TIFR graduate admissions in Mathematics
Syllabus, exam instructions and sample questions [1]

This page contains the syllabus and some sample questions for the nation-wide test that will be conducted in various centers in December, performance in which will be used to decide whether a student progresses to the second stage of the evaluation process, for each of the following programs:

- The PhD and the IntPhD programs at School of Mathematics, TIFR, Mumbai.
- The IntPhD program at TIFR CAM, Bengaluru.
- The PhD program at ICTS, Bengaluru.
- The CAM-ICTS PhD program in Applied and Computational Mathematics.

The selection to the PhD program at TIFR CAM, Bengaluru involves a different examination, syllabus and sample questions for which are provided elsewhere.

Syllabus for Stage I

The nation-wide test for the programs mentioned above is mainly based on mathematics covered in a reasonable B.Sc. course. This includes:

Algebra: Definitions and examples of groups (finite and infinite, commutative and non-commutative), cyclic groups, subgroups, homomorphisms, quotients. Group actions and Sylow theorems. Definitions and examples of rings and fields. Integers, polynomial rings and their basic properties. Basic facts about vector spaces, matrices, determinants, ranks of linear transformations, characteristic and minimal polynomials, symmetric matrices. Inner products, positive definiteness.

Analysis: Basic facts about real and complex numbers, convergence of sequences and series of real and complex numbers, continuity, differentiability and Riemann integration of real valued functions defined on an interval (finite or infinite), elementary functions (polynomial functions, rational functions, exponential and log, trigonometric functions), sequences and series of functions and their various types of convergence.

Geometry/Topology: Elementary geometric properties of common shapes and figures in 2 and 3 dimensional Euclidean spaces (e.g. triangles, circles, discs, spheres, etc.). Plane analytic geometry (= coordinate geometry) and trigonometry. Definition and basic properties of metric spaces, examples of subset Euclidean spaces (of any dimension), connectedness, compactness. Convergence in metric spaces, continuity of functions between metric spaces.

General: Pigeon-hole principle (box principle), induction, elementary properties of divisibility, elementary combinatorics (permutations and combinations, binomial coefficients), elementary reasoning with graphs, elementary probability theory.

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[1] except for the PhD program at TIFR CAM

Exam Instructions

1. The use of calculators, mobile phones, laptops, tablets, smart watches and other electronic devices, including those connecting to the internet, is NOT permitted.

2. Do not ask for clarifications from the invigilators regarding the questions. They have been instructed not to respond to any such queries from candidates.

3. **Test Structure:** The paper is divided into two sections – Part A and Part B.

   The evaluation process for the Integrated PhD program at TIFR CAM, Bengaluru, will only consider answers to Part A. For students applying to any other program, all questions are mandatory.

   Part A consists of multiple choice questions. Each question from this part has four options, of which exactly one is correct.

   Part B consists of true/false questions.

4. **Grading Scheme:** There will be negative marking for wrong answers. The grading scheme is as shown in the following table:

	Correct Answer	Wrong Answer	No Answer
Part A	+2	-1	0
Part B	+2	-2	0

5. The notation and conventions used in this test can be viewed any time after you begin the exam proper, by clicking on "Useful Data" at the top right corner of the display.

Sample questions for the nation-wide test

In addition to the following sample questions, you can also find some of the previous years' question papers at:
https://www.tifr.res.in/academics/past_question_papers.php

Sample multiple choice questions

1. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous bounded function. Then

   (a) f has to be uniformly continuous

   (b) there exists an $x \in \mathbb{R}$ such that $f(x) = x$

   (c) f can not be increasing

   (d) $\lim_{x \to \infty} f(x)$ exists.

2. Define a function

$$f(x) = \begin{cases} x + x^2 \cos(\frac{\pi}{x}), & \text{if} \quad x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

   Consider the statements:

   I. f is differentiable at $x = 0$ and $f'(0) = 1$.

   II. f is differentiable everywhere and $f'(x)$ is continuous at $x = 0$.

   III. f is increasing in a neighbourhood around $x = 0$.

   IV. f is not increasing in any neighbourhood of $x = 0$.

   Which one of the following combinations of the above statements is true.

   (a) **I.** and **II**.

   (b) **I.** and **III.**

   (c) **II.** and **IV.**

   (d) **I.** and **IV.**

Sample true/false questions

Note: The Integrated PhD program at TIFR CAM, Bengaluru will only evaluate answers to the multiple choice questions. In other words, students who apply only to this program can ignore the true/false questions.

1. If A and B are 3×3 matrices and A is invertible, then there exists an integer n such that $A + nB$ is invertible.

2. Let P be a degree 3 polynomial with complex coefficients such that the constant term is 2010. Then P has a root α with $|\alpha| > 10$.

3. The symmetric group S_5 consisting of permutations on 5 symbols has an element of order 6.

4. Suppose $f_n(x)$ is a sequence of continuous functions on the closed interval [0;1] converging to 0 pointwise. Then the integral

$$\int_0^1 f_n(x)dx$$

   converges to 0.

5. There are n homomorphisms from the group $\mathbb{Z}/n\mathbb{Z}$ to the additive group of rationals $\mathbb{Q}$.

6. A bounded continuous function on $\mathbb{R}$ is uniformly continuous.