2 = L_1, K_2 = K_1$
(b) $L_2 = L_1, K_2 > K_1$
(c) $L_2 > L_1. K_2 < K_1$
(d) $L_2 = L_1, K_2 < K_1$

18. Two solid spheres of the same metal but of mass M and $8\,M$ fall simultaneously in a viscous liquid. If their terminal velocities are v and nv, then the value of n will be

(a) 16 (b) 8 (c) 4 (d) 2

19. A piece of ice is tied using a string to the bottom of bucket A. The bucket is filled with water with ice completely submerged in it. Another bucket B is filled with water and a piece of ice is released in water. If floats on the surface of water (see Fig.). What would be the impact on the level of water in the two buckets, when ice pieces melt away completely?



(a) Level of water remain unchanged in both the buckets.
(b) Level of water will go down in bucket A, but will remain unchanged in bucket B.
(c) Level of water will go down in bucket, A but will go up in bucket B.
(d) Level of water will remain unchanged in bucket A, but will go up in bucket B.

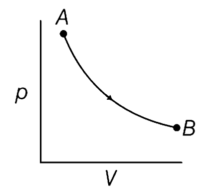
20. Consider the following thermodynamical variables

(i) Pressure (ii) Internal Energy
(iii) Volume (iv) Temperature

Out of these, the intensive variable(s) is (are)

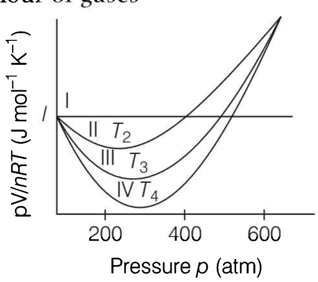
(a) (i) only
(b) (i), (iv)
(c) (i), (ii)
(d) (i), (ii), (iv)

21. A sample of an ideal gas undergoes an isothermal process as shown by the curve AB in the pV diagram. If ΔQ, ΔU and ΔW represent the amount of heat absorbed the change in internal energy and the work done respectively, then which of the following statement is correct?



(a) $\Delta Q = +$ ve, $\Delta U = 0, \Delta W = -$ ve
(b) $\Delta Q = +$ ve, $\Delta U = 0, \Delta W = +$ve
(c) $\Delta Q = +$ ve, $\Delta U = 0, \Delta W = 0$
(d) $\Delta Q = +$ ve, $\Delta U = +$ ve, $\Delta W = +$ve

22. Different curves in the figures show the behaviour of gases



(i) Curve I represent ideal gas behaviour
(ii) Curves II, III and IV also represents ideal gas behaviour at different temperatures T_2, T_3 and T_4.
(iii) Curves II, III and IV represents behaviour of a real gas at different temperatures T_2, T_3 and T_4 .
(iv) $T_2 > T_3 > T_4$
(v) $T_2 < T_3 < T_4$

The correct statements are

(a) (i), (ii), (iv)
(b) (i), (iii), (iv)
(c) (i), (iii), (v)
(d) (i), (ii), (v)

23. Sound waves from a loudspeaker reach a point *P via* two paths which differ in length by 1.8 m. When the frequency of sound is gradually increased, the resultant intensity at *P* is found to be maximum the frequency is 1000 Hz. At what next higher frequency will a maximum be detected?
(velocity of sound= 360 m/s)

(a) 1200 Hz (b) 1400 Hz (c) 1600 Hz (d) 1800 Hz

24. A rocket is moving at a speed of 200 m/s towards a stationary target. While moving it emits a wave of frequency 1000 Hz. Some of the sound reaching the target gets reflected back to the rocket as an echo. The frequency of the echo as detected by the rocket is (velocity of sound = 330 m/s)

(a) 1000 Hz (b) 1580 Hz (c) 2540 Hz (d) 4080 Hz

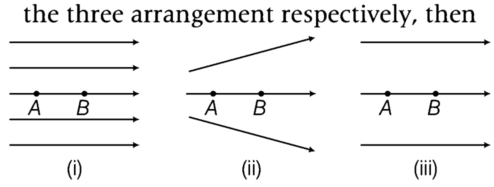
25. The air pressure at sea level is 101325 Pa. At the centre of a rarefaction of a sound wave in air, the pressure is 91000 Pa. Which is the most likely pressure at the centres of a compression of the same wave?

(a) 91000 Pa (b) 101000 Pa
(c) 111650 Pa (d) 121000 Pa

26. Four charges each equal to $(-Q)$ are placed at the four corners of a square and a charge q is placed at its centre. If the system of charges is in equilibrium, the value of q is

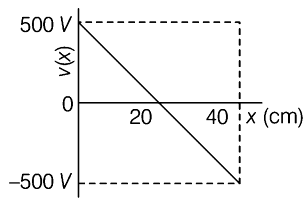
(a) $\frac{Q}{4}(2\sqrt{2}-1)$ (b) $\frac{Q}{4}(2\sqrt{2}+1)$
(c) $\frac{Q}{2}(2\sqrt{2}-1)$ (d) $\frac{Q}{2}(2\sqrt{2}+1)$

27. Figure shows three arrangements of electric field lines. In each arrangement, a proton is released from rest at point *A* and then accelerated through point *B* by the electric field. Points *A* and *B* have equal separations in the three arrangements. If p_1, p_2 and p_3 are linear momentum of the proton at point *B* in the three arrangement respectively, then



(a) $p_1 > p_3 > p_2$ (b) $p_1 > p_2 > p_3$
(c) $p_2 > p_1 > p_3$ (d) $p_1 = p_2 = p_3$

28. An electron is placed on *X*-axis where the electric potential depends on x as shown in figures (the potential does not depend on y and z). What is the electric force on the electron ?

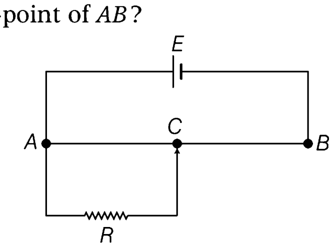


(a) 4.0×10^{-18} N (b) 8.0×10^{-18} N
(c) 3.2×10^{-16} N (d) 4.0×10^{-16} N

29. Two identical conducting spheres *A* and *B* carry equal charge. They are initially separated by a distance much larger than their diameters and the force between them is *F*. *A* third identical conducting sphere *C* is uncharged. Sphere *C* is first touched to *A*, then to *C* and removed. As a result, the force between *A* and *B* now is

(a) $F/16$ (b) $F/4$
(c) $3F/8$ (d) $F/2$

30. Figure shows a potentiometer. Length of the potentiometer wire *AB* is 100 cm and its resistance is 100 Ω . EMF of the battery *E* is 2 V. A resistance *R* of 50 Ω draws current from the potentiometer. What is the voltage across *R* when the sliding contact *C* is at the mid-point of *AB*?

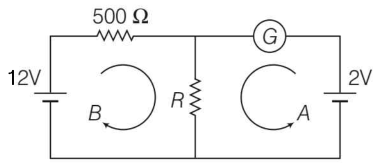


(a) 2/3 V (b) 1 V
(c) 4/3 V (d) 3/2 V

31. An inverter battery operated on 24 V and has negligible internal resistance. It is rated at 140 ampere-hour. What external resistance would have to be connected to the battery, if it were to be discharged in 14 hours.

(a) 1.6 Ω (b) 2.4 Ω
(c) 5.9 Ω (d) 10.0 Ω

32. In the circuit, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, then the value of R is

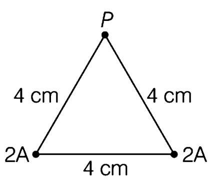


(a) 100 Ω (b) 200 Ω
(c) 500 Ω (d) 1000 Ω

33. A cyclotron's oscillator frequency is 10 MHz. What should be the operating magnetic field for accelerating protons?
(mass of the proton $= 1.67 \times 10^{-27}$ kg)

(a) 0.33 T (b) 0.66 T (c) 1.5 T (d) 3.0 T

34. Two long straight wires vertically pierced the plane of the paper at vertices of an equilateral triangle as shown in figure. They each carry 2 A, out of the paper. The magnetic field at the third vertex P has magnitude.



(a) 10^{-5} T (b) 1.4×10^{-5} T
(c) 1.7×10^{-5} T (d) 0.85×10^{-5} T

35. In a certain mass spectrometer, an ion beam passes through a velocity filter consisting of mutually perpendicular fields E and B. The beam then enters a region of another magnetic field B', perpendicular to the beam. The radius of curvature of the resulting ion beam is proportional to

(a) EB'/B (b) EB/B' (c) BB'/E (d) E/BB'

36. A magnetic field cannot

(a) change the velocity of a charged particle.
(b) change the momentum of a charged particle.
(c) change the kinetic energy of a charged particle.
(d) change the trajectory of a charged particle.

37. Two different coils have self-inductance $L_1 = 9$ mH and $L_2 = 3$ mH. At a certain instant, the current in the two coils is increasing at the same rate and the power supplied to the coils is also the same. The ratio of the energy stored in the two coils (U_1 / U_2) at that instant is

(a) 1/3 (b) 1
(c) 3 (d) 27

38. An alternating voltage $V = 200\sqrt{2} \sin (100\, t)$ volt is connected to a 1 μF capacitor through an AC ammeter. The reading of the ammeter is

(a) 40 mA (b) $20\sqrt{2}$ mA
(c) 20 mA (d) $10\sqrt{2}$ mA

39. In electromagnetic waves travelling in vacuum

(i) The electric field **E** is always perpendicular to the magnetic field **B**.
(ii) The cross product **E** × **B** always gives the direction in which the waves travel.
(iii) The field **E** and **B** vary sinusoidally.
(iv) There is a phase difference of $\frac{\pi}{2}$ between **E** and **B**.

The correct statement(s) is (are)

(a) (i), (iii) (b) (i), (iii), (iv)
(c) (i), (ii), (iii), (iv) (d) (i), (ii), (iii)

40. In Young's double slit experiment using monochromatic light of wavelength λ, the intensity of light at a point on the screen where path difference is λ, is k units. The intensity of light at a point, where path difference is $\lambda/3$ is

(a) $k/2$ (b) $k/3$
(c) $k/4$ (d) $2k/3$

41. The angle of prism and refractive index of the material of the prism are A and $\cot\frac{A}{2}$, respectively. The angle of minimum deviation of the prism is

(a) $\frac{\pi}{2} - A$ (b) $\pi - A$ (c) $\pi - \frac{A}{2}$ (d) $\pi - 2A$

42. Assume that the light of wavelength is 6000 Å coming from a star. What is the time resolution of a telescope whose objective has a diameter of 100 inch?

(a) 3.66×10^{-7} rad (b) 1.44×10^{-7} rad
(c) 2.9×10^{-7} rad (d) 5.8×10^{-7} rad

43. In a stack of three polarising sheets, the first and the third are crossed while the middle one has its axis at 45° to the axes of the other two. The fraction of intensity of an incident unpolarised beam of light that is transmitted by the stack is

(a) 1/2 (b) 1/3 (c) 1/4 (d) 1/8

44. When a beam of 10.6 eV photons of intensity 2.0 W/m^2 falls on a metallic surface of area 1×10^{-4} m^2, 0.53% of the incident photons eject photoelectrons. What is the number of photoelectrons emitted per second?

(a) 1.18×10^{16} (b) 6.25×10^{11}
(c) 6.25×10^{13} (d) 6.25×10^{15}

45. The wave nature of electron was first experimentally verified by

(a) Louis Victor de-Broglie
(b) James Frank and Gustav Hertz
(c) C.J. Davisson and L.H. Germer
(d) Hans Geiger and Ernst Marsden

46. Identify the correct statement(s) from among the following

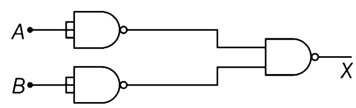
(i) The constancy of the binding energy per nucleon in the range $30 < A < 170$ is a consequence of the fact that the nuclear force is short ranged.
(ii) The nuclear force does not depend on the charge of nucleons.
(iii) The nuclear force is repulsive when distance between two nucleons is less than 0.8 fm.

(a) (i) only (b) (ii), (iii) (c) (i), (ii) (d) (i), (ii), (iii)

47. An alpha particle accelerated through V volts is fired towards a nucleus. It distance of closest approach is *r*. If a proton accelerated through the same potential is fired towards the same nucleus, its distance of closest approach will be

(a) r (b) $2r$ (c) $\frac{r}{2}$ (d) $\frac{r}{4}$

48. The combination of gates shown in figure yields



(a) NAND gate (b) OR gate
(c) NOT gate (d) XOR gate

49. For a CE transistor amplifier the audio signal voltage across the collector resistance of 2 kΩ is 2 V. The current amplification factor of the transistor is 100. If the base resistance is 1 kΩ, the input signal voltage and base current are respectively

(a) 0.01 V, 10 μA (b) 0.04 V, 10 μA
(c) 0.01 V, 10 mA (d) 0.04 V, 10 mA

50. Consider the following

(i) Submarine communications
(ii) A.M. radio
(iii) Shortwave ratio
(iv) Radar

Arrange the above in increasing frequency of the waves associated with them

(a) (iv), (iii), (ii), (i) (b) (i), (ii), (iii), (iv)
(c) (ii), (i), (iv), (iii) (d) (ii), (iii), (iv), (i)

Chemistry

51. Which of the following has the maximum vapour pressure?

(a) HCl (b) HBr
(c) HF (d) HI

52. Which of the following cations has the strongest tendency towards complex formation?

(a) Sm^{3+} (b) Lu^{3+} (c) Gd^{3+} (d) Yb^{3+}

53. Which of the following is most soluble in water?

(a) $CsClO_4$ (b) $NaClO_4$
(c) $KClO_4$ (d) $LiClO_4$

54. Which of the following metal ions is expected to be coloured?

(a) Zn^{2+} (b) Ti^{3+}
(c) Sc^{3+} (d) Ti^{4+}

55. The best reducing agent among the following is

(a) NH_3 (b) SbH_3
(c) PH_3 (d) AsH_3

56. The EAN value $y[\mathrm{Ti}(\sigma - C_6H_5)_2(\pi - C_5H_5)_2]^{\circ}$ is

(a) 32 (b) 33
(c) 34 (d) 35

57. Which of the following radioactive element is used in the treatment of cancer?

(a) Uranium (b) Thorium
(c) Cerium (d) Plutonium

58. Which of the following complexes is optically active?

(a) $[Co(NH_3)_5Cl]^+$ (b) $[Co(NH_3)Cl_5]^{3-}$
(c) *cis*-$[Co(en)_2Cl_2]$ (d) *trans*-$[Co(en)_2Cl_2]$

59. The crystal field splitting energy (CFSE) for $[CoCl_6]^{4-}$ is about 18000 cm^{-1}. What would be the CFSE value of $[CoCl_4]^{2-}$?

(a) 18000 cm^{-1} (b) 8000 cm^{-1}
(c) 16000 cm^{-1} (d) 2000 cm^{-1}

60. The following reactions show the H_2O_2 behaviour in I and II reactions as:

I. $PbS(s) + 4H_2O_2(aq) \longrightarrow PbSO_4(s) + 4H_2O(l)$

II. $HOCl + H_2O_2 \longrightarrow H_3O^+ + Cl^- + O_2$

(a) Oxidising in acidic medium and reducing in basic medium
(b) Reducing in acidic medium and oxidising in basic medium
(c) Oxidising in acidic medium and reducing in acidic medium
(d) Reducing in acidic medium and oxidising in acidic medium

61. Amongst the following interhalogen compounds which one is used for the production of UF_6 during ^{235}U enrichment process?

(a) ClF_3 (b) ClF_5
(c) IF_3 (d) IF_5

62. Products (X and Y) of the following reactions (I and II) are :

I. $\underset{\text{(Cold and dil.)}}{2NaOH + Cl_2} \longrightarrow NaCl + X + H_2O$

II. $\underset{\text{(Hot and conc.)}}{6NaOH + 3Cl_2} \longrightarrow NaCl + Y + 3H_2O$

(a) $X = NaClO_3$ and $Y = NaOCl$
(b) $X = NaClO$ and $Y = NaOCl_3$
(c) $X = NaHClO_3$ and $Y = NaOCl$
(d) $X = NaClO_3$ and $Y = NaHClO_3$

63. The intermediate product (X) formed in the following reaction is

$$B_2H_6 + 6NH_3 \rightarrow 3X \xrightarrow{\text{Heat}} 2B_3N_3H_6 + 12H_2$$

(a) $[BH(NH_3)_3]^+ [BH_4]^-$
(b) $[BH_2(NH_3)_4]^+ [BH_4]^-$
(c) $[BH(NH_3)_4]^+ [BH_4]^-$-
(d) $[BH_2(NH_3)_2]^+ [BH_4]^-$

64. An example of non-stoichiometric hydride is

(a) sodium hydride
(b) beryllium hydride
(c) lanthanum hydride
(d) diborane

65. Which one of the following is not the use of SO_2?

(a) Preservative (b) Anti-chlor
(c) Disinfectant (d) Insecticide

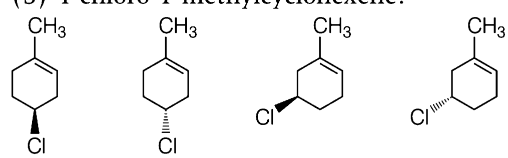
66. Which one of the following set of metals deposits an anode mud during the process of electrolytic refining of copper?

(a) Sn and Ag (b) Pb and Zn
(c) Ag and Au (d) Fe and Ni

67. Structure anions of acids, HNO_3, H_3PO_4 and H_2SO_4 are, respectively

(a) tetrahedral, tetrahedral and trigonal bipyramidal
(b) angular, tetrahedral and trigonal bipyramidal
(c) tetrahedral, tetrahedral and angular
(d) planar, tetrahedral and tetrahedral

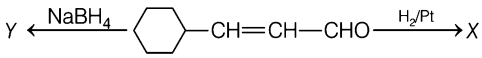
68. Which of the following compounds is (S)-4-chloro-1-methylcyclohexene?



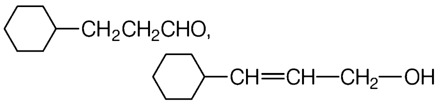
(a) (b) (c) (d)

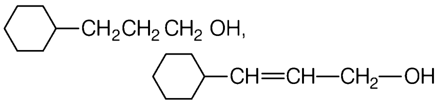
69. Aspartame is an

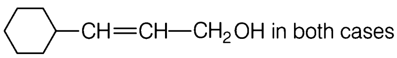
(a) alkaloid (b) insecticide
(c) artificial sweetener (d) antiseptic

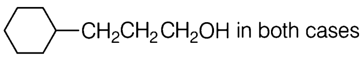
70. 

What are '*X*' and '*Y*' ?

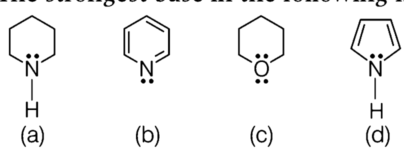
(a) 

(b) 

(c) 

(d) 

71. The strongest base in the following is



72. The most reactive to nucleophilic attack at the carbonyl group is

(a) $H_3C-\overset{O}{\overset{||}{C}}-OCH_3$ (b) $CH_3-\overset{O}{\overset{||}{C}}-Cl$

(c) $CH_3-\overset{O}{\overset{||}{C}}-H$

(d) $CH_3-\overset{O}{\overset{||}{C}}-O-\overset{O}{\overset{||}{C}}-CH_3$

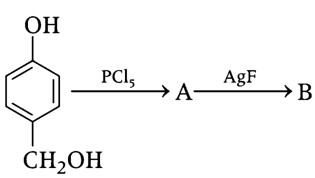
73. Name the end product in the following series of reactions

$$CH_3COOH \xrightarrow{NH_3} A \xrightarrow{\Delta} B \xrightarrow{P_2O_5} C.$$

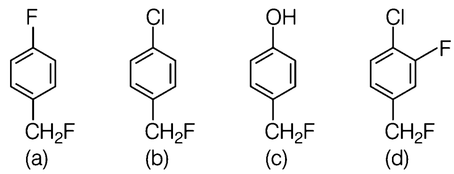
(a) Methane
(b) Methanol
(c) Acetonitrile
(d) Acetamide

74. Which one acts as refrigerant?

(a) CF_2Cl_2 (b) CF_4
(c) $CFCl_3$ (d) CF_3Cl

75. 

What is *B* in the above scheme?



76. The reactant '*P*' in the following reaction is

$$P \xrightarrow[\text{Dil. } H_2SO_4]{K_2Cr_2O_7} B \xrightarrow[H_3O^+]{CH_3MgBr} CH_3-\underset{OH}{\underset{|}{\overset{CH_3}{\overset{|}{C}}}}-CH_3$$

(a) $CH_3CHOHCH_3$ (b) CH_3COCH_3
(c) CH_3CH_2OH (d) CH_3COOH

77. A β-hydroxy carbonyl compound is obtained by the action of NaOH on

(a) R_3CCHO (b) C_6H_5CHO (c) CH_3CHO (d) HCHO

78. A compound $C_5H_{10}O$ (A) forms a phenylhydrazone and gives negative Tollen's test and a positive Iodoform reaction. It also gives *n*-pentane on reduction. The compound (A) is

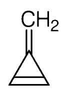
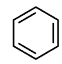
(a) pentanal (b) 2-pentanone
(c) 3-pentanone (d) allyl alcohol

79. Which of the following is used for the estimation of halogens in organic compounds?

(a) Carius method (b) Duma's method
(c) Kjeldahl's method (d) Newman method

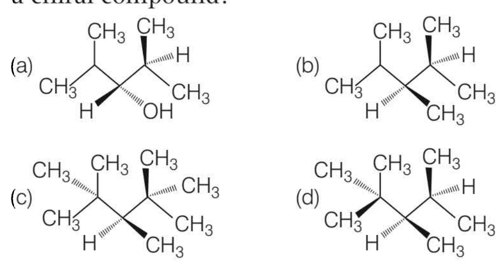
80. Which of the following structures contain *sp*-hybridised carbon atom(s)?

I. $HC\equiv CH$ II. $H_2C=C=CH_2$

III.  IV. 

(a) I, II and III (b) I, III and IV
(c) II, III and IV (d) I, II and IV

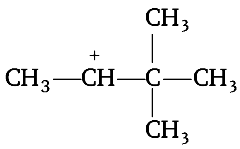
81. Which of the following structures represents a chiral compound?

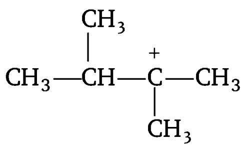


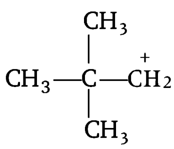
82. Which of the following is an example of thermosetting polymers?

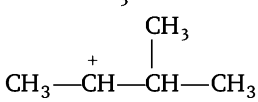
(a) Bakelite (b) PVC
(c) Nylon 6, 6 (d) Buna-S

83. Which of the following carbocations is most stable?

I. 

II. 

III. 

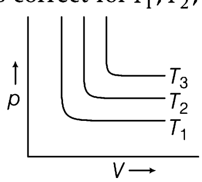
IV. 

(a) I (b) II (c) III (d) IV

84. On heating an aldehyde with Fehling's reagent, a reddish brown precipitate is obtained due to the formation of

(a) $RCOO^-$ (b) CuO
(c) Cu_2O (d) RCH_2OH

85. For the given pV isotherms, which of the following is correct for T_1, T_2, T_3?



(a) $T_1 < T_2 < T_3$ (b) $T_3 < T_2 < T_1$
(c) $T_2 < T_3 < T_1$ (d) $T_1 < T_3 < T_2$

86. What will be the pH of solution formed by mixing 10 mL 0.1 M NaH_2PO_4 and 15 mL 0.1 M Na_2HPO_4.
[Given: $pK_1 = 2.12$, $pK_2 = 7.2$]

(a) 7.0 (b) 6.9
(c) 7.4 (d) 7.5

87. The difference between $\bar{C}_p$ and $\bar{C}_V$ is

[$\bar{C}_p$ and $\bar{C}_V$ signify molar quantities]

(a) larger in case of gases in comparison to solids and liquids
(b) larger in case of liquids in comparison to gases and solids
(c) larger in case of solids in comparison to gases liquids
(d) equal in solids, liquids and gases

88. For the reaction,

$A + B \rightarrow P = -\frac{d[A]}{dt} = -\frac{d[B]}{dt} = k[A][B]$ and

$kt = \frac{1}{[A]_0 - [B]_0} \ln \frac{[A][B]_0}{[B][A]_0}$ when, $[A]_0 \neq [B]_0$

If, $[A]_0 = [B]_0$ then the integrated rate law will be

(a) $kt = \ln \frac{[A]}{[B]}$

(b) $\frac{1}{[B]} = \frac{1}{[A]_0} + kt$

(c) $\frac{1}{[A]} = \frac{1}{[B]_0} + kt$

(d) $\frac{1}{[A]} = \frac{1}{[A]_0} + kt$ or $\frac{1}{[B]} = \frac{1}{[B]_0} + kt$

89. The reaction quotient (Q) for the reduction of O_2 to H_2O in acid solution,

$O_2(g) + 4H^+(aq) + 4e^- \longrightarrow 2H_2O(l)$ is,

[where, α_{H^+} = activity of H^+, p_{O_2} = pressure of O_2 when it is present in the reaction, $p°$ = pressure of O_2 at standard state].

(a) $Q = \frac{p°}{\alpha^4_{H^+} + p_{O_2}}$ (b) $Q = \frac{p_{O_2}}{\alpha^4_{H^+} + p°}$

(c) $Q = \frac{\alpha^4_{H^+} + p_{O_2}}{p°}$ (d) $Q = \frac{\alpha^4_{H^+} + p°}{p_{O_2}}$

90. The standard emf of the cell ($E°_{cell}$ and equilibrium constant (K_{eq}) of the following reaction of 298 K

$$Cd^{2+} + 4NH_3 \rightleftharpoons Cd(NH_3)_4^{2+}$$

(a) $E°_{cell} = 1.0\,V$, $K_{eq} = 1.26 \times 10^7$
(b) $E°_{cell} = 0.21\,V$, $K_{eq} = 1.26 \times 10^7$
(c) $E°_{cell} = 1.0\,V$, $K_{eq} = 6.60 \times 10^{33}$
(d) $E°_{cell} = 0.21\,V$, $K_{eq} = 6.60 \times 10^{33}$

91. 0.002 M solution of a weak acid has an equivalent conductance (Λ) 60 ohm^{-1} cm^2 eq^{-1}. What will be the pH ?
(Given : $\Lambda° = 400$ ohm^{-1} cm^2 eq^{-1}]

(a) 3.52 (b) 2.52
(c) 1.87 (d) 2.7

92. The emf of the cell,
Cd | $CdCl_2$ (solution) (1 atm) |AgCl(*s*)| Ag is 0.675 at 25°C. The temperature coefficient of the cell is -6.5×10^{-4} V $degree^{-1}$. Find the change in heat content (kJ mol^{-1}) and entropy ($V\,deg^{-1}$) for the electrochemical reaction that occurs when 1 F of electricity is drawn for it

(a) 78.34, 83.83 (b) + 62.73, – 83.83
(c) – 62.73, – 83.83 (d) – 78.34, + 83.83

93. The rate constant, the activation energy and the Arrhenius parameter of a chemical reaction at 25°C are 3.0×10^{-4} s^{-1}, 104.4 kJ mol^{-1} and 6.0×10^{14} s^{-1}, respectively. The value of the rate constant at $T \to \infty$ is

(a) 2.0×10^{18} s^{-1}
(b) $6.0 \times 10^{14} s^{-1}$
(c) 3.6×10^{30} s^{-1}
(d) Infinity

94. An ideal gas initially at temperature, pressure and volume, 27°C, 1.00 bar and 10 L, respectively is heated at constant volume until pressure is 10.0 bar, it then undergoes a reversible isothermal expansion until pressure is 1.00 bar, what is the total work *W*, during this process?

(a) -23.02×10^3 J (b) -14.0×10^3 J
(c) 14.0×10^3 J (d) Zero

95. The ratio of the half-life time ($t_{1/2}$), to the three quarter life-time, ($t_{3/4}$), for a reaction that is second order

(a) depends directly on concentration of reactants
(b) is independent of concentration of reactant
(c) depends inversely on the concentration of reactants
(d) depends directly to the square of concentration of reactants

96. The electrode potential, $E°$, for the reduction of MnO_4^- to Mn^{2+} in acidic medium is + 1.51 V. Which of the following metal(s) will be oxidised? The reduction reactions and standard electrode potentials for Zn^{2+}, Ag^+, and Au^+ are given as

$Zn^{2+}(aq) + 2e \longrightarrow Zn(s)$, $E° = -0.762$ V
$Ag^+(aq) + e \rightleftharpoons Ag(s)$, $E° = +0.80$ V
$Au^+(aq) + e \rightleftharpoons Au(s)$, $E° + 1.69$ V

(a) Zn and Au (b) Ag and Au
(c) Au (d) Zn and Ag

97. For the reaction,

$$\frac{1}{2}H_2(g) + \frac{1}{2}Cl_2(g) \longrightarrow H^+(aq) + Cl^-(aq)$$
$$\Delta G°_{recation} = -131.23 \text{ kJ mol}^{-1}$$

The value of $\Delta G°_{formation}$ of $Ag^+(aq)$ shall be given by, (if $\Delta G°_f$ ($H^+ aq$) = 0)

(a) – 54.12 kJ mol^{-1} (b) – 131.23 kJ mol^{-1}
(c) + 77.11 kJ mol^{-1} (d) + 54.12 kJ mol^{-1}

98. The solubility of pure oxygen in water at 20°C and 1.0 atmosphere pressure is 1.38×10^{-3} mol/litre. What will be the concentration of oxygen at 20°C and partial pressure of 0.21 atmosphere?

(a) 2.9×10^{-4} mol/litre (b) 5.8×10^{-4} mol/litre
(c) 7.6×10^{-4} mol/litre (d) 11.6×10^{-4} mol/litre

99. If χ_1 and χ_2 represent the mole fractions of a component *A* in the vapour phase and liquid mixture respectively, and $p°_A$ and $p°_B$ represent vapour pressures of pure *A* and pure *B*, then total vapour pressure of liquid mixture is

(a) $\frac{p°_A \chi_1}{\chi_2}$ (b) $\frac{p°_A \chi_2}{\chi_1}$ (c) $\frac{p°_B \chi_1}{\chi_2}$ (d) $\frac{p°_B \chi_2}{\chi_1}$

100. The K_{sp} of $PbCO_3$ and $MgCO_3$ are 1.5×10^{-15} and 1×10^{-15}, respectively at 298 K. The concentration of Pb^{2+} ions in a saturated solution containing $MgCO_3$ and $PbCO_3$ is

(a) 1.5×10^{-8} M (b) 3×10^{-8} M
(c) 2×10^{-8} M (d) 2.5×10^{-8} M

Mathematics

101. The function $(x^2-9)|x^2-7x+12|+\cos(|x|)$ is not differentiable at

(a) 4 (b) 3 (c) – 3 (d) 0

102. Which statement is true for the line $\frac{x-4}{8}=\frac{y-2}{2}=\frac{z-3}{3}$ and plane having intercepts $-4, 2$ and 3 of the following

(a) line is orthogonal to the plane
(b) line lies in the plane
(c) line makes an acute angle $(\neq 0°)$ with the plane
(d) None of the above

103. Let * be a binary operation on the set R of real numbers defined by $a * b = \frac{3ab}{7}$, then the identity element in R for '*' is

(a) $\frac{3}{7}$ (b) $\frac{3}{14}$
(c) $\frac{2}{3}$ (d) None of these

104. The value of $\sin(2\sin^{-1}0.8)$ is

(a) 0.96 (b) 0.80 (c) 0.64 (d) 0.18

105. Let $f(x)=x^3+x$, then the equation $\frac{2}{y-f(2)}+\frac{3}{y-f(3)}+\frac{4}{y-f(4)}=0$, has

(a) both roots lying in $(f(2), f(3))$
(b) exactly one root lying in $(f(3), f(4))$
(c) exactly one root lying in $(-\infty, f(2))$
(d) exactly one root lying in $(f(4), \infty)$

106. The shortest distance between the parabolas $2y^2=2x-1, 2x^2=2y-1$ is

(a) $2\sqrt{2}$ (b) $\frac{1}{2\sqrt{2}}$ (c) 4 (d) $\sqrt{\frac{36}{5}}$

107. The image of the point $(1,-1,1)$ in the plane $x-2y+3z+1=0$ is

(a) (2, – 3, 4) (b) $\left(0, \frac{-1}{2}, \frac{-2}{3}\right)$
(c) $\left(\frac{-1}{6}, \frac{4}{3}, \frac{-5}{2}\right)$ (d) $\left(\frac{-7}{3}, \frac{-5}{6}, \frac{2}{3}\right)$

108. Let R and S be any two equivalence relations on a set X. Then which of the following is incorrect statement

(a) $R \cup S$ is an equivalence relation on X
(b) R^{-1} is an equivalence relation on X
(c) $R^{-1} \cap S^{-1}$ is an equivalence relation on X
(d) Δ is an equivalence relation on X, where Δ is the diagonal relation on X.

109. If $g(x)=x^2+x-2$ and $\frac{1}{2}(gof)\,x=2x^2-5x+2$, then $f(x)$ is equal to

(a) $2x-3$ (b) $2x+3$ (c) $3x-2$ (d) $2x-2$

110. The graph $y^2+2xy+50|x|=625$ divides the plane into regions. Then, the area of bounded regions is

(a) 500 sq units (b) 1250 sq units
(c) 2500 sq units (d) 800 sq units

111. If four whole numbers taken at random are multiplied together, then the probability that the last digit in the product is 1, 3, 7, or 9, is

(a) $\frac{81}{625}$ (b) $\frac{8}{625}$ (c) $\frac{32}{625}$ (d) $\frac{16}{625}$

112. A region in the xy-plane is bounded by the curve $y=\sqrt{25-x^2}$ and the line $y=0$. If the point $(a, a+1)$ lies in the interior of the region, then

(a) $a \in (-4, 3)$ (b) $a \in (-\infty, -1) \cup (3, \infty)$
(c) $a \in (-1, 3)$ (d) None of these

113. If $A=\begin{pmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & \alpha & 1\end{pmatrix}$, $A^{-1}=\begin{pmatrix}\frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\\ -4 & 3 & \beta\\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2}\end{pmatrix}$, then

(a) $\alpha=2, \beta=-\frac{1}{2}$ (b) $\alpha=1, \beta=-1$
(c) $\alpha=-1, \beta=1$ (d) $\alpha=\frac{1}{2}, \beta=\frac{1}{2}$

114. If A_i $(i=1,2,\dots,n)$ are n independent events, with $P(A_i)=1-\frac{1}{2^i}$, then the probability that atleast one of the n events occurs, is

(a) $\frac{1}{2^{n(n+1)/2}}$ (b) $\frac{1}{2^{(n+1)/2}}$
(c) $1-\frac{1}{2^{n(n+1)/2}}$ (d) $1-\frac{1}{2^{(n+1)/2}}$

115. A straight line has its extremities on two fixed straight lines and cuts off from them a triangle of constant area C^2. Then the locus of the middle point of the line is

(a) $2xy = C^2$ (b) $xy + C^2 = 0$
(c) $4x^2y^2 = C$ (d) None of these

116. Let $a_1, a_2, a_3, \dots, a_n$ be in AP. If

$$\frac{1}{a_1a_n} + \frac{1}{a_2a_{n-1}} + \dots + \frac{1}{a_na_1} = \frac{K}{a_1 + a_n}\left[\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n}\right]$$, then K is equal to

(a) 1 (b) 2 (c) 3 (d) 5

117. If both roots of the equation $x^2 - 2(a-1)x + (2a+1) = 0$ are positive, where a is a real number, then

(a) $a \in (4, \infty)$ (b) $a \in (-\infty, 0] \cup [4, \infty]$
(c) $a \in (1, \infty)$ (d) $a \in [4, \infty)$

118. Let A be an event that a family has children of both sexes and B be the event that the family has atmost one boy. If the family has 3 children then the events A and B are

(a) dependent (b) independent
(c) mutually exclusive (d) None of these

119. A ray of light passing through the point $(1, 2)$ reflects on the X-axis at point A and the reflected ray passes through the point $(5, 3)$. The coordinate of A is

(a) $\left(\frac{5}{13}, 0\right)$ (b) $\left(\frac{13}{5}, 0\right)$ (c) $\left(\frac{-5}{13}, 0\right)$ (d) $\left(\frac{-13}{5}, 0\right)$

120. The solution of the differential equation $\frac{dy}{dx} = \frac{2xy - y^2}{2xy - x^2}$, is

(a) $xy(x + y) = C$ (b) $xy(x - y) = C$
(c) $x^2y(x - y) = C$ (d) $x^3y(x - y) = C$

121. Consider the following relations in the real numbers

$R_1 = \{(x, y) \mid x^2 + y^2 \le 25\}$

$R_2 = \left\{(x, y)\ y \ge \frac{4x^2}{9}\right\}$,

then the range of $R_1 \cap R_2$ is

(a) [0, 5] (b) [– 3, 3] (c) [– 5, 5] (d) [– 3, 5]

122. The solution of the inequality $|x^2 - 4x| < 5$ is

(a) (– 1, 5) (b) (– 4, 5) (c) (– 5, 4) (d) (– 1, 4)

123. A five-digit number divisible by 3 is to be formed using the numbers 0, 1, 3, 4 and 5 without repetition. The total number of ways this can be done is

(a) 216 (b) 600 (c) 240 (d) 3125

124. The coefficient of x^{53} in the expansion $\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} \cdot 2^m$ is

(a) ${}^{100}C_{47}$ (b) ${}^{100}C_{53}$ (c) ${}^{-100}C_{53}$ (d) ${}^{-100}C_{100}$

125. A fair coin is tossed n times. Let the random variable X denote the number of times the head occurs. If $P[X = 1]$, $P[X = 2]$ and $P[X = 3]$ are in arithmetic progression (AP), then the number n of independent trial is

(a) 7 (b) 10 (c) 12 (d) 14

126. The minimum value of $z = 2x_1 + 3x_2$ subject to the constraints $2x_1 + 7x_2 \ge 22$, $x_1 + x_2 \ge 6, 5x_1 + x_2 \ge 10$ and $x_1, x_2 \ge 0$ is

(a) 14 (b) 20 (c) 10 (d) 16

127. $y = a\cos(\log x) + b\sin(\log x)$ is a solution of the differential equation

(a) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ (b) $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$
(c) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$ (d) $x\frac{d^2y}{dx^2} + x^2\frac{dy}{dx} + y = 0$

128. The amplitude of the complex number $1 + \sin\alpha - i\cos\alpha$ is

(a) $\frac{\pi}{4}$ (b) $\alpha - \frac{\pi}{4}$ (c) $\frac{\alpha}{2} - \frac{\pi}{4}$ (d) $\frac{\pi}{4} - \alpha$

129. The points $z_1 = x + iy$ and $z_2 = \frac{1}{-x + iy}$ in the complex plane lie on

(a) a circle with centre origin
(b) a straight line through origin
(c) axis of X
(d) axis of Y

130. The area of the region defined by $||x| - |y|| \le 1$ and $x^2 + y^2 \le 1$ in the xy-plane is

(a) π (b) 1
(c) 2 (d) None of these

131. The solution set of the inequality $\log_{\sin(\pi/3)}(x^2 - 3x + 2) \geq 2$ is

(a) $\left(\frac{1}{2}, 2\right)$ (b) $\left[\frac{1}{2}, 2\right]$

(c) $\left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$ (d) $\left(\frac{1}{2}, \frac{5}{2}\right)$

132. If S and S' are the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{26} = 1$, and P is any point on it then range of values of $SP \cdot S'P$ is

(a) $9 \leq f(\theta) \leq 16$ (b) $9 \leq f(\theta) \leq 25$

(c) $16 \leq f(\theta) \leq 25$ (d) $1 \leq f(\theta) \leq 16$

133. The length of perpendicular from the point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ is

(a) 6 (b) 7 (c) $\sqrt{17}$ (d) $\sqrt{14}$

134. Area of one loop formed by $|y| = |\sin x|$ is

(a) 0 (b) 2 (c) 4 (d) 2π

135. Let α_r be the *rth* term of an AP, whose first term is a and common difference is d. If for some positive integers $m, n, m \neq n$, $\alpha_m = \frac{1}{n}$ and $\alpha_n = \frac{1}{m}$, then $a - d$ equals

(a) $\frac{1}{mn}$ (b) 1 (c) 0 (d) $\frac{1}{m} + \frac{1}{n}$

136. If $\int_0^\infty [3e^{-x}]\, dx = l$, where [.] denotes the greatest integer function, then the value of l is

(a) 0 (b) ln3 (c) e^3 (d) $3e^{-1}$

137. If A and B are disjoint sets, then $B \cap A'$ where A' is complement of A is equal to

(a) A (b) B (c) A' (d) B'

138. Which of the following is an incorrect statement?

(a) $n^3 + 3n^2 + 5n + 3$ is divisible by 3 for all $n \in$ IN

(b) $n(n+1)(2n+1)$ is divisible by 6 for all $n \in$ IN

(c) $n^2 - n + 41$ is a prime number for all $n \in$ IN

(d) $7^n - 3^n$ is divisible by 4 for all $n \in$ IN

where IN denotes the set of all natural numbers.

139. The angle between the line $x - 2y + z = 0 = x + 2y - 2z$ and the plane $5x - 2y - z + 17 = 0$ is

(a) 30° (b) 60°

(c) 90° (d) 0°

140. The unit vector which is orthogonal to the vector $\hat{i} + \hat{j} + \hat{k}$ and is coplanar with vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $2\hat{i} + \hat{j} + 3\hat{k}$, is

(a) $\frac{\hat{i} + 5\hat{j} - 6\hat{k}}{\sqrt{62}}$ (b) $\frac{\hat{i} + 3\hat{j} - \hat{k}}{\sqrt{11}}$

(c) $\frac{\hat{i} + 7\hat{j}}{\sqrt{50}}$ (d) $\frac{\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{6}}$

141. If $\lim_{x \to 0} |x|^{[\cos x]} = l$; where [.] denotes the greatest integer function, then the value of l is

(a) 1 (b) – 1

(c) 0 (d) does not exist

142. If $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$, then $\sin x$ is equal to

(a) $\tan^2(\alpha/2)$ (b) $\cot^2(\alpha/2)$

(c) $\tan\alpha$ (d) $\cot(\alpha/2)$

143. The area bounded by the curve $y = \begin{cases} x^{1/\ln x}, & x \neq 1 \\ e, & x = 1 \end{cases}$ and $y = |x - e|$ is

(a) $e^2/2$ (b) e^2

(c) $2e^2$ (d) 1

144. The distance of point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ with the plane $x - y + z = 5$ from the point with position vector $\hat{i} - 2\hat{j} + 3\hat{k}$ is

(a) $\sqrt{14}$ (b) $\sqrt{42}$

(c) $3\sqrt{14}$ (d) $\sqrt{3}$

145. Let $0 < \alpha < \pi$, $0 < \beta < \pi$ and $\cos\alpha + \cos\beta - \cos(\alpha + \beta) = \frac{3}{2}$. Then the relation between α and β will be

(a) $\alpha = \beta$ (b) $\alpha > \beta$

(c) $\alpha < \beta$ (d) $\alpha = \frac{\beta}{3}$

146. The equation $|x| + \left|\frac{x}{x-1}\right| = \frac{x^2}{|x-1|}$ will be always true for x, belonging to

(a) [0, 1) (b) $\{0\} \cup (1, \infty)$
(c) (– 1, 1) (d) $(-\infty, \infty)$

147. Let $\frac{\sin(\theta-\alpha)}{\sin(\theta-\beta)} = \frac{a}{b}, \frac{\cos(\theta-\alpha)}{\cos(\theta-\beta)} = \frac{c}{d}$. Then the value of $\cos(\alpha-\beta)$ equals

(a) $\frac{ac-bd}{ad+bc}$ (b) $\frac{ac+bd}{ad+bc}$
(c) $\frac{ac+bd}{ab+cd}$ (d) $\frac{ac-bd}{ab+cd}$

148. If R is the set of real numbers and $f: R \to R$ is a function defined by $f(x) = \sin x$, then $f^{-1}([-1,1])$ is

(a) $\{x \mid x = n\pi, n \text{ is an integer}\}$
(b) $\{x \mid x = \pi/2 + 2n\pi, n \text{ is an integer}\}$
(c) R
(d) null set ϕ

149. If a, b, c are the integers between 1 and 9 and $a51, b41, c31$ are three-digit numbers and the value of determinant $D = \begin{vmatrix} 5 & 4 & 3 \\ a51 & b41 & c31 \\ a & b & c \end{vmatrix}$ is zero, then a, b, c are

(a) in GP
(b) in AP
(c) equal
(d) None of the above

150. If $A = \begin{bmatrix} k & l \\ m & n \end{bmatrix}$ and $k.n \neq lm$, then the value of $A^2 - (k+n)A + (kn - lm)I$ equals

(a) The zero matrix of order 2×2
(b) A
(c) $-A$
(d) $2A$

where I is the identify matrix of order 2×2.

Answers

Physics

1.	(a)	2.	(a)	3.	(b)	4.	(a)	5.	(a)	6.	(d)	7.	(a)	8.	(d)	9.	(b)	10.	(b)
11.	(d)	12.	(a)	13.	(b)	14.	(b)	15.	(b)	16.	(b)	17.	(b)	18.	(c)	19.	(b)	20.	(b)
21.	(b)	22.	(b)	23.	(a)	24.	(d)	25.	(c)	26.	(b)	27.	(b)	28.	(d)	29.	(c)	30.	(a)
31.	(b)	32.	(a)	33.	(b)	34.	(c)	35.	(d)	36.	(c)	37.	(a)	38.	(c)	39.	(d)	40.	(c)
41.	(d)	42.	(c)	43.	(d)	44.	(b)	45.	(a)	46.	(d)	47.	(a)	48.	(b)	49.	(a)	50.	(b)

Chemistry

51.	(a)	52.	(b)	53.	(d)	54.	(b)	55.	(b)	56.	(c)	57.	(b)	58.	(c)	59.	(b)	60.	(c)
61.	(a)	62.	(b)	63.	(d)	64.	(c)	65.	(d)	66.	(c)	67.	(d)	68.	(a)	69.	(c)	70.	(b)
71.	(a)	72.	(b)	73.	(c)	74.	(a)	75.	(c)	76.	(a)	77.	(c)	78.	(b)	79.	(a)	80.	(d)
81.	(d)	82.	(a)	83.	(b)	84.	(c)	85.	(a)	86.	(c)	87.	(a)	88.	(d)	89.	(a)	90.	(b)
91.	(a)	92.	(c)	93.	(b)	94.	(a)	95.	(b)	96.	(d)	97.	(b)	98.	(a)	99.	(b)	100.	(b)

Mathematics

101.	(a)	102.	(b)	103.	(d)	104.	(a)	105.	(b)	106.	(b)	107.	(*)	108.	(a)	109.	(a)	110.	(b)
111.	(d)	112.	(c)	113.	(b)	114.	(c)	115.	(a)	116.	(b)	117.	(d)	118.	(b)	119.	(b)	120.	(b)
121.	(a)	122.	(a)	123.	(a)	124.	(c)	125.	(a)	126.	(a)	127.	(a)	128.	(c)	129.	(b)	130.	(a)
131.	(c)	132.	(c)	133.	(b)	134.	(c)	135.	(c)	136.	(b)	137.	(b)	138.	(c)	139.	(d)	140.	(a)
141.	(a)	142.	(a)	143.	(b)	144.	(d)	145.	(a)	146.	(b)	147.	(b)	148.	(c)	149.	(b)	150.	(a)

Note : (*) None option is correct.

Answer *with* Solutions

Physics

1. ***(a)*** Since, we know that

1 light year = 9.46×10^{15} m

$\therefore$ 4.3 light year = $4.3 \times 9.46 \times 10^{15}$ m

and 1 Parsec = 3.1×10^{16} m

$$\therefore 4.3 \text{ light year} = \frac{4.3 \times 9.46 \times 10^{15}}{3.1 \times 10^{16}} \text{ Parsec}$$
$$= 13.12 \times 10$$
$$= 1.3 \text{ Parsec}$$

2. ***(a)*** Since, we know that,

Nuclear forces are responsible for keeping the nucleons (neutron and proton) bound in a nucleus. The range of nuclear forces very short 10^{-15} m (1-fm).

Nuclear forces are, one an average much stronger than electromagnetic forces 50-60 times stronger in their range.

Nuclear forces are strongest of all the four fundamental forces.

3. ***(b)*** As we know that from acceleration-time graph Speed = Area under acceleration-time graph. From the graph, as given in the question

Area is maximum for point 0 to point 2

Therefore, the speed of the body is maximum (or largest) at point 2.

4. ***(a)*** Given,

Displacement of the body at time 't'

$$x = 4t + 5t^3 \quad \ldots(i)$$

$\therefore$ $t = 1$ s

$$x_1 = 4 \times 1 + 5(1)^3 = 9$$

and $t = 2$s,

$$x = 4 \times 2 + 5 \times (2)^3 = 48$$

$\therefore$ Displacement of the body from $t = 1$s to $t = 2$s,

$$\Delta x = x_2 - x_1 = 48 - 9 = 39$$

$$\therefore \quad v_{avg} = \frac{\Delta x}{\Delta t} = \frac{39}{2-1} = 39 \text{ m/s}$$

From Eq. (i),

$$x = 4t + 5t^3$$

$$\therefore \quad \frac{dx}{dt} = v = 4 + 15t^2$$

v_{inst} at $t = 1$ s

$$\therefore \quad v_{inst} = 4 + 15 \times (1)^2 = 19 \text{ m/s}$$

Thus, $v_{avg} - v_{inst} = 39 - 19 = 20$ m/s

5. ***(a)*** Given that,

$$t = \alpha x^2 + \beta x$$

Differentiating with respect to 't'

$$1 = \alpha \cdot \frac{2x dx}{dt} + \beta \frac{dx}{dt}$$
$$1 = 2\alpha x v + \beta v$$
$$1 = v(2\alpha x + \beta)$$
$$v = \frac{1}{2\alpha x + \beta} \quad \ldots(i)$$

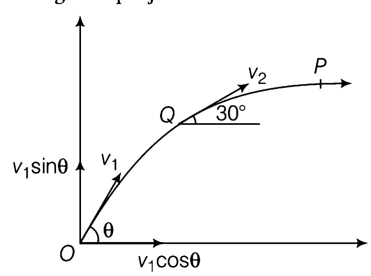
Retardation, $a = -\dfrac{dv}{dt}$

$$= -\frac{d}{dt}\left(\frac{1}{2\alpha x + \beta}\right)$$
$$= \left(-\frac{1}{(2\alpha x + \beta)^2}\right) \cdot \left(-2\alpha \frac{dx}{dt}\right)$$
$$= \frac{1}{(2\alpha x + \beta)^2} \cdot 2\alpha v$$

From Eq. (i), we get

$$= v^2 \cdot 2\alpha v$$
$$= 2\alpha v^3$$

6. ***(d)*** Let v_1 be the initial speed at 'O' and 'θ' be the initial angle of projection.



After two seconds particle reaches at Q and $\theta_1 = 30°$.

Hence, $v_2 \cos 30° = v_1 \cos\theta$

$$v_2 \cdot \frac{\sqrt{3}}{2} = v_1 \cos\theta$$
$$v_2 = \frac{2}{\sqrt{3}} v_1 \cos\theta \quad \ldots(i)$$

By equation of motion, when particle reaches from O to Q,

$$v_2 \sin 30^\circ = v_1 \sin\theta - 2g$$

$$\frac{v_2}{2} = v_1 \sin\theta - 2g$$

$$\frac{1}{2}\cdot\frac{2}{\sqrt{3}} v_1 \cos\theta = v_1 \sin\theta - 2g$$

$$v_1 \cos\theta = \sqrt{3} v_1 \sin\theta - 2\sqrt{3} g \quad \ldots\text{(ii)}$$

After 3 seconds, i.e. at top position, particle moves in horizontal direction, hence vertical component is zero.

$$\therefore \quad 0 = v_1 \sin\theta - g \times t$$

$$0 = v_1 \sin\theta - 3g$$

or
$$v_1 = \frac{3g}{\sin\theta} \quad \ldots\text{(iii)}$$

Putting the value of v_1 from Eq (iii) to Eq (ii), we have,

$$3g\frac{\cos\theta}{\sin\theta} = \sqrt{3}\frac{3g}{\sin\theta}\cdot\sin\theta - 2\sqrt{3}g$$

$$3g\cot\theta = 3\sqrt{3}g - 2\sqrt{3}g$$

$$3g\cot\theta = \sqrt{3}g$$

$$\cot\theta = \frac{\sqrt{3}g}{3g}$$

$$\cot\theta = \frac{1}{\sqrt{3}}$$

$$\cot\theta = \cot 60^\circ$$

$$\therefore \quad \theta = 60^\circ$$

$\therefore$ From Eq (iii),

$$v_1 = \frac{3g}{\sin 60^\circ} = \frac{3g}{\frac{\sqrt{3}}{2}}$$

$$= 2\sqrt{3}\,g$$

$$= 2\sqrt{3} \times 10$$

$$= 20\sqrt{3} \text{ m/s}$$

7. ***(a)*** Since, we know that from a position-time graph of a body.

Velocity of the body, v = slope of position-time graph.

From the graph, $v = \frac{\Delta x}{\Delta t} = \frac{3}{2}$

$$= \frac{3}{2} \text{ m/s}$$

Since, we know that

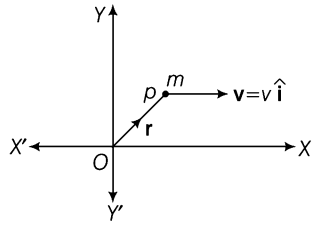
Impulse = Change in linear momentum

$$= m.v$$

$$= 4 \times \frac{3}{2} = 6 \text{ kg-ms}^{-1}$$

8. ***(d)*** According to the question,

Given, mass of the particle = m



$\therefore$ Linear momentum of the particle, $\mathbf{p} = m\mathbf{v}$

or $= mv\hat{\mathbf{i}}$

Position of particle at time t, $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$

$= vt\hat{\mathbf{i}} + y\hat{\mathbf{j}}$ $\quad (\because x = vt)$

$\therefore$ Angular momentum of the particle about O.

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = (vt\hat{\mathbf{i}} + y\hat{\mathbf{j}}) \times mv\hat{\mathbf{i}}$$

$$\mathbf{L} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ vt & y & 0 \\ v & 0 & 0 \end{vmatrix}$$

$$\mathbf{L} = \hat{\mathbf{i}}(0-0) - \hat{\mathbf{j}}(0-0) + \hat{\mathbf{k}}(0 - vy)$$

$$= -vy\hat{\mathbf{k}} \text{ (constant)}$$

($\because$ Particle is moving in +ve x-direction.)

Hence, angular momentum of particle w.r.t the origin remains constant for all positions of the particle.

9. ***(b)*** Given, mass of the ball, $m = 3$ kg, $\Delta v = 5$ m/s and the time taken the man to throw a ball,

$$\Delta t = \frac{2}{4} = 0.5 \text{ s}$$

$\therefore$ Change in momentum of the ball, $\Delta p = m\cdot\Delta v$

$$= 3 \times 5 = 15 \text{ N-s}$$

Since, we know,

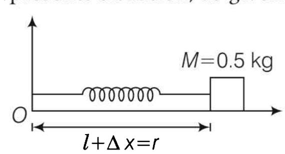
Force = Rate of change in linear momentum

$$F = \frac{\Delta p}{\Delta t} = \frac{15}{0.5} \quad \text{(Putting values)}$$

$$= \frac{15}{5} \times 10 = 30 \text{ N}$$

10. ***(b)*** Given, $M = 0.5$ kg, $\omega = 2$ rad/s, $l = 0.49$ m, and $k = 100$ N/m

Figure represents situation, as given in question.



Centripetal force on the blocks = Spring force

$$\Rightarrow \quad Mr\omega^2 = k\Delta x$$

$$\Rightarrow \quad 0.5(l + \Delta x)(2)^2 = 100\cdot\Delta x$$

$\Rightarrow \quad (0.49 + \Delta x)4 = 100 \cdot \Delta x \times 2$

$\Rightarrow \quad 0.49 + \Delta x = \frac{200}{4}\Delta x$

$\Rightarrow \quad 0.49 + \Delta x = 50\Delta x$

$\therefore \quad 49\Delta x = 0.49$

$\Delta x = 0.01 \text{ m} = 1 \text{ cm}$

11. ***(d)*** As we know, for a conservative force field

$$dU = -\mathbf{F}_{con}\, d\mathbf{r}$$

where, dU = change in potential energy,

$\mathbf{F}_{con}$ = conservative force (or $\mathbf{F}_{in}$)

and $d\mathbf{r}$ = change in position of the particle.

$$dU = -\mathbf{F}_{in} d\mathbf{r} \text{ or } \Delta U = -\int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F}_{in}\, d\mathbf{r}$$

$$U_2 - U_1 = -\int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F}_{in}\, d\mathbf{r} = -\text{ work done by } \mathbf{F}_{in} \text{ (or } W_{in})$$

For graph (i), $W_{in} = \frac{F_1 \cdot x_1}{2}$

For graph (ii), $W_{in} = F_1 \cdot x_1$

and for graph (iii), $W_{in} = \frac{-F_1 \cdot x_1}{2}$

Thus, change in potential energy

For graph (i), $\Delta U_1 = \frac{-F_1 x_1}{2}$ graph (ii) $\Delta U_2 = -F_1 x_1$

graph, (iii) $\Delta U_3 = \frac{F_1 x_1}{2}$

Thus, we have,

$\Delta U_2 < \Delta U_1 < \Delta U_3$ (according to the question)

or (ii) < (i) < (iii)

12. ***(a)*** Given, $m = 22$ kg,

$v_1 = 4$ m/s and $v_2 = 6$ m/s $\quad F = 5$ N

According to work-energy theorem,

$$\int \mathbf{F}_{net}\, d\mathbf{x} = \Delta\text{KE} = (\text{KE})_2 - (\text{KE})_1$$

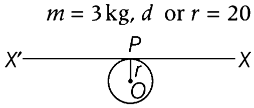
$$= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$= [(6)^2 - (4)^2]\frac{2}{2} \quad \text{[putting values]}$$

$$= [36 - 16] = 20 \text{ J}$$

13. ***(b)*** Given, $\quad T = 2\pi\sqrt{\frac{I}{mg.d}} \quad$ ...(i)

$m = 3$ kg, d or $r = 20$ cm $= 0.2$ m



Moment of inertia of the ring about XX'

$$I = I_0 + mr^2 = mr^2 + mr^2 = 2mr^2$$

$$= 2 \times 3 \times (0.2)^2 = 0.24 \text{ kg-m}^2$$

Putting values in Eq (i)

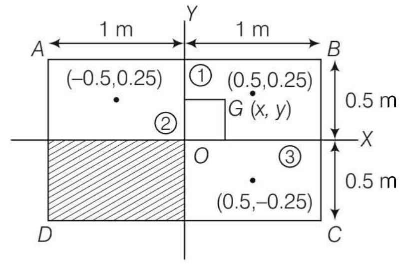
$$T = 2\pi\sqrt{\frac{0.24}{3 \times 10 \times 0.2}} = 2\pi\sqrt{0.04}$$

$$= 2\pi \times 0.2 = 1.2566 \text{ s}$$

$$T \approx 1.3 \text{ s}$$

14. ***(b)*** Given,

$AB = 2BC = 2$ m



$\therefore \quad BC = 1$ m

σ be the mass per unit area.

$$m_1 = m_2 = m_3 = (1 \times 0.5)\sigma = 0.5\sigma$$

If $G(\bar{x}, \bar{y})$ be the position of centre of mass, then

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{0.5\sigma \times 0.5 + 0.5\sigma \times (-0.5) + 0.5\sigma \times 0.5}{0.5\sigma + 0.5\sigma + 0.5\sigma}$$

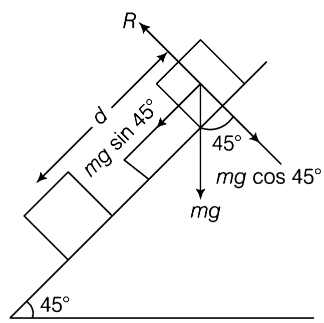
$$= \frac{0.5\sigma \times 0.5}{3 \times 0.5\sigma} = \frac{1}{6} \text{ m}$$

$$\bar{y} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$= \frac{0.5\sigma \times 0.25 + 0.5\sigma \times 0.25 + 0.5\sigma \times (-0.25)}{0.5\sigma + 0.5\sigma + 0.5\sigma}$$

$$= \frac{0.5\,\sigma \times 0.25}{3 \times 0.5\,\sigma} = \frac{1}{12} \text{ m}$$

15. ***(b)*** Figure represents situation, as given in question,



Smooth inclined plane

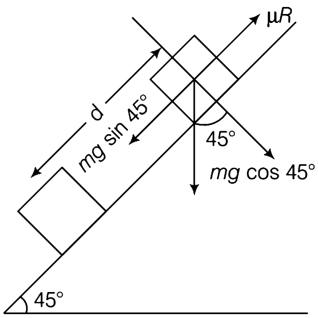
When body slides on smooth inclined plane at a distance 'd', then time taken by the body is t,

$$d = 0 + \frac{1}{2} g \sin 45^\circ t^2$$

$$d = \frac{gt^2}{2\sqrt{2}} \quad \ldots(i)$$

When body slides on rough incline plane, then time taken by body to slide same distance 'd' is nt,

Acceleration of body on rough plane,



$$a' = g \sin 45^\circ - \mu g \cos 45^\circ$$

$$= \frac{g}{\sqrt{2}} - \frac{\mu g}{\sqrt{2}} = \frac{g}{\sqrt{2}}(1 - \mu)$$

By equation of motion,

$$\therefore \quad d = 0 + \frac{1}{2} a' (nt)^2$$

$$d = \frac{1}{2} \cdot \frac{g}{\sqrt{2}} (1 - \mu) n^2 t^2 \quad \ldots(ii)$$

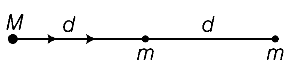
From Eqs. (i) and Eqs. (ii), we get

$$\frac{gt^2}{2\sqrt{2}} = \frac{g}{2\sqrt{2}} (1 - \mu) n^2 t^2$$

$$1 = (1 - \mu) n^2$$

$$\Rightarrow \quad \mu = 1 - \frac{1}{n^2}$$

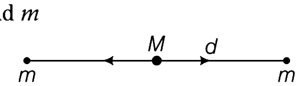
16. ***(b)*** For configuration (i), gravitational force on M due to m and m



$$F_1 = \frac{GMm}{d^2} + \frac{GMm}{(2d)^2} = \frac{GMm}{d^2}\left[1 + \frac{1}{4}\right]$$

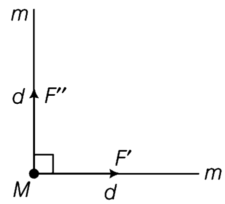
$$= \frac{GMm}{d^2} \cdot \frac{5}{4} \quad \ldots(i)$$

For configuration (ii), gravitational force on M due to m and m



$$F_2 = \frac{GMm}{d^2} - \frac{GMm}{d^2} = 0 \quad \ldots(iii)$$

For configuration (iii), gravitational force on M

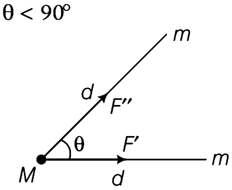


$$F_3 = \sqrt{(F')^2 + (F'')^2}$$

($\because$ angle between F' and F'' is 90°)

$$= \sqrt{\left(\frac{GMm}{d^2}\right)^2 + \left(\frac{GMm}{d^2}\right)^2} = \frac{GMm}{d^2}\sqrt{2} \quad \ldots(iii)$$

For configuration (iv), gravitational force on M, where $0 < \theta < 90^\circ$



Gravitational force on M,

$$F_4 = \sqrt{(F')^2 + (F'')^2 + 2F'F''\cos\theta}$$

$$= \sqrt{\left(\frac{GMm}{d^2}\right)^2 + \left(\frac{GMm}{d^2}\right)^2 + 2\frac{GMm}{d^2} \cdot \frac{GMm}{d^2}\cos\theta}$$

$$= \frac{GMm}{d^2}\sqrt{1 + 1 + 2\cos\theta}$$

$$= \frac{GMm}{d^2}\sqrt{2(1 + \cos\theta)}$$

$$= \left(\frac{GMm}{d^2}\sqrt{2}\right)\sqrt{1 + \cos\theta} \quad \ldots(iv)$$

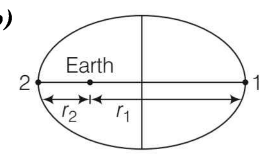
($\because$ *for* $0 < \theta < 90^\circ, \sqrt{1 + \cos\theta} >$)

From Eqs. (i), (ii), (iii) and (iv), we get

$$F_2 < F_1 < F_3 < F_4$$

Thus, the correct option is (b).

17. ***(b)***



$L = mvr$ (angular momentum)

$K = \frac{1}{2}mv^2$ (kinetic energy)

By Kepler's law, when a satellite is moving around the earth on elliptical path, then its angular momentum remains constant.

i.e. $L_1 = L_2$

$$m_1 v_1 r_1 = m_2 v_2 r_2$$

But, $m_1 = m_2 = m$

$\therefore$ $v_1 r_1 = v_2 r_2$

$$\frac{r_1}{r_2} = \frac{v_2}{v_1} \quad \dots\text{(i)}$$

Here, $r_1 > r_2$

$$\frac{r_1}{r_2} > 1$$

$\therefore$ From Eq. (i),

$$\frac{v_2}{v_1} > 1$$

$$v_2 > v_1$$

$$v_2^2 > v_1^2 \text{ or } \frac{1}{2}mv_2^2 > \frac{1}{2}mv_1^2$$

$$K_2 > K_1 \quad \left(\because K = \frac{1}{2}mv^2\right)$$

18. ***(c)*** If r be the radius of solid sphere having mass M and R be the radius of solid sphere having mass $8\,M$.

Mass of small solid sphere $= \frac{1}{8} \times$ Mass of large solid sphere

$$\frac{4}{3}\pi r^3 \rho = \frac{1}{8}\cdot\frac{4}{3}\pi R^3 \rho$$

$$r^3 = \left(\frac{R}{2}\right)^3$$

$$r = \frac{R}{2}$$

Terminal velocity of small solid sphere

$$v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta} \quad \dots\text{(i)}$$

Terminal velocity of large solid sphere

$$3nv = \frac{2}{9}\frac{R^2(\rho - \sigma)g}{\eta} \quad \dots\text{(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{v}{nv} = \frac{\left[\frac{2}{9}r^2(\rho - \sigma)g\right]/\eta}{\left[\frac{2}{9}R^2(\rho - \sigma)g\right]/\eta}$$

$$\frac{1}{n} = \frac{r^2}{R^2} = \frac{(R/2)^2}{R^2} = \frac{1}{4}$$

$$n = 4$$

19. ***(b)*** Since we know,

The density of ice < The density of water.

For the bucket A, when the piece of ice melt it's water level will decrease.

For the bucket B, when ice melts it's water level remain unchanged. Thus, the correct option is (b).

20. ***(b)*** An intensive property is that which does not depend on system size or amount of material in the system. So, pressure and temperature does not depend on system size, hence these are intensive properties.

21. ***(b)*** From the graph of an ideal gas given in the question, the process is an isothermal expansion because volume of the ideal gas increases.

Given, $V_B > V_A$ and $p_B < p_A$

$\because$ $\Delta T = 0$ (isothermal process)

$\therefore$ Change in internal energy,

$$\Delta U = 0 \quad (\because \Delta U \propto \Delta T)$$

Since we know, in isothermal process

$$\Delta Q = \Delta W = n\cdot RT \ln\left(\frac{V_f}{V_p}\right)$$

or $$\Delta Q = \Delta W = nRT \ln\left(\frac{V_B}{V_A}\right)$$

Thus, $\Delta Q = +$ ve and $\Delta W = +$ ve because $V_B > V_A$, (from the graph)

22. ***(b)*** In the question, the given curves represents the behaviour of gases at different temperatures.

where, (i) $T_2 > T_3 > T_4$

(ii) $\because$ For ideal gas, $\frac{pV}{nRT} = 1$, hence curve I represent the ideal gas behaviour of the gas.

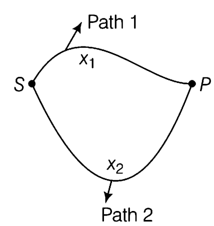
(iii) The curves II, III and IV represent the behaviour of real gases.

Thus, option (b) is correct.

23. ***(a)*** Given, path difference, $\Delta x = 1.8$ m

$\nu_1 = 1000$ Hz, $v_{sound} = 360$ m/s

According to the question,



For maximum intensity of sound at point P,
Path difference, $x_2 - x_1 = n\lambda$
or $\Delta x = n\lambda$ [where, λ = wavelength of the sound]

$$\Rightarrow \quad \Delta x = n\frac{v}{\nu} \quad \text{...(i)}$$

$$\left[\because \lambda = \frac{v}{\nu} \text{ (frequency)}\right]$$

For Ist case, $\nu_1 = n \times \frac{360}{1.8}$

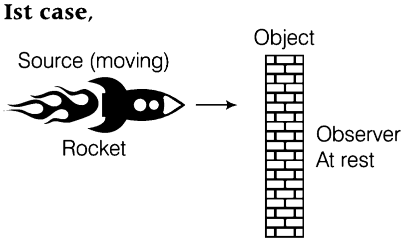
$$1000 = n \times \frac{360}{1.8} \quad \text{...(ii)}$$

For IInd case, $\nu_2 = (n+1)\frac{360}{1.8}$ ...(iii)

$$\nu_2 = n \cdot \frac{360}{1.8} + \frac{360}{1.8}$$

$$= 1000 + 200 = 1200 \text{ Hz [From Eq. (ii)]}$$

24. ***(d)*** Given, speed of rocket, $v_1 = 200$ m/s
Frequency emitted by the rocket, $\nu_1 = 1000$ Hz
Velocity of sound, $v_{\text{sound}} = 330$ m/s
In Ist case,



Apply Doppler's effect in sound's equation.

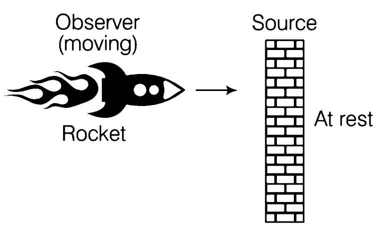
$$\nu = \frac{v + u_o}{v - u_s}$$

Putting values,

$$\nu_1 = \frac{330 + 0}{330 - 200} \times 1000$$

$$\nu_1 = \frac{33}{13} \times 1000 \text{ Hz} \quad \text{...(i)}$$

This frequency is reflected back to the rocket.
In IInd case.



Apply Doppler's effect in sound's equation

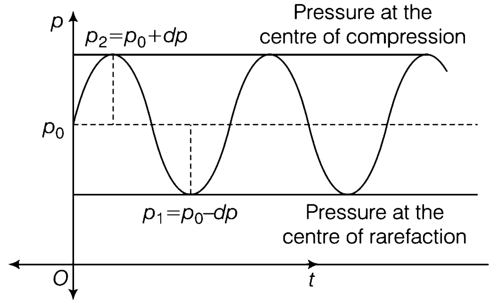
$$\nu_2 = \frac{330 + 200}{330}\nu_1$$

$$= \frac{530}{330} \times \frac{33}{13} \times 1000 \quad \text{[from Eq. (i)]}$$

$$= \frac{53}{13} \times 1000 = 4076.92 \text{ Hz}$$

Thus, the most appropriate option is (d).

25. ***(c)*** Given, air pressure, $p_0 = 10135\, p_a$ and pressure at the centre of rarefaction, $p_1 = 91000$ Pa
As we know that, during the propagation of sound wave in air pressure varies as



Thus, we have

$$p_1 = p_0 - dp$$

$$91000 = 101325 - dp \quad \text{(putting values)}$$

$$\therefore \quad dp = 101325 - 91000$$

$$= 10325 \text{ Pa}$$

Thus, the pressure at the centre of compression.

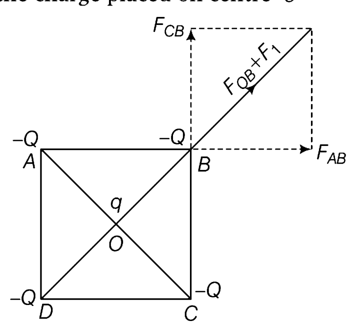
$$p_1 = p_0 + dp$$

$$= 101325 + 10325$$

$$= 111650 \text{ Pa}$$

Therefore, the most appropriate option is (c).

26. ***(b)*** Four charges having magnitude $(-Q)$ are placed on the four corner of square having side 'a' each.
q be the charge placed on centre 'O'



Electrostatic force between charges at A and B

$$F_{AB} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q^2}{a^2}$$

Similarly,

$$F_{BC} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{a^2}$$

and $$F_{DB} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{(\sqrt{2}a)^2}$$

$$[\because DB = \sqrt{a^2 + a^2} = \sqrt{2}a]$$

$$F_{DB} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{2a^2}$$

Total force on the charge $(-Q)$ at B due to charge at A, C and D.

$$F_2 = F_1 + F_{DB}$$

$\therefore$ (where, F_1 is the resultant force of F_{AB} and F_{BC})

$$= \sqrt{F_{AB}^2 + F_{BC}^2} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{2a^2}$$

$$F_2 = \sqrt{\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{a^2}\right)^2 + \left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{a^2}\right)^2} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{2a^2}$$

$$F_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\sqrt{2}Q^2}{a^2} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{2a^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{a^2}\left[\sqrt{2} + \frac{1}{2}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{a^2}\left[\frac{2\sqrt{2}+1}{2}\right] \quad \ldots(i)$$

Electrostatic force between charge q at O and $(-Q)$ at B

$$F_{OB} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{\left(\frac{\sqrt{2}a}{2}\right)^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{\frac{a^2}{2}}$$

$$F_{OB} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Qq}{a^2} \quad \ldots(ii)$$

Since, system of charge are in equilibrium, hence

$$F_{OB} = F_2$$

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{2Qq}{a^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{a^2}\left(\frac{2\sqrt{2}+1}{2}\right)$$

$$2q = Q\left(\frac{2\sqrt{2}+1}{2}\right)$$

$$q = \frac{Q}{4}(2\sqrt{2}+1)$$

27. ***(b)*** The region in which electric lines of forces are closer, have more value of electric field than the region in which electric lines of forces are farther.

(i) Electric lines of forces are closer and uniform, so acceleration on proton is increasing till B, hence proton has maximum velocity at B, therefore p_1 is maximum.

(ii) In this figure, electric lines of forces are going away from each other, hence electric field continuously decreases, so p_2 is moderate at B.

(iii) In this figure, electric lines of forces are farthest among all three figures, hence acceleration on proton is minimum due to weakest electric field, hence p_3 is minimum at B.

$\therefore$ $$p_1 > p_2 > p_3$$

28. ***(d)*** From the graph, given in the question.

The slope of the electric potential *versus* distance graph,

$$\frac{dV}{dx} = \frac{-500}{20\times10^{-2}} \text{ V/m}$$

$$= -2.5\times10^3 \text{ V/m}$$

Since, we know that

Electric field, $E = -\dfrac{dV}{dx}$ (at a point)

$$E = 2.5\times10^3 \text{ V/m}$$

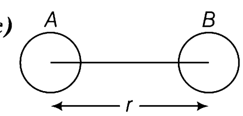
$\therefore$ The electric force on the electron at that point

$$F = eE$$

$$= 1.6\times10^{-19}\times2.5\times10^3$$

$$= 4\times10^{-16} \text{ N}$$

29. ***(c)***



r = distance between two identical spheres,

and Q = charge on spheres A and B.

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{r^2} \quad \ldots(i)$$

When a third identical sphere 'C' is first touches with A, then the value of charges on A and C will be $\frac{Q}{2}$.

Again, when C touches with B, then value of charge on B and $C = \dfrac{\frac{Q}{2}+Q}{2} = \dfrac{3Q}{4}$

Now, the electrostatic force between spheres A and B

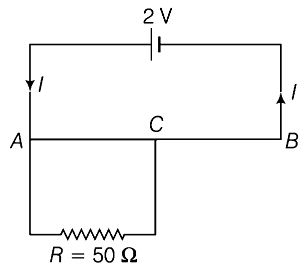
$$F' = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\frac{Q}{2}\cdot\frac{3Q}{4}}{r^2} = \frac{3}{8}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{r^2}$$

$$= \frac{3}{8}F \quad \text{[from Eq. (i)]}$$

30. ***(a)*** Given, $E = 2$ V, $R_{AB} = 100\,\Omega$

$l_{AB} = 100$ cm and $R = 50\,\Omega$

Circuit according to the question,



$$AC = CB = \frac{l_{AB}}{2} = 50\text{ cm}$$

$$R_{AB} = 100\,\Omega$$

$$R_{AC} = R_{CB} = \frac{R_{AB}}{2} = \frac{100}{2}$$

$\therefore$ $R_{AC} = R_{CB} = 50\,\Omega$

$\therefore$ Net resistance of the circuit, $R_{\text{net}} = R_{CB} + \frac{R \cdot R_{AC}}{R + R_{AC}}$

$$= 50 + \frac{50 \times 50}{50 + 50} = 50 + 25$$

$$R_{\text{net}} = 75\,\Omega$$

$\therefore$ Current from the battery, $I = \frac{E}{R_{\text{net}}} = \frac{2}{75}$ A

Using KVL in mesh 1,

$$E - V_{AC} - V_{CB} = 0$$

$\Rightarrow$ $V_{AC} + V_{CB} = 2$

$\Rightarrow$ $V_{AC} + IR_{CB} = 2$

$\Rightarrow$ $V_{AC} = 2 - \frac{2}{75} \times 50$

$\Rightarrow$ $V_{AC} = 2 - \frac{4}{3} = \frac{6-4}{3} = \frac{2}{3}$ V

31. *(b)* $V = 24$ V

$$Q = 140\text{ ampere-hour}$$
$$= 140 \times 3600\text{ A-s}$$
$$= 1.4 \times 3.6 \times 10^5\text{ C}$$
$$= 5.04 \times 10^5\text{ C}$$

If R be the external resistance, then

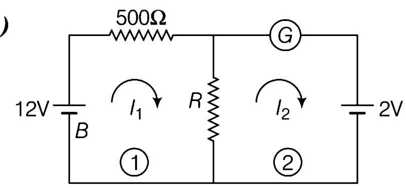
$$R = \frac{V}{I}$$
$$= \frac{24}{(Q/t)} = \frac{24 \times t}{5.04 \times 10^5}$$

$\because$ $t = 14$ hour $= 14 \times 3600$ s

Hence, $R = \frac{24 \times 14 \times 3600}{5.04 \times 10^5}$

$$= 2.4\,\Omega$$

32. *(a)*



According to given figure,

Applying KVL in loop (i)

$$500I_1 + R(I_1 - I_2) = 12$$

$$(500 + R)I_1 - RI_2 = 12 \quad \ldots\text{(i)}$$

Applying KVL in loop (ii), we have

$$R(I_2 - I_1) = -2$$

$$RI_2 = RI_1 - 2 \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$(500 + R)I_1 - (RI_1 - 2) = 12$$
$$500I_1 + RI_1 - RI_1 + 2 = 12$$
$$500I_1 = 10$$
$$I_1 = \frac{1}{50}$$

From Eq. (ii),

$$RI_2 = R \times \frac{1}{50} - 2$$

But given that galvanometer G shows zero deflection, hence $I_2 = 0$.

$\therefore$ $0 = \frac{R}{50} - 2$

$$\frac{R}{50} = 2$$

$$R = 100\,\Omega$$

33. *(b)* Given that

Frequency, $f = 10$ MHz

$$= 10 \times 10^6\text{ Hz}$$
$$= 10^7\text{ Hz}$$

For proton, $m = 1.67 \times 10^{-27}$ kg

$$q = 1.6 \times 10^{-19}\text{ C}$$

We know that,

$$f = \frac{Bq}{2\pi m}$$

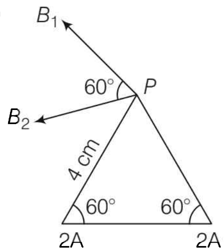
$$10^7 = \frac{B \times 1.6 \times 10^{-19}}{2 \times 3.14 \times 1.67 \times 10^{-27}}$$

$$10^7 = B \times 0.153 \times 10^8$$

$$B = \frac{10^7}{0.153 \times 10^8}$$

$$B = 0.66\text{ T}$$

34. *(c)*



Magnetic field due to first wire at P,

$$B_1 = \frac{\mu_0}{2\pi} \cdot \frac{I_1}{r_1} \qquad (r_1 = 4\,\text{cm} = 4 \times 10^{-2}\ \text{m})$$

$$= 2 \times 10^{-7} \times \frac{2}{4 \times 10^{-2}}$$

$$= 10^{-5}\ \text{T}$$

Similarly, magnetic field due to second wire at P,

$$B_2 = \frac{\mu_0}{2\pi} \frac{I_2}{r_2} \qquad [r_2 = 4\,\text{cm} = 4 \times 10^{-2}\ \text{m}]$$

$$= 2 \times 10^{-7} \times \frac{2}{4 \times 10^{-2}}$$

$$= 10^{-5}\ \text{T}$$

Resultant magnetic field at P,

$$B = \sqrt{B_1^2 + B_2^2 + 2B_1 B_2 \cos 60}$$

$$= B_1 \sqrt{3} \qquad (\because B_1 = B_2)$$

$$= 1.7 \times 10^{-5}\,\text{T}$$

35. *(d)* According to the question

In Ist case, velocity of the beam,

$$v = \frac{E}{B} \qquad \ldots\text{(i)}$$

As we know that, the radius of curvature of a charged particle in a magnetic field,

$$r = \frac{mv}{qB}$$

where, m = mass of the charged particle,

q = charge on the particle,

B = magnitude of the magnetic field

and v = velocity of the particle.

In IInd case,

$$r = \frac{mv}{qB'} = \frac{mE / B}{qB'} \qquad \text{[from Eq. (i)]}$$

$$= \frac{mE}{qBB'}$$

Thus, $\quad r \propto \dfrac{E}{BB'}$

36. *(c)* A magnetic field cannot change the kinetic energy of a charged particle, because charge particle moves on circular path in magnetic field, in which magnitude of velocity remains constant.

Hence, $K = \frac{1}{2}mv^2$ = constant.

37. *(a)* Given that $L_1 = 9\,\text{mH}$

$$L_2 = 3\,\text{mH}$$

$$\frac{dI_1}{dt} = \frac{dI_2}{dt} \qquad \ldots\text{(i)}$$

Power $\quad P_1 = P_2$

$$e_1 I_1 = e_2 I_2$$

$$\frac{e_1}{e_2} = \frac{I_2}{I_1}$$

$$\frac{L_1 \frac{dI_1}{dt}}{L_2 \frac{dI_2}{dt}} = \frac{I_2}{I_1}$$

Hence, $\quad \dfrac{L_1}{L_2} = \dfrac{I_2}{I_1}$

or $\quad \dfrac{L_2}{L_1} = \dfrac{I_1}{I_2} \qquad \ldots\text{(ii)}$

$$\therefore \quad \frac{U_1}{U_2} = \frac{\frac{1}{2}L_1 I_1^2}{\frac{1}{2}L_2 I_2^2}$$

$$= \frac{L_1}{L_2}\left(\frac{I_1}{I_2}\right)^2 \qquad \text{[from Eq. (ii)]}$$

$$= \frac{L_2}{L_1} = \frac{3}{9} = \frac{1}{3}$$

38. *(c)* Given,

Alternating voltage

$$V = 200\sqrt{2}\sin(100t)$$

$$\therefore \quad \omega = 100$$

$$C = 1\,\mu\text{F} = 1 \times 10^{-6}\ \text{F}$$

Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 10^{-6}} = 10^4\ \Omega$$

$$V_0 = 200\sqrt{2}$$

$$\therefore \quad V_{\text{rms}} = \frac{200\sqrt{2}}{\sqrt{2}}$$

$$V_{\text{rms}} = 200\,\text{V}$$

Reading of ammeter,

$$I = \frac{V_{rms}}{X_C} = \frac{200}{10^4}$$
$$= 0.02\,A$$
$$= 20\,mA$$

39. *(d)* Electromagnetic wave is a wave in which electric field vector and magnetic field vector are perpendicular to each other and also they are in same phase.

$\mathbf{E} \times \mathbf{B}$ always give the direction in which wave travel. The direction of $\mathbf{E} \times \mathbf{B}$ is also perpendicular to both **E** and **B**.

40. *(c)* In Young's double slit experiment, intensity at any point on the screen is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi \quad \ldots\text{(i)}$$

where, I_1 and I_2 are intensity of two sources.

But both sources are identical, hence

$$I_1 = I_2 = I_0 \quad [\therefore \text{ from Eq. (i)}]$$
$$I = I_0 + I_0 + 2\sqrt{I_0 I_0}\cos\phi$$
$$I = 2I_0 + 2I_0\cos\phi$$
$$= 2I_0(1 + \cos\phi) = 2I_0 \cdot 2\cos^2\frac{\phi}{2}$$
$$I = 4I_0\cos^2\frac{\phi}{2} \quad \ldots\text{(ii)}$$

When path difference is λ, then phase difference $\phi = 2\pi$

$\therefore$ From Eq. (ii),

$$I = 4I_0\cos^2\frac{2\pi}{2}$$
$$I = 4I_0 = k \text{ (given)} \quad \ldots\text{(iii)}$$

Again when path difference is $\frac{\lambda}{3}$, then phase difference $\phi = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{3} = \frac{2\pi}{3}$

$\therefore$ From Eq. (ii), $I = 4I_0\cos^2\left(\frac{2\pi/3}{2}\right) = 4I_0\cos^2\frac{\pi}{3}$

$$= 4I_0\cdot\frac{1}{4} = k\cdot\frac{1}{4} = \frac{k}{4}$$

41. *(d)* Given, angle of prism $= A$

Refractive index, $\mu = \cot\frac{A}{2}$

We know that, refractive index of prism is given by in the terms of angle of prism and minimum deviation

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\Rightarrow \quad \cot\frac{A}{2} = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\frac{\cos\frac{A}{2}}{\sin\frac{A}{2}}\cdot\sin\frac{A}{2} = \sin\left(\frac{A + \delta_m}{2}\right)$$

$$\cos\frac{A}{2} = \sin\left(\frac{A + \delta_m}{2}\right)$$

$$\sin\left(\frac{\pi}{2} - \frac{A}{2}\right) = \sin\left(\frac{A + \delta_m}{2}\right)$$

$$\frac{\pi}{2} - \frac{A}{2} = \frac{A + \delta_m}{2}$$

$$\pi - A = A + \delta_m$$

$$\pi - 2A = \delta_m$$

$$\Rightarrow \quad \delta_m = \pi - 2A$$

42. *(c)* Given, $\lambda = 6000\,Å$

$$= 6 \times 10^{-7}\,m$$

Diameter of objective lens, $a = 100$ inch

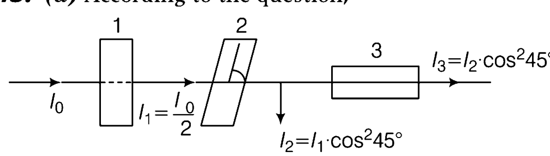
$$a = 254\,cm \quad (\because 1 \text{ inch} = 2.54\,cm)$$
$$= 2.54\,m$$

Since, we know that,

Resolution limit of a telescope,

$$d\theta = \frac{1.22\lambda}{a}$$
$$= \frac{1.22 \times 6 \times 10^{-7}}{2.54}$$
$$= 2.88 \times 10^{-7}$$
$$= 2.9 \times 10^{-7}\text{ rad}$$

43. *(d)* According to the question,



Thus, $I_3 = I_2\cos^2 45°$

$$= I_1\cos^2 45°\cos^2 45°$$
$$= \frac{I_0}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{I_0}{8} \quad \left(\because \cos^2 45° = \frac{1}{2}\right)$$

44. ***(b)*** Given, intensity, $I = 2\,\text{W/m}^2$

Area of metallic surface $(A) = 1 \times 10^{-4}\ \text{m}^2$

Hence, power incident over metallic surface area

$= I \times A$

$= 2 \times 1 \times 10^{-4}\ \text{W}$

$= 2 \times 10^{-4}\ \text{W}$

$\therefore$ Energy incident over metallic surface per second

$= 2 \times 10^{-4}\ \text{J}$

Energy required to produce photoelectrons per second

$= 0.53\%$ of 2×10^{-4}

$= \dfrac{0.53 \times 2 \times 10^{-4}}{100}$

$= 1.06 \times 10^{-6}\ \text{J}$

Hence, number of photoelectrons emitted per second

$= \dfrac{\text{Energy required for producing electron per second}}{\text{Energy of photon}}$

$= \dfrac{1.06 \times 10^{-6}}{10.6 \times 1.6 \times 10^{-19}}$

$= 6.25 \times 10^{11}$

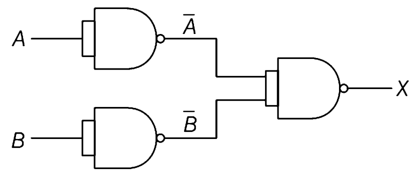
45. ***(a)*** French physicist Louis victor de-Broglie gives a hypothesis in 1924, when material particles are in motion, then they show wave like properties.

46. ***(d)*** The attraction force between nucleons is maximum, when they are separated at distance of 0.8 fm. When distance between two nucleons is less than 0.8 fm, then potential energy increases rapidly with distance, hence a strong repulsive force starts working.

Nuclear force does not depend upon charge. When atomic mass lies between 30-170, then due to saturation of nuclear forces, nuclear forces are short ranges.

47. ***(a)*** As initial and final parameters are same. Thus, the distance of closest approach will be equal to the initial value 'r'.

48. ***(b)***



Output $X = \overline{\bar{A} \cdot \bar{B}}$

$= \bar{\bar{A}} + \bar{\bar{B}}$ (By De-morgan's Law)

$= A + B$

= Output of OR gate

49. ***(a)*** Given, $R_c = 2 \times 10^3\ \Omega$, $V_c = 2\text{V}$

$\beta = 100$, $R_b = 10^3\ \Omega$

$V_b = ?$, $I_b = ?$

Since, we know that,

$\beta = \dfrac{I_c}{I_b}$

$\therefore \quad I_b = \dfrac{I_c}{\beta} = \dfrac{V_c / R_c}{\beta}$ $(\because I = V/R)$

$= \dfrac{\frac{2}{2} \times 10^{-3}}{100} = 10^{-5}\text{A}$

$I_b = 10^{-5}\text{A}$ or $10\ \mu\text{A}$

$\because \quad V_b = I_b \times R_b$

$= 10^{-5} \times 10^3 = 10^{-2}$ V or 0.01 V

50. ***(b)*** Thus, the order frequency (from low to high).

As we know that,

		Range of frequency
(i)	Submarine communication	3-30 kHz
(ii)	AM Radio	535-1605 kHz
(iii)	Shortwave radio	3-30 MHz
(iv)	Radar (VHf)	30-300 MHz

(i) < (ii) < (iii) < (iv)

Chemistry

51. ***(a)*** HCl is the most volatile compound among all hydrogen halides. The volatility of a compound is related to its high vapour pressure at ordinary temperature. HF due to hydrogen bonding is least volatile, as the molecular weight increases, intermolecular forces increases due to van der Waals' force of attraction. Thus, HI is liquid and HCl is gas with maximum vapour pressure among all hydrogen halides.

52. ***(b)*** The lanthanides do not have much tendency to form complexes due to low charge density because of their large size. However, the tendency to form complexes and their stability increases with increasing atomic number. Thus, among Sm^{3+}, Lu^{3+}, Gd^{3+}, Yb^{3+} and Lu^{3+} being the smallest in size with highest atomic number has the strongest tendency towards complex formation.

53. ***(d)*** Higher is the hydration energy, more will be the solubility in water. If the cations have different charge then the one with higher charge will have higher hydration energy. Among $CsClO_4$, $NaClO_4$, $KClO_4$ and $LiClO_4$, all of the cations have $+1$ charge, so for comparing hydration energy size factor will be considered. Smaller is the size more will be the hydration energy. Thus, among all cations Li^+ is smallest in size. Hence, $LiClO_4$ is most soluble in water.

54. ***(b)*** Ti^{3+} has one unpaired electron in *d*-orbital. In aqueous solution this electron undergoes *d-d* transition and impart colour. In Zn^{2+}, Sc^{3+} and Ti^{4+}, no unpaired electron is present in *d*-orbital hence, they won't impart colour.

55. ***(b)*** The reducing character of hydrides of group 15 elements increases down the group $NH_3 < PH_3 < AsH_3 < SbH_3$. As we move from NH_3 to SbH_3, the thermal stability of hydride complex decreases. In other words, their tendency to liberate hydrogen increases and hence their reducing character increases from NH_3 to SbH_3. Thus, among the given compounds SbH_3 is the best reducing agent.

56. ***(c)*** EAN = Atomic number of central atom − (oxidation state + electron gained by ligand)

Atomic number of Ti = 22

Contribution by $(\sigma - C_6H_5) = 1e^-$

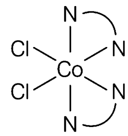
Contribution by $(\pi - C_5H_5) = 5e^-$

$\therefore$ EAN $= 22 - 0 + (1 \times 2) + (2 \times 5)$

$y = 34$

57. ***(b)*** Thorium is the radioactive element, belonging to the class of lanthanides. It is used in the treatment of cancer. It is formed by radioactive decay of uranium. Minerals (like monozite, thorite) are rich in thorium and mixed for it.

58. ***(c)***



cis$[Co(en)_2Cl_2]$ does not have a plane of symmetry hence, it is a chiral molecule and optically active.

59. ***(b)*** The crystal field energy of tetrahedral and octahedral complex are related as : $\Delta_t = \frac{4}{9}\Delta_O$

Given : $\Delta_O = 18000\,cm^{-1}$

$\therefore \quad \Delta_t = \frac{4}{9} \times 18000\,cm^{-1}$

$\Delta_t = 8000\,cm^{-1}$

60. ***(c)*** Hydrogen peroxide, H_2O_2 oxidises lead sulphide, $PbS(s)$ to lead sulphate, $PbSO_4(s)$ in acidic medium.

$$\underset{\text{Black}}{PbS(s)} + 4H_2O_2(aq) \longrightarrow \underset{\text{White}}{PbSO_4(s)} + 4H_2O(l)$$

It reacts with hypochlorous acid, HOCl to produce water, oxygen and hydrochloric acid, showing oxidising action in acidic medium.

$$HOCl + H_2O_2 \longrightarrow H_3O^+ + Cl^- + O_2$$

61. ***(a)*** Chlorine trifluoride, ClF_3 was first reported in 1931 and it is primarily used for the manufacture of uranium hexafluoride, UF_6 as part of nuclear fuel processing and reprocessing by the reaction.

$$U + 3ClF_3 \longrightarrow UF_6 + 3ClF$$

62. ***(b)*** Chlorine with cold and dilute alkali, NaOH forms a mixture of chloride and hypochlorite as

$$Cl_2 + 2NaOH(dil) \xrightarrow{\text{Cold}} NaCl + \underset{\text{Sodium hypochlorite }(X)}{NaOCl} + H_2O$$

Chlorine with hot and concentrated alkali, NaOH forms a mixture of chloride and chlorate as

$$3Cl_2 + 3NaOH(conc.) \xrightarrow{\text{Hot}} 5NaCl + \underset{\text{Sodium chlorate }(Y)}{NaClO_3} + 3H_2O$$

63. ***(d)*** B_2H_6 and NH_3 react in 1:2 to form borazine, $B_3N_3H_6$.

The intermediate product '*X*' is $[BH_2(NH_3)_2]^+[BH_4]^-$

$$3B_2H_6 + 6NH_3 \longrightarrow 3[BH_2(NH_3)_2]^+ [BH_4^-]$$

$$[BH_4]^- \xrightarrow{\text{Heat}} 2B_3N_3H_6 + 12H_2$$

64. ***(c)*** LaH_3 is a non-stoichiometric hydrides. These are hydrogen deficient compounds formed by the reaction of dihydrogen with *d*-block and *f*-block elements. These hydrides do not follow the law of constant composition.

65. ***(d)*** Sulphur dioxide, SO_2 is not used as an insecticide. It is used as food preservatives for dried apricots and other dried fruits. It is used as an antichlor as it decomposes to residual hypochlorite or chlorine. It is also used as a disinfectant, referigerant due to condensing properties.

66. ***(c)*** During electrolytic refining of copper, impurities of silver (Ag), gold (Au), platinum (Pt), selenium (Se) being less electropositive are not affected by electrolytic solution of acidified copper sulphate. Thus, these settle down under the anode as anode mud or anode sludge.

67. ***(d)*** Anions of the acids HNO_3, H_3PO_4 and H_2SO_4 are NO_3^-, PO_4^{3-} and SO_4^{2-} respectively.

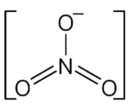
Their structures can be explained as follows:

NO_3^- has planar structure. It has nitrogen as a central atom without lone pair, which is bonded to three oxygen atoms. Its hybridisation can be predicted by $\frac{1}{2}[V+M-C+A]$

Where V = number of valence electrons
M = number of monovalent ion
C = cationic charge
A = anionic charge

$\therefore$ Hybridisation $= \frac{1}{2}[5+0-0+1]=3$

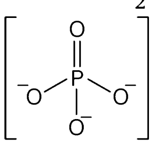
i.e. sp^2



Trigonal planar structure

PO_4^{3-} has tetrahedral structure. It has phosphorus as a central atom without lone pair which is bonded to four oxygen atoms. Its hybridisation can be predicted as $\frac{1}{2}[5+0-0+3]=3$

i.e. sp^3

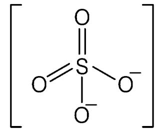


Tetrahedral structure

SO_4^{2-} has tetrahedral structure. It has phosphorus as a central atom without lone pair which is bonded to four oxygen atoms. Its hybridisation can be predicted as

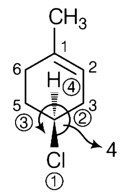
$\frac{1}{2}[6+0-0+2]=4$

i.e. sp^3



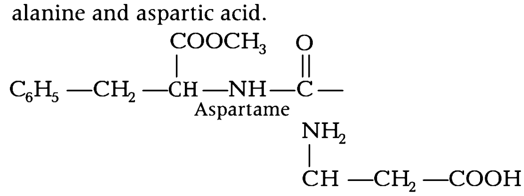
Tetrahedral structure

68. ***(a)***



is (S) - 4-chloro-1-methyl cyclohexene

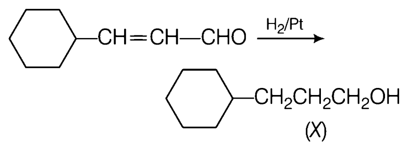
69. ***(c)*** Aspartame is an artificial sweetener. It is the methyl ester of dipeptide derived from phenyl alanine and aspartic acid.



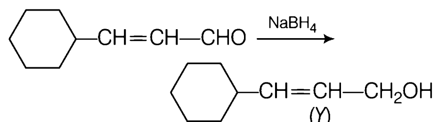
Aspartame

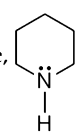
It is roughly 100 times as sweet as sucrose.

70. ***(b)*** H_2 / Pt reduces both alkene and aldehyde to alkane and alcohol respectively.

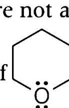


Whereas, $NaBH_4$ only reduces aldehyde to alcohol.



71. ***(a)*** The strongest base is pyrimidine,  because it's lone pair on N-atom are easily available for donation.

In case of pyrrole and pyridine, lone pair on nitrogen atom are involved in resonance and hence are not available for donation easily.

In case of , lone pairs are easily, available for donation but less readily than pyrimidine because oxygen is more electronegative than nitrogen.

72. ***(b)*** The case with which a nucleophile attacks the carbonyl group depends upon the electron deficiency, i.e. the magnitude of positive charge on carbonyl group carbon. Since, among all the groups attached

$(—OCH_3, —Cl, —H, —O—\overset{\overset{O}{||}}{C}—CH_3)$ — Cl has maximum electron withdrawing effect, electron deficiency will be maximum at carbonyl carbon of acetyl chloride. Hence, acetyl chloride, i.e.

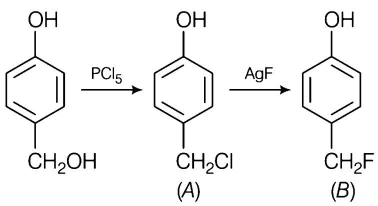
$CH—\overset{\overset{O}{||}}{C}—Cl$ is most reactive to nucleophilic attack at carbonyl group.

73. ***(c)*** $CH_3\overset{\overset{O}{||}}{C}—OH \xrightarrow{NH_3} \underset{\text{Ammonium acetate (A)}}{CH_3—\overset{\overset{O}{||}}{C}—O^- \overset{+}{N}H_4} \xrightarrow{\Delta}$

$$\underset{\text{Acetamide (B)}}{CH_3—\overset{\overset{O}{||}}{C}—NH_2} \xrightarrow{P_2O_5} \underset{\text{Acetonitrile (C)}}{CH_3C \equiv N}$$

74. ***(a)*** CF_2Cl_2, dichloro difluoro methane is used as a refrigerant and aerosal spray propellant. It is also known as freon.

75. ***(c)***



76. ***(a)*** $\underset{(P)}{CH_3—\overset{\overset{OH}{|}}{CH}—CH_3} \xrightarrow[\text{dil.}H_2SO_4]{K_2Cr_2O_7} \underset{(B)}{CH_3—\overset{\overset{O}{||}}{C}—CH_3}$

$$\xrightarrow{CH_3MgBr} CH—\overset{\overset{\overset{-}{O}\overset{+}{M}gBr}{|}}{\underset{\underset{CH_3}{|}}{C}}—CH_3 \xrightarrow{H_3O^+} CH_3—\overset{\overset{OH}{|}}{\underset{\underset{CH_3}{|}}{C}}—CH_3$$

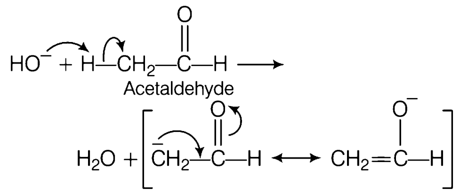
77. ***(c)*** β-hydroxy carbonyl compound is obtained by the action of NaOH on CH_3CHO as follows:

$$\underset{\text{Acetaldehyde}}{CH_3CHO + CH_3CHO} \xrightarrow{\text{Dil.NaOH}}$$

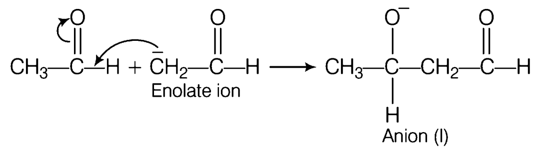
$$\underset{\beta\text{-hydroxy butanol}}{CH_3—\underset{\beta}{\overset{\overset{OH}{|}}{CH}}—\underset{\alpha}{CH_2}—\overset{\overset{O}{||}}{C}—H}$$

Mechanism

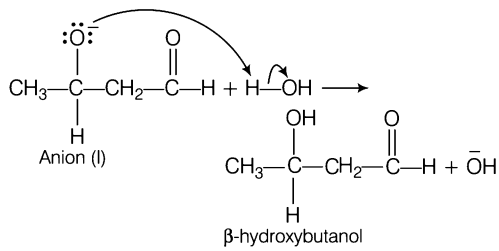
Step-I Formation of enolate ion



Step-II Attack of enolate ion on carbonyl group of second molecule of acetaldehyde.



Step-III Anion (I) abstracts a hydrogen from water to form β-hydroxy, carbonyl compound.



78. ***(b)*** Since, the compound (A) gives a positive iodoform test but negative Tollen's test, so it must be a methyl ketone. From the given molecular formula $C_5H_{10}O(A)$ can be 2-pentanone.

Following reactions are involved:

$$\underset{\text{2-pentanone}}{CH_3CH_2CH_2\overset{\overset{O}{||}}{C}—CH_3} + \underset{\text{Phenyl hydrazine}}{H_2NNHC_6H_5}$$

$$\xrightarrow{H^+} \underset{\text{Pentanone phenyl hydrazone}}{\overset{\overset{CH_3}{|}}{\underset{\underset{CH_2CH_2CH_3}{|}}{C}}=NNHC_6H_5}$$

Iodoform test

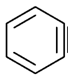
$$\underset{\text{2-pentanone}}{CH_3CH_2CH_2\overset{\overset{O}{||}}{C}—CH_3} + \underset{\text{Sodium hypoiodite}}{3NaOI} \longrightarrow \underset{\text{Iodoform}}{CHI_3\downarrow}$$

$$+ CH_3CH_2CH_2\overset{\overset{O}{||}}{C}—ONa + 2NaOH$$

$$\underset{\text{2-pentanone}}{CH_3CH_2CH_2—\overset{\overset{O}{||}}{C}—CH_3} \xrightarrow{H_2/Pt} \underset{n\text{-pentane}}{CH_3CH_2CH_2CH_2CH_3}$$

79. ***(a)*** Carius method is used for estimation of halogens in organic compound. A known mass of the organic substance is heated with fuming nitric acid and a few crystals of silver nitrate in a sealed tube called carius tube in furnace is washed , dried and weighed, knowing the mass of substance taken (W) and the mass of precipitate formed (X), the percentage of halogen is calculated as

$$\frac{\text{Atomic mass of } X}{108 + \text{atomic mass of } X} \times x \times \frac{100}{W}$$

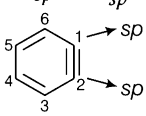
80. ***(d)*** $HC{\equiv}CH$, $HC_2{=}C{=}CH_2$ and 

contains sp^2-hybridised carbon.

In $\underset{sp}{\text{H—C}} {\equiv} \underset{sp}{\text{C}}\text{—H}$, both carbon are sp-hybridised.

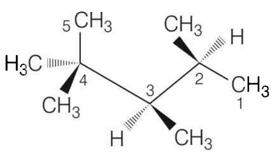
In $\underset{sp^2}{CH_2} = \underset{sp}{C} = \underset{sp^2}{CH_2}$, central carbon is sp- hybridised.

In



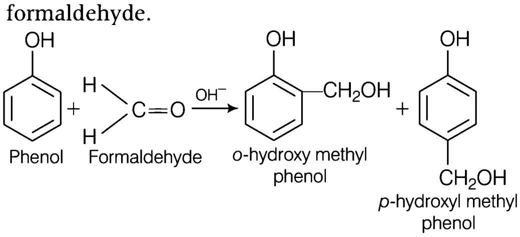
C_1 and C_2 are sp-hybridised.

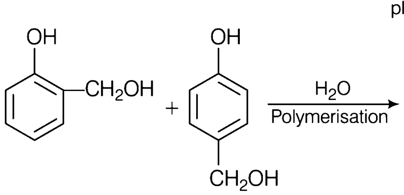
81. ***(d)***

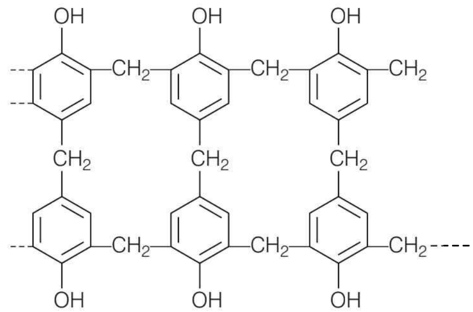


A chiral compound is the one that contains an asymmetric carbon atom, i.e. a carbon atom that has all the four substituent different. Here, C_3 contains all the different substituent.
Hence, it is a chiral compound.

82. ***(a)*** Bakelite is a thermosetting polymer. It is a phenol formaldehyde resin, formed from condensation reaction of phenol and formaldehyde.







Bakelite

83. ***(b)*** $CH_3—\underset{\text{(II)}}{\overset{\overset{\displaystyle CH_3}{|}}{CH}}—\overset{+}{\underset{\underset{\displaystyle CH_3}{|}}{C}}—CH_3$ is the most stable carbocation.

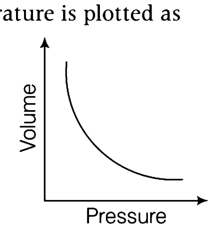
The stability of carbocations increases as we go from primary to secondary to tertiary carbons. It is because tertiary carbocations have more inductively donating groups as compared to secondary and primary carbocation. Secondly, in case of tertiary carbocation there are more hyper conjugative structures as compared to secondary and primary carbocation.

Hence, the order of stability of carbocations is $3° > 2° > 1°$.

84. ***(c)*** When an aliphatic aldehyde is heated with Fehling's solution, the latter is reduced to give a red ppt. of cuprous oxide. During this reduction, the following reactions occur.

$$\underset{\text{Aldehyde}}{RCHO} + \underset{\text{Fehling's solution}}{2\,Cu^{2+}} + 5\ OH^- \longrightarrow \underset{\text{Carboxylate ion}}{RCOO^-} + \underset{\text{Cuprous oxide (Red ppt.)}}{Cu_2O \downarrow} + 3\ H_2O$$

85. ***(a)*** According to Boyle's law, the product of volume and pressure of a given gas is constant at constant temperature. The curve between pressure and temperature is plotted as



On increasing temperature the curve shifts to higher value.

Hence, from the given options (according to the curve) $T_1 < T_2 < T_3$.

86. ***(c)*** Given, (For : NaH_2PO_4)

10 mL of 0.1 M NaH_2PO_4 and

$$pK_1 = 2.12$$

$$\because \quad pH_1 = pK_1 + \log\frac{[A^-]}{[H_A]} = \frac{[0.1 \times 2]}{1}$$

Here, $[H_A] = [NaH_2PO_4] = 0.1 \times 10 = 1$

$$[A^-] = [H^+]$$

$$\therefore \quad pH_1 = 2.12 + \log\left[\frac{0.2}{1}\right]$$

$$pH_1 = 2.12 + (-0.6989)$$

$$pH_1 = 1.421$$

For Na_2HPO_4,

15 mL of 0.1 M Na_2HPO_4 and

$$pK_2 = 7.2$$

$$\because \quad pH_2 = pK_2 + \log\left[\frac{A^-}{HA}\right]$$

Here $\quad [HA] = [NaHPO_4]$

$$[A^-] = [H^+]$$

$$pH_2 = 7.2 + \log\left[\frac{0.1}{1.5}\right]$$

$$pH_2 = 7.2 + (-1.1739)$$

$$pH_2 = 6.0261$$

Total pH = $pH_1 + pH_2$

$= 1.421 + 6.0261 = 7.4471$

87. ***(a)*** The difference between C_p and C_V is larger in case of gases as compared to solid and liquid.

In solids and liquids, the change in their volume, i.e. ΔV is negligible.

From the equation, $u = q + p\Delta V$. For constant volume, $dV = 0$, thus $u = q$.

For constant pressure also, $u = q$ (as change in volume is nearly zero for solids and liquids).

Since, for both the cases internal energy is same, hence heat capacity won't change for liquids and solids.

88. ***(d)*** For a second order reaction,

$$A + B \longrightarrow P$$

if $[A]_0 = [B]_0$, then integrated rate law will be calculated as follows

Then, $-\frac{d[A]}{dt} = -\frac{d[B]}{dt} = k[A][B] = k[A]^2$

$$-\frac{d[A]}{dt} = k[A]^2$$

$$\frac{d[A]}{[A]^2} = -kdt$$

Integrating between $t = 0$, $t = t$

$$\int_{[A]_0}^{[A]_t} \frac{d[A]}{[A]^2} = -k\int_o^t dt$$

On integrating the equation,

$$\frac{1}{[A]_t} = \frac{1}{[A]_0} + kt$$

Since, $[A]_0 = [B]_0$ above equation can also be written as

$$\frac{1}{[B]_t} = \frac{1}{[B]_0} + kt$$

89. ***(a)*** The half reaction for the reduction is given as

$$O_2(g) + 4H^+(aq) + 4e^- \longrightarrow 2H_2O(l)$$

We have, activity for H_2O (or any pure liquid or solid) $\propto H_2O = 1$

Activity for hydrogen ion $(H^+) = \alpha_{H^+}$

Activity for oxygen gas $(O_2) = \alpha_{O_2} = \frac{p_{O_2}}{p^\circ}$

Thus, reaction quotient (Q) is given as

$$Q = \frac{\alpha^2_{H_2O}}{\alpha^4_{H^+} \cdot \alpha_{O_2}} = \frac{1^2}{\alpha^4_{H^+} \cdot p_{O_2} / p^\circ} = \frac{p^\circ}{\alpha_{H^+} \cdot p_{O_2}}$$

90. ***(b)*** **Key point** (we assume that standard emf of the cell E°_{cell} is known)

For the given equilibrium,

$$Cd^{2+} + 4NH_3 \rightleftharpoons Cd(NH_3)_4^{2+}$$

At equilibrium,

$$E^\circ_{cell} = 0$$

Hence, we can calculate the equilibrium constant for reaction as follows :

$$E^\circ_{cell} = \frac{0.0591\ V}{n} \log K_C$$

Where, K_C is unknown

$$E^\circ_{cell} = 0.21\ V$$

$$0.21V = \frac{0.0591}{2} \log K_C$$

$$\log K_C = \frac{0.21 \times 2}{0.0591}$$

$$\log K_C = \frac{0.42}{0.0591} = 7.1065$$

$$K_C = 1.27 \times 10^7$$

91. ***(a)*** Degree of dissociation $(\alpha) = \frac{\Lambda^c_m}{\Lambda^\circ_m}$

Given $\Lambda^C_m = 60\ ohm^{-1} cm^2 eq^{-1}$

$\Lambda^\circ = 400\ ohm^{-1} cm^2 eq^{-1}$

On putting the values,

$$\alpha = \frac{60}{400} = 0.15$$

Concentration of H^+ in the solution will be $c\alpha$, where $C = 0.002$ M

$\therefore$ $[H^+] = (0.002 \times 0.15)$ M $= 0.0003$

Now, pH = log $[H^+]$

$= -\log(0.0003)$

$= 3.52$

92. ***(c)*** $\Delta S = nF\left(\frac{\delta E}{\delta T}\right)_p$

where, $\left(\frac{\delta E}{\delta T}\right)_p$ is temperature coefficient.

$\therefore$ $\Delta S = 2 \times 96500 \times (-6.5 \times 10^{-4}) V\deg^{-1}$

$\Delta S = -125.064\ V\ \deg^{-1}$

For 1F, $\Delta S = \frac{-125.064\ V\ \deg^{-1}}{2}$

$\Delta S = -62.53\ V\ \deg^{-1}$

$\Delta G = -nFE_{cell}$

For the given reaction, $n = 2$

$\Delta G = -2 \times 96500 \times 0.675$

$\Delta G = -130333$J

As, $\Delta G = \Delta H - T\Delta S$

On substituting values of ΔG, ΔS and T.

we get,

$\Delta H = -167.6$kJ

For 1F, $\Delta H = \frac{-167.6\text{kJ}}{2}$

$= -83.83$kJ

93. ***(b)*** Effect of temperature on the rate of reaction hence, rate constant k, can be calculated by using Arrhenius equation which is $k = Ae^{-E_a/RT}$

given : $A = 6.0 \times 10^{-14}\text{s}^{-1}$

$E_a = 104.4\text{kJmol}^{-1}$

$T = \infty$

$\because$ $k = e^{-E_a/RT}$

or $\log k = \log A - \frac{E_a}{2303\ RT}$

At $T = \infty$

$$\log k = \log A + \frac{E_a}{2303\ R \times 100}$$

$\log k = \log A + 0$

$\log k = \log 6 \times 10^{14}$

$k = 6 \times 10^{14}\text{s}^{-1}$

94. ***(a)*** Given, $p_i = 1$ bar

$V_i = 10$ L

$\because$ For ideal gas,

$pV = nRT$

$\therefore$ $n = \frac{pV}{RT} = \frac{1 \times 10}{RT}$

$\because$ Work done $= -p\Delta V$

$\therefore$ Work done at constant volume.

i.e when $\Delta V = 0$

$W = 0$

Work done in isothermal reversible process.

Now, $W = -2303\ nRT \log \frac{p_i}{p_t}$

$p_i = 10$ bar $n = \frac{10}{RT}$

$\therefore$ $W = 2303 \times \frac{10}{RT} RT \log \frac{10}{1}$

$W = -2303 \times 10 = -23.02$ atm L.

$\because$ 1 atm L $= 10^5$ N-m^2 and 1 N-m^2 = 1 J

$\therefore$ $W = -23.02 \times 10^5$ J

95. ***(b)*** Integrated rate law for the second order reaction is $\frac{1}{[A]_t} = \frac{1}{[A]_0} + kt$ ...(i)

For half-life time $t = t_{1/2}$, $[A]_t = [A]_0/2$

On putting the values in Eq. (i)

$$\frac{1}{[A]_{t/2}} = \frac{1}{[A]_0} + kt_{1/2}$$

$$kt_{1/2} = \frac{2}{[A]_0} - \frac{1}{[A]_0}$$

$$t_{1/2} = \frac{1}{k}\left[\frac{1}{[A]_0}\right] \quad \text{...(ii)}$$

For three quarter half-life time $t = t_{3/4}$, $[A]_t = [A]_0/4$

On putting the values in Eq. (i)

$$\frac{1}{[A]_0/4} = \frac{1}{[A]_0} + kt_{3/4}$$

$$t_{3/4} = \frac{1}{k}\left[\frac{3}{[A]_0}\right] \quad \text{...(iii)}$$

Now, ratio of $t_{1/2}$ to $t_{3/4}$ is given by

$$\frac{t_{1/2}}{t_{3/4}} = \frac{\frac{1}{k}\left[\frac{1}{[A]_0}\right]}{\frac{1}{k}\left[\frac{3}{[A]_0}\right]}$$

$t_{1/2} : t_{3/4} = 1 : 3$

Hence, $t_{1/2} : t_{3/4}$ is independent of the concentration of reactant.

96. ***(d)*** The reduction potential for MnO_4^- to Mn^{2+} is $+1.51\,V$.

It can oxidise only those metals, which have reduction potential lower than $+1.51\,V$. For Zn^{2+}/Zn and Ag^+/Ag, reduction potential given is $-0.762\,V$ and $+0.80\,V$ respectively. These values are lower than reduction potential of MnO_4^-/Mn^{2+}.

Hence, Zn and Ag will be oxidised. Since, for Au^+/Au, reduction potential is higher, hence it will not be oxidised by Mn.

97. ***(b)*** Given, $\Delta G^{\circ}_{(\text{Reaction})} = -131.23\,\text{kJ/mol}$

and, $\Delta G_f^{\circ}[H^+ aq) = 0$

$\because \quad \Delta G^{\circ}_{\text{reaction}} = \Sigma[G^{\circ}_{f(\text{product})}] - \Sigma[G^{\circ}_{f(\text{reactant})}]$

$$= \left[G_f^{\circ}(H^+) + G_f^{\circ}(Cl^-)\right] - [0]$$

$\because G_f^{\circ}$ for elements in free state, i.e

H_2 and O_2 = zero

$\because Ag^+$ ion will form AgCl with Cl^- ions and conc. of $[Ag^+{}_{(aq)}] = [Cl^-{}_{(aq)}]$

$\therefore$ both are in 1 : 1 ratio

$$G_f^{\circ}\,(Cl) = G_f^{\circ} = (Ag^+) - 131.25 = G_f^{\circ}[Cl^-{}_{(aq)}]$$

$$G_f[Ag^+{}_{(aq)}] = -131.25\,\text{kJ/mol}^{-1}$$

98. ***(a)*** The solubility of gas in liquid at particular temperature is given by Henry's law.

i.e. $m_A = K_H p_A$

where, m_A = mass of gas dissolved in a unit volume of solvent

p_A = pressure of the gas in equilibrium

K_H = Henry's law constant

On putting the given values in above expression

$1.38 \times 10^{-3}\,\text{mol/L} = K_H \times 1\,\text{atm}$

$$K_H = \frac{1.38 \times 10^{-3}\,\text{mol/L}}{1\,\text{atm}}$$

At 0.21 atm, m_A will be calculated as

$$m_A = \frac{1.38 \times 10^{-3}\,\text{mol/L}}{1\,\text{atm}} \times 0.21\,\text{atm}$$

$$m_A = 2.9 \times 10^{-4}\,\text{mol/L}$$

99. ***(b)*** Mole fraction in the vapour phase $(\chi_1) = \dfrac{p_A}{p_{\text{total}}}$

But, $p_A \times \chi_A \times p_A^{\circ} = \chi_2 p_A^{\circ}$

or $\quad \chi_1 \times p_A^{\circ} = \chi_2 p_A^{\circ}$ (as $\chi_A = \chi_1$)

$\therefore \quad \chi_1 = \dfrac{\chi_2 p_A^{\circ}}{p_{\text{total}}}$

or $\quad p_{\text{total}} = \dfrac{p_A^{\circ}\chi_2}{\chi_1}$

100. ***(b)*** When both $PbCO_3$ and $MgCO_3$ are present in solution.

Suppose solubility of $PbCO_3$ is $x\,\text{mol}\,L^{-1}$ and that of $MgCO_3$ is $y\,\text{mol}\,L^{-1}$. Then,

$$PbCO_3 \rightleftharpoons \underset{x}{Pb^{2+}} + \underset{x+y}{CO_3^{2-}}$$

$$MgCO_3 \rightleftharpoons \underset{y}{Mg^{2+}} + \underset{x+y}{CO_3^{2-}}$$

$$\frac{K_{sp}(PbCO_3)}{K_{sp}(MgCO_3)} = \frac{x(x+y)}{y(x+y)} = \frac{x}{y}$$

$$= \frac{1.5 \times 10^{-15}}{1.0 \times 10^{-15}} = 1.5$$

Thus, $x = 1.5y$

$$K_{sp}(PbCO_3) = x(x+y) = 1.5 \times 10^{-15}$$

$\therefore \quad 1.5y\,(1.5y + y) = 1.5 \times 10^{-15}$

or $\quad 3.75y^2 = 1.5 \times 10^{-15}$

$$y = \left(\frac{1.5 \times 10^{-15}}{3.75}\right)^{1/2} = 2 \times 10^{-8}$$

Now, $\quad x = 1.5y$

$$= 1.5 \times (2 \times 10^{-8}) = 3 \times 10^{-8}\,M.$$

Mathematics

101. ***(a)*** We have,

$$f(x) = (x^2-9)\left|x^2-7x+12\right| + \cos|x|$$

$\Rightarrow$ $$f(x) = (x-3)(x+3)\left|(x-3)(x-4)\right| + \cos|x|$$

$\cos|x|$ is differentiable for all values of x

$\therefore$ We check the differentiability of

$(x^2-9)\left|x^2-7x+12\right|$

Let $g(x) = (x^2-9)\left|(x-3)(x-4)\right|$

$$g(x) = \begin{cases} (x^2-9)(x-3)(x-4), & x<3 \\ -(x^2-9)(x-3)(x-4), & 3\le x<4 \\ (x^2-9)(x-3)(x-4), & x \ge 4 \end{cases}$$

Clearly, $g(x)$ is not differentiable at $x = 4$

Hence, $f(x)$ is not differentiable at $x = 4$.

102. ***(b)*** Equation of plane having intercepts -4, 2 and 3 is

$$\frac{x}{-4} + \frac{y}{2} + \frac{z}{3} = 1$$

$\Rightarrow$ $-3x + 6y + 4z = 12$

$\Rightarrow$ $3x - 6y - 4z + 12 = 0$

Given equation of line $\frac{x-4}{8} = \frac{y-2}{2} = \frac{z-3}{3}$

Direction ratio of line are $(8, 2, 3)$ and direction ratio of plane are $(3, -6, -4)$

$\therefore$ $8\times 3 + 2\times(-6) + 3(-4) = 0$

and point $4, 2, 3$ satisfies the plane.

$\therefore$ Line lies in the plane.

103. ***(d)*** We have,

$$a * b = \frac{3ab}{7}$$

Let e is the identity element.

$\therefore$ $a * e = a = e * a$

$\Rightarrow$ $a * e = \frac{3ae}{7} = a$

$\Rightarrow$ $e = \frac{7}{3}$

104. ***(a)*** $\sin(2\sin^{-1} 0.8)$

$= 2\sin\sin^{-1}(0.8)\cos(\sin^{-1} 0.8)$

$= 2\times 0.8 \times \cos(\cos^{-1}\sqrt{1-(0.8)^2})$

$= 2\times 0.8\times\sqrt{1-0.64}$

$= 2\times 0.8\times 0.6 = 0.96$

105. ***(b)*** We have, $f(x) = x^3 + x$

$$f(2) = 10,\ f(3) = 30,\ f(4) = 68$$

Given, $\frac{2}{y-f(2)} + \frac{3}{y-f(3)} + \frac{4}{y-f(4)} = 0$

$\Rightarrow$ $$\frac{2}{y-10} + \frac{3}{y-30} + \frac{4}{y-68} = 0$$

$\Rightarrow$ $2(y-30)(y-68) + 3(y-10)$

$(y-68) + 4(y-10)(y-30) = 0$

$\Rightarrow$ $9y^2 - 590y + 7320 = 0$

$$y = \frac{590 \pm \sqrt{(590)^2 - 4\times 9\times 7320}}{2\times 9}$$

$$y = \frac{590 \pm \sqrt{84580}}{18} = \frac{590 \pm 290.82}{18}$$

$y = 16.62$ or 48.93

Clearly, exactly one root lie between $(f(3), f(4))$.

106. ***(b)*** We have,

$2y^2 = 2x - 1$ and $2x^2 = 2y - 1$

$\Rightarrow$ $y^2 = x - \frac{1}{2}$ and $x^2 = y - \frac{1}{2}$

Now, the shortest distance always along common normal to curve.

Equation of normal to the curve $y^2 = x - \frac{1}{2}$ is

$$y = m\left(x - \frac{1}{2}\right) - 2\times\frac{1}{4}m - \frac{1}{4}m^3$$

$\Rightarrow$ $$y = mx - \frac{1}{2}m - \frac{1}{2}m - \frac{1}{4}m^3$$

$\Rightarrow$ $$y = mx - m - \frac{m^3}{4} \qquad \ldots\text{(i)}$$

Equation of normal to the curve $x^2 = y - \frac{1}{2}$ is

$$y - \frac{1}{2} = mx + 2\left(\frac{1}{4}\right) + \frac{1}{4m^2}$$

$\Rightarrow$ $$y = mx + 1 + \frac{1}{4m^2} \qquad \ldots\text{(ii)}$$

Since, Eqs. (i) and (ii) are same normal.

$\therefore$ $$-m - \frac{m^3}{4} = 1 + \frac{1}{4m^2}$$

$\Rightarrow$ $$\frac{-4m - m^3}{4} = \frac{4m^2 + 1}{4m^2}$$

$\Rightarrow$ $m^5 + 4m^3 + 4m^2 + 1 = 0$

$\Rightarrow$ $(m+1)(m^4 - m^3 + 5m^2 - m + 1) = 0$

$\Rightarrow$ $m = -1$

Hence, slope of tangent $= 1$

Equation of tangent to curve $y^2 = x - \frac{1}{2}$ is

$$y = \left(x - \frac{1}{2}\right) + \frac{1}{4} \Rightarrow y = x - \frac{1}{4}$$

Equation of tangent to curve $x^2 = y - \frac{1}{2}$ is

$$y = x + \frac{1}{4}$$

$\therefore$ Required distance $= \left| \frac{\frac{1}{4} + \frac{1}{4}}{\sqrt{2}} \right| = \frac{1}{2\sqrt{2}}$

107. ***(*)*** We know that if the image of the point (x_1, y_1, z_1) in the plane $ax + by + cz + d = 0$ is α, β, γ,

then $\frac{\alpha - x_1}{a} = \frac{\beta - y_1}{b} = \frac{\gamma - z_1}{c}$

$$= -\frac{2(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$$

Here, $(x_1, y_1, z_1) \equiv (1, -1, 1)$

$(a, b, c) \equiv (1, -2, 3)$

and equation of plane is $x - 2y + 3z + 1 = 0$

$\therefore \quad \frac{\alpha - 1}{1} = \frac{\beta + 1}{-2} = \frac{\gamma - 1}{3} = -\frac{2(1(1) - 2(-1) + 3(1) + 1)}{1^2 + (-2)^2 + (3)^2}$

$\Rightarrow \quad \frac{\alpha - 1}{1} = \frac{\beta + 1}{-2} = \frac{\gamma - 1}{3} = -\frac{2(1 + 2 + 3 + 1)}{1 + 4 + 9}$

$\Rightarrow \quad \frac{\alpha - 1}{1} = \frac{\beta + 1}{-2} = \frac{\gamma - 1}{3} = -1$

$\Rightarrow \quad \alpha = 0, \beta = 1, \gamma = -2$

$\therefore$ Image of point $(1, -1, 1)$ in the given plane is $(0, 1, -2)$.

108. ***(a)*** We know that the union of two equivalence relation on a set is not necessarily an equivalence relation on the set.

For example let $x = \{a, b, c\}$

$R = \{(a,a), (b,b)(c,c), (a,b), (b,a)\}$

$S = \{(a,a), (b,b), (c,c), (b,c), (c,b)\}$

Clearly, R and S are equivalence relation.

But $R \cup S$ is not transitive because $(a,b) \in R \cup S$ and $(b,c) \in R \cup S$ but $(a,c) \notin R \cup S$

Hence, $R \cup S$ is not equivalence relation.

109. ***(a)*** We have,

$$g(x) = x^2 + x - 2$$

and $\quad \frac{1}{2}(gof)x = 2x^2 - 5x + 2$

$\Rightarrow \quad \frac{1}{2}(gf(x)) = 2x^2 - 5x + 2$

$\Rightarrow \quad g(f(x)) = 4x^2 - 10x + 4 \quad \ldots(\text{i})$

and $\quad g(f(x)) = (f(x))^2 + f(x) - 2 \quad \ldots(\text{ii})$

Eqs. (i) and (ii) are equal.

$\therefore \quad (f(x))^2 + f(x) - 2 = 4x^2 - 10x + 4$

$\Rightarrow \quad (f(x))^2 + f(x) = 4x^2 - 10x + 6$

$\Rightarrow \quad \{f(x)\}^2 + f(x) + \frac{1}{4} = 4x^2 - 10x + 6 + \frac{1}{4}$

$\Rightarrow \quad \left(f(x) + \frac{1}{2}\right)^2 = 4x^2 - 10x + \frac{25}{4}$

$\Rightarrow \quad \left(f(x) + \frac{1}{2}\right)^2 = \left(2x - \frac{5}{2}\right)^2$

$\Rightarrow \quad f(x) + \frac{1}{2} = 2x - \frac{5}{2}$

$\Rightarrow \quad f(x) = 2x - 3$

110. ***(b)*** We have,

$$y^2 + 2xy + 50|x| = 625$$

Case I When $x \geq 0$,

$\therefore \quad y^2 + 2xy + 50x = 625$

$y^2 - 625 + 2xy + 50x = 0$

$(y + 25)(y - 25) + 2x(y + 25) = 0$

$(y + 25)(2x + y - 25) = 0$

$\Rightarrow y + 25 = 0$ or $2x + y - 25 = 0$

Case II When $x < 0$,

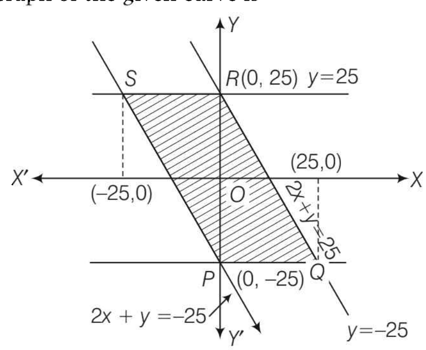
$y^2 + 2xy - 50x = 625$

$\Rightarrow \quad y^2 - 625 + 2xy - 50x = 0$

$\Rightarrow \quad (y - 25)(2x + y + 25) = 0$

$\Rightarrow \quad y = 25$ or $2x + y + 25 = 0$

Graph of the given curve is



Area of region $PQRS = 2 \times$ Area of ΔPRQ

$= 2 \times \frac{1}{2} \times PR \times PQ = 50 \times 25$

$= 1250$ sq units

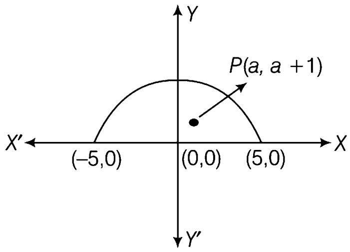
111. ***(d)*** All the 4 numbers should be odd numbers as their multiplication ends with odd number. So, probability of randomly choosing an odd number is 1/2. Also the product should not end with digit 5. So probability of choosing a multiple of 5 is 1/5. So, probability of choosing a non-multiple of 5 is 4/5. Thus probability of choosing a number which is odd and not multiple of 5 is $\frac{4}{5} \times \frac{1}{2} = \frac{2}{5}$.

So, probability of choosing 4 such numbers randomly whose product end with 1,3,7, 9 is

$$= \frac{2}{5}\times\frac{2}{5}\times\frac{2}{5}\times\frac{2}{5} = \frac{16}{625}$$

112. *(c)* If $(a, a+1)$ lies in the region $y=\sqrt{25-x^2}$ and line $y=0$

Then, $a+1>0$ and $a+1<\sqrt{25-a^2}$



$\Rightarrow a>-1$ and $(a+1)^2<25-a^2$

$\Rightarrow a>-1$ and $2a^2+2a-24<0$

$\Rightarrow a>-1$ and $a^2+a-12<0$

$\Rightarrow a>-1$ and $(a-3)(a+4)<0$

$\Rightarrow a>-1$ and $a\in(-4,3)$

$\therefore a\in(-1,3)$

113. *(b)* We have,

$$A=\begin{bmatrix}0&1&2\\1&2&3\\3&\alpha&1\end{bmatrix} \text{ and } A^{-1}=\begin{bmatrix}\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\-4&3&\beta\\\frac{5}{2}&-\frac{3}{2}&\frac{1}{2}\end{bmatrix}$$

We know that,

$$I=AA^{-1} \Rightarrow \begin{bmatrix}0&1&2\\1&2&3\\3&\alpha&1\end{bmatrix}\begin{bmatrix}\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\-4&3&\beta\\\frac{5}{2}&-\frac{3}{2}&\frac{1}{2}\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$\begin{bmatrix}1&0&\beta+1\\0&1&2\beta+2\\4-4\alpha&3\alpha-3&2+\alpha\beta\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$\therefore \alpha=1$ and $\beta=-1$

114. *(c)* Given, $A_i\,(i=1,2,3,\ldots,n)$ are n independent events and $P(A_i)=1-\frac{1}{2^i}$

Probability of atleast one of n events occurs

$= 1-$ Probability of none of n events occurs

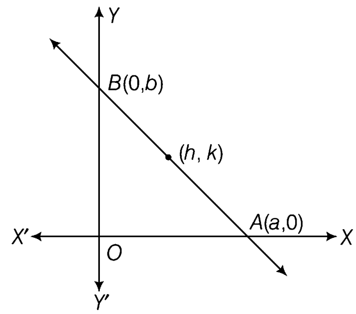
$$= 1-\prod_{i=1}^{n} P(\bar{A}_i)$$

$$= 1-\prod_{i=1}^{n}\left(1-1+\frac{1}{2^i}\right) \qquad [\because P(\bar{A})=1-P(A)]$$

$$= 1-\prod_{i=1}^{n}\frac{1}{2^i} = 1-\frac{1}{2^1\cdot 2^2\cdot 2^3\ldots 2^n}$$

$$= 1-\frac{1}{2^{1+2+3+\ldots n}} = 1-\frac{1}{2^{\frac{n(n+1)}{2}}}$$

115. *(a)* Let A and B are the extremities on two fixed straight line of the given line.



$\therefore$ Coordinates of $A(a,0)$ and $B(0,b)$

Let (h,k) is the mid-point of AB.

$\therefore \qquad h=\frac{a}{2}$ and $k=\frac{b}{2}$

Now, area of $\Delta OAB=\frac{1}{2}ab=c^2$

On putting the value of a and b, we get

$$\frac{1}{2}(2h)(2k)=c^2 \Rightarrow 2hk=c^2$$

$\therefore$ Locus of the mid-point of the line is $2xy=C^2$

116. *(b)* We have,

$a_1, a_2, a_3, a_4, \ldots, a_n$ are in AP.

We know that, if $a_1, a_2, a_3 \ldots a_n$ in AP, then

$a_1+a_n=a_2+a_{n-1}=a_3+a_{n-2}=\ldots=a_r+a_{n-(r-1)}$

Now, $\frac{1}{a_1a_n}+\frac{1}{a_2a_{n-1}}+\frac{1}{a_3a_{n-3}}+\ldots+\frac{1}{a_na_1}$

$$=\frac{1}{a_n+a_1}\left[\frac{a_n+a_1}{a_1a_n}+\frac{a_n+a_1}{a_2a_{n-1}}+\ldots+\frac{a_n+a_1}{a_na_1}\right]$$

$$=\frac{1}{a_n+a_1}\left[\frac{a_n+a_1}{a_1a_n}+\frac{a_{n-1}+a_2}{a_2a_{n-1}}+\ldots+\frac{a_n+a_1}{a_na_1}\right]$$

$$=\frac{1}{a_n+a_1}\left[\frac{1}{a_n}+\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_{n-1}}+\ldots+\frac{1}{a_n}+\frac{1}{a_1}\right]$$

$$=\frac{1}{a_n+a_1}\left[\frac{2}{a_1}+\frac{2}{a_2}+\frac{2}{a_3}+\ldots+\frac{2}{a_n}\right]$$

$$= \frac{2}{a_n + a_1}\left[\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots + \frac{1}{a_n}\right]$$

$\therefore \quad K = 2$

117. *(d)* If both roots of the equation $x^2 - 2(a-1)x + (2a+1) = 0$ are positive.

Then following conditions must be true

(i) $D \geq 0$ (ii) $2(a-1) > 0$ (iii) $2a+1 > 0$

Case I $D = (-2(a-1))^2 - 4(2a+1) \geq 0$

$$4(a^2 - 2a + 1) - 4(2a+1) \geq 0$$
$$a^2 - 4a \geq 0$$
$$a(a-4) \geq 0$$
$$a \in (-\infty, 0] \cup [4, \infty) \quad \ldots(\text{i})$$

Case II $2(a-1) > 0$

$$a > 1$$
$$a \in (1, \infty) \quad \ldots(\text{ii})$$

Case III $2a + 1 > 0$

$$a > \frac{-1}{2} \quad \ldots(\text{iii})$$

From Eqs. (i), (ii) and (iii),
we get, $a \in [4, \infty)$

118. *(b)* Given, A family has 3 children.

$\therefore$ Sample space

$= \{BBB, BBG, BGB, GBB, GGG, GGB, GBG, BGG\}$

A is the event that family has children of both sexes.

$\therefore A = \{BBG, BGB, GBB, GGB, GBG, BGG\}$

B be the event that family has atmost one boy.

$\therefore \quad B = \{GGG, GGB, GBG, BGG\}$

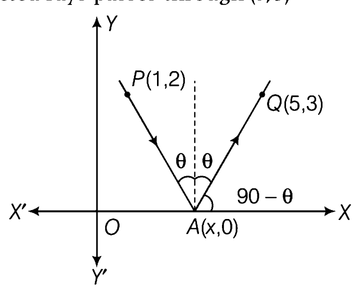
$A \cap B = \{GGB, GBG, BGG\}$

$$P(A) = \frac{6}{8},\ P(B) = \frac{4}{8},\ P(A \cap B) = \frac{3}{8}$$

$$P(A) \times P(B) = \frac{6}{8} \times \frac{4}{8} = \frac{3}{8} = P(A \cap B)$$

$\therefore$ A and B are independent events.

119. *(b)* Given a ray of light passing through the points $(1, 2)$ reflects on the X-axis at point $A(x, 0)$ and the reflected rays passes through $(5, 3)$



Slope of line PA

i.e. $\quad \tan(90 + \theta) = \dfrac{-2}{x-1}$

$$\Rightarrow \quad \cot\theta = \frac{2}{x-1} \quad \ldots(\text{i})$$

Again slope of line QA

i.e., $\tan(90 - \theta) = \dfrac{-3}{x-5}$

$$\cot\theta = \frac{-3}{x-5} \quad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we get

$$\frac{2}{x-1} = \frac{-3}{x-5}$$

$$\Rightarrow \quad 2x - 10 = -3x + 3$$
$$\Rightarrow \quad 5x = 13$$
$$\Rightarrow \quad x = 13/5$$

$\therefore$ Coordinates of A is $\left(\dfrac{13}{5}, 0\right)$.

120. *(b)* We have,

$$\frac{dy}{dx} = \frac{2xy - y^2}{2xy - x^2} \quad \ldots(\text{i})$$

Let $\quad y = vx \quad \ldots(\text{ii})$

$$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx} \quad \ldots(\text{iii})$$

On putting the values from Eqs. (ii) and (iii) in Eq. (i), we get

$$v + x\frac{dv}{dx} = \frac{2v - v^2}{2v - 1}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{2v - v^2}{2v - 1} - v$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{2v - v^2 - 2v^2 + v}{2v - 1}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{3v - 3v^2}{2v - 1}$$

$$\Rightarrow \quad \frac{2v - 1}{v^2 - v}dv = \frac{-3}{x}dx$$

$$\Rightarrow \quad \int\left(\frac{2v - 1}{v^2 - v}\right)dv = -3\int\frac{dx}{x}$$

$$\Rightarrow \quad \log|v^2 - v| = -3\log x + \log c$$

$$\Rightarrow \log(v^2 - v)x^3 = \log c$$

$$\Rightarrow \quad \left(\frac{y^2}{x^2} - \frac{y}{x}\right)x^3 = c$$

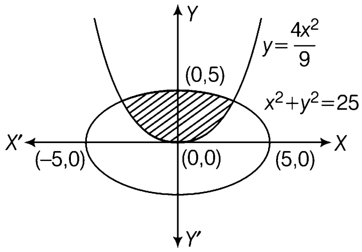
$$\Rightarrow \quad x(y^2 - xy) = c$$
$$\Rightarrow \quad xy(y - x) = c$$
$$\Rightarrow \quad xy(x - y) = c$$

121. ***(a)*** We have,

$$R_1 = \{(x, y), x^2 + y^2 \le 25\}$$

$$R_2 = \{(x, y), y \ge \frac{4x^2}{9}\}$$

Graph of R_1 and R_2 are



Clearly, from graph

range of $R_1 \cap R_2$ is $[0, 5]$.

122. ***(a)*** We have,

$$|x^2 - 4x| < 5$$

$\therefore \quad -5 < x^2 - 4x < 5$

Case I $\quad x^2 - 4x < 5$

$\Rightarrow \quad x^2 - 4x - 5 < 0$

$\Rightarrow \quad (x-5)(x+1) < 0$

$\Rightarrow \quad x \in (-1, 5) \quad \ldots$(i)

Case II $\quad x^2 - 4x > -5$

$\Rightarrow \quad x^2 - 4x + 5 > 0,$

which is true $\quad \forall\, x \in R$

$\Rightarrow \quad x \in (-\infty, \infty) \quad \ldots$(ii)

From Eqs. (i) and (ii),

$$x \in (-1, 5)$$

123. ***(a)*** A number is divisible by *b* if sum of its digit is divisible by 3. Now, out of 0, 1, 2, 3, 4, 5 if we take 1, 2, 3, 4, 5 or 0, 1, 2, 4, 5. Then, 5-digit number will be divisible by 3.

Case I Total number of five-digit number formed using the digits 1, 2, 3, 4, 5 is $5! = 120$

Case II Taking 0, 1, 2, 4, 5

Total number is $4 \times 4! = 4 \times 24 = 96$

Hence, total number divisible by 3 is $120 + 96 = 216$

Note There is a correction in question. The five-digit number is formed using 0, 1, 2, 3, 4, 5.

124. ***(c)*** We have,

$$\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} 2^m = [(x-3)+2]^{100}$$

$$\left[\because \sum_{r=0}^{n} {}^{n}C_r x^{n-r} y^r = (x+y)^n\right]$$

$$= (x-1)^{100}$$

$\therefore \quad T_{r+1} = {}^{100}C_r x^{100-r} (-1)^r$

For coefficient of x^{53}, $r = 47$.

$\therefore$ Coefficient of x^{53} in $(x-1)^{100} = {}^{100}C_{47}(-1)^{47}$

$$= {}^{-100}C_{47} = {}^{-100}C_{53}$$

125. ***(a)*** Here, $n = n$

p = Probability of head $= \frac{1}{2}$

$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

Given, $P(X=1)$, $P(X=2)$ and $P(X=3)$ are in AP.

$\therefore 2P(X=2) = P(X=1) + P(X=3)$

$$\Rightarrow 2\,{}^{n}C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{n-2} = {}^{n}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{n-1} + {}^{n}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{n-3}$$

$$\Rightarrow \frac{2 \cdot n(n-1)}{2} \left(\frac{1}{2}\right)^n = n \cdot \left(\frac{1}{2}\right)^n + \frac{n(n-1)(n-2)}{3!} \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad n - 1 = 1 + \frac{(n-1)(n-2)}{6}$$

$$\Rightarrow \quad 6n - 6 = 6 + n^2 - 3n + 2$$

$$\Rightarrow \quad n^2 - 9n + 14 = 0$$

$$\Rightarrow \quad (n-7)(n-2) = 0$$

$\therefore \quad n = 7 \quad [\because n \ne 2]$

126. ***(a)*** We have, $\quad z = 2x_1 + 3x_2$

Subject to the constraints

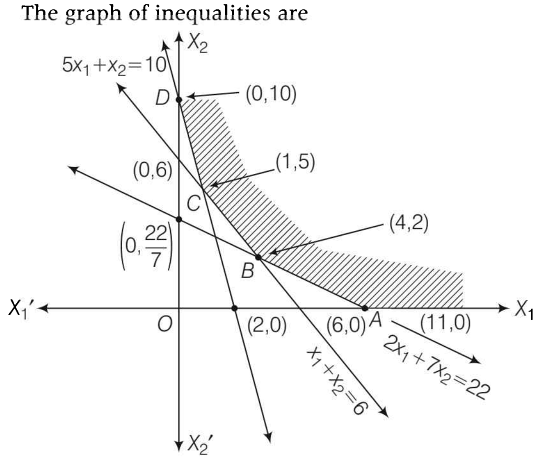
$$2x_1 + 7x_2 \ge 22$$

$$x_1 + x_2 \ge 6$$

$$5x_1 + x_2 \ge 10$$

$$x_1, x_2 \ge 0$$

The graph of inequalities are



The feasible region are $ABCD$

Corner points	$Z = 2x_1 + 3x_2$
$A(11,0)$	$22+0=22$
$B(4,2)$	$8+6=14$
$C(1,5)$	$2+15=17$
$D(0,10)$	$0+30=30$

Minimum value of Z is 14.

127. ***(a)*** We have,

$$y = a\cos(\log x) + b\sin(\log x)$$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = -\frac{a\sin(\log x)}{x} + \frac{b\cos(\log x)}{x}$$

$$\Rightarrow \quad x\frac{dy}{dx} = -a\sin(\log x) + b\cos(\log x)$$

Again differentiating w.r.t x, we get

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = -\frac{a\cos\log x}{x} - \frac{b\sin\log x}{x}$$

$$\Rightarrow \quad x^2\frac{d^2y}{dx^2} + \frac{xdy}{dx} = -(a\cos(\log x) + b\sin(\log x))$$

$$\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

128. ***(c)*** Let $z = 1 + \sin\alpha - i\cos\alpha$

$$\text{amp}(z) = \tan^{-1}\left(\frac{-\cos\alpha}{1+\sin\alpha}\right)$$

$$= -\tan^{-1}\left(\frac{\cos\alpha}{1+\sin\alpha}\right)$$

$$= -\tan^{-1}\left(\frac{\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}}{\left(\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}\right)^2}\right)$$

$$= -\tan^{-1}\left(\frac{\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}}\right)$$

$$= -\tan^{-1}\left(\tan\frac{\pi}{4} - \frac{\alpha}{2}\right)$$

$$= -\left(\frac{\pi}{4} - \frac{\alpha}{2}\right) = \frac{\alpha}{2} - \frac{\pi}{4}$$

129. ***(b)*** We have,

$$z_1 = x + iy \text{ and } z_2 = \frac{1}{-x+iy}$$

$$\therefore \quad z_2 = -\frac{1}{x-iy} \times \frac{x+iy}{x+iy}$$

$$z_2 = -\frac{(x+iy)}{x^2+y^2}$$

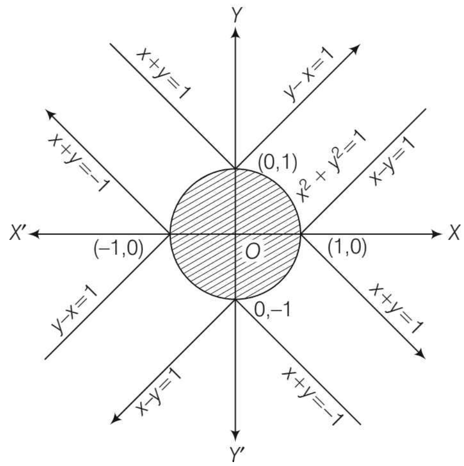
$$\Rightarrow \quad z_2 = -\frac{z_1}{|z_1|^2}$$

Clearly, the point z_1 and z_2 lie on a straight line passes through origin.

130. ***(a)*** We have,

$$\big||x| - |y|\big| \le 1 \text{ and } x^2 + y^2 \le 1$$

The graph of given curve



Required area is area of circle whose radius is 1.

$\therefore$ Area $= \pi(1)^2 = \pi$

131. ***(c)*** We have,

$$\log_{\left(\sin\frac{\pi}{3}\right)}(x^2 - 3x + 2) \ge 2$$

$$\Rightarrow \quad \log_{\frac{\sqrt{3}}{2}}(x^2 - 3x + 2) \ge 2$$

$$\Rightarrow \quad (x^2 - 3x + 2) \le \left(\frac{\sqrt{3}}{2}\right)^2$$

and $\quad x^2 - 3x + 2 > 0$

$$\Rightarrow \quad x^2 - 3x + 2 \le \frac{3}{4} \text{ and } x^2 - 2x - x + 2 > 0$$

$$\Rightarrow \quad 4x^2 - 12x + 5 \le 0 \text{ and } (x-2)(x-1) > 0$$

$$\Rightarrow (2x-5)(2x-1) \le 0 \text{ and } x \in (-\infty, 1) \cup (2, \infty)$$

$$x \in \left[\frac{1}{2}, \frac{5}{2}\right] \text{ and } x \in (-\infty, 1) \cup (2, \infty)$$

$$\therefore \quad x \in \left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$$

132. ***(c)*** We have, $\quad \dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$

Let $P(5\cos\theta, 4\sin\theta)$ be any point on the ellipse.

Then, $SP = 5 + 5e\cos\theta$ and $S'P = 5 - 5e\cos\theta$

$$SP \cdot S'P = 25(1 - e^2 \cos^2 \theta)$$

$$= 25\left(1 - \frac{9}{25}\cos^2\theta\right) \qquad \left[\because e^2 = 1 - \frac{16}{25} = \frac{9}{25}\right]$$

$$= 25 - 9\cos^2\theta = 25 - 9(1 - \sin^2\theta)$$

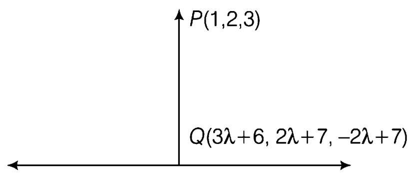
$$= 16 + 9\sin^2\theta = f(\theta)$$

$\therefore \quad f(\theta) \in [16, 25]$

Hence, $16 \le f(\theta) \le 25$.

133. *(b)* Let $Q(3\lambda + 6, 2\lambda + 7, -2\lambda + 7)$ be any point lie on line

$$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2} = \lambda$$



$$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$$

$\therefore$ Direction ratio of line $PQ\,(3\lambda + 5, 2\lambda + 5, -2\lambda + 4)$

Since, line PQ is perpendicular to given line.

$\therefore 3(3\lambda + 5) + 2(2\lambda + 5) - 2(-2\lambda + 4) = 0$

$\Rightarrow \quad 17\lambda + 17 = 0 \Rightarrow \lambda = -1$

$\therefore Q(3, 5, 9)$

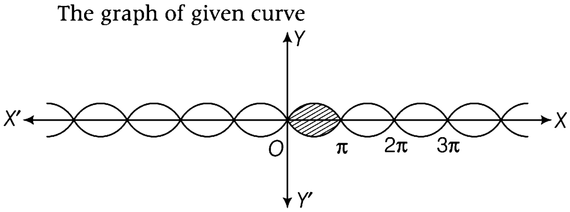
Distance $(PQ) = \sqrt{(3-1)^2 + (5-2)^2 + (9-3)^2}$

$$= \sqrt{4 + 9 + 36}$$

$$= \sqrt{49} = 7$$

134. *(c)* We have, $|y| = |\sin x|$

The graph of given curve



The area of one loop

$$= 2\int_0^{\pi} \sin x \, dx$$

$$= 2[-\cos x]_0^{\pi} = 2[-\cos\pi + \cos 0]$$

$$= 2(1 + 1) = 4$$

135. *(c)* Given, a, b, α_r are the first term, common difference and rth term of an AP, respectively.

$$\therefore \quad \alpha_m = a + (m-1)d = \frac{1}{n} \quad \dots\text{(i)}$$

$$\text{and} \quad \alpha_n = a + (n-1)d = \frac{1}{m} \quad \dots\text{(ii)}$$

On subtracting Eq. (ii) from Eq. (i), we get

$$d(m - 1 - n + 1) = \frac{1}{n} - \frac{1}{m}$$

$$\Rightarrow \quad d(m-n) = \frac{m-n}{mn}$$

$$\Rightarrow \quad d = \frac{1}{mn}$$

On putting the value of d in Eq.(i), we get

$$a = \frac{1}{mn}$$

$$\therefore \quad a - d = 0$$

136. *(b)* Let $I = \int_0^{\infty} [3e^{-x}]\,dx$

$3e^{-x}$ is decreasing function for $x \in [0, \infty)$.

When $x \in (0, \ln 3)$, then $[3e^{-x}] = 1$ And when $x \in (\ln 3, \infty)$, then $[3e^{-x}] = 0$

$$\therefore \quad I = \int_0^{\ln 3} [3e^{-x}]\,dx + \int_{\ln 3}^{\infty} [3e^{-x}]\,dx$$

$$\Rightarrow \quad I = \int_0^{\ln 3} dx + 0 \Rightarrow I = \ln 3$$

137. *(b)* Given A and B are disjoint sets

$\therefore \quad A \cap B = \phi$

Now, $B \cap A' = B - A = B - (A \cap B) = B - \phi = B$

138. *(c)* $n^2 - n + 41$ is not a prime number $\forall\, n \in N$

Put $n = 41$

$\therefore n^2 - n + 41 = 41^2 - 41 + 41 = 41^2$

Which is not prime.

139. *(d)* The vector parallel to the line $x - 2y + z = 0$ $= x + 2y - 2z$ is given by

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 1 & 2 & -2 \end{vmatrix} = (4-2)\hat{i} - (-2-1)\hat{j} + (2+2)\hat{k}$$

$$= 2\hat{i} + 3\hat{j} + 4\hat{k}$$

and normal vector of plane $5x - 2y - z + 17 = 0$ is $5\hat{i} - 2\hat{j} - \hat{k}$

$\therefore$ Angle between line and plane is

$$\sin\theta = \frac{(2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (5\hat{i} - 2\hat{j} - \hat{k})}{|2\hat{i} + 3\hat{j} + 4\hat{k}||5\hat{i} - 2\hat{j} - \hat{k}|}$$

$$\sin\theta = 0$$

$$\therefore \quad \theta = 0^\circ$$

140. *(a)* Let vector $\vec{a}$ is coplanar with $\hat{i} + 2\hat{j} - \hat{k}$ and $2\hat{i} + \hat{j} + 3\hat{k}$.

$$\therefore \quad \vec{a} = \lambda(\hat{i} + 2\hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 3\hat{k})$$

$$\vec{a} = (\lambda + 2\mu)\hat{i} + (2\lambda + \mu)\hat{j} + (-\lambda + 3\mu)\hat{k}$$

Since, a is orthogonal to $\hat{i}+\hat{j}+\hat{k}$

$\therefore \quad \lambda+2\mu+2\lambda+\mu-\lambda+3\mu=0$

$\Rightarrow \quad 2\lambda+6\mu=0 \Rightarrow \lambda=-3\mu$

$\therefore \quad \vec{a}=-\mu\hat{i}-5\mu\hat{j}+6\mu\hat{k}$

$\vec{a}=-\mu(\hat{i}+5\hat{j}+6\hat{k})$

Again, $|a|=1$

$\therefore \quad \mu^2(1+25+36)=1$

$\Rightarrow \quad \mu=\pm\sqrt{\dfrac{1}{62}}$

$\therefore \quad a=\dfrac{\hat{i}+5\hat{j}+6\hat{k}}{\sqrt{62}}$

141. ***(a)*** We have,

$$l=\lim_{x\to 0}|x|^{[\cos x]}$$

LHL $\quad l=\lim_{x\to 0^-}|x|^{[\cos x]}$

$\Rightarrow \quad l=\lim_{h\to 0}|0-h|^{[\cos(0-h)]}$

$\Rightarrow \quad l=\lim_{h\to 0}(h)^{\circ}=1 \qquad [\because [\cos(0-h)]=0]$

RHL $\quad l=\lim_{x\to 0^+}|x|^{[\cos x]}$

$l=\lim_{h\to 0}|0+h|^{[\cos(0+h)]}$

$l=\lim_{h\to 0}(h)^{\circ}=1$

LHL = RHL

$\therefore \lim_{x\to 0}|x|^{[\cos x]}=1$

142. ***(a)*** We have,

$$\cot^{-1}(\sqrt{\cos\alpha})-\tan^{-1}(\sqrt{\cos\alpha})=x$$

$$\frac{\pi}{2}-\tan^{-1}\sqrt{\cos\alpha}-\tan^{-1}\sqrt{\cos\alpha}=x$$

$\Rightarrow \quad 2\tan^{-1}(\sqrt{\cos\alpha})=\dfrac{\pi}{2}-x$

$\Rightarrow \quad \cos^{-1}\left(\dfrac{1-\cos\alpha}{1+\cos\alpha}\right)=\dfrac{\pi}{2}-x$

$$\left[\because 2\tan^{-1}x=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$$

$$\frac{2\sin^2\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}}=\cos\left(\frac{\pi}{2}-x\right)$$

$\Rightarrow \quad \tan^2\dfrac{\alpha}{2}=\sin x$

$\therefore \quad \sin x=\tan^2\dfrac{\alpha}{2}$

Note There is a correction in question. It is $\sin x$ not $\sin n$.

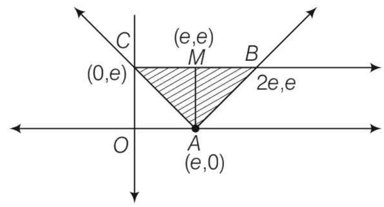
143. ***(b)*** We have,

$$y=\begin{cases}x^{\frac{1}{\ln x}}, & x\neq 1\\ e, & x=1\end{cases} \text{ and } y=|x-e|$$

$$\Rightarrow \quad y=\begin{cases}e^{\ln\left(x^{\frac{1}{\ln x}}\right)} & x\neq 1\\ e, & x=1\end{cases} \text{ and } y=\begin{cases}-x+e, & x<e\\ x-e, & x\geq e\end{cases}$$

$$\Rightarrow \quad y=e,\ x\in R^+ \text{ and } y=\begin{cases}e-x, & x<e\\ x-e, & x\geq e\end{cases}$$

The graph of given curve is



$\therefore$ Area of shaded region

$$=\frac{1}{2}\times BC\times MA$$

$$=\frac{1}{2}\times 2e\times e=e^2$$

144. ***(d)*** Let $P(3\lambda+2, 4\lambda-1, 12\lambda+2)$ be any point lie on line $\dfrac{x-2}{3}=\dfrac{y+1}{4}=\dfrac{z-2}{12}$

Since, line intersect the plane

$$x-y+z=5$$

$\therefore P(3\lambda+2, 4\lambda-1, 12\lambda+2)$ lie on plane

$\therefore 3\lambda+2-(4\lambda-1)+12\lambda+2=5$

$\Rightarrow \quad 11\lambda+5=5$

$\Rightarrow \quad \lambda=0$

$\therefore P(2,-1,2)$

Distance from $(1,-2,3)$ and $(2,-1,2)$ is

$\sqrt{(2-1)^2+(-1+2)^2+(2-3)^2}$

$=\sqrt{1+1+1}=\sqrt{3}$

145. ***(a)*** We have,

$$\cos\alpha+\cos\beta-\cos(\alpha+\beta)=\frac{3}{2}$$

It is possible when

$$\cos\alpha=\cos\beta=-\cos(\alpha+\beta)=\frac{1}{2}$$

$\therefore \quad \alpha=60^\circ, \beta=60^\circ$ and $\alpha+\beta=120^\circ$

$\therefore \quad \alpha=\beta$

146. ***(b)*** We have,

$$|x| + \left|\frac{x}{x-1}\right| = \frac{x^2}{|x-1|}$$

We know that,

$|a| + |b| = |a+b|$. Then, $ab \geq 0$

Here, $$|x| + \left|\frac{x}{x-1}\right| = \left|x + \frac{x}{x-1}\right|$$

$$|x| + \left|\frac{x}{x-1}\right| = \left|\frac{x^2}{x-1}\right| = \frac{x^2}{|x-1|}$$

$\therefore$ $$x \cdot \frac{x}{x-1} \geq 0 \Rightarrow \frac{x^2}{x-1} \geq 0$$

$\Rightarrow$ $$x \in \{0\} \cup (1, \infty)$$

147. ***(b)*** We have,

$$\frac{\sin(\theta-\alpha)}{\sin(\theta-\beta)} = \frac{a}{b} \text{ and } \frac{\cos(\theta-\alpha)}{\cos(\theta-\beta)} = \frac{c}{d}$$

$\Rightarrow$ $$\frac{\sin(\theta-\alpha)+\sin(\theta-\beta)}{\sin(\theta-\alpha)-\sin(\theta-\beta)} = \frac{a+b}{a-b}$$

and $$\frac{\cos(\theta-\alpha)+\cos(\theta-\beta)}{\cos(\theta-\alpha)-\cos(\theta-\beta)} = \frac{c+d}{c-d}$$

$$\Rightarrow \frac{2\sin\left(\frac{2\theta+(\alpha-\beta)}{2}\right)\cos\left(\frac{\beta-\alpha}{2}\right)}{2\cos\left(\frac{2\theta+\alpha-\beta}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right)} = \frac{a+b}{a-b} \quad \ldots\text{(i)}$$

$$\text{and } \frac{2\cos\left(\frac{2\theta+(\alpha-\beta)}{2}\right)\cos\left(\frac{\beta-\alpha}{2}\right)}{2\sin\left(\frac{2\theta+\alpha-\beta}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right)} = -\left(\frac{c+d}{c-d}\right) \ldots\text{(ii)}$$

On multiplying Eqs. (i) and (ii), we get

$$\cot^2\left(\frac{\alpha-\beta}{2}\right) = \frac{(a+b)(c+d)}{(a-b)(d-c)}$$

$\Rightarrow$ $$\tan^2\left(\frac{\alpha-\beta}{2}\right) = \frac{(a-b)(d-c)}{(a+b)(c+d)}$$

$$\cos(\alpha-\beta) = \frac{1-\tan^2\frac{\alpha-\beta}{2}}{1+\tan^2\frac{\alpha-\beta}{2}}$$

$$= \frac{1-\frac{(a-b)(d-c)}{(a+b)(c+d)}}{1+\frac{(a-b)(d-c)}{(a+b)(c+d)}} = \frac{ac+bd}{ad+bc}$$

148. ***(c)*** We have,

$$f(x) = \sin x$$

Range of $f(x)$ is $[-1, 1]$, $\forall\ x \in R$

$\therefore$ $\quad f^{-1}[-1, 1]$ is R.

149. ***(b)*** We have,

$$\begin{vmatrix} 5 & 4 & 3 \\ a51 & b41 & c31 \\ a & b & c \end{vmatrix} = 0$$

$$\begin{vmatrix} 5 & 4 & 3 \\ 100a+51 & 100b+41 & 100c+31 \\ a & b & c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 5 & 4 & 3 \\ 100a & 100b & 100c \\ a & b & c \end{vmatrix} + \begin{vmatrix} 5 & 4 & 3 \\ 51 & 41 & 31 \\ a & b & c \end{vmatrix} = 0$$

$$\Rightarrow \quad 0 + \begin{vmatrix} 5 & 4 & 3 \\ 51 & 41 & 31 \\ a & b & c \end{vmatrix} = 0$$

Apply, $R_2 \to R_2 - 10R_1$

$$\begin{vmatrix} 5 & 4 & 3 \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix} = 0$$

$\Rightarrow$ $\quad 5(c-b) + 4(a-c) + 3(b-a) = 0$

$\Rightarrow$ $\quad 5c - 5b + 4a - 4c + 3b - 3a = 0$

$\Rightarrow$ $\quad 2b = a + c$

So, a, b, c are in AP.

150. ***(a)*** We have,

$$A = \begin{bmatrix} k & l \\ m & n \end{bmatrix}$$

$$A^2 = \begin{bmatrix} k & l \\ m & n \end{bmatrix}\begin{bmatrix} k & l \\ m & n \end{bmatrix} = \begin{bmatrix} k^2+lm & kl+ln \\ km+mn & lm+n^2 \end{bmatrix}$$

$$(k+n)A = \begin{bmatrix} k^2+kn & kl+ln \\ km+mn & kn+n^2 \end{bmatrix}$$

$$(kn-ml)I = \begin{bmatrix} kn-ml & 0 \\ 0 & kn-ml \end{bmatrix}$$

$$A^2 - (k+n)A + (kn-lm)I = \begin{bmatrix} k^2+lm & kl+ln \\ km+mn & lm+n^2 \end{bmatrix}$$

$$-\begin{bmatrix} k^2+kn & kl+ln \\ km+mn & kn+n^2 \end{bmatrix} + \begin{bmatrix} kn-ml & 0 \\ 0 & kn-ml \end{bmatrix} = O$$