Section A

A.1 A surface is given by

$$4x^2y - 2xy^2 + 3z^3 = 0$$

Which of the following is a vector normal to it at the point (2,3,1)?

(a) $30\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$

(b) $15\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 18\hat{\mathbf{k}}$

(c) $30\hat{\mathbf{i}} + 8\hat{\mathbf{j}} - 9\hat{\mathbf{k}}$

(d) $30\hat{\mathbf{i}} - 8\hat{\mathbf{j}} - 9\hat{\mathbf{k}}$

A.2 The value of the first derivative of the function

$$f(x) = \frac{2}{\sqrt{3}} e^{-\sqrt{3}\,x^2|x|}$$

at $x = 0$ is $f'(0) =$

(a) 0

(b) 2

(c) $2/\sqrt{3}$

(d) undefined

A.3 Consider a symmetric matrix

$$M = \begin{pmatrix} 1/3 & 0 & 2/3 \\ 0 & 1 & 0 \\ 2/3 & 0 & 1/3 \end{pmatrix}$$

An orthogonal matrix O which can diagonalize this matrix by an orthogonal transformation $O^T M O$ is given by $O =$

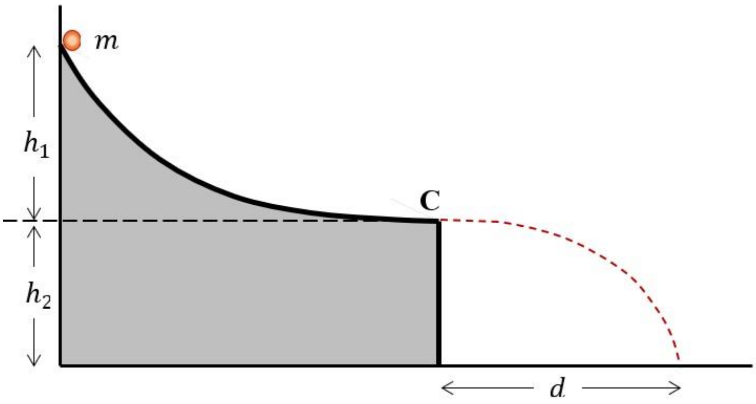
(a) $\begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \end{pmatrix}$

(b) $\begin{pmatrix} \sqrt{2/3} & 0 & \sqrt{1/3} \\ 0 & 1 & 0 \\ \sqrt{1/3} & 0 & -\sqrt{2/3} \end{pmatrix}$

(c) $\begin{pmatrix} \sqrt{1/3} & 0 & \sqrt{2/3} \\ 0 & 1 & 0 \\ \sqrt{2/3} & 0 & -\sqrt{1/3} \end{pmatrix}$

(d) $\begin{pmatrix} 1/\sqrt{2} & 0 & i/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -i/\sqrt{2} \end{pmatrix}$

A.4 A small body of mass m is released from rest at the top of a frictionless curved surface as shown in the figure, and permitted to slide down the curve. At the endpoint C, the tangent to the curve is horizontal. The mass then falls on the ground at a distance d as shown in the figure below when the experiment is carried out on the surface of the Earth. The heights h_1 and h_2 are also shown in the figure.



Suppose the same experiment is repeated on the surface of the Moon, where the acceleration due to gravity is $g' = g/6$, where g is the value on Earth. The corresponding distance d' at which the mass will fall on the ground in the Moon is

(a) d

(b) $6d$

(c) $d\sqrt{h_1/h_2}$

(d) dependent on the shape of the curve

A.5 A particle is executing simple harmonic motion in a straight line. When the distance of the particle from the equilibrium position is $x_1$and x_2, the corresponding values of its velocity arev_1 and v_2 respectively. The time period of oscillation is given by

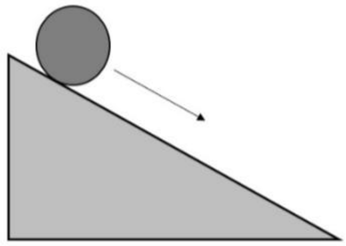
(a) $2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$

(b) $2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_2^2 - v_1^2}}$

(c) $2\pi\frac{x_2 - x_1}{v_2 - v_1}$

(d) $2\pi\frac{x_2 - x_1}{v_1 - v_2}$

A.6 A solid cylinder of uniform mass density rolls down a fixed inclined plane without slipping (see figure).



The fraction of the total kinetic energy of the cylinder associated with its rotation about its centre of mass is

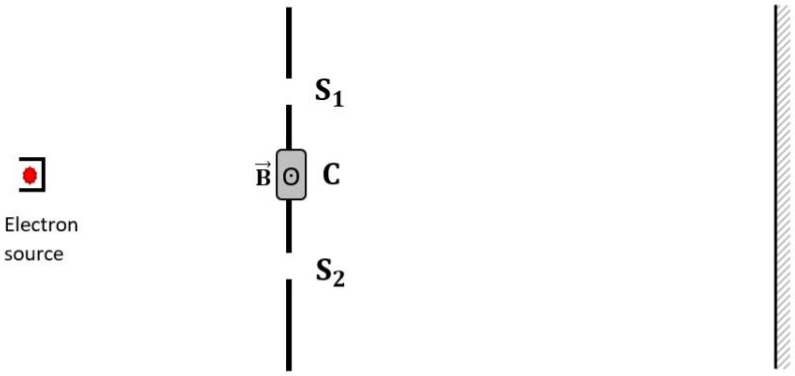
(a) 1/3

(b) 1/6

(c) 1/4

(d) 1/2

A.7 Consider an electron double slit experiment as shown in the figure below, with slits S_1 and S_2.



If now, within the shaded region marked C, a constant uniform magnetic field pointing outside the page is turned on, the fringe pattern

(a) will get shifted.

(b) will disappear.

(c) will remain unchanged.

(d) will become dimmer.

A.8 An atom of mass M at rest emits or absorbs a photon of frequency ν and recoils with a momentum p. The frequency of the internal transition of electronic levels is ν_0 without accounting for recoil. Assuming the process is nonrelativistic, the fractional differences between the photon frequency for emission and absorption $(\nu - \nu_0)/\nu$, respectively, are given by

(a) $-\frac{h\nu}{2Mc^2}$ (emission), $+\frac{h\nu}{2Mc^2}$ (absorption)

(b) $+\frac{2h\nu}{Mc^2}$ (emission), $-\frac{2h\nu}{Mc^2}$ (absorption)

(c) $-\frac{2h\nu_0}{Mc^2}$ (emission), $+\frac{2h\nu_0}{Mc^2}$ (absorption)

(d) $+\frac{h\nu_0}{2Mc^2}$ (emission), $-\frac{h\nu_0}{2Mc^2}$ (absorption)

A.9 Consider an electron with mass m_e, charge $-e$ and spin $1/2$, whose spin angular momentum operator is given by

$$\hat{\vec{\boldsymbol{S}}} = \frac{\hbar}{2}\vec{\boldsymbol{\sigma}}$$

This electron is placed in a magnetic field $\vec{\boldsymbol{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$, where all three components (B_x, B_y, B_z) are nonvanishing.

At time $t = 0$, the electron is at rest in the $S_z = \hbar/2$ state. The earliest time when the state of the spin will be orthogonal to the initial state is

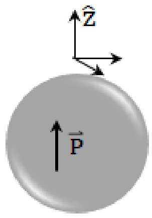
(a) infinity, i.e., it will never be orthogonal.

(b) $\dfrac{2m_e}{ge|\vec{\boldsymbol{B}}|}$

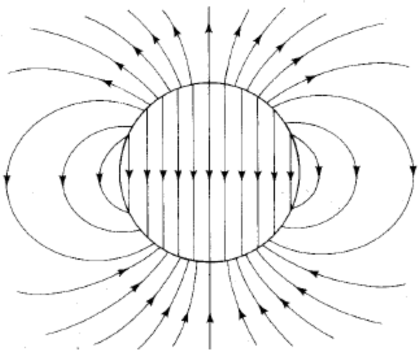
(c) $\dfrac{4m_e}{ge|\vec{\boldsymbol{B}}|}$

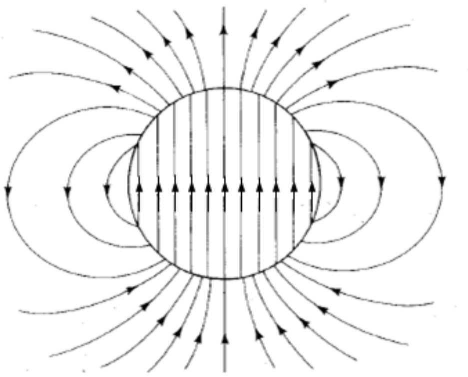
(d) dependent on the direction of the magnetic field $\vec{\boldsymbol{B}}$

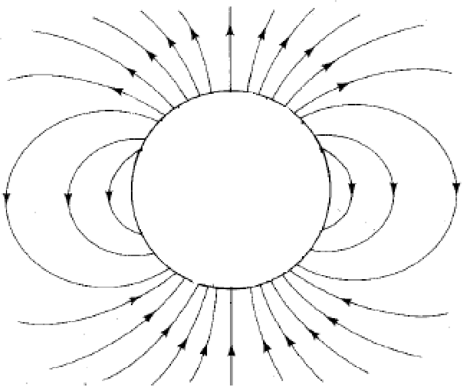
A.10 The electric field lines due to a uniformly polarized dielectric sphere (polarization along the positive z-axis as shown in the figure)

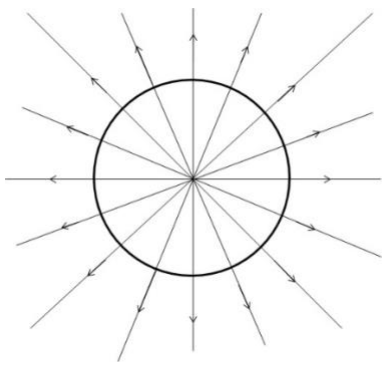


will look like

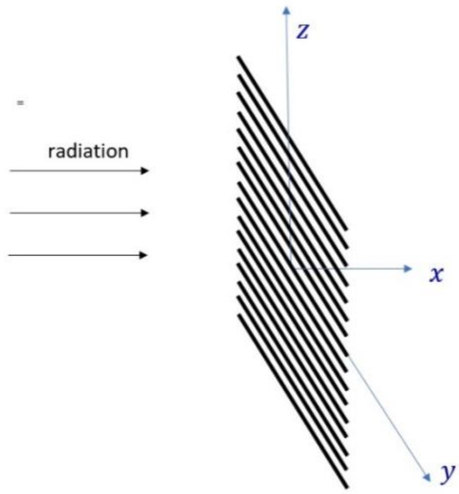
(a) 

(c) 

(b) 

(d) 

A.11 A beam of unpolarized microwave radiation is incident along the x-axis on a grid of metal wires in the yz-plane with wires running along the y-axis (see figure below).



If the width of each wire and the spacing between the adjacent wires is less than the wavelength of the microwave, the observation would be that

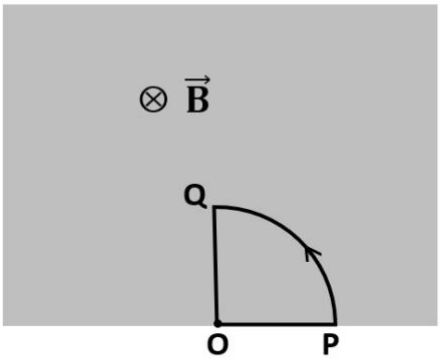
(a) the transmitted wave would be polarized along the z-axis.

(b) the transmitted wave would be polarized along the y-axis.

(c) the transmitted wave would be unpolarized.

(d) no wave will pass through as the spacing is smaller than the wavelength.

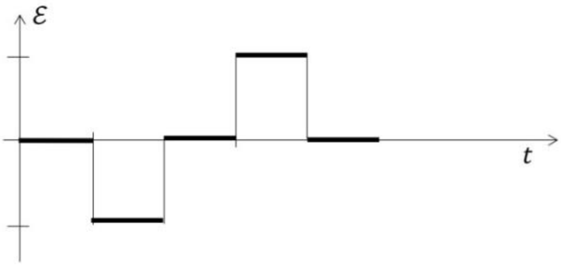
A.12 Consider the following situation. A uniform magnetic field $\vec{B}$ pointing into the plane of the paper is present everywhere inside the rectangular region shown shaded in the adjoining figure. Outside the rectangular region, there is no magnetic field.



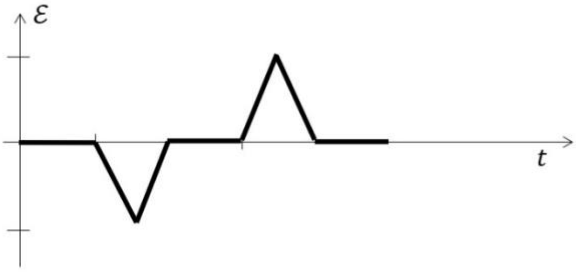
A closed loop of conducting wire is placed inside the rectangular region as shown in the figure at time $t = 0$. The loop is then rotated counterclockwise with a uniform angular velocity ω about an axis perpendicular to the paper passing through the point O.

If the direction along PQOP is taken to be positive, then a correct graph for the EMF $\mathcal{E}$ generated in the loop is

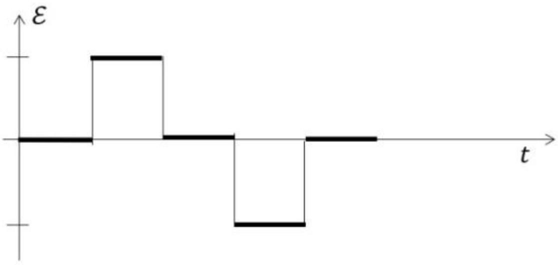
(a)



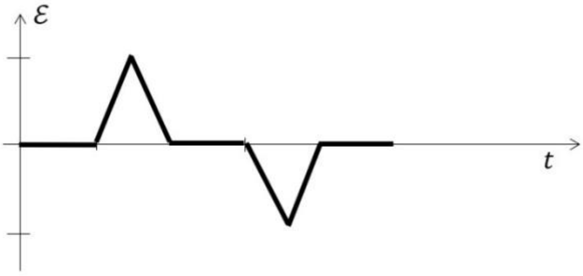
(b)



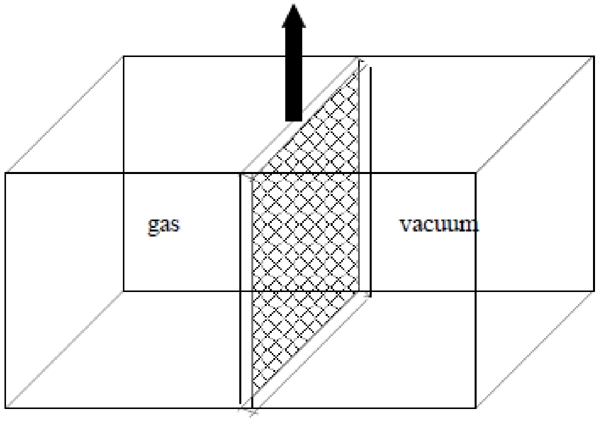
(c)



(d)



A.13 Consider a sealed but thermally conducting container of total volume V, which is in equilibrium with a thermal bath at temperature T. The container is divided into two equal chambers by a thin partition, which is thermally conducting but impermeable to particles. One of the chambers contains an ideal gas, while the other is a vacuum.



If the partition is removed suddenly and the ideal gas is allowed to expand and fill the entire container, then, once equilibrium has been reached, the entropy per molecule will increase by an amount

(a) $+k_B \ln 2$

(b) $-k_B \ln 2$

(c) $2k_B$

(d) $\frac{1}{2}k_B \ln 2$

A.14 Five identical bosons are distributed in energy levels E_1 and E_2 with degeneracy of 2 and 3, respectively. Find the number of microstates if there are three bosons in the energy level E_1 and two bosons in the energy level E_2.

(a) 24

(b) 1024

(c) 120

(d) 6

A.15 A two-level system with zero ground state energy is in equilibrium at a nonzero finite temperature. If α is defined as the ratio

$$\alpha = \frac{\langle E^2 \rangle}{\langle E \rangle^2}$$

where $\langle E \rangle$ denotes the mean energy and $\langle E^2 \rangle$ denotes the mean squared energy, then

(a) $2 < \alpha < \infty$

(b) $1 < \alpha \leq 2$

(c) $\frac{1}{2} < \alpha < 1$

(d) $0 < \alpha < \frac{1}{2}$

A.16 The pulse train at the output of an XNOR gate with the three inputs

A = 00011011

B = 10100011

C = 00101110

will be

(a) 01101001

(b) 10010110

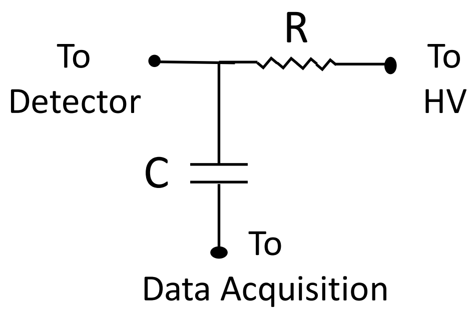
(c) 01010111

(d) 10101000

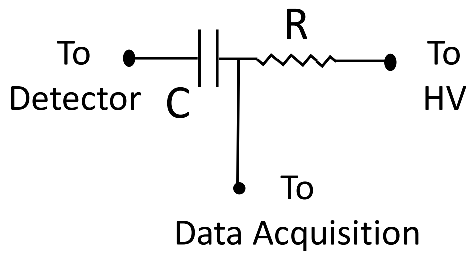
A.17 In an experimental setup, positively charged particles are detected by a detector which requires a negative DC high voltage of -2000 V. Every time a charged particle is detected by the detector, it gives a transient pulse of height 10 mV.

The data collection system used for the experiment needs to detect this pulse; however, it cannot operate at -2000 V. Which of the following circuits can be used to connect the data collection system to the detector to obtain these pulses?

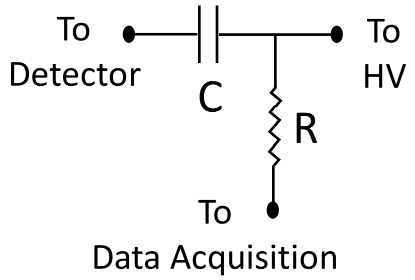
(a)



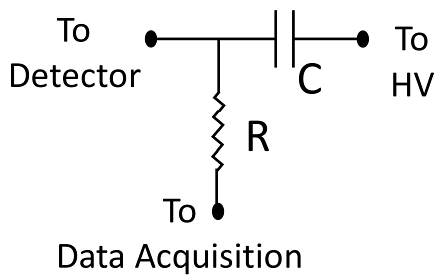
(b)



(c)



(d)



A.18 A faint star is known to emit light of a given frequency at an average rate of 10 photons per minute. An astronomer plans to measure this rate using a photon-counting detector. If she wants to measure the rate to an accuracy of 5%, approximately how long should be the exposure time?

(a) 40 minutes

(b) 1 hour

(c) 20 minutes

(d) 10 minutes

A.19 In a mercury vapour lamp an electric arc passing through mercury vapour produces light. When the lamp is switched on, the arc is struck, and the liquid mercury starts evaporating as the temperature gradually increases.

In a certain experiment, a Michelson interferometer is set up with a mercury vapour lamp as the light source, and the lamp is switched on. Which of the following effects will be observed?

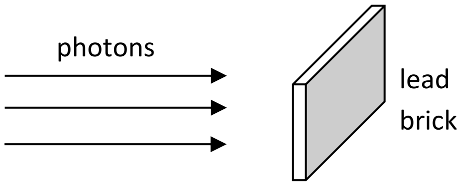
(a) Initially, fringes will appear with high contrast but low intensity, which will be reduced in contrast over the period of time as the light intensity builds up.

(b) Initially, no fringes will be observed, but then fringes will emerge with high contrast as the lamp heats up.

(c) No fringes will be observed as the source is incoherent and has many frequencies.

(d) High contrast fringes will appear as soon as the lamp is switched on and will remain thus so long as the lamp is on.

A.20 A beam of photons of 1 MeV energy each is shot at a 10 mm thick lead brick (see figure).



Given that the density of lead is 11.29 g-cm^{-3}, its atomic mass is 207.2 a.m.u., and also that the interaction cross-section for these photons with a lead atom is 10^{-23}cm^2, the fraction of the incident photons that will cross the brick without losing any energy is

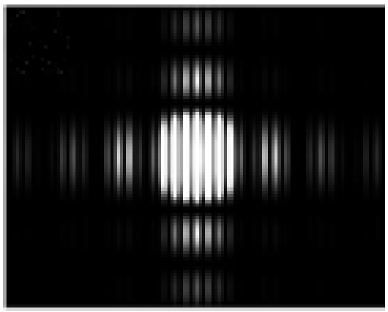
(a) 72 %

(b) 28 %

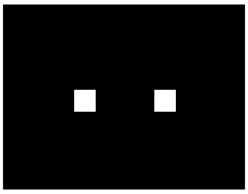
(c) 33 %

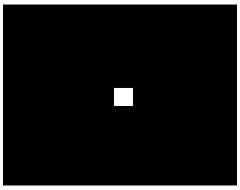
(d) 67 %

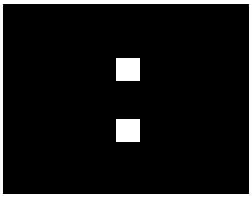
A.21 The following Fraunhofer diffraction pattern was observed in an experiment.

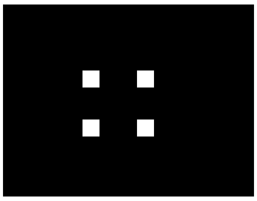


The aperture arrangement that would yield such a fringe pattern is

(a) 

(b) 

(c) 

(d) 

A.22 An electromagnetic wave is described by the following expression

$$\vec{E}(z,t) = E_0 \sin kz \left\{ \hat{\boldsymbol{\imath}} \cos \omega t + \hat{\boldsymbol{\jmath}} \cos\left(\omega t - \frac{\pi}{2}\right) \right\}$$

Which of the following correctly describes this waveform?

(a) A left circular-polarised standing wave along the positive z-axis.

(b) A right circular-polarised standing wave along the positive z-axis.

(c) A left circular-polarised travelling wave along the positive z-axis.

(d) A right circular-polarised travelling wave along the positive z-axis.

A.23 The minimum energy required to dissociate a hydrogen molecule (H_2) into two atoms is 4.5 eV. If the electron affinity of the hydrogen atom is 0.75 eV, the minimum energy required to dissociate the hydrogen molecule into H^+ and H^- would be

(a) 17.35 eV

(b) 14.35 eV

(c) 18.85 eV

(d) 5.25 eV

A.24 The binding energy $\mathcal{E}_b$ of a nuclide ${}^{Z}_{A}\mathrm{X}$ with atomic number Z and mass number A is given by the semi-empirical formula

$$\mathcal{E}_b = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} + a_A \frac{(A-2Z)^2}{A}$$

where the constant parameters and source of effect for each term are

Volume term	Surface term	Coulomb term	Asymmetry term
a_V	a_S	a_C	a_A
15.56 MeV	17.8 MeV	0.7 MeV	23.29 MeV

What is the mass difference between the two-mirror nuclei ${}^{13}_{6}C$ and ${}^{13}_{7}N$? It is known that both of them are spherical in shape and have a uniform charge distribution.

(a) 2.62 MeV

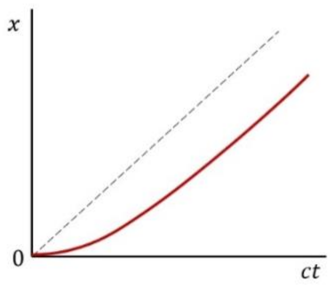
(b) 3.40 MeV

(c) 0.78 MeV

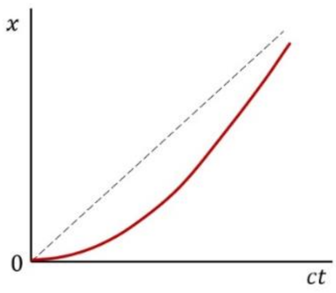
(d) 1.84 MeV

A.25 A relativistic particle, moving in one dimension x, starts from rest at $x = 0$ and is subjected to a uniform and constant force field along the positive x-direction. If the dashed line corresponds to $x = ct$, which of the following curves (red line) would best represent the position $x(t)$ of the particle?

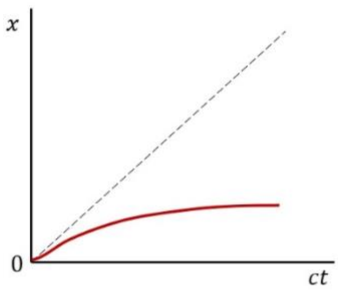
(a)



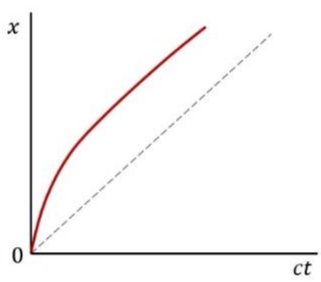
(b)



(c)



(d)



Section B

B.1 A random positive variable x follows an exponential distribution

$$p(x) \propto e^{-\lambda x}$$

with $\lambda > 0$. The probability of observing an event $x > 3\langle x \rangle$, where $\langle x \rangle$ represents the average value of x, is

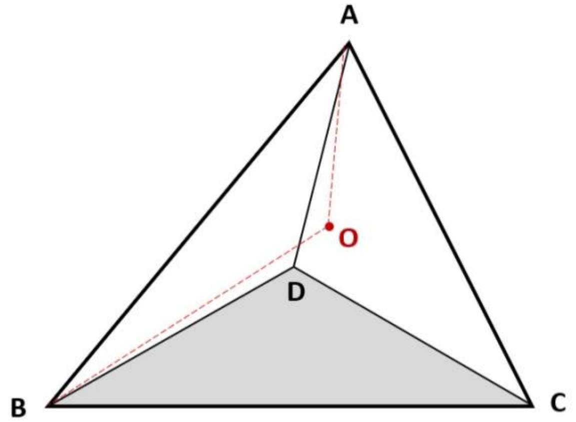
(a) $1/e^3$

(b) $1/e^4$

(c) $1/e$

(d) $1/e^2$

B.2 Consider a regular tetrahedron ABCD, as shown in the figure below. Let the point O be its centre.



The value of the angle AOB must be

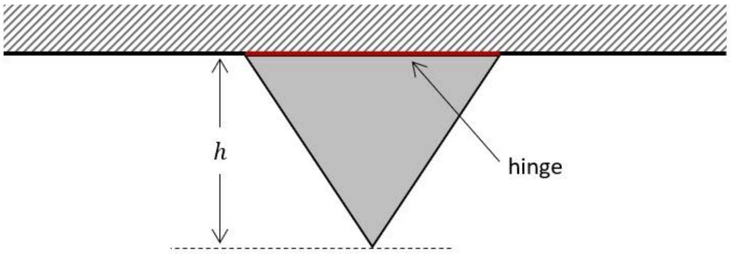
(a) $\cos^{-1}(-1/3)$

(b) $\cos^{-1}(-4/5)$

(c) $\cos^{-1}\left(-\sqrt{4/5}\right)$

(d) $2\pi/3$

B.3 A thin equilateral triangular plate of uniform mass density is attached to a fixed horizontal support along one of its sides through a frictionless hinge, as shown in the figure below. The vertical distance between the rod and the lower tip of the plate is h.



If the pointed tip of the plate is displaced (out of the plane of the paper) so that its plane forms a small angle with the vertical plane passing through the rod, the angular frequency ω of the resultant motion is $\omega =$

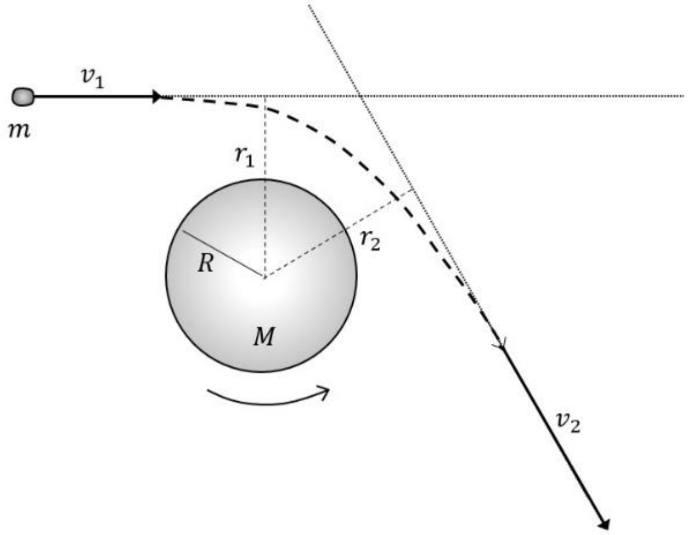
(a) $\sqrt{\frac{2g}{h}}$

(b) $\sqrt{\frac{2\sqrt{3}g}{h}}$

(c) $\sqrt{\frac{2g}{\sqrt{3}h}}$

(d) $\sqrt{\frac{\sqrt{3}g}{2h}}$

B.4 A spherical planet of mass M, radius R and uniform density is rotating anticlockwise about an axis passing through its centre, which, in the figure below, is normal to the plane of the paper. The duration of a 'day' on this planet is T.



A small asteroid of mass m approaches the above planet from far away with a uniform speed v_1 along a straight line at a perpendicular distance r_1 from the centre of the planet (see figure). This path gets distorted by the gravitational field of the planet, and the asteroid leaves with a final uniform speed v_2 along a straight line at a perpendicular distance r_2 from the centre of the planet.

It is observed that after the passage of the asteroid, the length of the day on the planet has changed by $\delta T =$

(a) $\dfrac{5T^2}{4\pi}\dfrac{m(v_2r_2 - v_1r_1)}{MR^2}$

(b) $\dfrac{4\pi}{5}\dfrac{MR^2}{m(v_2r_2 - v_1r_1)}$

(c) $\dfrac{5}{4\pi}\dfrac{MR^2}{m(v_2r_1 - v_1r_2)}$

(d) 0

B.5 A particle is in the ground state of a one-dimensional box

$$-\frac{L}{2} \le x \le +\frac{L}{2}$$

The uncertainty product $\Delta x \Delta p$ for this state satisfies

(a) $\frac{\hbar}{2} < \Delta x \Delta p \le \hbar$

(b) $\hbar < \Delta x \Delta p \le \frac{3\hbar}{2}$

(c) $\frac{3\hbar}{2} < \Delta x \Delta p \le 2\hbar$

(d) $\Delta x \Delta p = \frac{\hbar}{2}$

B.6 Consider a diatomic molecule with two atoms of masses $m_1 = 1$ a.m.u. and $m_2 = 8$ a.m.u. which are separated by a distance r and bound by an effective interaction potential of the form

$$V(r) = \epsilon\left(\frac{a^4}{4r^4} - \frac{b^2}{2r^2}\right)$$

where $\epsilon = 4 \times 10^{-18}$ J, $a = b = 1$ Å and 1 a.m.u. $\approx 1.6 \times 10^{-27}$ kg.

Making a small oscillations approximation, the transition frequency corresponding to the vibrational spectra of the molecule is approximately

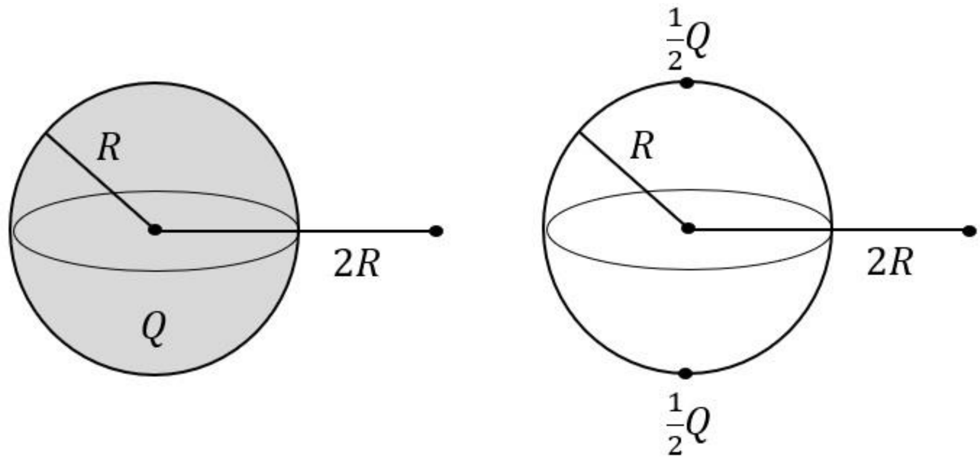
(a) 1.2×10^{14} Hz

(b) 0.4×10^{14} Hz

(c) 7.5×10^{14} Hz

(d) 3.6×10^{14} Hz

B.7 Consider a solid sphere of radius R with a total charge Q distributed uniformly throughout its volume (see figure, left). The electric field measured at a distance $x = 2R$ from the centre of the sphere along the equatorial plane is found to be E_1.



Next, the same charge is distributed differently, such that Q/2 is concentrated at the north pole, and the remaining Q/2 is concentrated at the south pole (see figure, right). The electric field is measured again at the same point on the equatorial plane and found to be E_2.

The value of E_2/E_1 is

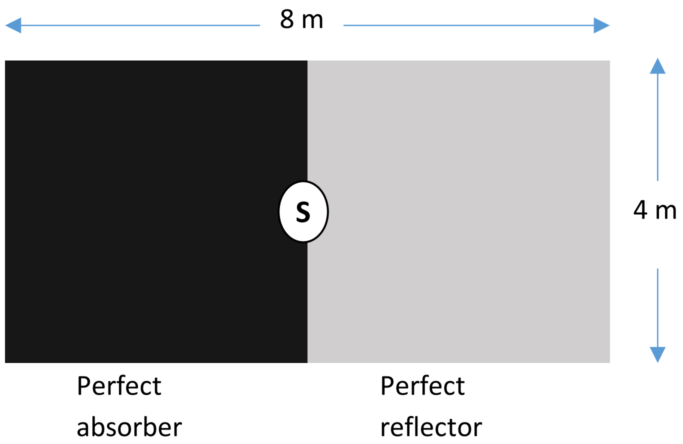
(a) $\frac{8}{5\sqrt{5}}$

(b) 1

(c) $\frac{2}{\sqrt{5}}$

(d) $\frac{4}{5}$

B.8 A small satellite S has a rectangular solar sail of dimensions 8 m × 4 m, which propels the satellite upon receiving sunlight. One half of the sail is a perfect reflector, while the other half is a perfect absorber, as shown in the figure.



Assuming uniform sunlight incident normally on the sail with an intensity $1370\ \mathrm{W\,m^{-2}}$ and ignoring the satellite's shadowing effects, the instantaneous torque experienced by the satellite is

(a) 1.46×10^{-4} N-m

(b) 2.92×10^{-4} N-m

(c) 0.73×10^{-4} N-m

(d) 2.19×10^{-4} N-m

B.9 The equilibrium temperature (T_0) on the surface of a planet results from the balance between the energy received from their host star and the energy they emit back into space. In the case of the Earth, $T_0 = 300$ K and the orbit is almost circular. We may safely assume that planets absorb and emit radiation like perfect blackbodies

Now consider an exoplanet of the same size as the Earth, which orbits a fainter star having a power output only 25% of that of the Sun, in an almost-circular orbit of radius 25% that of the Earth-Sun distance.

The equilibrium temperature T_0' on the surface of this exoplanet will be about

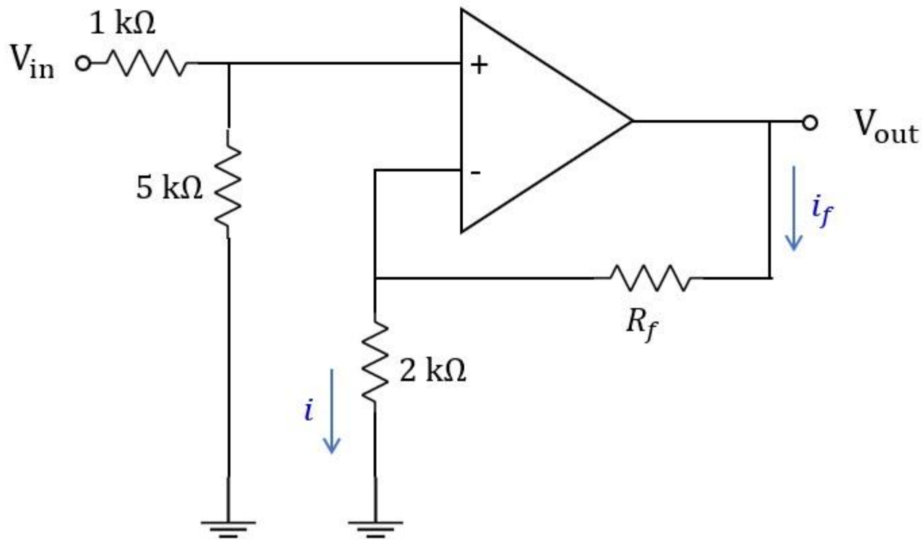
(a) 424 K

(b) 212 K

(c) 300 K

(d) 600 K

B.10 At what value of R_f will the ideal op-amp shown in the figure provide a gain of 6 ?



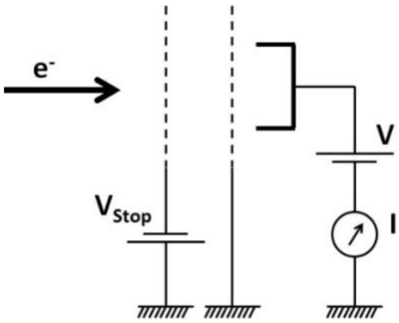
(a) 12.4 kΩ

(b) 19.5 kΩ

(c) 22.5 kΩ

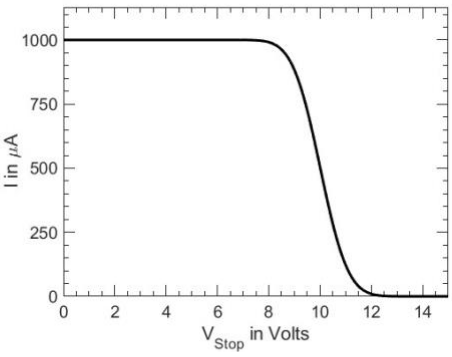
(d) 14.4 kΩ

B.11 A well-collimated constant-current electron beam of Gaussian energy distribution centred at 10 eV with FWHM of 2 eV is detected using a metal cup connected to an ammeter, as shown in the figure below. The entire apparatus is kept in vacuum.

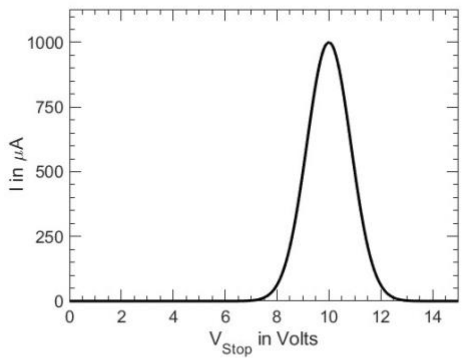


To measure the energy width of the electron beam, a grid is introduced with a voltage source V_{stop} connected to it, as shown in the figure. The current measured in the cup is plotted as a function of the value of V_{stop}. The graph of the current I vs V_{stop} would be

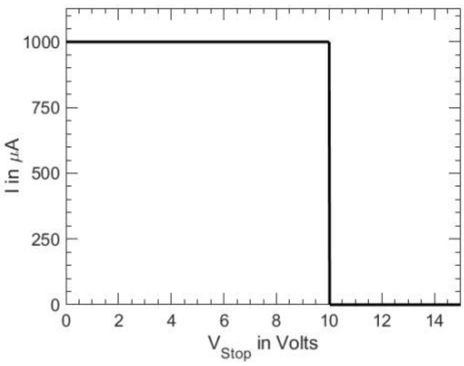
(a)



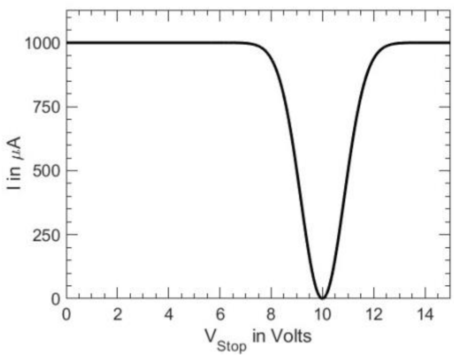
(b)



(c)



(d)



B.12 A diffraction grating spectrograph is used to resolve the two sodium D lines (589 and 589.6 nm) in the first order of diffraction. If the width of the grating is 2 cm and the focal length of the spectrograph camera is 20 cm, what the linear separation at the focal plane of the two D lines will be about

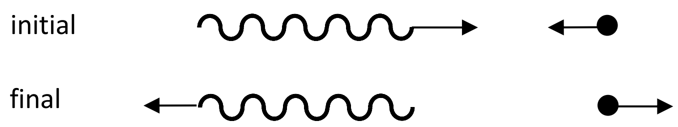
(a) 6 μm

(b) 6 mm

(c) 60 μm

(d) 60 nm

B.13 A photon of frequency ν_i collides "head on" with an electron of mass m having speed u_i and the photon scatters off in a direction exactly opposite to its initial momentum (see figure).



It is found that the frequency of the scattered photon is the same as that of the incident photon. Which of the following conditions must hold for this to happen?

(a) The magnitude of the initial momentum of the electron is $p_i^e = h\nu_i/c$

(b) The initial energy of the electron is $E_i^e = h\nu_i$

(c) The magnitude of the initial momentum of the electron is $p_i^e = 2h\nu_i/c$

(d) The initial energy of the electron is $E_i^e = 2h\nu_i$

B.14 The number of hyperfine states found in the ^{3}He atom for the electronic configuration

$$1s^1 2s^0 2p^1$$

would be

(a) 7

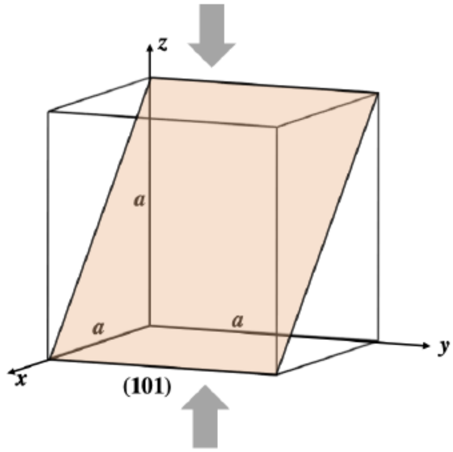
(b) 2

(c) 4

(d) 1

B.15 An X-ray of wavelength λ, when incident on the (101) plane of a cubic lattice with lattice constant a produces a first-order Bragg's reflection at $\theta = 30°$ (θ is measured from the lattice plane).

Suppose this cubic lattice is compressed along the z axis such that its lattice parameters along the x and y axes remain the same while that along the z axis becomes $\frac{1}{\sqrt{3}}a$ (see figure).



The first-order reflection for the (101) plane of the compressed lattice occurs at:

(a) $\theta = 45°$

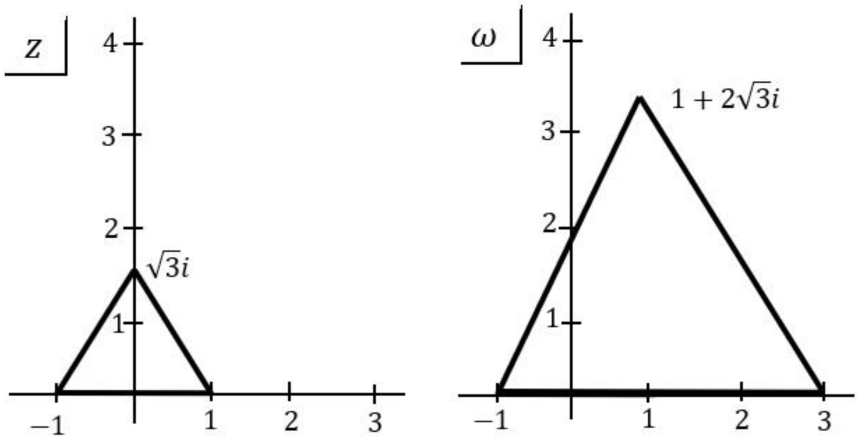
(b) $\theta = 15°$

(c) $\theta = 30°$

(d) $\theta = 60°$

Section C

C.1 A complex analytic function $\omega = f(z)$ transforms an equilateral triangle in the complex z-plane to another equilateral triangle in the complex ω-plane as shown in the figure.



Which of the options below **CANNOT** be $f(z)$?

(a) $f(z) = e^{5\pi i/6}\, z + 2i\sqrt{3}$

(b) $f(z) = 2e^{2\pi i/3}\, z + 2 + i\sqrt{3}$

(c) $f(z) = 2ie^{5\pi i/6}\, z + i\sqrt{3}$

(d) $f(z) = 2z + 1$

C.2 Calculate the integral

$$\int_0^\infty \frac{dx}{\sqrt{x}(x^2+1)}$$

(a) $\frac{\pi}{\sqrt{2}}$

(b) $\pi\sqrt{2}$

(c) 2π

(d) $\frac{\pi}{2}$

C.3 Consider the Hamiltonian for a one-dimensional classical oscillator

$$H = \frac{1}{2m}(p^2 + m^2\omega^2 q^2)$$

A canonical transformation to variables (P, Q) via the generating function

$$F = \frac{m\omega q^2}{2}\cot Q$$

leads to which of the following Hamiltonians in the new coordinates?

(a) $H = \omega P$

(b) $H = P^2 + \omega^2 Q^2$

(c) $H = 2\omega P$

(d) $H = 2\omega Q$

C.4 A simple pendulum is oscillating freely in the vertical plane. If the string is shortened very slowly to half its length, the angular amplitude $\theta_{\max}$ will change by a factor

(a) $2^{3/4}$

(b) $\sqrt{2}$

(c) 2

(d) Does not change.

C.5 Consider a particle of mass m in a quartic potential

$$H = \frac{p^2}{2m} + ax^4$$

If we take a variational wavefunction

$$\psi(x, \lambda) = e^{-\lambda x^2}$$

with $\lambda > 0$ and try to estimate the ground state energy, the value of λ should be chosen as

[You may use the integral

$$\int_{-\infty}^{+\infty} dx\ (A + Bx^2 + Cx^4)e^{-\lambda x^2} = A\sqrt{\frac{\pi}{\lambda}} + \frac{B}{2}\sqrt{\frac{\pi}{\lambda^3}} + \frac{3C}{4}\sqrt{\frac{\pi}{\lambda^5}}$$

where A, B, C and $\lambda > 0$ are all constants.]

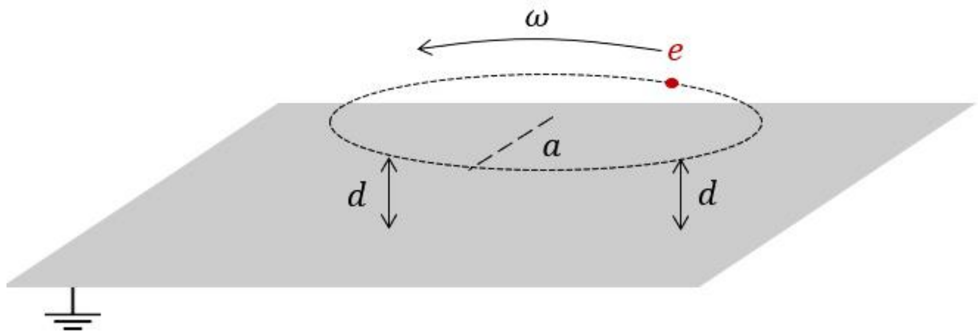
(a) $\left(\frac{3ma}{4\hbar^2}\right)^{1/3}$

(b) $\left(\frac{5ma}{3\pi^2\hbar^2}\right)^{1/3}$

(c) $\left(\frac{15ma}{8\hbar^2}\right)^{1/3}$

(d) $\left(\frac{ma}{2\pi\hbar^2}\right)^{1/3}$

C.6 A charge e is moving with an angular frequency ω along a circle of radius a always keeping a small distance d $(d \ll a)$ from a grounded infinite conducting plane.



The leading dependence of the radiated power $P(\omega)$ at a distance r $(r \gg a)$ will be

(a) $P(\omega) \propto \omega^4$

(b) $P(\omega) \propto \omega^6$

(c) $P(\omega) \propto \omega^3$

(d) $P(\omega) \propto \omega^2$

C.7 For an electromagnetic wave propagating through a rectangular waveguide, the electric and magnetic fields

(a) are never perpendicular to each other

(b) are always perpendicular to each other

(c) are perpendicular to each other only in the TE mode

(d) are perpendicular to each other only in the TM mode

C.8 Consider a fermionic system with a Hamiltonian

$$\hat{H} = \begin{bmatrix} 0 & E_0 & 0 \\ E_0 & 0 & 2E_0 \\ 0 & 2E_0 & 0 \end{bmatrix}$$

Consider the grand canonical ensemble of this system at temperature T and zero chemical potential, where k_B is the Boltzmann constant. The grand canonical partition function of the system is given by

(a) $4 + \cosh\left(\sqrt{5}\frac{E_0}{k_B T}\right)$

(b) $\cosh\left(\sqrt{5}\frac{E_0}{k_B T}\right)$

(c) $\dfrac{1}{4 + \cosh\left(\sqrt{5}\frac{E_0}{k_B T}\right)}$

(d) $\text{sech}\left(\sqrt{5}\frac{E_0}{k_B T}\right)$

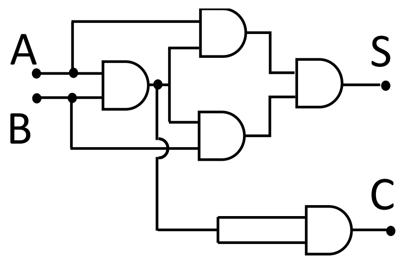
C.9 A half-adder circuit is defined as a two-input, two-output logic circuit where the output S gives the sum of inputs up to a single bit, and the output C gives carryover in a single bit.

The expected truth table of the half-adder is given as

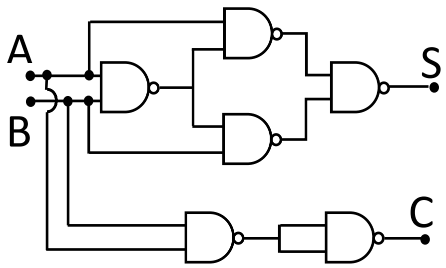
Input		Output	
A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Which of the following circuits **does NOT** behave like a half adder?

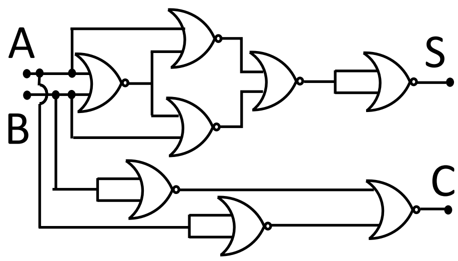
(a)



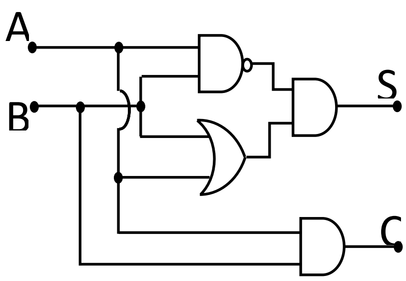
(c)



(b)



(d)



C.10 In an experiment that measures the angular distribution of the emission of particles, the angular distribution function is defined as

$$f(\theta) = \frac{n(\theta)}{n(\pi/2)}$$

where $n(\theta)$ is the number of particles detected at an angle θ.

If, for a certain sample, we count

$$n(\pi/4) = 16265 \qquad n(\pi/2) = 8192$$

then the uncertainty

$$\left.\frac{\Delta f}{f}\right|_{\theta=\pi/4}$$

in this measurement at $\theta = \pi/4$ would be approximately

(a) 1.350 %

(b) 0.013 %

(c) 0.707 %

(d) 0.018 %

C.11 The energy gap between the $n = 1$ and the $n = 2$ energy levels of a hydrogen atom is denoted E_0.

Now, consider a muonic carbon ion C^{5+}, i.e., a carbon nucleus ($^{12}_{6}C$) orbited by a muon μ ($q = -e, M_\mu = 210\ m_e$). The energy of the photon emitted in the transition of the muon from the $n = 3$ level to the $n = 2$ level of this ion will be approximately

(a) $1400\ E_0$

(b) $235\ E_0$

(c) $1050\ E_0$

(d) $7560\ E_0$

C.12 A lattice in the three-dimensional space has N sites, each occupied by an atom whose magnetic moment is μ. The lattice is in contact with a heat reservoir at a fixed temperature T. The atoms interact with an applied magnetic field

$$\vec{H} = H(\vec{x})\,\hat{\boldsymbol{z}}$$

Ignoring the interactions between the atoms, the average magnetic susceptibility per lattice site is given by

(a) $\dfrac{\mu^2}{3k_BT}$

(b) $\dfrac{\mu^2}{9k_BT}$

(c) $\dfrac{\mu}{3k_BT}$

(d) $\dfrac{\mu H}{3k_BT}$

C.13 The angular position of a star is found to change by an amount of 0.2 arc seconds (relative to the very distant background stars) when measured by a telescope on the Earth on two different nights separated by exactly six months. Note that the distance between the Earth and Sun is known to be approximately 1.5×10^{13} cm.

If the energy flux received from the star is $F = 10^{-7}$ erg s^{-1} cm^{-2}, what is the approximate value of its luminosity?

(a) 10^{33} erg s^{-1}

(b) 10^{31} erg s^{-1}

(c) 10^{35} erg s^{-1}

(d) 10^{29} erg s^{-1}

C.14 The binding energy $\mathcal{E}_b$ of a nuclide ${}^{Z}_{A}\mathrm{X}$ with atomic number Z and mass number A is given by the semi-empirical formula

$$\mathcal{E}_b = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} + a_A \frac{(A-2Z)^2}{A}$$

where the constant parameters and source of effect for each term are

Volume term	Surface term	Coulomb term	Asymmetry term
a_V	a_S	a_C	a_A
15.56 MeV	17.8 MeV	0.7 MeV	23.29 MeV

For a spherical neutron star consisting of only neutrons and having uniform nuclear density throughout its volume, the Coulomb term is replaced by gravitational energy. What would be the smallest radius of this neutron star?

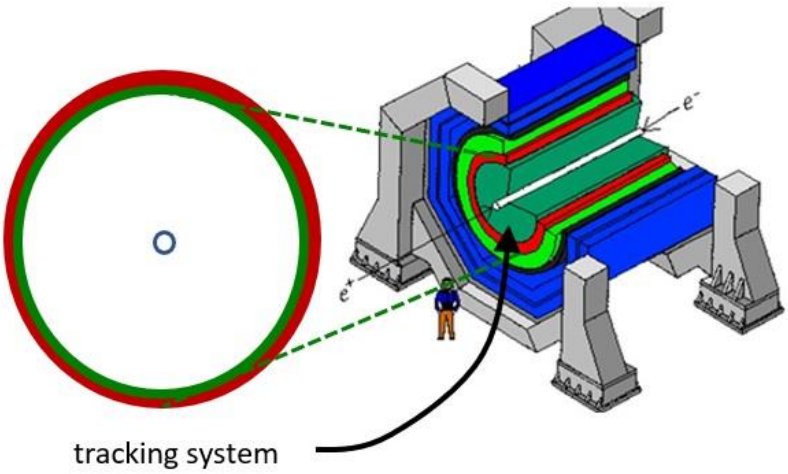
(a) 4.345 km

(b) 10.435 km

(c) 2.165 km

(d) 4.345 m

C.15 The figure below shows on the right a sketch of an electron-positron collider experiment where the innermost detector (shaded dark green) is a tracking system which records the tracks of charged particles which pass through it. On the left of the figure, a cross-sectional view of the same tracking system is shown. The narrow (white) pipe in the centre is where electrons and positrons are injected as shown and collide in the centre. (On the left it appears as a small central circle.) Inside the tracking system there is a strong uniform magnetic field collinear with the e^+ direction.

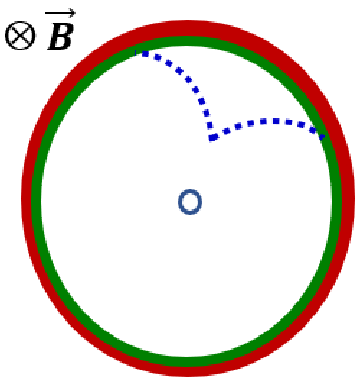


In one of the e^+e^-collisions, a high-energy K_S^0 meson is produced that subsequently decays as follows

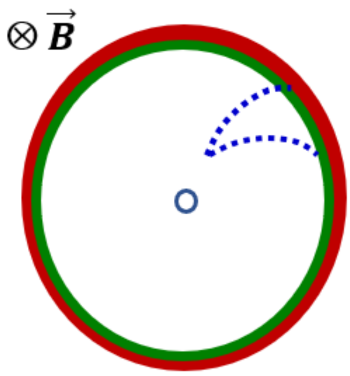
$$K_S^0 \rightarrow \pi^+ + \pi^-$$

A possible representation of the tracks (dotted lines) of the pions $\pi^\pm$ in the tracking system would be

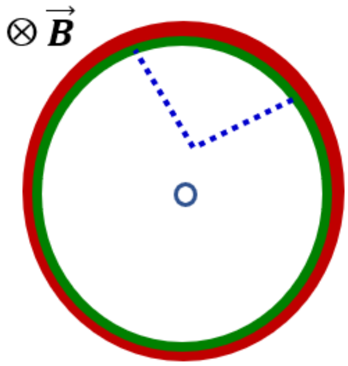
(a)



(b)



(c)



(d)