7. PROBABILITY DISTIBUTIONS

I. MCQ (2 marks each)

(i) Let the p.m.f. of a random variable X be

P (x) = $\frac{3-x}{10}$, for x = -1 ,0,1,2

= 0 , otherwise

Then E (X) is …..

(A) 1 (B) 2 (C) 0 (D) -1

(ii) c.d.f of a discrete random variable X is

(A) an identity function. (B) a step function.

(B) an even function. (D) an odd function.

(iii) If X denotes the number on the uppermost face of cubic die when

it is tossed, then E(x) is ….

(A) $\frac{2}{7}$ (B) $\frac{7}{2}$ (C) 1 (D) $\frac{1}{2}$

(iv) A random variable X has the following probability distribution

X	2	3	4
P(x)	0.3	0.4	0.3

Then the variance of this distribution is

(A) 0.6 (B) 0.7

(C) 0.77 (D) 0.66

(v) For the random variable X , if V(X) = 4 , E(X) = 3 , then E(X^2) is …

(A) 9 (B) 13

(c) 12 (D) 7

(vi) If a d.r.v. *X* takes values 0, 1, 2, 3, . . . with probability

$P(X = x) = k(x + 1) \times 5^{-x}$, where k is a constant then $P(X = 0) =$…

(A) $\frac{7}{25}$ (B) $\frac{16}{25}$ (C) $\frac{18}{25}$ (D) $\frac{19}{25}$

(vii) The p.m.f of a d.r.v. X is $P(X = x) = \begin{cases} \frac{\binom{5}{x}}{2^5}, \text{for } x = 0, 1, 2, 3, 4, 5 \\ 0, \text{ otherwise} \end{cases}$

If $a = P(X \leq 2)$ and $b = P(X \geq 3)$ then

(A) $a < b$ (B) $a > b$ (C) $a = b$ (D) $a + b = 2$

(viii) If the p.m.f of a d.r.v. X is $P(X = x) = \begin{cases} \frac{x}{n(n+1)}, \text{for } x = 1, 2, 3, \dots, n \\ 0, \text{ otherwise} \end{cases}$ then

$E(X) = \cdots$

(A) $n + \frac{1}{2}$ (B) $\frac{n}{3} + \frac{1}{6}$ (C) $\frac{n}{2} + \frac{1}{5}$ (D) $n + \frac{1}{3}$

(ix) If the p.m.f of a d.r.v. X is $P(X = x) = \begin{cases} \frac{c}{x^3}, \text{for } x = 1, 2, 3 \\ 0, \text{ otherwise} \end{cases}$

then $E(X) = \cdots$

(A) $\frac{343}{297}$ (B) $\frac{294}{251}$ (C) $\frac{297}{294}$ (D) $\frac{294}{297}$

(x) If a d.r.v. X has the following probability distribution :

X	−2	−1	0	1	2	3
P(X=x)	0·1	k	0·2	2k	0·3	k

then $P(X=-1)$ is …

(A) $\frac{1}{10}$ (B) $\frac{2}{10}$ (C) $\frac{3}{10}$ (D) $\frac{4}{10}$

(xi) If a d.r.v. X has the following probability distribution:

X	1	2	3	4	5	6	7
P(X=x)	k	2k	2k	3k	k^2	2 k^2	$7k^2$+k

then $k=\ldots$

(A) $\frac{1}{7}$ (B) $\frac{1}{8}$ (C) $\frac{1}{9}$ (D) $\frac{1}{10}$

xii) The $p.d.f.$ of random variable X is $f(x)=\begin{cases}-kx\ , & 0<x<2\\ 0\ , & \text{otherwise}\end{cases}$,then $k=$

(A)-2 (B) 2 (C)$-\frac{1}{2}$ (D)$\frac{1}{2}$

xiii) If X is $c.r.v.$ and $F(x)$ denote $c.d.f$ of X, then which of the following is not true?

(A)$P(a<X<b)=F(b)-F(a)$ (B)$P(X<a)=F(a)$

(C)$P(X>a)=1-F(a)$ (D)$P(X>b)=F(b)$

xiv) If X is $c.r.v.$ with $p.d.f.\ f(x)=\begin{cases}\frac{1}{16}(4-x)\ , & -2<x<2\\ 0\ , & \text{otherwise}\end{cases}$,

then $P\left(X<-\frac{1}{2}\ or\ X>\frac{1}{2}\right)=$

(A)$\frac{1}{4}$ (B)$\frac{3}{4}$ (C)$\frac{1}{16}$ (D)$\frac{3}{16}$

xv) If the cumulative distribution function of X is $F(x)=\frac{1}{2}(3x^2-x^3)$, $0\le x\le 1$ then $P(X>1)$ is

(A)$\frac{1}{4}$ (B)$\frac{1}{2}$ (C) 1 (D) 0

xvi) The $p.d.f.$ of random variable X is $f(x) = \begin{cases} \frac{1}{2} , & 0 < x < 2 \\ 0 , & \text{otherwise} \end{cases}$

If $a = P\left(X < \frac{1}{2}\right)$ and $b = P\left(X > \frac{3}{2}\right)$ then

(A) $a = b$ (B)$a = 3b$ (C)$a = 2b$ (D) $b = 2a$

II. Very Short Answers (1 mark)

1. Let X represent the difference between number of heads and number of tails obtained when a coin is tossed 6 times. What are possible values of X?

2. An urn contains 5 red and 2 black balls. Two balls are drawn at random. X denotes number of black balls drawn. What are possible values of X?

3. State which of the following are not probability mass function of random variable.Give reasons of your answers.

(i)

X	0	1	2
P (X)	0.4	0.4	0.2

(ii)

X	0	1	2	3	4
P (X)	0.1	0.5	0.2	-0.1	0.2

(iii)

X	0	1	2
P (X)	0.1	0.6	0.3

(iv)

Y	-1	0	1
P (Y)	0.6	0.1	0.2

4. Find mean for the following probability distribution

X	0	1	2	3
P(X=4)	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$

5. State whether the following is not the probability mass function of random variable. Give reasons for your answer.

X	3	2	1	0	-1
P(X=x)	0.3	0.2	0.4	0	0.05

6. If X is $c.r.v.$ and $F(x)$ denote $c.d.f$ of $X$and$F(0) = 0 \cdot 3$, $F(3) = 0 \cdot 7$then find$P(0 \leq X \leq 3)$

III. Short Answers (2 marks)

(1) Find the expected value and variance of r.v. X whose p.m.f. is given below.

X	1	2	3
P(X=x)	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

(2) Find the probability distribution of number of heads in two tosses of a coin .

(3) The probability distribution of X is as follows:

X	0	1	2	3	4
P($X=x$)	0·1	k	2k	2k	k

Find k and P [X < 2].

(4) The following probability distribution of r.v. *X*

X	-3	-2	-1	0	1	2	3
P($X=x$)	0.05	0.1	0.15	0.20	0.25	0.15	0.1

Find the probability that (i) X is positive. (ii) X is odd

(5) In the p.m.f. of r.v. *X*

X	1	2	3	4	5
P($X=x$)	$\frac{1}{20}$	$\frac{3}{20}$	a	$2a$	$\frac{1}{20}$

Find a and obtain c.d.f. of *X*.

IV. Short answers (3 Marks)

(1) Find the probability distribution of the number of successes in tosses of a die, where a success is defined as number greater than 4 appears on at least one die.

(2) A coin is biased so that the head is 3 times as likely to occur as a tail. If the coin is tossed twice, find the probability distribution of tails..

(3) A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2\,k^2$	$7k^2+k$

Determine : (i) k ii)P($X < 3$) iii)$P(X > 4)$

(4) Find the probability distribution of the number of doublets in three throws of a pair of dice.

(5) Find the mean and variance of the number randomly selected from 1 to 15.

(6) Let the p.m.f. of r.v. X be $f(x) = \begin{cases} \frac{3-x}{10}, & \text{for } x = -1, 0, 1, 2 \\ 0, & \text{otherwise} \end{cases}$

Calculate $E(X)$ and $Var(X)$.

(7) Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as six appears on at least one die.

(8) If the $p.d.f.$ of random variable X is $f(x) = \begin{cases} \frac{k}{\sqrt{x}}, & 0 < x < 4 \\ 0, & \text{otherwise} \end{cases}$

then find the cumulative distribution function of a random variable X.

(9) If the $p.d.f.$ of random variable X is $f(x) = \begin{cases} \frac{k}{x^2+1}, & 0 < x < \infty \\ 0, & \text{otherwise} \end{cases}$

then find k and cumulative distribution function of a random variable X.

(10) If X is $c.r.v.$ with $p.d.f.\, f(x) = \begin{cases} 4x^3, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$,

then find $P\left(X \leq \frac{3}{4} / X > \frac{1}{4}\right)$

(11) Find k , if the $p.d.f.$ of random variable X is $f(x) = \begin{cases} ke^{-3x} & , \ 0 \leq x < \infty \\ 0 & , \ \text{otherwise} \end{cases}$

and $P(0 < X < k) = \frac{1}{2}$.

(12) If X is $c.r.v.$ with $p.d.f.\ f(x) = \begin{cases} kx & , \ 0 \leq x < 1 \\ k & , \ 1 < x < 2 \\ (3-x) & , \ 2 \leq x \leq 3 \\ 0 & , \ \text{otherwise} \end{cases}$,

then find k .

V. Long answers (4 Marks)

1.Let a pair of dice be thrown and the random variable *X* be the sum of the numbers that appear on the two dice. Find the mean or expectation of *X* and variance of *X*.

2. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled of pack of 52 cards. Find the mean, variance and standard deviation of the number of kings drawn.

3. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find E (X).

4. In a meeting,70% of the members favor and 30% oppose a certain proposal. A member is selected at random and we take X=0 if he opposed, and X=1 if he is in favor. Find E(X) and Var (X) .

5. The following is the c.d.f. of r.v. X

X	-3	-2	-1	0	1	2	3	4
F(x)	0.1	0.3	0.5	0.65	0.75	0.85	0.9	1

Find p.m.f. of X. (i) $P\ (-1 \leq X \leq 2)$ (ii) $P\ (X \leq 3/X > 0)$.

6. A player tosses two coins he wins Rs.10 if 2 heads appears , Rs. 5 if 1 head appears and Rs.2 if no head appears. Find the expected winning amount and variance of winning amount.

7.From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

8. Let X denote the sum of the number obtained when two fair dice are rolled. Find the standard deviation of X.