Trigonometry

Que.) 1 A) .Choose the correct alternative from those given below each question : (1 mark for each MCQ)

1. $\cos\theta \cdot \sec\theta = ?$

A) 1 B) 0 C) $\frac{1}{2}$ D) $\sqrt{2}$

2. $\sec 60^0 = ?$

A) $\frac{1}{2}$ B) 2 C) $\frac{2}{\sqrt{3}}$ D) $\sqrt{2}$

3. $1+ \cot^2\theta = ?$

A) $\tan^2\theta$ B) $\sec^2\theta$ C) $\text{cosec}^2\theta$ D) $\cos^2\theta$

4. $\cot\theta \cdot \tan\theta = ?$

A) 1 B) 0 C) 2 D) $\sqrt{2}$

5. $\sec^2\theta - \tan^2\theta = ?$

A) 0 B) 1 C) 2 D) $\sqrt{2}$

6. $\sin^2\theta + \sin^2(90 - \theta) = ?$

A) 0 B) 1 C) 2 D) $\sqrt{2}$

7. $\frac{1+\cot^2 A}{1+\tan^2 A} = ?$

A) $\tan^2\theta$ B) $\sec^2\theta$ C) $\text{cosec}^2\theta$ D) $\cot^2\theta$

8. $\sin\theta = \frac{1}{2}$ then $\theta = ?$

A) 30^0 B) 45^0 C) 60^0 D) 90^0

9. tan (90-θ) = ?

A) sin θ B) cos θ C) cot θ D) tan θ

10. cos 45^0 = ?

A) sin 45^0 B) sec 45^0 C) cot 45^0 D) tan 45^0

11. If sin θ = $\frac{3}{5}$ then cos θ = ?

A) $\frac{5}{3}$ B) $\frac{3}{5}$ C) $\frac{4}{5}$ D) $\frac{5}{4}$

12. Which is not correct formula ?

A) $1 + \tan^2\theta = \sec^2\theta$

B) $1 + \sec^2\theta = \tan^2\theta$

C) $\text{cosec}^2\theta - \cot^2\theta = 1$

D) $\sin^2\theta + \cos^2\theta = 1$

13. If $\angle$ A = 30^0 then tan 2A = ?

A) 1 B) 0 C) $\frac{1}{\sqrt{3}}$ D) $\sqrt{3}$

Que.) 1 B). Solve the following questions : (1 mark each)

1. $\frac{1-\tan^2 45^0}{1+\tan^2 45^0}$ = ?

2. If tan θ = $\frac{13}{12}$ then cot θ = ?

3. Prove that cosec θ X $\sqrt{1-\cos^2\theta}$ = 1.

4. If tan θ = 1 then sin θ . cos θ = ?

5. If 2 sin θ = 3 cos θ then tan θ = ?

6. If cot (90 – A) = 1 then $\angle$ A = ?

7. If $1 - \cos^2\theta = \frac{1}{4}$ then θ = ?

8. Prove that $\frac{\cos(90 - A)}{\sin A} = \frac{\sin(90 - A)}{\cos A}$.

9. If tan θ X ☐ = sin θ then ☐ = ?

10. (sec θ + tan θ) . (sec θ - tan θ) = ?

11. $\frac{\sin 75^0}{\cos 15^0}$ = ?

Que.) 2 A). Complete the following activities (2 marks each)

*** (Write complete answers, don't just fill the boxes)**

1. Prove that $\cos^2\theta \,.\, (1 + \tan^2\theta) = 1$. Complete the activity given below.

Activity ⇨ L . H . S. = ☐

= $\cos^2\theta$ X ☐ ...($1 + \tan^2\theta$ = ☐)

= (cos θ X ☐)^2

= 1^2

= 1

= R .H .S.

2. $\frac{5}{\sin^2\theta} - 5\cot^2\theta$, Complete the activity given below.

Activity ⇨ $\frac{5}{\sin^2\theta} - 5\cot^2\theta$

$= \square \left(\frac{1}{\sin^2\theta} - \cot^2\theta\right)$

$= 5\ (\ \square\ - \cot^2\theta\ )$ ..... $(\frac{1}{\sin^2\theta} = \square$)

$= 5\ (\ 1\ )$

$= \square$

3. If $\sec\theta + \tan\theta = \sqrt{3}$. Complete the activity to find the value of $\sec\theta - \tan\theta$

Activity ⇨ $\square = 1 + \tan^2\theta$ ……(Fundamental trigonometric identity)

$\square - \tan^2\theta = 1$

$(\sec\theta + \tan\theta) \cdot (\sec\theta - \tan\theta) = \square$

$\sqrt{3}\ \cdot (\sec\theta - \tan\theta) = 1$

$(\sec\theta - \tan\theta) = \square$

4. If $\tan\theta = \frac{9}{40}$. Complete the activity to find the value of $\sec\theta$.

Activity ⇨ $\sec^2\theta = 1 + \square$ ..... (Fundamental trigonometric identity)

$\sec^2\theta = 1 + \square^2$

$\sec^2\theta = 1 + \square$

$\sec\theta = \square$

Que.) 2 B). Solve the following questions : (2 marks each)

1. If $\cos\theta = \frac{24}{25}$ then $\sin\theta$ = ?

2. Prove that $\frac{\sin^2\theta}{\cos\theta} + \cos\theta = \sec\theta$.

3. Prove that $\frac{1}{\operatorname{cosec}\theta - \cot\theta} = \operatorname{cosec}\theta + \cot\theta$.

4. If $\cos(45^0 + x) = \sin 30^0$ then x = ?

5. If $\tan\theta + \cot\theta = 2$ then $\tan^2\theta + \cot^2\theta$ = ?

6. Prove that $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \text{ X } \operatorname{cosec}^2\theta$.

7. Prove that $\cot^2\theta \text{ X } \sec^2\theta = \cot^2\theta + 1$.

8. If $3\sin\theta = 4\cos\theta$ then $\sec\theta$ = ?

9. If $\sin 3A = \cos 6A$ then $\angle A$ = ?

10. Prove that $\sec^2\theta - \cos^2\theta = \tan^2\theta + \sin^2\theta$.

11. Prove that $\frac{\tan A}{\cot A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A}$.

12. Prove that $\frac{\sin\theta + \tan\theta}{\cos\theta} = \tan\theta\,(1 + \sec\theta)$.

13. Prove that $\frac{\cos^2\theta}{\sin\theta} + \sin\theta = \operatorname{cosec}\theta$.

14. Prove that $\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$.

Que.) 3 A). Complete the following activities (3 marks each)

*** (Write complete answers, don't just fill the boxes)**

1. $\sin^4 A - \cos^4 A = 1 - 2\cos^2 A$, For proof of this complete the activity given below.

Activity ⇨ L . H . S. = [ ]

= ($\sin^2 A + \cos^2 A$) ([ ])

= 1 ([ ]) ..... ($\sin^2 A$ + [ ] = 1)

= $\square$ - $\cos^2 A$ ..... ($\sin^2 A = 1 - \cos^2 A$)

= $\square$

= R. H. S.

2. $\tan^2\theta - \sin^2\theta = \tan^2\theta$ X $\sin^2\theta$ .For proof of this complete the activity given below.

Activity ⇨ L . H . S. = $\square$

= $\square$ ($1 - \frac{\sin^2\theta}{\tan^2\theta}$)

= $\tan^2\theta$ ($1 - \frac{\square}{\frac{\sin^2\theta}{\cos^2\theta}}$)

= $\tan^2\theta$ ($1 - \frac{\sin^2\theta}{1}$ X $\frac{\cos^2\theta}{\square}$)

= $\tan^2\theta$ ($1 - \square$)

= $\tan^2\theta$ X $\square$ ..... ($1 - \cos^2\theta = \sin^2\theta$)

= R. H. S.

3. If $\tan\theta = \frac{7}{24}$ then To find value of cos θ complete the activity given below.

Activity ⇨ $\sec^2\theta$ = 1+ $\square$ .....(Fundamental tri. identity)

$\sec^2\theta$ = 1+ $\square^2$

$\sec^2\theta$ = 1+ $\frac{\square}{576}$

$\sec^2\theta$ = $\frac{\square}{576}$

sec θ = $\square$

$\cos\theta =$ $\square$ .....($\cos\theta = \frac{1}{\sec\theta}$)

4. To prove $\cot\theta + \tan\theta = \text{cosec}\,\theta \times \sec\theta$. Complete the activity given below.

Activity $\Rightarrow$ L . H . S. = $\square$

$= \frac{\square}{\sin\theta} = \frac{\sin\theta}{\cos\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\square}$

$= \frac{1}{\sin\theta \,.\, \cos\theta}$ .....($\cos^2\theta + \sin^2\theta =$ $\square$)

$= \frac{1}{\sin\theta} \times \frac{1}{\square}$

= $\square$

= R. H. S.

Que.) 3 B). Solve the following questions : (3 marks each)

1. If $\sec\theta = \frac{41}{40}$ then find values of $\sin\theta$, $\cot\theta$, $\text{cosec}\,\theta$.

2. If $5\sec\theta - 12\,\text{cosec}\,\theta = 0$ then find values of $\sin\theta$, $\sec\theta$.

3. Prove that $\frac{\tan(90-\theta) + \cot(90-\theta)}{\text{cosec}\,\theta} = \sec A$.

4. Prove that $\cot^2\theta - \tan^2\theta = \text{cosec}^2\theta - \sec^2\theta$.

5. Prove that $\frac{1+\sin\theta}{1-\sin\theta} = (\sec\theta + \tan\theta)^2$.

6. Prove that $\frac{\sin\theta}{\sec\theta + 1} + \frac{\sin\theta}{\sec\theta - 1} = 2\cot\theta$.

7. Prove that $\frac{\sec A}{\tan A + \cot A} = \sin A$.

8. Prove that $\frac{\sin\theta + \text{cosec}\,\theta}{\sin\theta} = 2 + \cot^2\theta$.

9. Prove that $\frac{\cot A}{1-\cot A} + \frac{\tan A}{1-\tan A} = -1$.

10. Prove that $\sqrt{\frac{1+\cos A}{1-\cos A}}$ = cosec A + cot A .

11. Prove that $\sin^4 A - \cos^4 A = 1 - 2\cos^2 A$.

12. Prove that $\sec^2\theta - \cos^2\theta = \tan^2\theta + \sin^2\theta$.

13. Prove that cosec θ – cot θ = $\frac{\sin\theta}{1+\cos\theta}$.

14. In Δ ABC, cos C = $\frac{12}{13}$ and BC = 24 then AC = ?

15. Prove that $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$.

16. If sin A = $\frac{3}{5}$ then show that 4 tan A + 3 tan A = 6 cos A

17. Prove that $\frac{1+\sin B}{\cos B} + \frac{\cos B}{1+\sin B}$ = 2 sec B .

Que. 4 Solve the following questions : (Challenging questions, 4 marks each)

1. Prove that
   $\sin^2 A$. tan A + $\cos^2 A$. cot A + 2 sin A . cos A = tan A + cot A
   .

2. Prove that $\sec^2 A - \operatorname{cosec}^2 A = \frac{2\sin^2 A - 1}{\sin^2 A \,.\, \cos^2 A}$.

3. Prove that $\frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} = \frac{1+\cos A}{\sin A}$.

4. Prove that sin θ (1 – tan θ) − cos θ (1 − cot θ) = cosec θ − sec θ
.

5. If cos A = $\frac{2\sqrt{m}}{m+1}$ then Prove that cosec A = $\frac{m+1}{m-1}$.

6. If sec A = $x + \frac{1}{4x}$ then show that sec A + tan A = $2x$ or $\frac{1}{2x}$.

7. In Δ ABC , $\sqrt{2}$ AC = BC, sin A = 1, $\sin^2 A + \sin^2 B + \sin^2 C = 2$ then ∠A = ? ∠B = ? ∠C= ?

8. Prove that $\sin^6 A + \cos^6 A = 1 - 3 \sin^2 A . \cos^2 A$.

9. Prove that $2 (\sin^6 A + \cos^6 A) - 3 (\sin^4 A + \cos^4 A) + 1 = 0$.

10. Prove that $\frac{\cot A}{1-\tan A} + \frac{\tan A}{1-\cot A}$ = 1+ tanA + cotA = secA . cosecA + 1

Que. 5 Solve the following questions : (Creative questions, 3 marks each)

1. If 3 sin A + 5 cos A = 5 then show that 5 sin A – 3 cos A = ±3.

2. If cos A + $\cos^2 A = 1$ then $\sin^2 A + \sin^4 A$ = ?

3. If cosec A – sin A = p आणि sec A – cos A = q then prove that

$(p^2 q)^{\frac{2}{3}} + (pq^2)^{\frac{2}{3}} = 1$

4. Show that tan 7^0 X tan 23^0X tan 60^0X tan 67^0X tan 83^0 = $\sqrt{3}$.

5. If sin θ + cos θ = $\sqrt{3}$ then show that tan θ + cot θ = 1 .

6. If tan θ - $\sin^2\theta = \cos^2\theta$ then show that $\sin^2\theta = \frac{1}{2}$.

7. Prove that

$(1 - \cos^2 A) . \sec^2 B + \tan^2 B (1 - \sin^2 A) = \sin^2 A + \tan^2 B$