3 Circle.

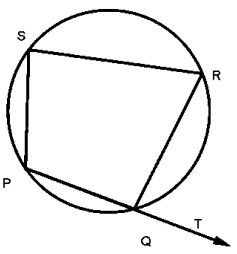
Q.1. Four alternative answers for each of the following questions are given. Choose the correct alternative.

1) Two circles intersect each other such that each circle passes through the centre of the other. If the distance between their centres is 12, what is the radius of each circle ?
   (A) 6 cm (B) 12 cm (C) 24 cm (D) can't say
2) A circle touches all sides of a parallelogram. So the parallelogram must be a, ...
   (A) rectangle (B) rhombus (C) square (D) trapezium

3) ∠ACB is inscribed in arc ACB of a circle with centre O. If ∠ ACB = 65°, find m(arc ACB).
   (A) 65° (B) 130° (C) 295° (D) 230°
4) In a cyclic □ ABCD, twice the measure of ∠A is thrice the measure of ∠C. Find the measure of ∠C?
   (A) 36 (B) 72 (C) 90 (D) 108
5) How many circles can drawn passing through three non -collinear points?
   (A) 0 (B) Infinite (C) 2 (D) One and only one(unique)
6) Two circles of radii 5.5 cm and 4.2 cm touch each other externally. Find the distance between their centres
   (A)9.7 (B) 1.3 (C) 2.6 (D) 4.6
7) What is the measurement of angle inscribed in a semicircle?
   (A) 90° (B) 120° (C) 100° (D) 60°

8) Two circles having diameters 8 cm and 6 cm touch each other internally. Find
   the distance between their centres.
   (A) 2 (B) 14 (C) 7 (D) 1
9) Points A, B, C are on a circle, such that m(arc AB) = m(arc BC) = 120°. No point, except point B, is common to the arcs. Which is the type of Δ ABC?
   (A) Equilateral triangle (B) Scalene triangle
   (C) Right angled triangle (D) Isosceles triangle

10) In □ PQRS if ∠RSP = 80° then find ∠RQT ?

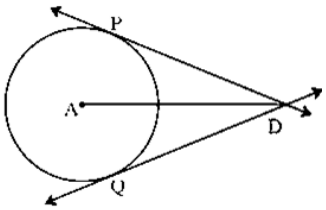


(A) 100° (B) 80°
(C) 70° (D) 110°

Q.2 Solve the following sub-questions. (1 mark question)

1) How many circles can be drawn passing through a point?

2) Segment DP and segment DQ are tangent segments to the circle with center A,

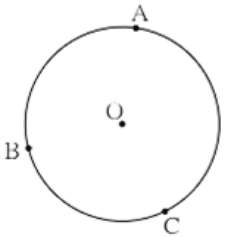


If DP = 7 cm. So find the length of the segment DQ?

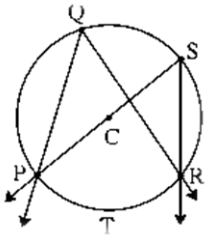
3) Two circles having radii 3.5 cm and 4.8 cm touch each other internally. Find the distance between their centres.

4) What is the measure of a semi circular arc?

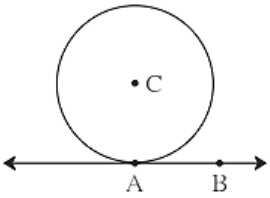
5) A, B, C are any points on the circle with centre O. If m arc (BC) = 110° and m arc (AB) = 125°, find measure arc AC



6) In the figure if ∠PQR = 50° then find ∠PSR

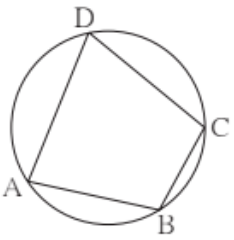


7)

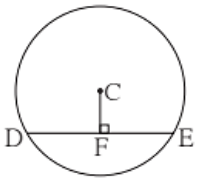


In the adjoining figure the radius of a circle with centre C is 6 cm, line AB is a tangent at A. What is the measure of ∠ CAB? Why?

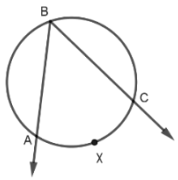
8) In the figure quadrilateral ABCD is a cyclic , if ∠DAB = 75° then find measure of ∠DCB



9) In the adjoining figure, seg DE is the chord of the circle with center C. seg CF⊥ seg DE and DE = 16 cm, then find the length of DF?

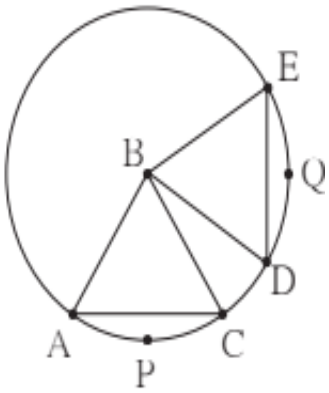


10) In the figure, if ∠ABC = 35° then find m(arc AXC) ?



Q.3 Complete the following activities (2 marks each).

The chords corresponding to congruent arcs of a circle are congruent.Prove the theorem by completing following activity.



Given : In a circle with centre B

arc APC ≅ arc DQE

To Prove : Chord AC ≅chord DE

Proof : In Δ ABC and Δ DBE,

side AB ≅ side DB [ ]

side BC ≅ side [ ] [ ]

∠ ABC ≅ ∠ DBE (measure of congruent arcs)

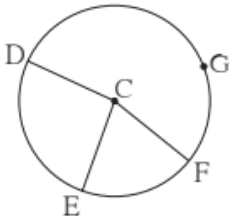
Δ ABC ≅ Δ DBE [ ]

2) In figure , points G, D, E, F

are concyclic points of a circle with centre C.

∠ ECF = 70°, m(arc DGF) = 200°

find m(arc DEF) by completing activity.



m(arc EF) = ∠ECF ..... (Definition of measure of arc)

∴ m(arc EF) = ☐

But; m(arc DE)+ m(arc EF)+ m(arc DGF) = ☐ (measure of a complete circle)

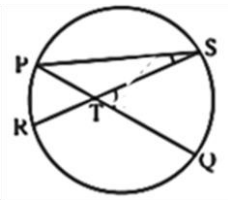
∴ m(arc DE) = ☐

∴ m(arc DEF) = m(arc DE) + m(arc EF)

∴ m(arc DEF) = ☐

3)

In the figure if the chord PQ and chord RS intersect at point T Prove that : **m**∠STQ = $\frac{1}{2}$ [m(arc PR) + m(arc SQ)] for any measure of ∠STQ by filling out the boxes.

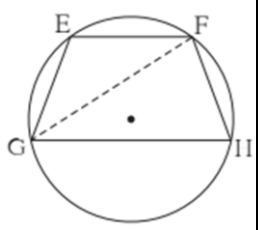


Proof: **m∠STQ** = **m∠SPQ** + ☐ .. (Theorem of the external angle of a triangle)

= $\frac{1}{2}$ m(कंस SQ) + ☐ ..... (inscibed angle theorem)

$$= \frac{1}{2}[\quad + \quad]$$

4) In figure, chord EF || chord GH. Prove that,
chord EG≅ chord FH. Fill in the blanks and write the proof.
Proof : Draw seg GF.



$\angle EFG = \angle FGH$ ..... ☐ (I)

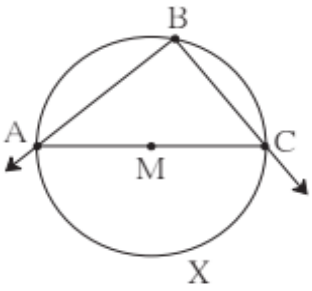
$\angle EFG =$ ☐ .....(inscribed angle theorem) (II)

$\angle FGH =$ ☐ .....(inscribed angle theorem) (III)

∴ m(arc EG) = ☐ .....[By (I) , (II) व (III)]

chord EG ≅ chord FH .....(corresponding chords of congruent arcs)

The angle inscribed in the semicircle is a right angle Prove the result by completing the following activity .



Given: ∠ABC is inscribed angle in a semicircle with center M.

To prove : ∠ABC is a right angle.

Proof: segment AC is a diameter of the circle.

∴ m(arc AXC) = ☐

Arc AXC is intercepted by the inscribed angle ∠ABC .

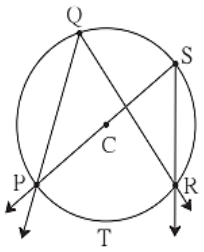
∠ABC = ☐ **.....(**Inscribed angle theorem**)**

$= \frac{1}{2} \times$ ☐

∴ m ∠ABC = ☐

∴ ∠ABC is a right angle.

6) Prove that angles inscribed in the same arc are congruent**.**



Given: In a circle with centre C, ∠PQR and ∠PSR is inscribed in same arc PQR.Arc PTR is intercepted by the angles.
To prove : ∠PQR ≅ ∠PSR.

Proof : $m\angle PQR = \frac{1}{2} \times [m(\text{arc PTR})]$ **.....** (i) ☐

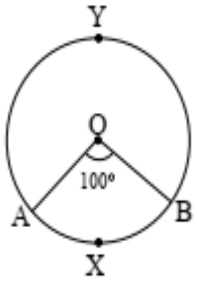
$m\angle$ ☐ $= \frac{1}{2} \times [m(\text{arc PTR})]$ **.....** ☐

$m\angle$ ☐ = $m\angle PSR$ .....By(i) &(ii)

∴ ∠PQR ≅ ∠PSR

7) If O is the center of the circle in the figure alongside , then complete the table from the given information.

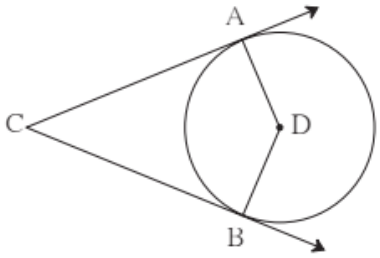
The type of arc



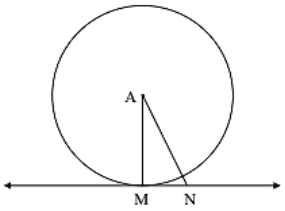
Type of circular arc	Name of circular arc	Measure of circular arc
Minor arc		
Major arc		

Q.4. Solve the following sub-questions. (2 marks question)

1) In the adjoining figure circle with Centre D touches the sides of ∠ACB at A and B. If ∠ ACB = 52°, find measure of ∠ ADB.

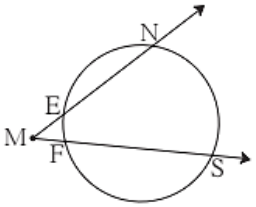


2) In the adjoining figure, the line MN touches the circle with center A at point M. If AN = 13 and MN = 5 then find the radius of the circle?



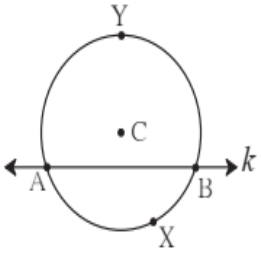
3) What is the distance between two parallel tangents of a circle having radius 4.5 cm? Justify your answer.

4) In figure, m(arc NS) = 125°, m(arc EF) = 37°, find the measure ∠NMS.

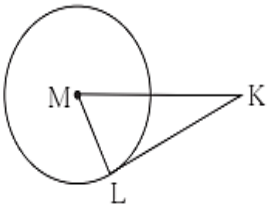


5) Length of a tangent segment drawn from a point which is at a distance 15 cm from the centre of a circle is 12 cm, find the diameter of the circle?

6) In the figure a circle with center C has m (arc AXB) = 100° then find central ∠ACB and measure m (arc AYB).



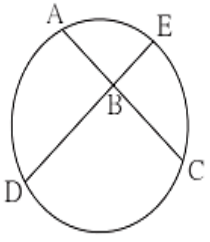
7)



In figure , M is the centre of the circle and seg KL is a tangent segment. If MK = 12, KL = $6\sqrt{3}$ then find (1) Radius of the circle.

(2) Measures of ∠K and ∠M.

8) In figure, chords AC and DE intersect at B.
If ∠ ABE = 108°, m(arc AE) = 95°, find m(arc DC) .

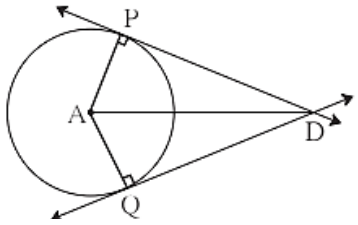


Q. **5. Complete the following activity. (3 marks each)**

1) Tangent segments drawn from an external point to a circle are congruent , prove this theorem.Complete the following activity.

Given : ☐

To Prove: ☐



Proof : Draw radius AP and radius AQ and complete the following proof of the theorem.

In ΔPAD and ΔQAD **,**

Seg PA ≅ ☐ **.... (** radii of the same circle. **)**

Seg AD ≅ Seg AD **.... (** ☐ **)**

$\angle APD \cong \angle AQD = 90°$ **....(**tangent theorem **)**

$\therefore \Delta PAD \cong \Delta QAD$ **....** **(** ☐ **)**

∴seg DP ≅ seg DQ **.... (** ☐ **)**

2) ☐MRPN is cyclic, ∠R = (5x - 13) °, ∠ N = (4x + 4) °. Find measures of ∠ R and ∠ N, by completing the following activity.

Solution : ☐ MRPN is cyclic

The opposite angles of a cyclic square are ☐

$\angle R + \angle N$ = ☐

$\therefore (5x-13)° + (4x+4°)$ = ☐

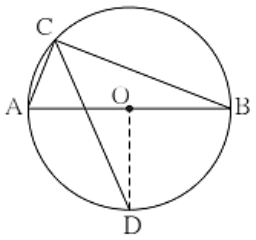
$\therefore 9x = 189$

$\therefore x$ = ☐

$\therefore \angle R = (5x-13)°$ = ☐

$\therefore \angle N = (4x+4)°$ = ☐

3) In figure , seg AB is a diameter of a circle with centre O . The bisector of ∠ACB intersects the circle at point D. Prove that, seg AD ≅ seg BD. Complete the following proof by filling in the blanks.



Proof Draw seg OD.

∠ACB = [ ] ..... angle inscribed in semicircle

∠DCB = [ ] ..... CD is the bisector of ∠C

m(arc DB) = [ ] ..... inscribed angle theorem

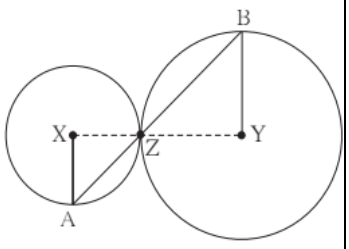
∠DOB = [ ] ..... definition of measure of an arc (I)

seg OA ≅ seg OB ..... [ ] (II)

∴line OD is [ ] of seg AB ..... From (I) and (II)

∴seg AD ≅ seg BD

4) In the adjoining figure circles with centres X and Y touch each other at point Z. A secant passing through Z intersects the circles at points A and B respectively.
Prove that , radius XA || radius YB.
Fill in the blanks and complete the proof.



Construction : Draw segments XZ and YZ.

Proof : By theorem of touching circles, points X, Z, Y are [ ]

∴ ∠XZA ≅ [ ] ..... opposite angles

Let ∠XZA = ∠BZY = *a* ..... (I)

Now, seg XA ≅ seg XZ ..... (radii of the same circle.)

∴ ∠XAZ = [ ] = *a* ..... (isosceles triangle theorem) (II)

similarly, seg YB ≅ seg YZ ..... (radii of the same circle.)

∴ ∠BZY = [ ] = *a* ..... (isosceles triangle theorem.) (III)

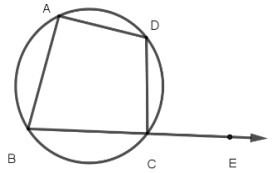
∴from (I), (II), (III),
∠XAZ = [ ]

∴ radius XA || radius YB ……. [ ])

5) An exterior angle of a cyclic quadrilateral is congruent to the angle opposite to its adjacent interior angle, to prove the theorem complete the activity .

Given : □ ABCD is cyclic ,

[ ] is the exterior angle of □ABCD



To prove : ∠DCE ≅ ∠BAD

Proof : [ ] + ∠BCD = [ ]....(Angles in linear pair) (I)

□ ABCD is a cyclic .

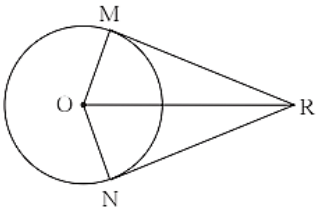
[ ] + ∠BAD [ ] .....(Theorem of cyclic quadrilateral) (II)

By (I) and (II)

∠DCE + ∠BCD [ ] + ∠BAD

∠DCE ≅ ∠BAD

6) Seg RM and seg RN are tangent segments of a circle with centre O. Prove that seg OR bisects ∠MRN as well as ∠MON with the help of activity.



Proof : In ΔRMO and ΔRNO **,**

$\angle RMO \cong \angle RNO = 90°$ **....(** ☐ **)**

hypt OR ≅ hypt OR **.... (** ☐ **)**

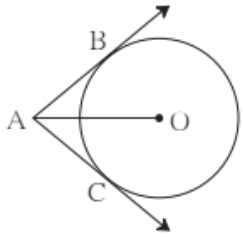
seg OM ≅ seg☐ **.... (** radii of the same circle **)**

$\therefore \Delta RMO \cong \Delta RNO$ **....** **(** ☐ **)**

$\angle MOR \cong \angle NOR$

Similarly $\angle MRO \cong$ ☐ **....** **(** ☐ **)**

7) In figure , O is the centre of the circle.
Seg AB, seg AC are tangent segments.
Radius of the circle is r and ℓ(AB) = r ,
Prove that, □ABOC is a square.



Proof : Draw segment OB and OC.

ℓ(AB) = r ..... (Given) **(**I)

AB=AC .....(☐) (II)

But OB = OC = r ..... (☐) (III)

From (I),(II) and (III)

AB= ☐ = OB = OC = r

$\therefore$ Quadrilateral ABOC is ☐ .

Similarly $\angle OBA =$ ☐ ....(Tangent Theorem)

If one angle of ☐ is right angle ,then it is a square.

$\therefore$ Quadrilateral ABOC is a suqare**.**

Q.6. Solve the following sub-questions. (3 marks question)

1) Prove the following theorems:

i) Opposite angles of a cyclic quadrilateral are supplementry.

ii) Tangent segments drawn from an external point to a circle are congruent.

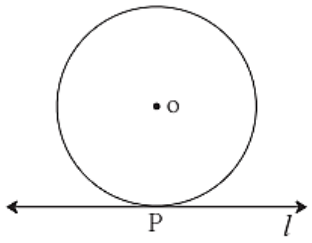
iii) Angles inscribed in the same arc are congruent**.**

2) Line ℓ touches a circle with centre O at point P. If radius of the circle is 9 cm, answer the following.

(i) What is d(O, P) = ? Why ?

(ii) If d(O, Q) = 8 cm, where does the point Q lie ?

(iii) If d(PQ) = 15 cm, How many locations of point R are line on line ℓ? At what distance will each of them be from point P ?

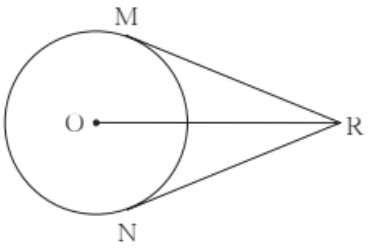


3) In the adjoining figure, O is the centre of the circle. From point R, seg RM and seg RN are tangent segments touching the circle at M and N. If (OR) = 10 cm and radius of the circle = 5 cm, then

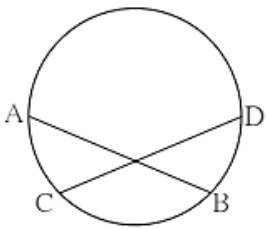
(1) What is the length of each tangent segment ?

(2) What is the measure of ∠MRO ?

(3) What is the measure of ∠ MRN ?



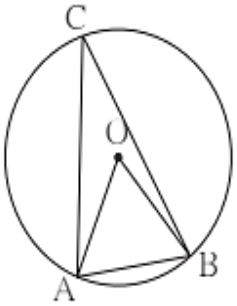
4) In figure ,chord AB ≅ chord CD,
Prove that, arc AC ≅ arc BD



5) In figure , in a circle with centre O, length of chord AB is equal to the radius of the circle. Find measure of each of the following.

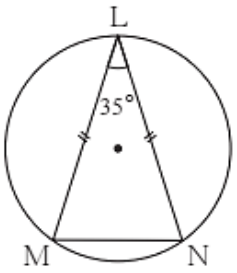
(1) ∠AOB (2) ∠ACB

(3) arc AB



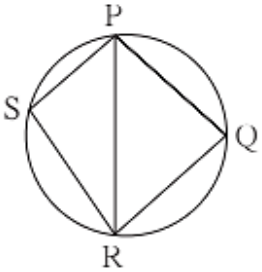
6) In figure , chord LM ≅ chord LN , ∠L = 35° find (i) m(arc MN)

(ii) m(arc LN)

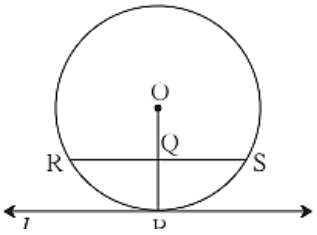


7) Prove that, any rectangle is a cyclic quadrilateral.

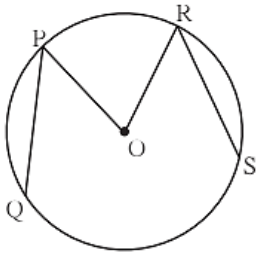
8) In figure , PQRS is cyclic.
side PQ ≅ side RQ. ∠ PSR = 110°,
Find- (1) measure of ∠ PQR
(2) m(arc PQR)
(3) m(arc QR)



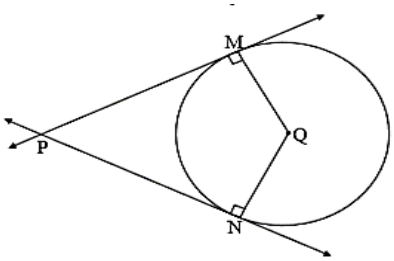
9) In figure , line ℓ touches the circle with centre O at point P. Q is the mid point of radius OP. RS is a chord through Q such that chords RS || line ℓ. If RS = 12 find the radius of the circle



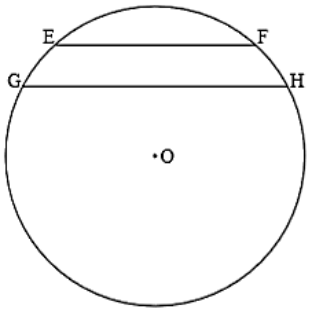
10) In figure , O is the centre of a circle, chord PQ ≅chord RS If ∠ POR = 70° and (arc RS) = 80°, find (1) m(arc PR) (2) m(arc QS) (3) m(arc QSR)



11) In the adjoining figure circle with Centre Q touches the sides of ∠MPN at M and N. If ∠ MPN = 40°, find measure of ∠ MQN.

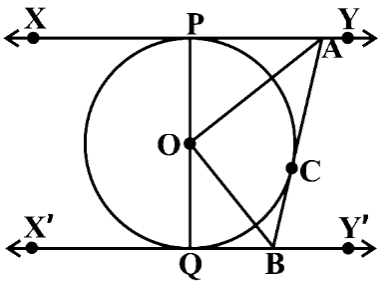


12) In the figure if O is the center of the circle and two chords of the circle EF and GH are parallel to each other. Show that $\angle EOG \cong \angle FOH$



Q. 7. Solve the following sub-questions. (4 marks question)

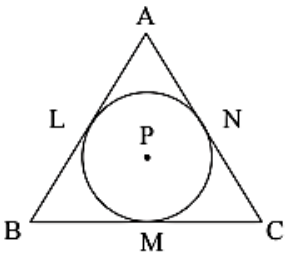
1) In the figure segment PQ is the diameter of the circle with center O. The tangent to the tangent circle drawn from point C on it , intersects the tangents drawn from points P and Q at points A and B respectively , prove that $\angle AOC = 90°$



2) The chords AB and CD of the circle intersect at point M in the interior of the same circle then prove that CM × BD = BM × AC.

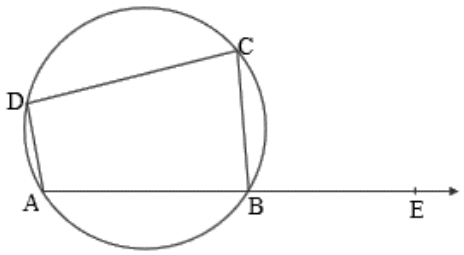
3) A circle with centre P is inscribed in the ΔABC. Side AB, side BC and side AC touches the circle at points L, M and N respectively. Radius of the circle is r.

Prove that : $A(\Delta ABC) = \frac{1}{2}(AB + BC + AC) \times r$



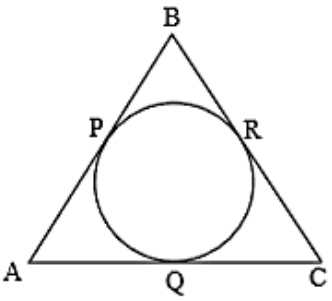
4)

In the figure $\square$ABCD is a cyclic quadrilateral. If m(arc ABC) = 230°.then find $\angle ABC$, $\angle CDA$, $\angle CBE$

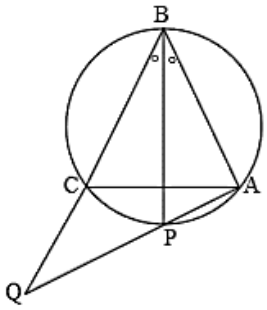


5)

The figureΔABC is an isosceles triangle with a perimeter of 44 cm. The sides AB and BC are congruent and the length of the base AC is 12 cm. If a circle touches all three sides as shown in the figure, then find the length of the tangent segment drawn to the circle from the point B



6)

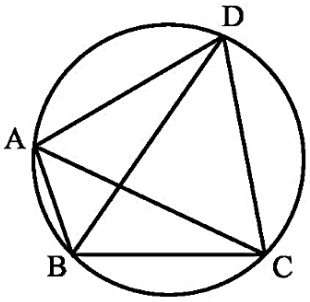


In the figure ΔABC is an equilateral triangle.The angle bisector of $\angle$**B** will intersect the circumcircle ΔABC at point P.

Then prove that : CQ = CA.

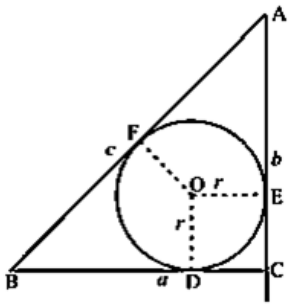
7)

In the figure quadrilateral ABCD is cyclic , If m(arc BC) = 90° and $\angle DBC$ = 55°. Then find the measure of $\angle BCD$.



8) Given : A circle inscribed in a right angled ΔABC. If $\angle ACB = 90°$ and the radius of the circle is r.

To prove : $2r = a + b - c$



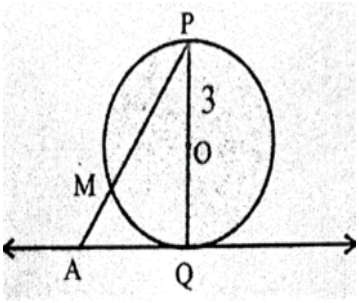
9) In a circle with centre P , chord AB is parallel to a tangent and intersects the radius drawn from the point of contact to its midpoint. If AB = $16\sqrt{3}$ then find the radius of the circle.

10) In the figure, O is the center of the circle.

Line AQ is a tangent. If OP = 3

m(arc PM) = 120°

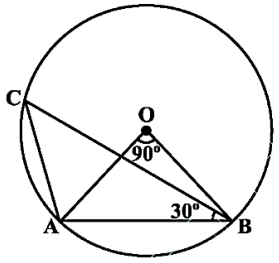
then find the length of AP?



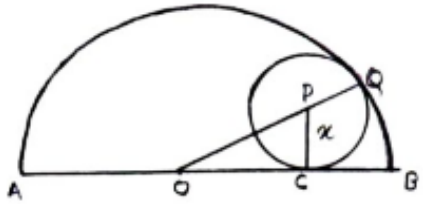
Q. 8. Solve the following sub-questions (3 marks each)

1) In the figure, O is the centre of the circle and $\angle AOB = 90°$, $\angle ABC = 30°$

Then find $\angle CAB$**?**



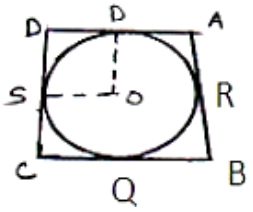
2)



In the figure a circle with center P touches the semicircle at points Q and C having center O. if diameter AB = 10, AC = 6 then find the radius x of the smaller circle?

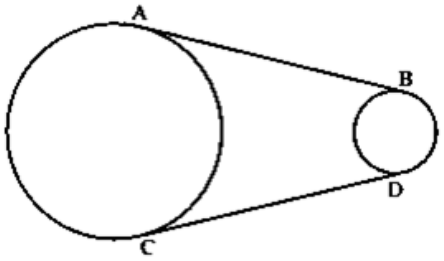
3)

In the figure a circle touches all the sides of quadrilateral ABCD from the inside. The center of the circle is O. If AD⊥ DC and BC = 38 , QB = 27, DC = 25 then find the radius of the circle?



4)

If AB and CD are the common tangents in the circles of two unequal (different) radii then show that seg AB ≅ seg CD



5) Circles with centres A, B and C touch each other externally. If AB = 36, BC = 32, CA = 30, then find the radii of each circle.