Question Bank

Council of Higher Secondary Education, Odisha

Mathematics

One Mark Questions

1. The number of relations from the Set A to the Set B with $|A| = 3 \; and \; |B| = 4$ is _______.
2. The number of relations on set A with $|A| = n$ is _______.
3. Is ϕ a relation from A to B?
4. Is the relation R = {(1, 1), (2, 2)} on {1, 2, 3} symmetric?
5. Give an example of a relation R on any set A such that $R = R^{-1}$.
6. Is the relation R = $\{(m,n): 2|m+n\}$ on set of integers transitive?
7. Is the relation R = $\{(x, y): |x-y|^2\}$ on Z a function?
8. The number of non-empty relations on A with $|A| = n$ is _______.
9. The largest equivalence relation on any set A is ___________.
10. If R is relation on A such that $R = R^{-1}$, then R is a __________ relation.
11. If R = $\{(x, x^3)$: x is a prime number less than 5} be a relation then the range of R is ________.
12. If $f(x) = (3 - x^3)^{1/3}$ then $fof(x)$ = ____________.
13. Let $f: R_+ \to A$ defined by $f(x) = 9x^2 + 6x - 5$. Find 'A' if f is Bijective.
14. Define even and odd functions.
15. Is $g(x) = e^x + e^{-x}$ even?
16. Is the function $f(x) = 2x^2 + 5$ on R invertible?
17. $f(x) = x^3 + 1$, $g(x) = \sqrt{\sin x}$ find fog = ?
18. Is $fog = gof$. ?
19. $f(x) = \sqrt{9 - x^2}$ find range of f.
20. $f(x) = \log|x| + \log x$ find dom$f$.
21. Find domain of fog and gof where $f(x) = \sqrt{x}$ and $g(x) = 1 - x^2$.
22. If $|A| = 5 \; and \; |B| = 3$, then number of one-one function from A to B is ______.
23. If $|A| = 3$, $|B| = 2$ then how many relations are not functions from A to B.
24. If $f(x) = \dfrac{x-1}{x+1} \; then \; f(2x)$ is

    (a) $\dfrac{f(x)+1}{f(x)+3}$

    (b) $\dfrac{3f(x)+1}{f(x)+3}$

    (c) $\dfrac{f(x)+3}{f(x)+1}$

    (d) $\dfrac{f(x)+3}{3f(x)+1}$

25. The range of the function $f(x)=\dfrac{x^2-x+1}{x^2+x+1},\ x\in R$ is ________.

26. The domain of $f(x)=e^{\sqrt{5x-3-2x^2}}$ is ______.

27. Which of the following relation is not a function

(a) $f=\{(x,x):x\in R\}$

(b) $g=\{(x,3):x\in R\}$

(c) $h=\left\{\left(n,\dfrac{1}{n}\right):n\in Z\right\}$

(d) $t=\{(n,n^2):n\in N\}$

28. The Domain of $f(x)=\sqrt{x^2-5x+6}+\sqrt{2x+8-x^2}$ is _________.

29. Write the Domain of $f(x) = \sin^{-1}x + \cos x$.

30. If $f(x) = \sin x + 2$ in $\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]$ then what is the greatest value of $f(x)$.

31. If $\sin^{-1}x=\operatorname{cosec}^{-1}\left(\dfrac{1}{x}\right)$ then x = _________.

32. If x + y = 4 and xy = 1 then $\tan^{-1}x + \tan^{-1}y$ = ________.

33. Find the principle value of $\tan^{-1}\left(\tan\dfrac{3\pi}{4}\right)$.

34. Find 'x' if $\sin^{-1}x+\cos^{-1}\dfrac{1}{2}=\dfrac{\pi}{2}$.

35. $\cot^{-1}(-\sqrt{3})$ is in the second Quadrant write true/flase.

36. If $A = (a_{ij})_{2\times 3}$ such that $a_{ij} = i - j$ find A = ?

37. If $A=\begin{pmatrix}1&1\\1&1\end{pmatrix}$ then A^{100} = _______.

38. If det A= 3 and A is a determinate of order 3 then find the value of $a_{11}c_{11} + a_{12}c_{22} + a_{13}c_{23}$ = ________.Where Cij is the co factor of the element in the ith row & jth Col.

39. Define Singular matrix .

40. If A is square matrix of order 2 and $|A|=5$, then the matrix A. (adj A) = ________.

41. Find x and y when $\begin{pmatrix}1&3\\2&-1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}4\\1\end{pmatrix}$.

42. Given that $\begin{pmatrix}1&w&w^2\\w&w^2&1\\w^2&1&w\end{pmatrix}\begin{pmatrix}k&1&1\\1&1&1\\1&1&1\end{pmatrix}=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}$ find k = ________.

43. If A = B + C where $A=\begin{pmatrix}1&4\\2&3\end{pmatrix}$ find B and C.

44. If A and B are matrix of same order, then $AB^1 - BA^1$ is a ________ matrix.

(a) Skew symmetric

(b) Null matrix

(c) Symmetric matrix

(d) Unit matrix

45. If A is a square matrix of order 3. Write the value of n, when $|3A|=n|A|$.

46. If A is a 2 × 3 matrix and B is a matrix such that A^TB and BA^T both defined then find the order of B.

47. Find 'x' if (x – 1)$\begin{pmatrix} 1 & 0 \\ -2 & 0 \end{pmatrix}$ = (0).

48. If (4 5 6)A = (0) find the order of A.

49. Transform the matrix $\begin{pmatrix} 6 & 3 \\ 2 & 2 \end{pmatrix}$ into unit matrix.

50. Find the value of 'λ' for which the system of equations x – y + z = 0, x + λy – z = 0 and –x + y + z = 0 has infinitely many solutions.

51. The value of the determinat $\begin{vmatrix} \cos^2 54 & \cos^2 36 & \cot 135 \\ \sin^2 53 & \cot 135 & \sin^2 37 \\ \cot 135 & \cos^2 25 & \cos^2 65 \end{vmatrix} =$ ________.

52. If $\begin{vmatrix} a & 5x & p \\ b & 10y & 5 \\ c & 15z & 15 \end{vmatrix} = 125$, then the value of $\begin{vmatrix} 3a & 3b & c \\ x & 2y & z \\ p & 5 & 5 \end{vmatrix} =$ ______.

53. If a system of equation –ax + y + z = 0, x – by + z = 0 and x + y – cz = 0, (a, b, c ≠ -1) has a non zero solution then $\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c} =$ _______________.

54. If the first and third rows of a determinate are inter changed then the value of the det is __________. (Changed, remain same)

55. The region satisfying the set of constrants and the non-negative restriction called _______.

56. Every point of the feasible region is called _________.

57. The maximum value of z = 3x + 4y s.t. x + y ≤ 4, x, y ≥ 0 is ________.

58. The minimum value of z = 3x + 5y s.t. x + 3y ≥ 3, x + y ≥ 2, x, y ≥ 0 is _________.

59. The optimal value of the objective function is attained at _________, in feasible region.

60. When a LPP has infinitely many solutions?

61. Find the unit vector along $\sqrt{2}i + j - \hat{k}$.

62. If $|\vec{a}| = 7, |\vec{b}| = 1 \, and \, |\vec{a}+\vec{b}| = 10\sqrt{3} \, then \, |\vec{a}-\vec{b}| =$ _________.

63. If the position vector of three points are $\vec{a} - 2\vec{b} + 3\vec{c}, 2\vec{a} + 3\vec{b} - 4\vec{c}, -7\vec{b} + 10\vec{c}$, then the three points are _____.
(a) collinear
(b) non-collinear
(c) coplanner
(d) none

64. If $|\vec{a}| = |\vec{b}| = |\vec{a}-\vec{b}| = 1$, then write the measure of the angle between $\vec{a} \, and \, \vec{b}$.

65. If $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k} \, and \, \vec{b} = \alpha\hat{i} - \hat{j} + 2\hat{k}$ are parallel then α = _______.

66. The vector projection of the vector $2\hat{i} - 3\hat{j} - 6\hat{k} \, on \, 2\hat{i} + 2\hat{j} - \hat{k}$ is _________.

67. If $\vec{a} = 2\hat{i} - \hat{j} - \hat{k} \, and \, \vec{b} = \lambda\hat{i} + \hat{j} + 5\hat{k}$ are perpendicular then λ = __________.

68. Let $\vec{a} \, and \, \vec{b}$ related by $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$ then find the angle between $\vec{a} \, and \, \vec{b}$.

69. A vector $\perp_r$ to the vectors $\hat{i} + \hat{j} \, and \, \hat{i} + \hat{k}$ is ________.

70. If $\vec{a}, \vec{b}, \vec{c}$ are non-zero vectors and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ then
(i) $\vec{b} = \vec{c}$
(ii) $\vec{a} \parallel (\vec{b} - \vec{c})$
(iii) $\vec{b} \parallel \vec{c}$

(iv) $\vec{b} \perp_r \vec{c}$

71. Find the unit vector $\perp_r$ to the vectors $2\hat{i}+3\hat{k}, \hat{i}-2\hat{j}$.

72. The value of $(-\vec{a}).\vec{b}\times(-\vec{c}) =$ ________.

(i) $\vec{a}\times\vec{b}.\vec{c}$

(ii) $-\vec{a}.(\vec{b}\times\vec{c})$

(iii) $\vec{a}\times\vec{c}.\vec{b}$

(iv) $\vec{a}.(\vec{c}\times\vec{b})$

73. Find the value of λ, so that the vectors $\hat{i}-2\hat{j}+2\hat{k}, \lambda\hat{i}+4\hat{j}+5\hat{k}, -2\hat{i}+4\hat{j}-4\hat{k}$ are co-planer.

74. If $[\vec{a}\ \vec{b}\ \vec{c}] = 5$, the value of $[\vec{a}+\vec{b}\ \vec{b}+\vec{c}\ \vec{c}+\vec{a}] =$ __________.

75. If $(\vec{a}+\vec{b})(\vec{a}-\vec{b}) = 12$ *and* $|\vec{a}| = 2|\vec{b}|$ *then* $|\vec{a}| =$ ____________.

76. If $\vec{a} = 2\vec{b}$ *and* $\vec{c} = -3\vec{b}$ then angle between $\vec{a}$ *and* $\vec{c}$ is __________.

77. If θ is the angle between $\vec{a}$ *and* $\vec{b}$ *and* $|\vec{a}\times\vec{b}| = |\vec{a}.\vec{b}|$ then find θ.

78. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c} = 0$ then what is the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is ________.

79. If $|\alpha\vec{a}| = 2$, then the value of α = ________.

80. If $[\vec{a}\ \vec{b}\ \vec{c}] = 3$ then the value of $[\vec{a}\times\vec{b}\ \vec{b}\times\vec{c}\ \vec{c}\times\vec{a}] =$ ______.

81. The projection of (-1, 2, 3) on yz-plane is ________.

82. The projection of (1, -2, 3) on X-axis is __________.

83. A line is perpendicular to yz-plane. Write the angle between the line with x-axis.

84. The Direction ratios of a line is unique. Write true or false.

85. The ratio in which the line joining the points (2, 3, 4) and (-3, 5 -4) is divided by xy-plane.

86. The projection of the line joining the points (2, 1, 3) and (3, 2, 4) on z-axis is _______.

87. The distance between the planes x + 2y + 3z + 1 = 0 and 2x + 4y + 6z + 5 = 0 is ________.

88. The components of the unit vector perpendicular to the plane $\vec{r}.(2\hat{i}+3\hat{j}-6\hat{k}) - 6 = 0$.

89. The equation of the plane parallel to z-axis and with intercepts 3 and 4 on x and y –axes respectively is ________.

90. Are the two points (1, 2, -1) and (2, -1, 3) lie on the same side of the plane x + 3y + z -1 = 0?

91. Write the equation of the plane 3x – 2y + z -1 = 0 in normal form.

92. If the lines $\frac{x+3}{2} = \frac{y+5}{3} = \frac{7-z}{3}$ *and* $\frac{x+1}{\lambda} = \frac{y+1}{5} = \frac{z-1}{1}$ are perpendicular, find λ = ____?

93. Write the axis to which the plane ax + by + c = 0 is parallel.

94. Find the value of k such that the line $\frac{x-4}{1} = \frac{y-z}{1} = \frac{z-k}{2}$ lies in the plane 2x – 4y + z = 7.

95. The Direction ratios of two parallel lines are equal. (write true or false)

96. Find the dis of a line which is $\perp_r$ to the lines whose direction ratios are $\langle 1,-2,3\rangle$ *and* $\langle 2,2,1\rangle$.

97. If a line is perpendicular to x-axis and makes an angle 30^0 with y-axis, then the angle it makes with z-axis is ________.

98. The line $\frac{x+1}{2}=\frac{y-6}{0}=\frac{z-4}{1}$ is ______.(parallel x-axis, perpendicular to y-axis, perpendicular to z-axis, none of these)

99. If the line $\frac{x-k}{2}=\frac{y-2}{-1}=\frac{z+1}{-5}$ lies on the plane 2x – y + z -7 = 0 then the value of k is ____.

100. The symmetric form of the line 2x + 3y + 5 = 0, 6z = 0 is ______.

101. Find the angle between the lines whose drs are proportional to a, b, c and b – c, c – a, a – b.

102. The equation of the line in vector form whose Cartesian form $\frac{x+1}{2}=\frac{x-1}{-1}=\frac{x+3}{5}$ is _____.

103. The point of intersection of the line $\frac{x-1}{2}=\frac{y-3}{1}=\frac{z+2}{3}$ with the plane x – 2y + 4z = 11 is _________.

104. If $2P(A)=P(B)=\frac{5}{13}$ *and* $P(A\setminus B)=\frac{2}{5}$ *then* $P(A\cup B)$ is ________.

105. One card is drawn from a pack of 52 cards. Write the probability that the card drawn is either a king or spade.

106. A binomial distribution has mean 4 and variance 3, then the number of trials is _______.

107. If P(A) = P(B) = x and $P(A\cap B)=P(A'\cap B')=\frac{1}{3}$ then x = _________.

108. $y=2^{2^x}$ *then* $\frac{dy}{dx}=$

(i) $y\left(\log_{10}^{2}\right)^2$

(ii) $y\left(\log_{e}^{2}\right)^2$

(iii) $y\,2^x\left(\log_{x}^{2}\right)^2$

(iv) $y\log_{e}^{2}$

109. If $f(x)=\log_e\left(\log_e^x\right)$ *then* $f^1(e)=$______.

110. If $f(x)=x.\tan^{-1}x$ *then* $\lim_{x\to1}\frac{f(x)-f(1)}{x-1}=$__________.

111. $x=\frac{1-\sqrt{y}}{1+\sqrt{y}}\Rightarrow\frac{dy}{dx}=$___________.

112. If $xy=(x+y)^n$ *and* $\frac{dy}{dx}=\frac{y}{x}$ then n = _______.

113. If g is the inverse of a function *f* and $f'(x)=\frac{1}{1+x^2}$ then g'(x) is equal to

114. If *f* is an even function and $f^1(x)$ exists then $f^1(0)$ = _________.

115. y = sin (7 $\sin^{-1}$x) then $(1-x^2)y_2 - xy_1$ = _________.

116. Is the function *f*(x) = x – [x] continuous at x = 2?

117. If $f(x)=\begin{cases}\frac{\sin 3x}{x} & , x\neq 0\\ \frac{k}{2} & , x=0\end{cases}$ is continuous at x = o then k = _______.

118. If $f(x)=\begin{cases}\dfrac{1-\sqrt{2}\sin x}{\pi-4x} & , x\neq\dfrac{\pi}{4}\\ a & , x=\dfrac{\pi}{4}\end{cases}$ is continuous at $x=\dfrac{\pi}{4}$ then a = ______.

119. If f:R → R is defined by $f(x)=\begin{cases}\dfrac{1+3x^2-\cos 2x}{x^2} & , x\neq 0\\ k & , x=0\end{cases}$ is continuous at x = 0 then k = _________.

120. If f:R → R is continuous such that f(x + y) = f(x) + f(y) $\forall\ x, y\in R$ and f(1) = 2 then f(100) = ______.

121. If f(x) is increasing at x = 0 then $f^1(0) > 0$ write true or false.

122. The point on the curve y = x^2 where the tangent parallel to the chord joining (0, 0) and (1, 1) is ________.

123. The interval where the function f(x) = $x^3(x-2)^2$ is decreasing is __________.

124. Find the interval where $f(x)=x+\dfrac{1}{x}$ is increasing.

125. The interval where $f(x)=\dfrac{x}{\ln x}$, $x>0$ is decreasing is ______.

126. If f(x) = $2x^2 + 3x - 5$, x = 2, $\delta x = 0.01$ *then* δf = __________.

127. If the lines y = -4x + b are tangents to the curve $y=\dfrac{1}{x}$ then b = ________.

128. The distance between the origin and the normal to the curve y = $e^{2x} + x^2$ at x = 0 is _______.

129. The angle between the curves y = x^2, x = y^2 at (1, 1) is ________.

130. If the curves $y^2 = 4ax$, $xy = c^2$ cut orthogonally then the relation between a and c is ______.

131. The constant 'C' of lagrange's mean value theorem for f(x) = 2 sin x + sin 2x in [0, π] is _____.

132. The condition that f(x) = $ax^3 + bx^2 + cx + d$ has no extreme values is _____.

133. $f(x)=(x-\alpha)(x-\beta)$ then the minimum value of f(x) = ________.

134. If $l^2 + m^2 = 1$ then the minimum value of l + m is _________.

135. $\int |x|dx =$ ________.

136. $\int e^{\log(1+\cot^2 x)}dx =$ ________.

137. $\int \dfrac{\sqrt{\cot x}}{\sin x.\cos x}dx = f(x)+C$ *then* $f(x) =$ __________.

138. $\int \dfrac{1}{\sqrt{x}+x}dx =$ _________.

139. $\int e^x \operatorname{cosec}(1-\cot x)dx =$ _________.

140. $\int \dfrac{dx}{(x^2+1)2} = a\tan^{-1}x + b.\dfrac{x}{x^2+1} + C$ then a and b = ______.

141. $\int_0^b (a^{-x}-b^{-x})dx =$ _______.

142. $\int_{-\pi}^{\pi} x^6 \sin x dx =$ ________.

143. $\int_{-2}^{2} |[x]|dx =$ _____________.

144. $\int_0^{\pi} \frac{1}{2}\left(\cos x + |\cos x|\right)dx =$ ____________.

145. The area cut off by the parabola $y^2 = 4ax$ and its latus rectum is ______.

146. The area bomded by the curve y = x and $y = x^3$ is _______.

147. Find the value of C for which the area bunded by the curve $y = 8x^2 - x^5$, the lines x = 1, x = c and x – axis is $\frac{16}{3}$ is ________.

148. The area bunded by y = sin x, y = cos x and y – axis is _________.

149. Area between the curves $y = 1 - |x|$ and x – axis is __________.

150. The degree of $\left[5 + \left(\frac{dy}{dx}\right)^2\right]^{5/3} = 7.\frac{d^2y}{dx^2}$ is _________.

151. The solution of $\frac{dy}{dx} + \frac{y}{3} = 1$ is ___________.

152. The order of the differential equation of the family of all concentric circles centered at (h, k) is ________.

153. The solution of $\frac{dy}{dx} + 1 = e^{x+y}$ is ___________.

154. Integrating factor of $\left(x + 2y^3\right)\frac{dy}{dx} = y^2$ is _________.

155. The solution of $(x + y + 1)\frac{dy}{dx} = 1$ is _________.

156. If A and B are two independent events such that $P(A^1 \cap B) = \frac{2}{15}$ *and* $P(A \cap B^1) = \frac{1}{6}$ then P(B) = _________.

157. Three persons A, B, C in order toss a die the person who first throws 1 or 2 wins. The ratio of probabilities of their success is ______.

158. The random variable X has the following distribution

X	1	2	3	4
P(X = x)	C	2C	3C	4C

Then the value of C = __________.

159. For a Binomial distribution with mean 6 and variance 2, then first term is ________.

160. If A and B are two independent events such that

$P(B) = \frac{2}{7}$ *and* $P(A \cup B^C) = 0.8$ *then* $P(A) =$ __________.

ANSWERS

1. 2^{12}
2. 2^{n^2}
3. Yes
4. Yes
5. R = {(x, y) : x = y} on {1, 2, 3}
6. No
7. No
8. 2^{n^2} - 1
9. A × A
10. equivalence
11. {8, 9}

12. $fof\,(x) = x$
13. $[-5,\infty)$
14. $f(-x) = f(x)$ – even & $f(-x) = -f(x)$ - odd
15. Yes
16. No
17. $(\sin x)^{3/2} + 1$
18. No
19. $[0,\infty)$
20. $(0,\infty)$
21. [-1, 1] = Domfog, dom gof = R
22. No one – one function
23. $2^6 - 8$
24. B
25. $\left[\frac{1}{3},3\right]$
26. $(-\infty,1)\cup\left(\frac{3}{2},\infty\right)$
27. (c)
28. $[-2,2]\cup[3,4]$
29. $[-1,1]$
30. 3
31. $|x| \geq 1$
32. $\frac{\pi}{2}$
33. $\frac{\pi}{4}$
34. $x = \frac{1}{2}$
35. True
36. $\begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & 1 \\ & & \end{pmatrix}$
37. 99 A
38. 0
39. A is a square matrix and $|A| = 0$
40. $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$
41. x = y = 1
42. k = 1
43. $B = \begin{pmatrix} 1 & 3 \\ 3 & 3 \end{pmatrix} C = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
44. (a)
45. n = 9
46. Order of B = 2 × 3
47. x = 2
48. Order of A = 3 × 1
49. Use elementary row/ coloumb operation
50. λ = - 1

51. 0
52. 25
53. 1
54. changed
55. feasible region
56. feasible solution
57. 16
58. 9
59. Corner points
60. LPP has two optional solution
61. $\frac{1}{2}\left(\sqrt{2}i + j - \hat{k}\right)$
62. 10
63. (a)
64.
65. -2/3
66. $\frac{4}{9}\left(2\hat{i} + 2\hat{j} - \hat{k}\right)$
67. λ = 3
68. $\vec{a} \perp_r \vec{b}$
69. $\frac{i - j - k}{\sqrt{3}}$
70. (ii)
71. $\frac{6\hat{i} + 3\hat{j} - 4\hat{k}}{61}$
72. i
73. λ = 3
74. 10
75. 4
76. π
77. $\theta = \frac{\pi}{4}$
78. -3/2
79. $\alpha = \pm\frac{1}{|\vec{a}|}$
80. 9
81. (-1, 2, 0)
82. (-1, 2, 3)
83. 0
84. false
85. 1 : 1
86. 1
87. $\frac{3}{2\sqrt{14}}$
88. $\left(\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}\right)$
89. $\frac{x}{3} + \frac{y}{4} = 1$
90. Yes

91. $-\frac{3}{\sqrt{14}}x+\frac{2}{\sqrt{14}}y-\frac{1}{\sqrt{14}}z=\frac{1}{\sqrt{14}}$
92. $\lambda = -9$
93. z - mix
94. k = 7
95. false
96. $\left(-\frac{8}{5\sqrt{5}},\frac{1}{\sqrt{5}},\frac{6}{5\sqrt{5}}\right)$
97. 60^0
98. y = mis $\left(\perp_r\right)$
99. k = 3
100. $\frac{x}{3}=\frac{y+\frac{5}{3}}{-2}=\frac{z-6}{0}$
101. 90^0
102. $\left(\hat{i}+\hat{j}-3\hat{k}\right)+\lambda\left(2\hat{i}-\hat{j}+5\hat{k}\right)=0$
103. (5, 5, 5)
104. $\frac{11}{26}$
105. $\frac{4}{13}$
106. n = 16
107. x = 1/2
108. (iii)
109. 1/e
110. $\frac{\pi+2}{4}$
111. $\frac{4(x-1)}{(1+x)^3}$
112. 2
113. $1 + \{g(x)\}^5$
114. 1
115. -49 y
116. No
117. 6
118. 1/4
119. 5
120. 200
121. false
122. $\left(\frac{1}{2},\frac{1}{4}\right)$
123. $\left(\frac{6}{5},2\right)$
124. $[1,\infty)$
125. (0, e)
126. 0.1102
127. ± 4
128. $\frac{2}{\sqrt{5}}$

129. $\tan^{-1}\left(\frac{3}{4}\right)$
130. $c^4 = 32a^4$
131. $\frac{\pi}{3}$
132. $b^2 < 3ac$
133. $-\frac{1}{4}(\alpha - \beta)^2$
134. $\sqrt{2}$
135. $\frac{x.|x|}{2} + C$
136. - cot x + C
137. $2\sqrt{\cot x} + C$
138. $2\log\left(1+\sqrt{x}\right) + C$
139. e^x cosec x + C
140. a = b = ½
141. $\frac{1}{\ln a} - \frac{1}{\ln b}$
142. 0
143. 4
144. 2
145. $\frac{8a^2}{3}$
146. ½
147. - 1
148. $\sqrt{2} - 1$
149. 1
150. 3
151. $y = 3 + ce^{-\frac{x}{3}}$
152. 1
153. $e^{-(x+y)} + x + c = 0$
154. $e^{\frac{1}{y}}$
155. x = - (y + 2) + ce^y
156. $\frac{1}{6}\ or\ \frac{4}{5}$
157. 9 : 6 : 4
158. $\frac{1}{10}$
159. $\frac{1}{3^9}$
160. 0.3

4 Marks:

Relation and Function

1. $R = \{(m,n) \in N^2 / m+n \geq 50\}$ is a relation on the set of counting number N. Verify the relation for reflexive symmetric or transitive.
2. Test whether the relation $R = \{(m,2)|2|(m+n)\}$ on Z is reflexive, symmetric or transitive.
3. Let R be a relation on Z such that *a* – b is an integer test whether R is an equivalence relation.
4. Show that *f*: R → R defined by $f(x) = \dfrac{3x+5}{2}$ is invertible and find f^{-1}.
5. Show that *f*: N → N given by $f(n) = \begin{cases} n+1 & if\ \ n\ is\ odd \\ n-1 & if\ \ n\ is\ even \end{cases}$ is bijective.
6. If R and S are two equivalence relations on any set then prove that $R \cap S$ is also an equivalence relation.
7. Find number of relations and equivalence relations on the set A = {1, 2, 3}. Also find all the equivalence relations on 'A'.
8. Show that *f*: R → R defined by $f(x) = |x|$ is neither one – one nor on to.
9. Prove that *f*: R → R given by $f(x) = \dfrac{2x^2}{x^2+1}$ is neither one – one nor on to.
10. Show that the relation $f = \{(a,b)/ a \leq b^3, a, b \in\}$is neither reflexive nor symmetric nor transitive.

Inverse Trigonmetric Function

1. Evaluate: $\tan\left[\cos^{-1}\dfrac{4}{5} + \tan^{-1}\dfrac{2}{3}\right]$.
2. Solve: $3\tan^{-1}\dfrac{1}{2+\sqrt{3}} - \tan^{-1}\dfrac{1}{x} = \tan^{-1}\dfrac{1}{3}$.
3. Show that: $\tan\left(\dfrac{\pi}{4} + \dfrac{1}{2}\cos^{-1}\dfrac{a}{b}\right) + \tan\left(\dfrac{\pi}{4} - \dfrac{1}{2}\cos^{-1}\dfrac{a}{b}\right) = \dfrac{2b}{a}$.
4. Show that: $\tan^{-1}\dfrac{4}{5} + 2\tan^{-1}\dfrac{1}{3} = \dfrac{\pi}{2}$.
5. Show that: $\sin^{-1}\sqrt{\dfrac{x-q}{p-q}} = \cos^{-1}\sqrt{\dfrac{p-x}{p-q}} = \cot^{-1}\sqrt{\dfrac{p-x}{x-q}}$.
6. Solve: $\sin^{-1}\dfrac{2a}{1+a^2} + \sin^{-1}\dfrac{2b}{1+b^2} = 2\tan^{1} x$.
7. Solve: $\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\dfrac{3}{4}\right)$.
8. Show that: $\tan\left(2\tan^{-1}\dfrac{1}{5} - \dfrac{\pi}{4}\right) = -\dfrac{7}{17}$.
9. Prove that: $\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18 = \cot^{-1} 3$.
10. Solve: $\tan^{-1} x + 2\cot^{-1} x = \dfrac{2\pi}{3}$.
11. Solve: $\tan^{-1}\left(\dfrac{1-x}{1+x}\right) = \dfrac{1}{2}\tan^{-1} x$.

12. Show that: $\tan^2 \cos^{-1}\frac{1}{\sqrt{3}} + \cot^2 \sin^{-1}\frac{1}{\sqrt{5}} = 6.$

13. Solve: $\cos\left(2\sin^{-1} x\right) = \frac{1}{9}, x > 0.$

14. Prove: $\sin^{-1}\sqrt{\frac{x-q}{p-q}} = \cos^{-1}\sqrt{\frac{p-x}{p-q}} = \tan^{-1}\sqrt{\frac{x-q}{p-x}}.$

15. Show that: $\tan^{-1}\frac{a-b}{1+ab} + \tan^{-1}\frac{b-c}{1+bc} + \tan^{-1}\frac{c-a}{1+ca} = 0.$

16. Solve: $2\tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$.

17. If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, show that $x + y + z = xyz$.

18. Show that $\cos^{-1} x + \sin^{-1} x = \frac{\pi}{6}.$

Matrices & Determinants

1. If $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, then determine A^{-1} and show that $AA^{-1} = I$.

2. If $A = \begin{bmatrix} 1 & -2 & 2 \\ 3 & 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ 1 & 2 \\ 3 & -1 \end{bmatrix}$ show that $(AB)^T = B^T A^T$.

3. Find the inverse of the matrix $\begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$ using elementary row transformation.

4. If $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ -2 & 5 & 3 \end{bmatrix}$ then verify that $A + A^T$ is symmetric and $A - A^T$ is skew symmetric.

5. Show that $\begin{bmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{bmatrix} = (a+b+c)^2.$

6. If the matrix A is such that $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} A = \begin{bmatrix} -4 & 1 \\ 7 & 7 \end{bmatrix}$ find A.

7. Find the inverse of the matrix $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}.$

8. Show that $(a + 1)$ is a factor of $\begin{bmatrix} a+1 & 2 & 3 \\ 1 & a+1 & 3 \\ 3 & -6 & a+1 \end{bmatrix}.$

9. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ *and* $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, show that for what value of α, $A^2 = B$.

10. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ show that $A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} k \in N.$

11. If $A=\begin{bmatrix} 0 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 0 \end{bmatrix}$ then prove that $(1 + A)(1 - A)^{-1} = I$.

12. Find the value of $\begin{vmatrix} 17 & 58 & 97 \\ 19 & 60 & 99 \\ 19 & 59 & 98 \end{vmatrix}$ without expanding.

13. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$.

14. Solve for x: $\begin{vmatrix} 15-2x & 11 & 10 \\ 11-3x & 17 & 16 \\ 7-x & 14 & 13 \end{vmatrix} = 0$.

15. If $A=\begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ find $A^3 - A^2$.

16. Prove that $A^2 - 5A + 7I = 0$ if $A=\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$.

17. Show that inverse of a matrix is unique.

18. Find the adjoint of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$.

19. Find the matrix which when added to $\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$ *gives* $\begin{bmatrix} 4 & -1 \\ 3 & 2 \end{bmatrix}$.

20. Verify that $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ satisfies the equation $x^2 - (a + d)x + (ad - bc)I_2 = 0$.

Unit – 3

1. Prove that $y = \ln\tan\left(\frac{\pi}{4}+\frac{x}{2}\right) \Rightarrow \frac{dy}{dx} = \sec x$.

2. Find $\frac{dy}{dx}$ *if* $y = 2^{x^2} + \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$.

3. Find the derivative of $x^{\sin x}$ with respect to x.

4. Differentiate $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ *w.r.t* $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$.

5. Find the slope of the tangent to the curve $x = 2(t - \sin t)$, $y = 2(1 - \cos t)$ at $t = \frac{\pi}{4}$.

6. If $\cos y = x \cos (a + y)$, then show that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.

7. Find $\frac{dy}{dx}$, *if* $x^m y^n = \left(\frac{x}{y}\right)^{m+n}$.

8. Differentiate $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ *w.r.t* $\sqrt{1-x^2}$.

9. Find y_2 if $y = x^4e^{2x}$.

10. Differentiate y = $(\sin y)^{\sin 2x}$.

11. Differentiate $y = \tan^{-1}\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}$.

12. Differentiate y = $(\sin y)^{\sin 2x}$.

13. Examine the continuity of the following function at x = 0, 1

$$f(x) = \begin{cases} 2x+1 & if \quad x \le 0 \\ x & if \quad 0 < x < 1. \\ 2x-1 & if \quad x \ge 1 \end{cases}$$

14. If sin (x + y) = y cos (x + y) then prove that $\frac{dy}{dx} = -\frac{1+y^2}{y^2}$.

15. If $x^y = y^x$ find $\frac{dy}{dx}$.

16. If x = $a\cos^3 t$, y = $a\sin^3 t$, then find $\frac{dy}{dx}$.

17. If $e^{xy} = x^2 + y^2$, then find $\frac{dy}{dx}$.

18. Examine the continuity of the function $f(x) = \begin{cases} \frac{1}{x+[x]} & if \ x < 0 \\ -1 & if \ x \ge 0 \end{cases}$ at x = 0.

19. The function $f(x) = \begin{cases} \frac{x^2-3x+2}{(x-1)^2} & if \ x \ne 0 \\ k & if \ x = 0 \end{cases}$ is continous for all x then what is the values of k.

20. Discuss the continuity of the function $f(x) = \begin{cases} \frac{\sin 3x}{2x} & when \ x \ne 0 \\ \frac{3}{2} & when \ x = 0 \end{cases}$ at x = 0.

21. Examine the continuity of the function $f(x) = \begin{cases} \frac{e^{\frac{1}{x}-1}}{e^{\frac{1}{x}+1}} & , x \ne 0 \\ 0 & , x = 0 \end{cases}$ at x = 0.

22. Show that the function *f*(x) = $x^3 - 3x^2 + 3x - 100$ is increasing for all real values of x.

23. Examine the continuity of the function at x = 0, 1, $f(x) = \begin{cases} 2x+1 & if \quad x \le 0 \\ x & if \quad 0 < x < 1. \\ 2x-1 & if \quad x \ge 1 \end{cases}$

24. Find the intervals in which the function $f(x) = \frac{\ln x}{x}$ x>0 is increasing and decreasing.

25. Find the interval where the function is increasing y = sin x + cos x, $x \in [0, 2\pi]$.

26. Find the minimum distance of a point on the curve $\frac{4}{x^2}+\frac{1}{y^2}=1$ from the origin.

27. Write the maximum value of the function y = x^5 in the interval [1, 5].

28. A ballon is pumped at the rate of 2 cm^{-3}/minute. What is the rate of increase of the surface area when radius is 0.5 cm.

29. Find the interval in which $f(x)=x^{\frac{1}{x}}, x>0$ is decreasing.

30. What is the radius of a sphere if the rate of increasing of its volume is twice that of the surface area?

31. Find the maximum value of $y=(1+\cos x)\sin x, x \in \left[0, \frac{3\pi}{4}\right]$.

32. Find the maximum value of the function $f(x)=\left(\frac{1}{x}\right)^x$.

33. Integrate $\int \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} dx.$

34. Evaluate $\int_0^{\frac{1}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx.$

35. Evaluate $\int \frac{2x+1}{\sqrt{x^2+10x+29}} dx.$

36. Evaluate $\int \frac{dx}{(1+x)\sqrt{1-x^2}}.$

37. Evaluate $\int (\log x)^2 dx.$

38. Evaluate $\int \frac{2x+9}{(x+3)^2} dx.$

39. Evaluate $\int_0^{\frac{3}{2}} [x^2] dx.$

40. Integrate $\int \frac{xe^x}{(1+x)^2} dx.$

41. Integrate $\int \frac{1+x^2}{x\sqrt{x^4+1}} dx.$

42. Evaluate $\int_0^{\frac{\pi}{2}} \log(\tan x+\cot x) dx.$

43. Integrate $\int \frac{\sin 6x+\sin 4x}{\cos 6x+\cos 4x} dx.$

44. Integrate $\int \sin^{-1} x \, dx.$

45. Find the value of $\int \frac{dx}{\cos^2 x . \sin^2 x}.$

46. If $\int_0^1 f(1-x)dx=2$, then find the value of $\int_0^{1/2} f(2t)dt.$

47. If $\int_1^2 f(x)dx = \lambda$, then what is the value of $\int_1^2 f(3-x)dx$?

48. Find $\int \frac{\cos 3x.\cos x}{1+\cos 2x} dx.$

49. Find $\int \frac{x^5}{\left(x^3+1\right)^4} dx.$

50. Integrate $\int \frac{1}{1+\sin x} dx.$

51. Evaluate $\int_0^2 \left[x^2\right]dx.$

52. Integrate $\int \sin^4 x.\cos^3 x \, dx.$

53. Evaluate $\int_0^4 \left[\sqrt{x}\right]dx.$

54. Evaluate $\int_0^1 [3x]dx.$

55. Evaluate $\int_0^4 \left([x]+|x|\right)dx.$

56. Integrate $\int e^x\left(\cot x + \log \sin x\right)dx.$

57. Evaluate $\int_0^4 |8-3x|dx.$

58. Evaluate $\int \frac{dx}{x \ln x \sqrt{(\ln x)^2 - 4}}.$

59. Find the area of the region bounded by the parabola $y^2 = x$ and the ordinate $x = 4$.
60. Find the area of the trapezium bounded by the sides $y = x$, $y = 0$ and $x = 2$, $x = 4$.
61. If the area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, find the value of a.
62. Find the area of the region banded by the curve $y = 6x - x^2$ and the x-axis.
63. Find the area of the region enclosed by the two parabolas $y^2 = 4ax$ and $x^2 = 4ay$.
64. Find the area of the circle $x^2 + y^2 = 2ax$.
65. Find the area of the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1.$
66. Find the area of the region bounded by the curve $y = 6x - x^2$, the x-axis and the ordinates $x = 0$ and $x = 9$.
67. Find the area bounded by the straight lines $y = 0$, $y = x$ and $x + 2y = 3$.
68. Find the area enclosed by the curve $y^2 = x$ and the straight lines $x = 0$, $y = 1$.
69. Find the differential equation whose general solution is $c_1x^2 + c_2y = 1$ where c_1, c_2 are arbitration constants.
70. Solve: $\frac{dy}{dt} = e^{2t+3y}.$
71. Solve: $\frac{dy}{dx} + y = e^{-x}.$
72. Find the differential equation whose general solution is $y = a \cos x + b \sin x$.
73. Find the integrating factor of the differential equation $\left(1+y^2\right)dx + \left(x - e^{-\tan^{-1} y}\right)dy = 0.$
74. Solve: $(x + y)dy + (x - y)dx = 0$.

75. Solve: $x^2(y - 1)dx + y^2(x - 1)dy = 0$.

76. Solve: $ydy + e^{-y} x \sin sdx = 0$.

77. Solve: $\cos ecx \frac{d^2y}{dx^2} = x$.

78. Form the differential equation whose general solution is $y = a \sin t + be^t$.

79. Solve: $(x^2 + 7x + 12)dy + (y^2 - 6y + 5) dx = 0$.

80. Solve: $(x^2 + y^2)dx - 2xy dy = 0$.

81. Solve: $\ln\left(\frac{dy}{dx}\right) = 3x + 4y$.

82. Solve: $x\frac{dy}{dx} + y = y^2 \ln x$.

3 – D

1. Prove that two lines whose d.c.s are connected by the equations l + 2m + 3n = 0, 3ln – 4ln + mn = 0 are perpendicular to each other.
2. Find the co-ordinates of the foot of the perpendicular drawn from the point A(1, 3, 4) with the line joining the points B(3, 0, -1) and C(0, 1, -2).
3. If A, B, C, D are the points (6, 3, 2), (3, 5, 7), (2, 3, -1) and (3, 5 -3) respectively then find the projections of AB on CD.
4. Prove that the points P(3, 2, -4), Q(5, 4, -6) and R(9, 8, -10) are collinear.
5. Prove that measure of the angle between two main diagonals of the cube is $\cos^{-1}\frac{1}{3}$.
6. Find d.c.s of a line passing through origin and lying in the first octent, making equal angles with the three co-ordinate axis.
7. Write the vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$.
8. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect.
9. Find the symmetric form of equation of the line x + 2y + z -3 = 0 = 6x + 8y + 3z – 10.
10. Find the image of the point (3, 5, 7) w.r.t the plane 2x + y + z = 6.
11. Find the shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$.
12. Obtain the equation of the line through the point (1, 2, 3) and parallel to the line x – y + 2z – 5 = 0 = 3x + y + z.
13. Find the angle between the lines whose d.r.s are proportional and b – c, c – a, a – b.

Vectors

1. Find the unit vector in the direction of the sum of the vectors $\hat{i} - \hat{j} + \hat{k}, 2\hat{i} + \hat{j} - 3\hat{k}$ *and* $\hat{i} + \hat{j} + \hat{k}$.
2. Find the vector joining the points (2, -3) and (-1, 1). Find its magnitude and the unit along the same direction. Also determine the scalar components and component vectors along the co-ordinate axes.
3. Using vectors show that points (3, 2, 1), (5, 5, 2) and (-1, -4, -1) are collinear.
4. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then find the angle between $\vec{a}$ *and* $\overleftarrow{b}$.
5. Find the scalar and vector projections of the vector $2\hat{i} - 3\hat{j} - 6\hat{k}$ on the line joining the points (3, 4, -2) and (5, 6, -3).

6. If $\vec{a} = 2\hat{i} - 2\hat{j} - 3\hat{k}, \vec{b} = -\hat{i} - 2\hat{j} + \hat{k}$ *and* $\vec{c} = 3\hat{i} + \hat{j}$ such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$. Find the value of λ.
7. Prove that two vectors are perpendicular iff $\left|\vec{a} + \vec{b}\right|^2 = \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2$.
8. If $\vec{a} = 5\hat{i} - \hat{j} + 7\hat{k}$ *and* $\vec{b} = \hat{i} - \hat{j} + t\hat{k}$, find 't' such that $\vec{a} + \vec{b}$ *and* $\vec{a} - \vec{b}$ are perpendicular to each other.
9. Find a unit vector perpendicular to each of the vectors $2\hat{i} - \hat{j} + \hat{k}$ *and* $3\hat{i} + 4\hat{j} - \hat{k}$. Find the sine of angle between the two vectors.
10. Show that the points A, B and C with position vectors $3\hat{i} - 4\hat{j} - 4\hat{k}, 2\hat{i} - \hat{j} + \hat{k}$ *and* $\hat{i} - 3\hat{j} - 5\hat{k}$ respectively form the vertices of a right angle.
11. Obtain the area of the parallelogram whose adjacent sides are vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ *and* $-3\hat{i} - 2\hat{j} + \hat{k}$.
12. Show that the vector area of the triangle whose vertices have position vectors $\vec{a}, \vec{b}, \vec{c}$ is $\frac{1}{2}\left(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right)$.
13. Prove that the four points with position vectors $4\hat{i} + 5\hat{j} + \hat{k}, -\hat{j} - \hat{k}, 3\hat{i} + 9\hat{j} + 4\hat{k}$ and $-4\hat{i} + 4\hat{j} + 4\hat{k}$ are coplanar.
14. Prove by vector method that altitudes of a triangle are concurrent.
15. Find the value of the following $\hat{i}.\left(\hat{j} + \hat{k}\right) + \hat{j}.\left(\hat{j} \times \hat{k}\right) + \hat{k}.\left(\hat{i} \times \hat{j}\right)$.
16. Prove by vector method that an angle in scribed in a semi-circle is a right angle.

<u>LPP</u>

1. Solve the following graphically,
   Maximize Z = 20x + 30y
   Subject to 3x + 5y ≤ 15
   x, y ≥ 0
2. Find the feasible region of the following system:
   2y – x ≥ 0
   6y – 3x ≤ 21
   x ≥ 0, y ≥ 0
3. Shade the feasible region satisfying the in equations 2x + 3y ≤ 6, x ≥ 0, y ≥ 0 in a rough sketch.
4. Let an LPP be as follows:
   Maximize Z = 3x + 5y
   Subject to 5x + 3y ≤ 30
   x + 2y ≤ 12
   2x + 5y ≤ 20
   and x, y ≥ 0
   Test whether the points (2, 3) and (-3, 4) are feasible solutions or not.
5. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two food A and B are available at a cost of Rs. 4 and Rs. 3 per unit respectively. If one unit of A contains 200 units of vitamin, 1 unit of mineral and 40 colories and one units of B contains 100 units of vitamins, 2 units of minerals and 40 colories. Formulate the LPP to minimize the cost of foods.
6. Solve the following LPP graphically:
   Minimize Z = 4x + 3y
   Subject to 2x + 5y ≥ 10
   x, y ≥ 0

7. Find the feasible solution region of the following system: $-x + y \geq -1$, $x + y \leq 6$, $y \leq 5$, $x \geq 0$, $y \geq 0$.
8. A farmer has 5 acres of land on which he wishes to grow two crops X and Y. He has to use 4 cart loads and 2 cart load of manure per acre for crops X and Y respectively. But not more than 18 cart loads of manure is available other expenses are Rs. 200 and Rs. 500 per acre for the crops X and Y respectively. He estimate profit from crops X and Y at the rates Rs. 1000 and Rs. 800 per acre respectively. Formulate the LPP as to how much land he should allocate to each crop for maximum profit.
9. Write the general form of an LPP and its component.
10. Solve the following system graphically:

    $x + 2y < 2$

    $2x - y + 2 \geq 0$.

Probability

1. A pair of dice is thrown. Find the probability of getting a sum of at least 9 if 5 appears on at least one of the dice.
2. Four cards are drawn successively with replacement from a well – shuffled pack of 52 cards. Find the probability distribution of the number of aces. Calculate the mean and variance of the number of aces.
3. There are 25 girls and 15 boys in class XI and 30 boys and 20 girls in class XII. If a student from a class, selected at random, happens to be a boy, find the probability that he has been chosen from class XII.
4. If A and B are independent events, show that
   (i) A^C and B^C are independent
   (ii) A and B^C are independent
5. If A, B are events such that P(A) = 0.6, P(B) = 0.4 and $P(A \cap B)$ = 0.2 then find
   (i) P(B/A) (ii) $P(B/A^C)$
6. 10% of the bulbs produced in a factory are of red colour and 2% are red and defective. If one bulb is picked up at random, determine the probability of its being defective if it is red.
7. A committee of 4 students is selected at random from a group consisting 8 boys and 4 girls. Given that there is at least one girl on the committee, calculate the probability that there are exactly 2 girls on the committee.
8. Two cards are drawn simultaneously for successively without replacement for a well shuffled pack of 52 cards. Find the mean and variance of the no. of aces.
9. Find the probability distribution of number of heads in 3 tosses of a fair coin.
10. A person takes 4 tests in succession. The probability of his passing the first test is P, that of his passing each succeeding test is P or $\frac{P}{2}$, depending on his passing or failing the preceding test. Find the probability of his passing just 3 tests.

6 Marks:

Relation and Function

1. Prove that congruence modulo relation on set of integers is an equivalence relation.
2. Show that the relation ~is given by ~= {(a, b) : $\frac{a}{b}$ is power of 5} on Z – {0} is an equivalence relation.
3. Show that the relation R = $\{(m,n) | 2 | (m+n)\}$ on Z is an equivalence relation.
4. Show that f : R → R defined by $f(x) = x^2 - 1$ is not invertible in general. Find the domain and codomain where f is invertible. Also find f^{-1}.

5. Prove that the function f: (-1, 1) → R given by $f(x) = \frac{x}{1-x^2}$ is invertible & find its inverse.

6. Let f: x → y if there exists a map g : y → x such that $g_o f = id_x$ and $f_0 g = id_y$, then show that f is bijective and $g = f^{-1}$.

7. Show that the function f: R → R defined by $f(x)$ = cosx is neither one-one nor onto.

8. If f: N → N defined by $f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$ for all n $\in$ N then find whether the function is bijective.

9. Let R be defined by (m, n) R (p, q) if mq = np where m, n, p, q $\in$ z – {0}. Show that R is an equivalence relation.

10. Let n $\in z_+$ and f be a function defined as $f(n) = \begin{cases} 0, & \text{when } n = 1 \\ f\left(\left[\frac{n}{2}\right]\right) + 1, & \text{when } n > 1 \end{cases}$ then find $f(35)$.

Inverse Trigonometric Functions

1. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$.

2. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, prove that $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$.

3. If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, show that xy + yz + zx = 1.

4. Solve $\tan^{-1} \frac{1}{2x+1} + \tan^{-1} \frac{1}{4x+1} = \tan^{-1} \frac{2}{x^2}$.

5. Solve $\cot^{-1} \frac{1-x^2}{2x} = \operatorname{cosec}^{-1} \frac{1+a^2}{2a} - \sec^{-1} \frac{1+b^2}{1-b^2}$.

6. Solve $\cos^{-1}\left(x+\frac{1}{2}\right) + \cos^{-1} x + \cos^{-1}\left(x-\frac{1}{2}\right) = \frac{3\pi}{2}$.

7. If $r^2 = x^2 + y^2 + z^2$, prove that $\tan^{-1} \frac{yz}{xr} + \tan^{-1} \frac{zx}{yr} + \tan^{-1} \frac{xy}{zr} = \frac{\pi}{2}$.

8. In a triangle ABC if $m\angle A = 90^0$, prove that $\tan^{-1} \frac{b}{a+c} + \tan^{-1} \frac{c}{a+b} = \frac{\pi}{4}$ where a, b, c are sides of the triangle.

9. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, prove that $x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$.

10. Prove that $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right] = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x, \left(-\frac{1}{\sqrt{2}} \le x \le 1\right)$.

Matrices & Determinants

1. Prove that a square matrix can be uniquely expressed as a sum of symmetric and a skew symmetric matrix.

2. Find the inverse of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 1 & 0 & 2 \end{pmatrix}$.

3. Solve the system x + 2y + 3z = 8, 2x + y + z = 8, x + y + 2z = 6 by matrix inversion method.

4. Applying elementary operations, find the inverse of the matrix $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$.

5. If A, B and C are matrices of order 2 × 2 each and 2A + B + C = $\begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$, A + B + C = $\begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix}$, A + B - C = $\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$ then find A, B and C.

6. Show that the system of equations $\begin{matrix} x-y+z=4 \\ 2x+y-3z=0 \\ x+y+z=2 \end{matrix}$ are consistant.

7. Examine whether the following system has any nontrivial solution. If so, find it.

$$x+2y+z=0$$
$$3x+5y+2z=0.$$
$$4x+3y-z=0$$

8. Express $\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix}$ in the form of a perfect square.

9. Factorize the determinant $\begin{vmatrix} x^3-a^3 & x^2 & x \\ b^3-a^3 & b^2 & b \\ c^3-a^3 & c^2 & c \end{vmatrix}$ without expands.

10. If A + B + C = π, show that $\begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix} = 0$.

11. Prove that $\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$.

12. If A + B + C = π, prove that $\begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix} = 0$.

13. Eliminate x, y, z from $a=\frac{x}{y-z}, b=\frac{y}{z-x}, c=\frac{z}{x-y}$.

14. If $A=\begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{pmatrix}$, show that $A^3 - 23A - 40I = 0$.

15. If $A=\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{pmatrix}$ then find the value of $A^3 - A^2 + I_3$.

16. Prove that $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix}$ = (x – y)(y – z)(z – x) (xy + yz + zx).

17. If $A = \begin{pmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 2 & 0 & -3 \end{pmatrix}$ then find A^{-1} and hence solve the system of equations x – 2y + z = 0, -y + z = -2 and 2x – 3z = 10.

18. Verify that $(AB)^T = B^T A^T$ where $A = \begin{pmatrix} 1 & 2 & 3 \\ 6 & 7 & 8 \\ 6 & -3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 2 \\ 5 & 6 & 1 \end{pmatrix}$.

19. If $A = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}, B = \begin{pmatrix} -1 & 3 \\ -3 & 1 \end{pmatrix}$ then show that $(A+B)^2 \neq A^2 + 2AB + B^2$.

20. Find the value of x, y & z if $A = \begin{pmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{pmatrix}$ satisfies $A^T = A^{-1}$.

Continuity and Differentiability

1. Find the values of k so that the function f defined by $f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x} & when\ x \neq \dfrac{\pi}{2} \\ 0 & at\ x = \dfrac{\pi}{2} \end{cases}$ is continuous at $x = \dfrac{\pi}{2}$.

2. If the function f(x) given by $f(x) = \begin{cases} 3ax + b & if\ x > 1 \\ 3ax + b & if\ x = 1 \\ 5ax - 2b & if\ x < 1 \end{cases}$ is continuous at x = 1, then find the values of a and b.

3. Find the value of k for which $\begin{cases} \sqrt{1+kx} - \sqrt{1-kx} & when\ 1 < x < 0 \\ \dfrac{2x+1}{x-1} & if\ when\ 0 \le x < 1 \end{cases}$ is continuous at $x = \dfrac{\pi}{2}$.

4. Find all points of discontinuity of 'f' where f is defined as follows
$$f(x) = \begin{cases} |x| + 3 & when & x \le -3 \\ -2x & when & -3 < x < 3. \\ 6x + 2 & when & x \ge 3 \end{cases}$$

5. If $(\cos x)^y = (\cos y)^x$ then find $\dfrac{dy}{dx}$.

6. If $y = x^{\sin^{-1} x} + x^3 \dfrac{\sqrt{x^2+4}}{\sqrt{x^3+3}}$ then find $\dfrac{dy}{dx}$.

7. If $e^{y/x} = \frac{x}{a+bx}$ then show that $x^3 \frac{d}{dx}\left(\frac{dy}{dx}\right) = \left(x\frac{dy}{dx} - y\right)^2$.

8. If $x = \frac{1-\cos^2\theta}{\cos\theta}, y = \frac{1-\cos^{2n}\theta}{\cos^n\theta}$ then show that $\left(\frac{dy}{dx}\right)^2 = n^2\left(\frac{y^2+4}{x^2+4}\right)$.

9. Differentiate $y = \frac{(x-1)^2\sqrt{3x-1}}{x^7(6-7x^2)^{3/2}}$.

10. If $\sqrt{1-x^4} + \sqrt{1-y^4} = k(x^2 - y^2)$ then show that $\frac{dy}{dx} = \frac{x\sqrt{1-y^4}}{y\sqrt{1-x^4}}$.

11. Find the greatest and least value of $x^4 - 2x^2 + 3$ in [-2, 2].

12. Find the maxima and minima of $f(x) = \sin x + \cos x$, $x \in [0, 2\pi]$.

13. Show that the rectangle of maximum area that can be inscribed in a circle is a square.

14. Show that the semi vertical angle of a cone of given slant height is $\tan^{-1}\sqrt{2}$ when the volume is maximum.

15. Find the coordinates of the point on the curve $x^2y - x + y = 0$ where the slope of the tangent is maximum.

16. Determine the points of extreme values on the curve $y^3 = (x-1)^2(x+2)$.

17. Evaluate $\int \frac{2\cos x + 7}{4 - \sin x} dx$.

18. Show that $\int_0^1 \frac{\log(1+x)}{1+x^2} dx = \frac{\pi}{8}\log 2$.

19. Integrate $\int \frac{x^2}{x^4 + x^2 + 1} dx$.

20. Determine $\int \frac{dx}{(x-2)\sqrt{3x^2 - 16x + 24}}$.

21. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$.

22. Evaluate $\int \left\{\frac{1}{\log x} - \frac{1}{(\log x)^2}\right\} dx$.

23. Integrate $\int_0^{\pi/2} \ln \sin x\, dx$.

24. Integrate $\int_0^1 \frac{\ln(1+x)}{1+x^2} dx$.

25. Determine the area included between the parabola $y^2 = x$ and the circle $x^2 + y^2 = 2x$.

26. Find the area enclosed by $y = 4x - 1$ and $y^2 = 2x$.

27. Find the area of the portion of the ellipse $\frac{x^2}{12} + \frac{y^2}{16} = 1$ bounded by the major axis and the double ordinate $x = 3$.

28. Determine the area of the region between the curve $y = \cos x$ and $y = \sin x$ bounded by $x = 0$.

29. Find the particular solutions of the differential equations $\frac{d^2y}{dx^2} = 6x$ given by $y = 1$ and $\frac{dy}{dx} = 2$ when $x = 0$.

30. Solve $(1 + y^2)dx + xdy = \tan^{-1}y dy$.

31. Solve $\frac{dy}{dx} + \frac{y}{x} = xy^2$.

32. Solve $\left(x + 2y^3\right)\frac{dy}{dx} = y$.

33. Solve $\left(x + y + 1\right)\frac{dy}{dx} = 1$.

34. Solve $\frac{dy}{dx} = \frac{y - x + 1}{y + x + 5}$.

35. Solve $(4x + 6y + 5)dx - (2x + 3y + 4)dy = 0$.

Vectors and Three – Dimensional Geometry

Vectors

1. Three vectors of magnitude a, 2a and 3a act along the diagonals of three adjacent faces OABC, OCDG, OAFG of a cube. Find their sum and its direction cosines.
2. Prove by vector method that altitudes of a triangle are concurrent.
3. Prove by vector method that median to the base of an isosceles triangle is perpendicular to the base.
4. Prove by vector method that measure of the angle between two diagonals of a cube is $\cos^{-1}\frac{1}{3}$.
5. Prove by vector method that in any triangle ABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
6. If $\vec{a} = 2\hat{i} + \hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ & $\vec{c} = 4\hat{i} - 3\hat{j} + 7\hat{k}$ then find the vector $\vec{r}$ which satisfies $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ *and* $\vec{r} \cdot \vec{a} = 0$.
7. Prove by vector method that in a triangle ABC, $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$.
8. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitude prove that $\vec{a} - b - \vec{c}$ is equally inclined to $\vec{b}$ & $\vec{c}$.
9. If the vertices A, B, C of a triangle ABC are at (1, 1, 20, (2, 2, 3) and (3, -1, -1) respectively, then using vector method, find the area of the triangle.
10. Prove that $\left[\vec{a} \times \vec{b}\ \vec{b} \times \vec{c}\ \vec{a} \times \vec{c}\right] = \left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$.
11. Show that $\vec{a}, \vec{b}$ *and* $\vec{c}$ are coplanar iff $\vec{a} + \vec{b}, \vec{b} + \vec{c}$ *and* $\vec{c} + \vec{a}$ are coplanar.

Three-dimensional Geometry

1. A line makes angles α, β, γ, δ with the four main diagon of a cube. Prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$.
2. Prove that the lwo lines whose direction cosines are connected by the equations l + 2m + 3n = 0, 3lm – 4ln + mn = 0 are perpendicular to each other.
3. Show that th measure of the angles between the four diagonals of a rectangular parallelepiped whose edges are a, b, c are $\cos^{-1}\left(\frac{a^2 \pm b^2 \pm c^2}{a^2 + b^2 + c^2}\right)$.

4. Find the shortest distances between the following two lines:
$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}; \frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$. Find also the equations of the line of shortest distance.

5. Find the distance of the point (3, -4, 5) from the plane 2x + 5y – 6z -19 = 0 measured parallel to the line $\frac{x-1}{2}=\frac{y}{1}=\frac{z+3}{-2}$.

6. Find the image of the point (3, 5, 7) with respect to the plane 2x + y + z = 6.

7. Find the foot of the perpendicular drawn from the point (5, 7, 3) to the line $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$. Find the length of the perpendicular and its equation.

8. Find the equation of the line given by the pair of equations x – y + z + 1 = 0 and x – 2y + z + 1 = 0 in symmetrical form.

9. Prove that the line joining (1, 2, 3) and (2, 1, -1) intersect the line joining (-1, 3, 1) and (3, 1, 5).

10. Find the equation of line passing through the point (1, 0, -1) and intersecting the lines x = 2y = 2z; 3x + 4y – 1 = 0 = 4y + 5z – 2.

Linear Programming

1. Solve the following LPP graphically,
Optimize z = 20x + 40y
Subject to 6x – y ≥ -6
x + 4y ≥ 8
2x + y ≥ 4
x , y ≥ 0

2. Solve the following LPP graphically,
Minimize z = 6x + 9y
Subject to x + 12y ≤ 65
7x - 2y ≤ 25
2x + 3y ≥ 10
x , y ≥ 0

3. Solve the following LPP graphically,
Maximize z = 14x - 4y
Subject to x + 12y ≤ 65
7x - 2y ≤ 25
2x + 3y ≥ 10
x , y ≥ 0
Also find two other points which maximize z.

4. Solve the following LPP graphically method
Minimize z = x + 3y
Subject to x + 2y ≥ 2
3x + y ≥ 3
4x + 3y ≥ 6
x , y ≥ 0

5. A company produces three types of cloth A, B and C. Three kinds of wool, say red, green and blue are required for the cloth. One unit length of type A cloth needs 2 metres of red and 3 metres of blue wool, one unit of length of B cloth needs 3 metres of red, 2 metres of green and 2 metres of blue wool and one unit length of type C cloth needs 5 metres of green and 4 metres of blue wool. The firm has a stock of only 80 metres of red, 100 metres of green and 150 metres of blue wool. Assuming that income obtained from one unit length of cloth is Rs. 30, Rs. 50 and Rs. 40 of types A, B and C respectively. Formulate the LPP so as to maximize income.

Probability

1. If a pair of dice is thrown thrice then find the mean & the variance of the number of doublets.
2. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of kings. Also determine the mean and the variance of the number of kings.
3. State and prove Bayes theorem.
4. There are two bags B_1 and B_2 containing 4 red, 3 black balls and 2 red, 4 black balls respectively. If the ball drawn from a bag selected at random, is red, find the probability that the ball is drawn from the bag B_1.
5. Four cards are drawn successively from a well-shuffled pack of 52 cards with replacement after each draw. Find the probability that
   (i) all four cards are diamonds
   (ii) only two cards are diamonds
   (iii) none is a diamond.
6. A box containing 20 electric bulbs includes 5 defective bulbs. Four bulbs are drawn at random with replacement. Find the probability distribution of the number of non-defective bulbs. Calculate also the mean and the variance.
7. A man is known to speak the truth 3 out of 5 times. He throws a die and report that it is 1. Find the probability that it is actually 1.
8. From a lot of 10 bulbs which includes 3 defectives, a sample of 2 bulbs is drawn at random. Find the probability distribution of number of defective bulbs also determine the man.
9. A speaks the truth is 80% of cases and B in 70% of the cases. In what percentage of cases they are likely to contradict each other in reporting the same fact.
10. Find the probability distribution of total number of heads obtained in 4 tosses of a balanced coin & also find the mean.

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