

3. INDEFINITE INTEGRATION

I. MCQ (2marks)

Q1) $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx = \dots$

a) $\frac{1}{2}\sqrt{\tan x} + c$

b) $2\sqrt{\tan x} + c$

c) $\sqrt{2} \tan x + c$

d) $\frac{1}{\sqrt{2\tan x}} + c$

Q2) $\int \frac{1}{\sqrt{x} + \sqrt{x^3}} dx = \dots$

a) $2\tan^{-1}(\sqrt{x}) + c$

b) $\tan^{-1}(2x - 1) + c$

c) $2\tan \sqrt{x} + c$

d) $2\tan^{-1}(x) + c$

Q3) $\int \frac{1}{\sqrt{(x-3)(x-2)}} dx = \dots$

a) $\frac{1}{2} \log[(2x - 1) + \sqrt{x^2 - x - 6}] + c$

b) $\tan^{-1}(2x - 1) + c$

c) $\log\left[\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x - 6}\right] + c$

d) $\log\left[\left(x - \frac{1}{2}\right) + \sqrt{x^2 + x + 6}\right] + c$

Q4) $\int \frac{1}{\cos x - \sin x} dx = \dots$

a) $\frac{1}{\sqrt{2}} \log\left[\operatorname{cosec}\left(x + \frac{\pi}{4}\right) - \cot\left(x + \frac{\pi}{4}\right)\right] + c$

b) $\sqrt{2} \log\left[\operatorname{cosec}\left(x + \frac{\pi}{4}\right) + \cot\left(x + \frac{\pi}{4}\right)\right] + c$

c) $\frac{1}{\sqrt{2}} \log\left[\sec\left(x + \frac{\pi}{4}\right) + \tan\left(x + \frac{\pi}{4}\right)\right] + c$

d) $\sqrt{2} \log\left[\sec\left(x + \frac{\pi}{4}\right) - \tan\left(x + \frac{\pi}{4}\right)\right] + c$

Q5) $\int \frac{x^2}{\sqrt{1-x^6}} dx = \dots$

a) $-\sin^{-1}(x^3) + c$

b) $\cos^{-1}(x^3) + c$

c) $\sin^{-1}(x^3) + c$

d) $\frac{1}{3} \sin^{-1}(x^3) + c$

Q6) $\int \frac{e^x}{x} [x(\log x)^2 + 2 \log x] dx = \dots$

a) $e^x \log x + c$

b) $e^x (\log x)^2 + c$

c) $e^{2x} \log x + c$

d) $e^{2x} (\log x)^2 + c$

Q7) $\int \sqrt{x^2 + 2x + 5} \, dx = \dots$

- a) $(x + 1)\sqrt{x^2 + 2x + 5} + \log[(x + 1) + \sqrt{x^2 + 2x + 5}] + c$
- b) $(x + 2)\sqrt{x^2 + 2x + 5} + \log[(x + 2) + \sqrt{x^2 + 2x + 5}] + c$
- c) $(\frac{x+2}{2})\sqrt{x^2 + 2x + 5} + \frac{1}{2}\log[(x + 2) + \sqrt{x^2 + 2x + 5}] + c$
- d) $(\frac{x+1}{2})\sqrt{x^2 + 2x + 5} + 2\log[(x + 1) + \sqrt{x^2 + 2x + 5}] + c$

Q8) If $f(x) = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, $g(x) = e^{\sin^{-1} x}$, then $\int f(x)g(x)dx = \dots$

- a) $e^{\sin^{-1} x}(\sin^{-1} x - 1) + c$
- b) $e^{\sin^{-1} x}(1 - \sin^{-1} x) + c$
- c) $e^{\sin^{-1} x}(\sin^{-1} x + 1) + c$
- d) $-e^{\sin^{-1} x}(\sin^{-1} x + 1) + c$

Q9) $\int \frac{\sin^m x}{\cos^{m+2} x} \, dx = \dots$

- a) $(m + 2) \tan^{m+1} x + c$
- b) $\frac{\tan^m x}{m} + c$
- c) $(m + 1) \tan^{m+1} x + c$
- d) $\frac{\tan^{m+1} x}{m+1} + c$

Q10) $\int \frac{2}{\sqrt{x} - \sqrt{x+3}} \, dx = \dots$

- a) $-\frac{2}{3} \left[x^{\frac{3}{2}} + (x + 3)^{\frac{3}{2}} \right] + c$
- b) $\frac{2}{3} \left[x^{\frac{3}{2}} - (x + 3)^{\frac{3}{2}} \right] + c$
- c) $\frac{4}{9} \left[x^{\frac{3}{2}} - (x + 3)^{\frac{3}{2}} \right] + c$
- d) $-\frac{4}{9} \left[x^{\frac{3}{2}} + (x + 3)^{\frac{3}{2}} \right] + c$

Q11) $\int \cos \sqrt{x} \, dx = \dots$

- a) $2[\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}] + c$
- b) $\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x} + c$
- c) $2[\sqrt{x} \cos \sqrt{x} + \sin \sqrt{x}] + c$
- d) $\frac{1}{2}[\sqrt{x} \sin \sqrt{x} - \cos \sqrt{x}] + c$

Q12) $\int \frac{2(\cos^2 x - \sin^2 x)}{\cos^2 x + \sin^2 x} \, dx = \dots$

- a) $\sin 2x + c$
- b) $\cos 2x + c$
- c) $\tan 2x + c$
- d) $2\sin 2x + c$

Q13) $\int \frac{\log x}{(\log ex)^2} dx = \dots$

a) $x(1 + \log x) + c$

b) $\frac{x}{1+\log x} + c$

c) $\frac{1}{1+\log x} + c$

d) $\frac{1}{1-\log x} + c$

Q14) If $\int \frac{1}{x+x^5} dx = f(x) + c$, then $\int \frac{x^4}{x+x^5} dx = \dots$

a) $f(x) - \log x + c$

b) $f(x) + \log x + c$

c) $\log x - f(x) + c$

d) $\frac{1}{5}x^5 f(x) + c$

Q15) $\int \frac{e^x(x-1)}{x^2} dx = \dots$

a) $xe^{-x} + c$

b) $\frac{e^x}{x^2} + c$

c) $\left(x - \frac{1}{x}\right)e^x + c$

d) $\frac{e^x}{x} + c$

II. Very Short Answers (1 mark)

Q1) $\int \sqrt{1 + \sin 2x} \, dx$

Q2) $\int \frac{\sin 4x}{\cos 2x} \, dx$

Q3) $\int \frac{e^{3x}}{e^{3x}+1} \, dx$

Q4) $\int \frac{1}{\sqrt{3x^2+8}} \, dx$

Q5) $\int \frac{\log x}{x} \, dx$

Q6) $\int (2 + \cot x - \operatorname{cosec}^2 x)e^x \, dx$

Q7) $\int e^x \left[\frac{(x+3)}{(x+4)^2} \right] \, dx$

Q8) $\int \frac{e^{2x}+e^{-2x}}{e^x} \, dx$

Q9) $\int x^x(1 + \log x) \, dx$

Q10) $\int \frac{1}{x \sin^2(\log x)} \, dx$

$$\text{Q11)} \int \sqrt{x} \sec(x)^{\frac{3}{2}} \tan(x)^{\frac{3}{2}} dx$$

$$\text{Q12)} \int \frac{\cos 2x}{\sin^2 x} dx$$

$$\text{Q13)} \int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$$

$$\text{Q14)} \int \cot^2 x dx$$

$$\text{Q15)} \int \frac{x}{x+2} dx$$

III. Short Answers (2 marks)

$$\text{Q1)} \int \frac{\sin x}{1 + \sin x} dx$$

$$\text{Q2)} \int \frac{(2x-7)}{\sqrt{4x-1}} dx$$

$$\text{Q3)} \int \frac{1}{4x+5x^{-11}} dx$$

$$\text{Q4)} \int e^{3 \log x} (x^4 + 1)^{-1} dx$$

$$\text{Q5)} \int \frac{\sin(x-a)}{\cos(x+b)} dx$$

$$\text{Q6)} \int \cos^7 x dx$$

$$\text{Q7)} \int \frac{x^7}{(1+x^4)^2} dx$$

$$\text{Q8)} \int \frac{1}{\sqrt{2x^2-5}} dx$$

$$\text{Q9)} \int \frac{\log(\log x)}{x} dx$$

$$\text{Q10)} \int x^2 \sqrt{a^2 - x^6} dx$$

$$\text{Q11)} \int \sqrt{4^x(4^x + 4)} dx$$

$$\text{Q12)} \int [\operatorname{cosec}(\log x)][1 - \cot(\log x)] dx$$

$$\text{Q13)} \int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$$

$$\text{Q14)} \int \sin 4x \cos 3x dx$$

$$\text{Q15)} \int \frac{e^x \log(\sin e^x)}{\tan e^x} dx$$

IV. Short answers (3 Marks)

$$\text{Q1)} \int \frac{1}{x(x^3-1)} dx$$

$$\text{Q2)} \text{If } f'(x) = x - \frac{3}{x^3}, f(1) = \frac{11}{2} \text{ find } f(x)$$

$$\text{Q3)} \int \frac{(x^2+2)}{x^2+1} a^{x+\tan^{-1}x} dx$$

$$\text{Q4)} \int \frac{7+4x+5x^2}{(2x+3)^{\frac{3}{2}}} dx$$

$$\text{Q5)} \int \sqrt{\frac{9+x}{9-x}} dx$$

$$\text{Q6)} \int \frac{1}{4x^2-20x+17} dx$$

$$\text{Q7)} \int \frac{\sin x}{\sin 3x} dx$$

$$\text{Q8)} \int \frac{1}{2+\cos x - \sin x} dx$$

$$\text{Q9)} \int \sec^3 x dx$$

$$\text{Q10)} \int \sin(\log x) dx$$

$$\text{Q11)} \int \sec^2 x \sqrt{\tan^2 x + \tan x - 7} dx$$

$$\text{Q12)} \int e^{\sin^{-1}x} \left[\frac{x+\sqrt{1-x^2}}{\sqrt{1-x^2}} \right] dx$$

$$\text{Q13)} \int e^x \frac{(1+x^2)}{(1+x)^2} dx$$

$$\text{Q14)} \int \frac{x^2+x-1}{x^2+x-6} dx$$

$$\text{Q15)} \int \frac{6x^3+5x^2-7}{3x^2-2x-7} dx$$

V. Long answers (4 Marks)

$$\text{Q1)} \int \frac{dx}{2+3\tan x}$$

$$\text{Q2)} \int \sqrt{\tan x} + \sqrt{\cot x} dx$$

$$\text{Q3)} \int \frac{3x+4}{\sqrt{2x^2+2x+1}} dx$$

$$\text{Q4)} \int \sqrt{\frac{e^{3x}-e^{2x}}{e^x+1}} dx$$

$$\text{Q5)} \int x^3 \tan^{-1} x \, dx$$

$$\text{Q6)} \int x \sin 2x \cos 5x \, dx$$

$$\text{Q7)} \int \frac{x+\sin x}{1-\cos x} dx$$

$$\text{Q8)} \int \frac{x^2}{(x^2+1)(x^2-2)(x^2+3)} dx$$

$$\text{Q9)} \int \frac{dx}{x^3-1}$$

$$\text{Q10)} \int \frac{5e^x}{(e^x+1)(e^{2x}+9)} dx$$

$$\text{Q11)} \int \frac{1}{\sin x(3+2\cos x)} dx$$

$$\text{Q12)} \int x \cos^3 x \, dx$$

$$\text{Q13)} \int \frac{\sin 2x}{3\sin^4 x - 4\sin^2 x + 1} dx$$

$$\text{Q14)} \int \frac{3e^{2x}+5}{4e^{2x}-5} dx$$

$$\text{Q15)} \int \frac{(2\log x+3)}{x(3\log x+2)[(\log x)^2+1]} dx$$

Q16) Integrate the functions w. r. t. x

i) $(3x-2)\sqrt{x^2+x+1}$

ii) $x\sqrt{1+x-x^2}$

iii) $(x-5)\sqrt{x^2-1}$

iv) $(x+1)\sqrt{x^2+x+1}$

v) $(2x+3)\sqrt{3x^2+2x-5}$