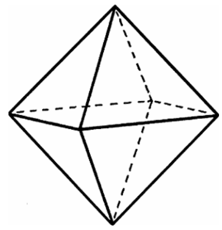
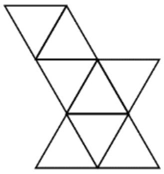
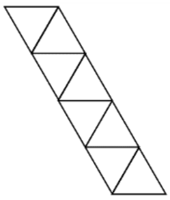
GS 2024 – Physics question paper (IPhD)

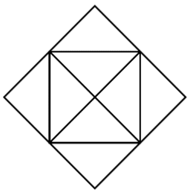
Section A

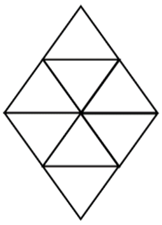
A1 Which of the following sheets of paper can be turned into a regular octahedron (a three-dimensional regular polyhedron with eight triangular faces, as shown on the right) by folding along the marked lines?



(a) 

(b) 

(c) 

(d) 

A2 A surface is given by

$$2x^3z + 4y^2z + 3z^2 = 81$$

Which of the following is a vector tangential to it at the point on the surface with coordinates $(x, y, z) = (1,2,3)$?

(a) $2\hat{\imath} - 3\hat{\jmath} + 3\,\hat{k}$

(b) $18\hat{\imath} + 48\hat{\jmath} + 36\,\hat{k}$

(c) $-3\hat{\imath} + 2\hat{\jmath} + 6\,\hat{k}$

(d) $-3\hat{\imath} - 2\hat{\jmath} + 6\,\hat{k}$

A3 Consider the following differential equations:

$$\frac{dx}{dt} = ay(t), \quad \frac{dy}{dt} = a$$

where a is a positive constant. The solutions to these equations define a family of curves in the x,y plane. What are these curves?

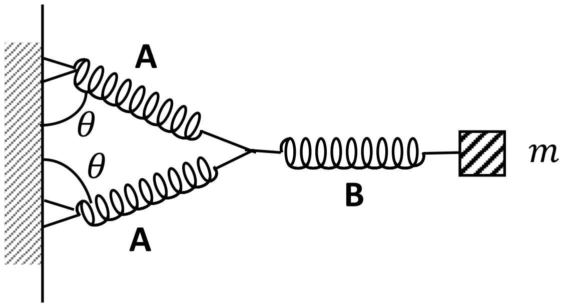
(a) Parabolas

(b) Circles

(c) Hyperbolas

(d) Ellipses

A4 Consider a mass m connected to a network of massless springs shown in the figure below.



The spring constant of spring A is k_A, and that of spring B is k_B. The springs are shown in a relaxed position, and the angle θ in this position is $\pi/3$. The mass is displaced horizontally by a small distance. What is the angular frequency of small oscillations of m? (Ignore gravity and friction.)

(a) $\sqrt{(3k_A\, k_B)/[m(2k_B + 3k_A)]}$

(b) $\sqrt{(k_A\, k_B)/[m(k_B + k_A)]}$

(c) $\sqrt{(2k_A\, k_B)/[m(k_B + 2k_A)]}$

(d) $\sqrt{(\sqrt{3}\, k_A k_B)/[m(k_B + \sqrt{3}\, k_A)]}$

A5 Consider an object falling in air. In addition to gravity, it experiences an air resistance force, R, given by $R = bv$, where v is the speed and b is a constant. If the object is dropped from rest ($v = 0$ at $t = 0$), the distance traversed by the object at $t = m/b$ is:

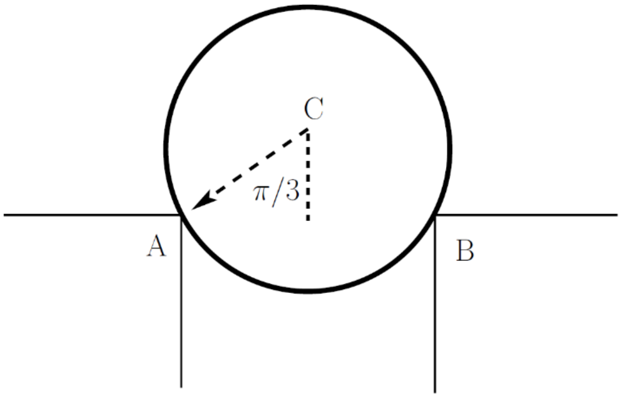
(a) $\left(\frac{m^2 g}{b^2}\right)\left(\frac{1}{e}\right)$

(b) $\left(\frac{m^2 g}{b^2}\right)\left(1 - \frac{1}{e}\right)$

(c) $\left(\frac{m^2 g}{b^2}\right)(e - 1)$

(d) $\left(\frac{m^2 g}{b^2}\right)\left(2 - \frac{1}{e}\right)$

A6 A frictionless disk of mass m is balanced at rest on the edges of two platforms at points A and B that are at equal height as shown below. The angle made by the line joining the centre to point A (line CA) with the vertical is $\pi/3$.



What is the magnitude of the force exerted by point A on the disk?

(a) mg

(b) $\frac{mg}{2}$

(c) $\frac{mg}{\sqrt{3}}$

(d) $\frac{mg\sqrt{3}}{2}$

A7 Consider a particle of mass m moving in a one-dimensional potential of the form

$$V(x) = \begin{cases} \frac{1}{2}kx^2 & \text{for } x > 0 \\ \infty & \text{for } x \leq 0 \end{cases}$$

In a quantum mechanical treatment, what is the ground state energy of the particle?

(a) $\frac{3}{2}\hbar\sqrt{\frac{k}{m}}$

(b) $\frac{1}{2}\hbar\sqrt{\frac{k}{m}}$

(c) $\hbar\sqrt{\frac{k}{m}}$

(d) $\frac{5}{2}\hbar\sqrt{\frac{k}{m}}$

A8 The un-normalized energy eigenfunction of a one-dimensional simple quantum harmonic oscillator in dimensionless units $(m = \hbar = \omega = 1)$ is

$$\psi_a(x) = (2x^3 - 3x)e^{-x^2/2}$$

Which of the following are two other (un-normalized) eigenfunctions which are closest in energy to ψ_a?

(a) $(2x^2 - 1)e^{-x^2/2}$; $(4x^4 - 12x^2 + 3)e^{-x^2/2}$

(b) $e^{-x^2/2}$; $(2x^2 - 1)e^{-x^2/2}$

(c) $x\, e^{-x^2/2}$; $(4x^5 - 20x^3 + 15x)e^{-x^2/2}$

(d) $(2x^2 - 1)\, e^{-x^2/2}$; $(4x^5 + 20x^3 + 15x)e^{-x^2/2}$

A9 A quantum-mechanical state of a particle, with Cartesian coordinates x, y and z, is described by the normalized wave function

$$\psi(x,y,z) = \frac{\alpha^{5/2}}{\sqrt{\pi}}\ ze^{-\alpha\sqrt{x^2+y^2+z^2}}$$

For this state what are the angular quantum number ℓ, L^2 and L_Z respectively?

(a) $1;\ 2\hbar^2;\ 0$

(b) $0;\ 0;\ 0$

(c) $1;\ 2\hbar^2;\ \hbar$

(d) $2;\ 6\hbar^2;\ 0$

A10 A thin spherical shell of radius R has a constant surface charge density σ. This shell is cut symmetrically into two pieces. What is the electrostatic force between the two halves?

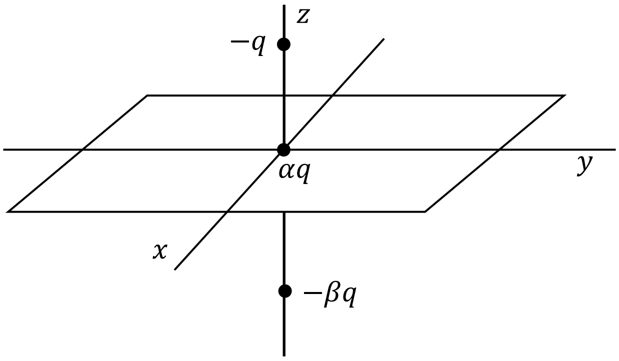
(a) $\frac{\pi}{2}\frac{\sigma^2 R^2}{\epsilon_0}$

(b) $\frac{\pi}{4}\frac{\sigma^2 R^2}{\epsilon_0}$

(c) $\pi\frac{\sigma^2 R^2}{\epsilon_0}$

(d) $2\pi\frac{\sigma^2 R^2}{\epsilon_0}$

A11 Consider a system of three electric charges: (i) a charge $-q$ placed at the point $(x, y, z) = (0,0, d)$, (ii) a charge $+\alpha q$ placed at the origin and (iii) a charge $-\beta q$ placed at the point $(x, y, z) = (0,0, -d)$.



The values of α and β are such that the monopole and dipole terms vanish in the multipole expansion of the electrostatic potential.

What is the quadrupole term of the potential at a point $(x, y, 0)$?

(a) $\frac{q}{4\pi\epsilon_0} \frac{d^2}{(x^2 + y^2)^{3/2}}$

(b) $\frac{q}{2\pi\epsilon_0} \frac{d^2}{(x^2 + y^2)^{3/2}}$

(c) 0

(d) $\frac{q}{4\pi\epsilon_0} \frac{1}{(x^2 + y^2)^{1/2}}$

A12 In an infinite fluid of density ρ there are two spherical gas bubbles of radii r_1 and r_2 respectively. The gas has density $\rho_g < \rho$. The centres of the bubbles are separated by a distance $R \gg r_1, r_2$. If the space has no other forces than gravity, the bubbles will:

(a) Move towards each other due to an attractive gravitational force

$$F = G(\rho - \rho_g)^2 \left(\frac{4\pi}{3}\right)^2 \frac{r_1^3 r_2^3}{R^2}$$

(b) Move towards each other due to an attractive gravitational force

$$F = G(\rho - \rho_g)^2 \left(\frac{4\pi}{3}\right) \frac{r_1^3 r_2^3}{R^2}$$

(c) Move away from each other due to a repulsive gravitational force

$$F = G(\rho - \rho_g)^2 \left(\frac{4\pi}{3}\right)^2 \frac{r_1^3 r_2^3}{R^2}$$

(d) Move away from each other due to a repulsive gravitational force

$$F = G(\rho - \rho_g)^2 \left(\frac{4\pi}{3}\right) \frac{r_1^3 r_2^3}{R^2}$$

A13 A mass of M kg of water at temperature T_a is isobarically and adiabatically mixed with an equal mass of water at temperature T_b. The specific heat of water at constant pressure is C_p. What is the entropy change (ΔS) of the system?

(a) $\Delta S = MC_p \ln\left\{1 + \frac{(T_a - T_b)^2}{4T_aT_b}\right\}$

(b) $\Delta S = MC_p \ln\left\{1 - \frac{(T_a + T_b)^2}{4T_aT_b}\right\}$

(c) $\Delta S = MC_p \ln\left\{1 + \frac{4T_aT_b}{(T_a - T_b)^2}\right\}$

(d) $\Delta S = MC_p \ln\left\{\frac{T_a + T_b}{\sqrt{T_aT_b}}\right\}$

A14 A string has 8 beads in a row, with n identical red beads and $(8-n)$ identical blue beads. When one of the red beads is replaced by a blue one, the entropy of the given system changes from S to $S + k_B \ln 2$. All configurations of the beads are equally probable. What is the value of n ?

(a) 6

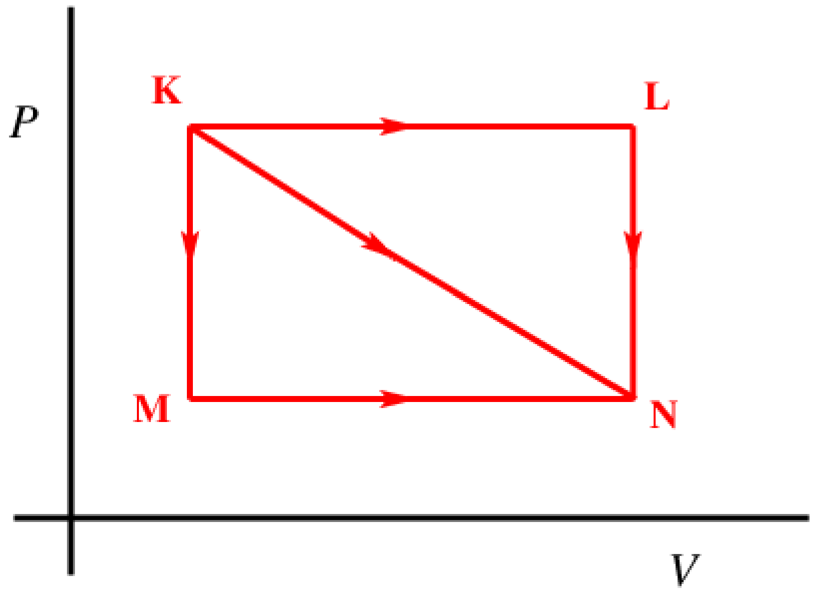
(b) 2

(c) 4

(d) 8

A15 **[In the previous version, the arrow on the segment MN was incorrectly marked. This small typographical error has been fixed. The question and the correct answer were clear from the text.]**

An ideal gas on the Pressure (P)-Volume (V) diagram can be taken from point K to point N along three different paths, as shown below. K → L → N, K → N, and K → M → N. Which of the following options is a true statement?



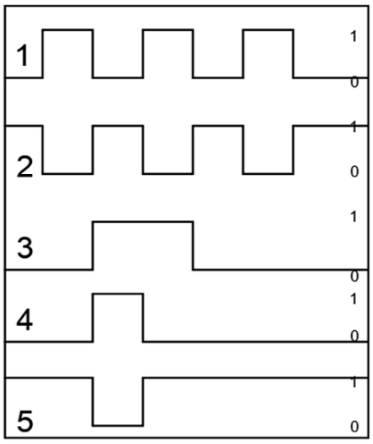
(a) The change in internal energy is the same along each path

(b) The same work is done along each path

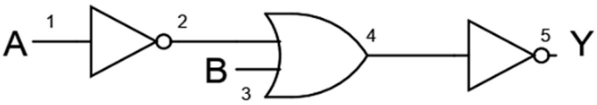
(c) The same amount of heat is added to each the gas along each path

(d) There is no work done along the path K → N

A16 A technician receives an electronic instrument on which the following circuit diagram is drawn. Based on the shown timing diagram (binary values at pins 1, 2, 3, 4, 5 as a function of time) measured by the technician, identify the fault in the instrument.



Timing diagram



Electronic circuit diagram

(a) An AND gate is used where an OR gate should have been used

(b) Input inverter acts like an OR gate

(c) Pin 4 shorted to ground

(d) Output inverter is faulty

A17 The minimum number of two input NAND gates required to obtain the output $Y = \bar{A}B + \bar{C}$ from three inputs A, B and C is:

(a) 3

(b) 7

(c) 4

(d) 6

A18 A student measures the radioactive decay of a material with a half-life of 13,000 years with a Geiger counter. In the laboratory notebook, the student records the following number of decays every 10 seconds:

158, 146, 145, 163, 154, 163, 160, 160, 152, 157, 154, 156, 149, 168, 152

The teacher suspects that the experiment was not done properly and the student created the numbers manually.

Why would the teacher have such a suspicion?

(a) The variance is much less than the mean, unlike what is expected for a Poisson distribution.

(b) The standard deviation is much less than the variance, as expected for a Poisson distribution.

(c) The median is less than the mean, unlike what is expected for a Poisson distribution.

(d) The median is greater than the mean, as expected for a Poisson distribution.

A19 Unpolarised light of intensity 200 W/m^2 is incident on a set of two perfect polarisers arranged one behind the other. The first polariser has its transmission axis at +55^0 with respect to the vertical and the second polariser has its transmission axis at +100^0 with respect to the vertical. What is the intensity of the transmitted light?

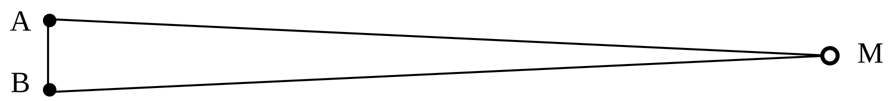
(a) 50 W/m^2

(b) 100 W/m^2

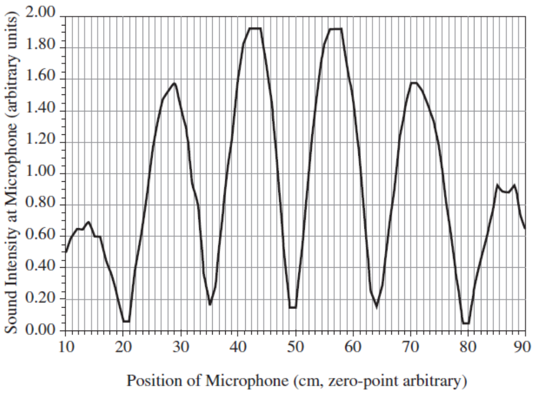
(c) 1.98 W/m^2

(d) 3.01 W/m^2

A20 Two small loudspeakers A and B, separated by 15 cm, were pointed toward a small microphone M at a distance 1.5 m away from the centre of the line AB, in the perpendicular direction as shown in the sketch below.



The following sound intensity pattern was observed as a function of the position of the microphone as it is moved parallel to AB.



The dips in the signal were repeated at the interval of 14.5 cm. The speed of sound in the experiment's background condition is 343 m/s. What can we conclude from this information?

(a) The two loudspeakers are vibrating at frequency 23.65 kHz and they are out of phase.

(b) The two loudspeakers are vibrating at frequency 23.65 kHz and they are in phase.

(c) The two loudspeakers are vibrating at frequency 47.3 kHz and they are in phase.

(d) The two loudspeakers are vibrating at frequency 47.3 kHz and they are out of phase.

A21 The ionization potential of the H atom is 13.598 eV. If the mass of a proton is 1.673×10^{-27} kg, the mass of an electron is 9.109×10^{-31} kg and the mass of the D nucleus is 3.344×10^{-27} kg, the ionization potential of the D atom is given by:

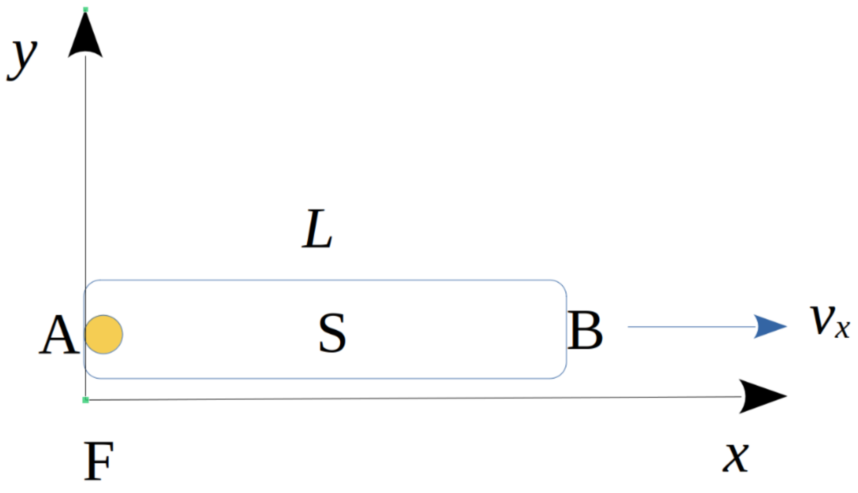
(a) 13.602 eV

(b) 13.594 eV

(c) 13.598 eV

(d) 27.188 eV

A22 Consider a spaceship S of length L is moving relativistically in the x direction with a speed v_x relative to an inertial reference frame F as shown in the figure. In S, a light bulb is placed at the left end (point A) and a detector is placed at the right end (point B). What is the time taken for light to travel from A to B in the reference frame F?



(a) $\frac{L}{c}\sqrt{\frac{1+v_x/c}{1-v_x/c}}$

(b) $\frac{L}{c}\sqrt{1-\frac{v_x^2}{c^2}}$

(c) $\frac{L}{c}\sqrt{\frac{1-v_x/c}{1+v_x/c}}$

(d) $\frac{L}{c}\frac{1}{\sqrt{1-v_x^2/c^2}}$

A23 A beam of neutrons is incident normally upon a thick sheet of Cadmium. The mass density of Cadmium is $\rho = 8.6$ g cm^{-3}. The absorption cross-section of neutrons on Cadmium nuclei is 2.5×10^{-20} cm^{2}. The atomic weight of Cadmium is known to be $112.40\ \text{g/mol}$. You may take $N_A = 6.02 \times 10^{23}$.

At what depth is the intensity of the beam reduced by a factor $1/e$?

(a) 9 μm

(b) 9 fm

(c) 9 nm

(d) 900 fm

A24 An electron confined in a two-dimensional square box, is in the ground state. The length of the side of this square is unknown, but it is seen that the electron jumps to the first excited energy state by absorbing electromagnetic radiation of wavelength 4,040 nm. What is the length of one side of the square well?

(a) 1.91 nm

(b) 1.68 nm

(c) 2.55 nm

(d) 3.82 nm

A25 A student designed a new semiconductor with lattice constant a that crystallizes in the face-centered cubic (fcc) structure. The conduction band minimum of this semiconductor lies at all momentum points equivalent to $\vec{k} = (0.5, 0,0)\ \pi/a$. How many conduction band minimum points are inside the first Brillouin zone?

(a) 6

(b) 4

(c) 3

(d) 1

Section B

B1 The following series

$$S = \sum_{n=1}^{\infty} (-1)^{1+n} \frac{1}{n\, 2^{4n}}$$

has the sum

(a) $S = \ln\left(\frac{17}{16}\right)$

(b) $S = \sqrt{\frac{17}{16}}$

(c) S is not convergent

(d) $S = \frac{1}{1 + \sqrt{\frac{1}{16}}}$

B2 Consider a particle of mass m orbiting around a central potential $V(r) = -\frac{a}{r} - \frac{b}{r^3}$ with $a, b > 0$. What is the smallest angular momentum it must have to be in a stable orbit?

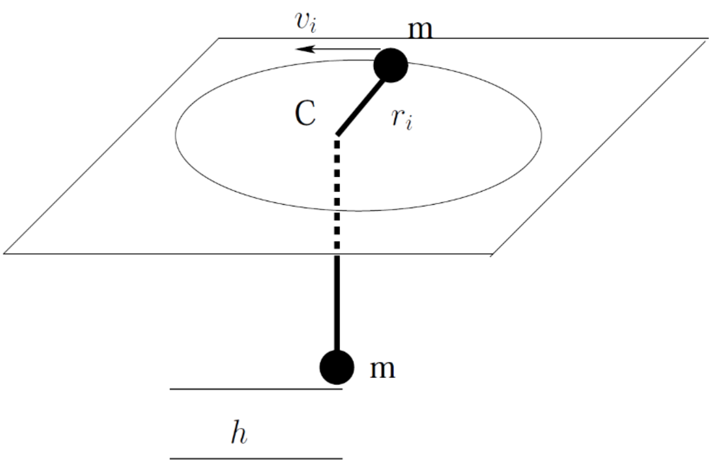
(a) $(12abm^2)^{1/4}$

(b) $(4abm^2)^{1/4}$

(c) $(3abm^2)^{1/4}$

(d) $(6abm^2)^{1/4}$

B3 Two equal masses m are connected with a massless string. The first mass is initially set in a uniform circular motion with speed v_i at a radius r_i on top of a table while the second mass hangs vertically on the string which passes through a hole in the centre of the table (C), as shown in the figure below.



The system is released with this initial configuration and the second mass starts falling. What is the net speed of the first mass when the second mass has fallen a height h (smaller than r_i)?

(Assume that there is no friction and that the string always remains tight.)

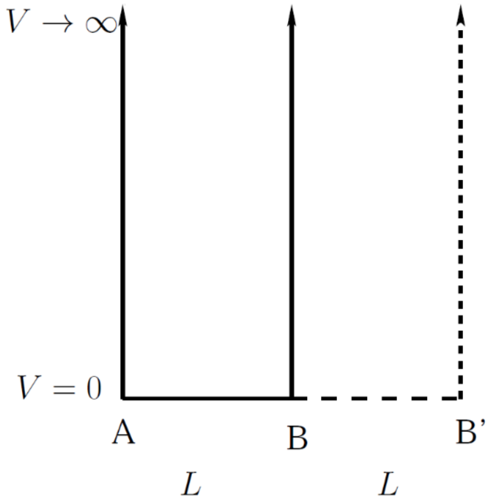
(a) $\sqrt{gh + \frac{1}{2}v_i^2\left[1 + \frac{r_i^2}{(r_i - h)^2}\right]}$

(b) $\sqrt{gh + \frac{1}{2}v_i^2\left[1 + \frac{r_i}{(r_i - h)}\right]}$

(c) $\sqrt{gh + \frac{1}{2}v_i^2\left[1 + \frac{(r_i - h)^2}{r_i^2}\right]}$

(d) $\sqrt{gh + \frac{1}{2}v_i^2\left[1 + \frac{(r_i - h)}{r_i}\right]}$

B4 Consider a particle of mass m moving in an infinite one-dimensional potential well of length L between points A and B. In a quantum-mechanical treatment, the particle is in the ground state with an energy E_g. The wall at B is suddenly shifted to B′ where AB′ has length $2L$. If we measure the energy again, what is the most probable result?



(a) E_g

(b) $E_g/4$

(c) $4E_g$

(d) $9E_g/4$

B5 A particle of mass m is subjected to a force $\vec{F}(\vec{r})$ such that the wavefunction $\phi(\vec{p},t)$ satisfies the momentum-space Schrödinger equation

$$\left(\frac{\vec{p}^2}{2m} - a\vec{\nabla}_p^2\right)\phi(\vec{p},t) = i\hbar\frac{\partial}{\partial t}\phi(\vec{p},t)$$

where a is a real constant and

$$\vec{\nabla}_p^2 = \frac{\partial^2}{\partial p_x^2} + \frac{\partial^2}{\partial p_y^2} + \frac{\partial^2}{\partial p_z^2}$$

It follows that $\vec{F}(\vec{r}) =$

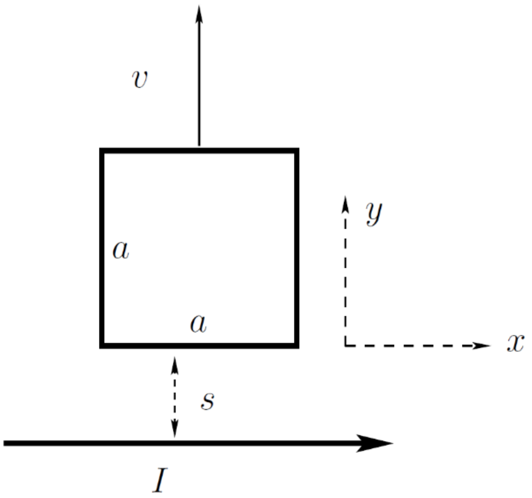
(a) $-\frac{2a}{\hbar^2}\vec{r}$

(b) $\frac{2a}{\hbar^2}\vec{r}$

(c) $-a\hbar^2\frac{\vec{r}}{r^3}$

(d) $a\hbar^2\frac{\vec{r}}{r^3}$

B6 A conducting square loop of wire (side a) lies on a table, at a distance s from an infinite straight wire as shown in the figure. The infinite wire carries a current I in the x direction as shown. If one now pulls the loop directly away from the wire in the y direction at a constant speed v, what is the generated EMF in the loop? In what direction (clockwise or counterclockwise) does the current flow in the loop?



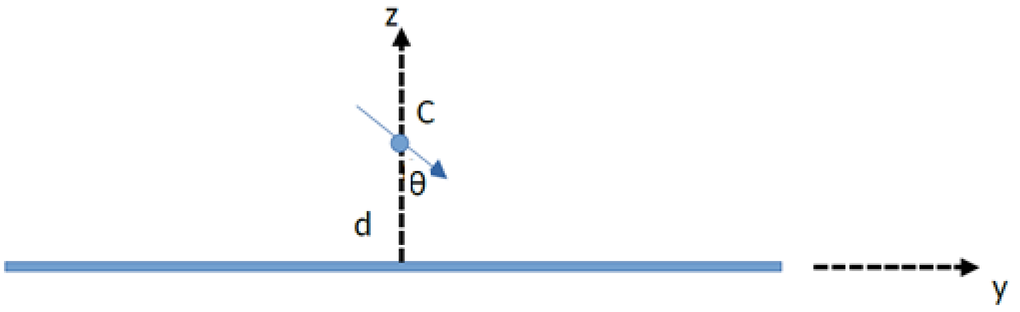
(a) $\frac{a^2 v I \mu_0}{2\pi(as+s^2)}$, anticlockwise

(b) $\frac{a^2 v I \mu_0}{2\pi(as+s^2)}$, clockwise

(c) $\frac{a\, v I \mu_0}{2\pi(a+s)}$, anticlockwise

(d) $\frac{a\, v I \mu_0}{2\pi(a+s)}$, clockwise

B7 Consider an electric dipole of strength *p* placed near a grounded infinite conducting sheet in the *xy* plane at a distance *d* from it in the *z* direction as shown below. The centre of the dipole (C) is fixed to a pivot but the dipole is free to rotate about the *x* axis (coming out of the page).



What is the magnitude of the torque on the dipole when the angle between the dipole and the positive *z* axis is $(\pi - \theta)$ as shown?

(a) $\tau = \frac{p^2}{(64\pi\epsilon_0 d^3)}\sin(2\theta)$

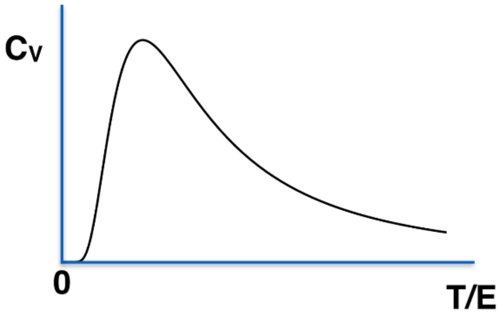
(b) $\tau = \frac{p^2}{(16\pi\epsilon_0 d^3)}\cos(2\theta)$

(c) $\tau = \frac{p^2}{(64\pi\epsilon_0 d^3)}\cos(\theta)$

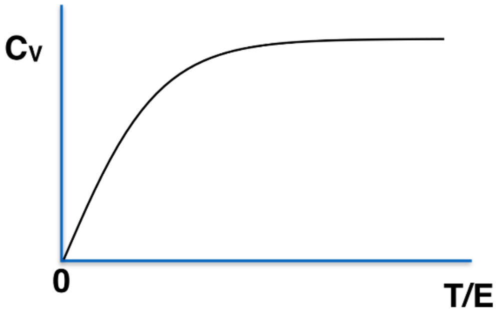
(d) $\tau = \frac{p^2}{(16\pi\epsilon_0 d^3)}\sin(\theta)$

B8 A collection of N spin-½ objects, with individual energy eigenvalues $0, E$ are kept at a temperature T. Which of the following curves accurately represent the temperature dependence of specific heat of the system?

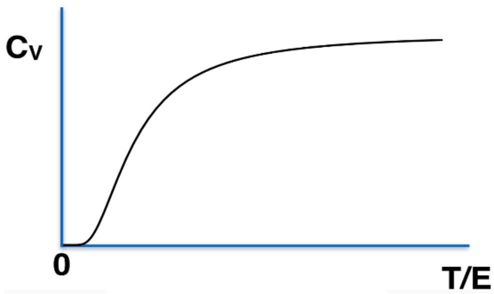
(a)



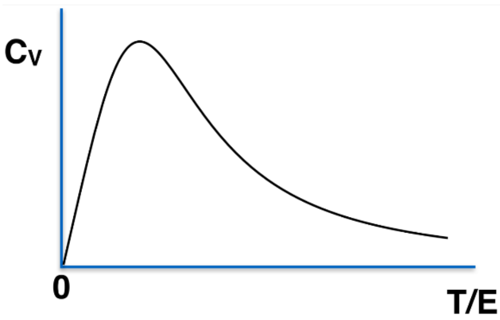
(b)



(c)



(d)



B9 An ideal heat pump delivers heat at a rate $\beta(T_H - T_S)$ from the colder surroundings (S) at a temperature T_S to a heater (H) kept at a constant temperature T_H. Here β is a constant. What is the power consumption of the heat pump?

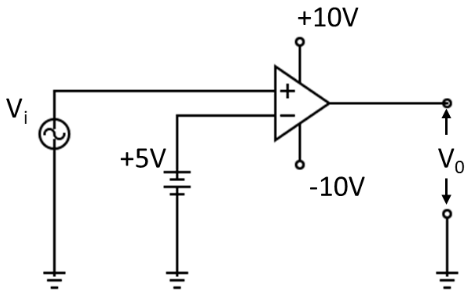
(a) $\dfrac{\beta(T_H - T_S)^2}{T_H}$

(b) $\beta(T_H - T_S)$

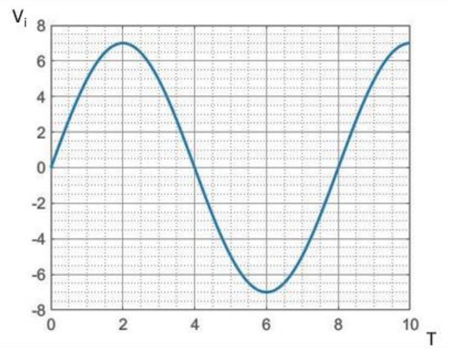
(c) $\dfrac{\beta(T_H - T_S)^2}{T_S}$

(d) $\dfrac{\beta(T_H - T_S)T_H}{T_S}$

B10 In the following ideal Op-Amp circuit

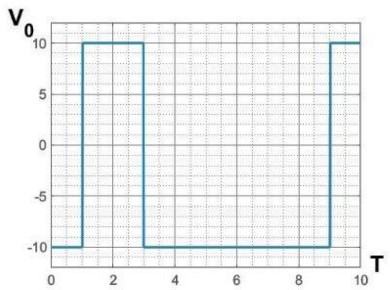


if the input voltage V_i is

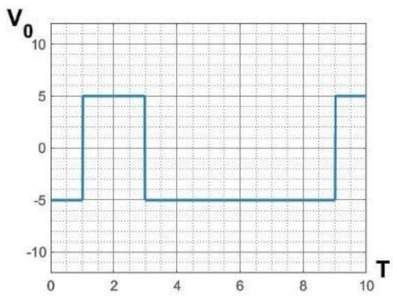


The output waveform would be

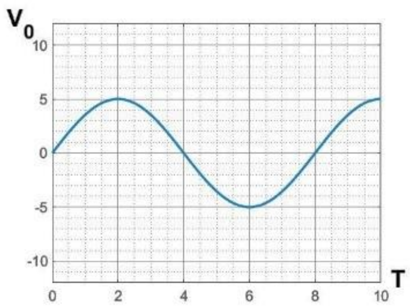
(a)



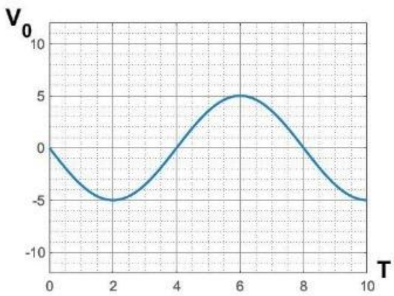
(b)



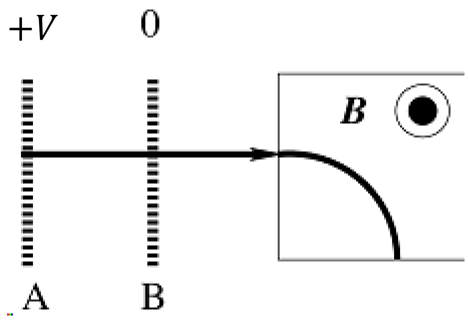
(c)



(d)



B11 A mass spectrometer consists of a parallel plate capacitor with plates A and B that accelerates ions through an electric potential V, which is followed by a box carrying a uniform magnetic field $\boldsymbol{B}$ of magnitude 0.2 Tesla (coming out of the page as shown).



This setup is used to separate two isotopes of Uranium:

$$^{235}U \quad (\text{mass} = 3.93 \times 10^{-25} \text{ kg})$$

$$^{238}U \quad (\text{mass} = 3.98 \times 10^{-25} \text{ kg})$$

Singly charged ions (charge $+e$) of the two isotopes are created at the plate A and pass without energy loss through plate B into the box. In order to separate the isotopes, their radii of curvature in the box must differ by 2 mm. What is the approximate V through which the ions must be accelerated in order to achieve this?

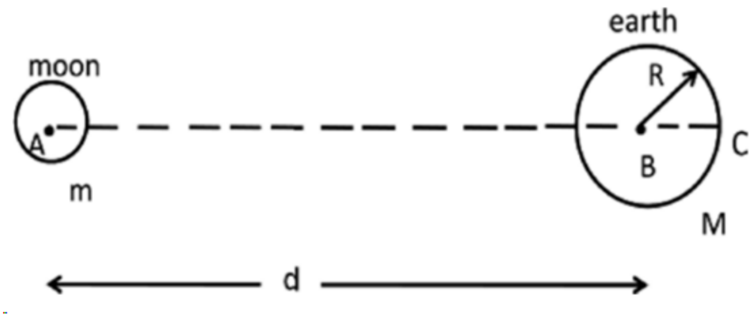
(a) 800 Volts

(b) 8000 Volts

(c) 80 Volts

(d) 8 Volts

B12 A tidal force is exerted on the oceans by the Moon. This can be estimated by the differential acceleration (Δg) which is the difference between the gravitational acceleration at B and C due to the moon (see figure below).



If R and M are the radius and mass of Earth, d the distance of separation of the centre of Earth and the moon, and m the mass of moon, which of the following answers is closest to the magnitude of Δg?

(a) $\frac{2GmR}{d^3}$

(b) $\frac{4GmR}{d^3}$

(c) $\frac{GmR}{2d^3}$

(d) $\frac{GmR^2}{d^4}$

B13 Incident unpolarized light is reflected from a glass ($n_g = 1.65$) plate immersed in ethyl alcohol ($n_a = 1.36$) and this reflection is found to be completely linearly polarised. Find the angle at which the incident light would be transmitted through the plate.

(a) 39.5^0

(b) 31.6^0

(c) 50.5^0

(d) 69.3^0

B14 A massive particle moving at a speed of $4c/5$ collides with an identical particle at rest. What would be the speed of the second particle in the center-of-mass frame after the collision?

(a) $c/2$

(b) $2\,c/5$

(c) $c/\sqrt{2}$

(d) $2\sqrt{2}\,c/5$

B15 The energy dispersion in the conduction band of a one-dimensional metal with lattice spacing a is given by,

$$E(k) = E_0(1 - \cos ka)$$

where $k \in (-\pi/a, +\pi/a]$. Suppose that each site contributes one conduction electron to the conduction band. What is the Fermi energy of the system?

(a) E_0

(b) $\frac{E_0}{2}$

(c) $2E_0$

(d) $\frac{E_0}{4}$