4. DEFINITE INTEGRATION

I. MCQ (2 marks each)

(1) $\int_1^9 \frac{x+1}{\sqrt{x}}\, dx$

(A) $\frac{80}{3}$ *(B)* $\frac{64}{3}$ *(C)* $\frac{17}{3}$ *(D)* $\frac{15}{3}$

(2) $\int_0^1 \sqrt{\frac{1-x}{1+x}}\, dx =$

(A) $\frac{\pi}{2} - 1$ (B) $\frac{\pi}{2} + 1$ (C) $\frac{\pi}{2} - 2$ (D) $\frac{\pi}{2} + 2$

(3) $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}\, dx =$

(A)$2\sqrt{e}(1+\sqrt{e})$ (B) $\sqrt{e}(1-\sqrt{e})$ (C) $\sqrt{e}(\sqrt{e}-1)$ (D) $\sqrt{e}(1+\sqrt{e})$

(4) $\int_0^{\frac{\pi}{4}} \sqrt{1+sin2x}\, dx =$

(A) $\frac{1}{\sqrt{2}}$ (B)$\sqrt{2}$+1 (C)$2\sqrt{2}$ (D) 1

(5) If $\int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}} = \frac{k}{3}$ then $k = \cdots$

(A) $\sqrt{2}(2\sqrt{2}-2)$ (B) $\frac{\sqrt{2}}{3}(2-2\sqrt{2})$ (C) $\frac{2\sqrt{2}-2}{3}$ (D) $4\sqrt{2}$

(6)$\int_{\frac{\pi}{5}}^{\frac{3\pi}{10}} \frac{sinx}{sinx+cosx} dx =$

(A) $\frac{\pi}{10}$ (B) $\frac{\pi}{20}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{12}$

(7) $\int_0^1 \frac{x^2-2}{x^2+1} dx =$

(A) $1 - \frac{3\pi}{4}$ (B) $2 - \frac{3\pi}{4}$ (C) $1 + \frac{3\pi}{4}$ (D) $2 + \frac{3\pi}{4}$

(8) Let $I_1 = \int_e^{e^2} \frac{1}{\log x}\, dx$ and $I_2 = \int_1^2 \frac{e^x}{x}\, dx$ then

(A) $I_1 = \frac{1}{3} I_2$ (B) $I_1 + I_2 = 0$ (C) $I_1 = 2I_2$ (D) $I_1 = I_2$

(9)$\int_0^4 \frac{1}{\sqrt{4x-x^2}}\,dx =$

(A) 0 (B)2π (C) π (D)4π

(10) $\int_0^{\frac{\pi}{2}} \log(tanx)\,dx =$

(A) $\frac{\pi}{8}(log2)$ (B) 0 (C) $-\frac{\pi}{8}(log2)$ (D) $\frac{\pi}{2}(log2)$

II. Very Short Answers (1 mark)

(11) $Evaluate$: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} cosx\,dx.$

(12) $Evaluate$: $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x^3 \sin^4 x\,dx$

(13) $Evaluate$: $\int_0^1 \frac{1}{1+x^2}\,dx$

(14) $Evaluate$: $\int_0^{\frac{\pi}{4}} \sec^2 x\,dx$

(15) $Evaluate$: $\int_0^1 |x|dx$

(16) $Evaluate$: $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$

(17) $Evaluate$: $\int_1^2 \frac{x}{1+x^2}\,dx$

(18) $Evaluate$: $\int_0^1 \frac{e^x}{\sqrt{e^x-1}}\,dx$

(19) $Evaluate$: $\int_0^{\frac{\pi}{2}} \frac{sin2x}{1+\sin^2 x}\,dx$

(20) $Evaluate$: $\int_0^1 (x+1)^2 dx$

III. Short Answers (2 marks)

(21) Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} sin^2 x\,dx$

(22) Evaluate: $\int_0^{\pi/2} \sqrt{1-cos4x}\,dx$

(23) Evaluate: $\int_0^{\frac{\pi}{2}} \cos^3 x\ dx$

(24) Evaluate: $\int_0^{\pi} cos^2 x.dx$

(25) Evaluate: $\int_0^{\frac{\pi}{4}} \frac{\tan^3 x}{1+\cos 2x}\ dx$

(26) Evaluate: $\int_0^{\frac{\pi}{4}} \frac{cosx}{4-sin^2 x} dx$

(27) Evaluate: $\int_1^3 \frac{\cos(logx)}{x} dx$

(28) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{(1+\cos x)^2}\ dx$

(29) Evaluate: $\int_0^9 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}}\ dx$

IV. Short Answers (3 marks)

(30) Prove that : $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$where $a < c < b$

(31) Prove that: $\int_a^b \boldsymbol{f(x)\,dx} = \int_a^b \boldsymbol{f(a+b-x)\,dx}$

(32) Prove that: $\int_0^a \boldsymbol{f(x)\,dx} = \int_0^a \boldsymbol{f(a-x)\,dx}$ **. Hence find** $\int_0^{\frac{\pi}{2}} \boldsymbol{sin^2 x\,dx}$.

(33) Evaluate: $\int_0^{\pi/2} \frac{sin^4 x}{sin^4 x + cos^4 x} dx$

(34) Evaluate: $\int_3^8 \frac{(11-x)^2}{x^2+(11-x)^2} dx$

(35) Evaluate: $\int_{-1}^1 |5x-3|\ dx$

(36) Evaluate: $\int_{-4}^2 \frac{1}{x^2+4x+13} dx$

(37) Evaluate: $\int_0^1 \frac{1}{\sqrt{3+2x-x^2}} dx$

(38) Evaluate: $\int_0^1 x.tan^{-1}x\,dx$

(39) Evaluate: $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{(1-x^2)^{\frac{3}{2}}}\, dx$

(40) Evaluate: $\int_0^{\frac{\pi}{4}} sec^4 x\, dx$

(41) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{1}{5+4\,cosx}\, dx$

(42) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{cosx}{(1+sinx)(2+sinx)}\, dx$

(43) Evaluate: $\int_{-1}^{1} \frac{1}{a^2e^x + b^2e^{-x}}\, dx$

(44) Evaluate: $\int_0^a \frac{1}{x+\sqrt{a^2-x^2}}\, dx$

(45) Evaluate: $\int_0^3 x^2(3-x)^{\frac{5}{2}}\, dx$

(46) Evaluate: $\int_0^1 t^2\sqrt{1-t}\ \, dt$

V. Long answers (4 Marks)

(47) $Prove\ that$: $\int_0^{2a} f(x)\, dx = \int_0^a f(x)\, dx + \int_0^a f(2a-x)\, dx$

(48) Prove that: $\int_{-a}^{a} f(x)\, dx = 2\ \int_0^a f(x)\, dx$, if $f(x)$ is even function.

$= 0$, if $f(x)$ is odd function

(49) $Evaluate$: $\int_{\mathbf{0}}^{\mathbf{1/2}} \frac{\mathbf{1}}{(\mathbf{1-2x^2})\sqrt{\mathbf{1-x^2}}}\, \boldsymbol{dx}$

(50) Evaluate: $\int_0^{\frac{\pi}{4}} \frac{sec^2 x}{3tan^2 x + 4tanx + 1}\, dx$

(51) Evaluate: $\int_{\frac{1}{\sqrt{2}}}^{1} \frac{(e^{cos^{-1}x})(\sin^{-1} x)}{\sqrt{1-x^2}}\, dx$

(52) Evaluate: $\int_0^1 \frac{\log(x+1)}{x^2+1}\, dx$

(53) Evaluate: $\int_0^{\pi} x \cdot sinx \cdot cos^2 x\ \cdot dx$

(54) Evaluate: $\int_0^{\frac{\pi}{2}} x\ sinx\, dx$

(55) Evaluate: $\int_{-1}^{1} \frac{1+x^3}{9-x^2}\ dx$

(56) Evaluate: $\int_{0}^{1} \left(\frac{1}{1+x^2}\right) \sin^{-1}\left(\frac{2x}{1+x^2}\right)\ dx$

(57) Evaluate: $\int_{0}^{\frac{\pi}{4}} \frac{cos2x}{1+\cos 2x+\sin 2x}\ dx$

(58) Evaluate: $\int_{0}^{\pi/4} log\ (1+tanx)\, dx$

(59) Evaluate: $\int_{0}^{\pi} \frac{1}{3+2sinx+cosx} dx$

60) Evaluate integral using limit of sum

i) $\int_{0}^{3}(x+4)dx$

ii) $\int_{0}^{2}(2x+1)dx$

iii) $\int_{0}^{1} 2^x dx$

iv) $\int_{0}^{4}(x-x^2)dx$

v) $\int_{0}^{4}(8x^3)dx$