Bio-SET B -2024

- There are 20 multiple choice questions in this question paper.
- Only one answer is correct in each question
- Difficulty level score: 52; average difficulty:2.6

1. A researcher was trying to amplify a DNA fragment *in vitro* using a polymerase chain reaction (PCR). All the required components for the reaction except the *Taq* DNA polymerase were available. The experimenter decided to use DNA polymerase isolated from *E. coli* instead of *Taq* DNA polymerase. The *E. coli* polymerase was added to the reaction at the extension step of each cycle of the PCR. In this scenario, from the options given below, the optimal temperature to be set in the extension step of the PCR is:

   A. 72 ^{0}C

   B. 37 ^{0}C

   C. 25 ^{0}C

   D. 92 ^{0}C

2. The correct statement with respect to arteries, veins and capillaries is:

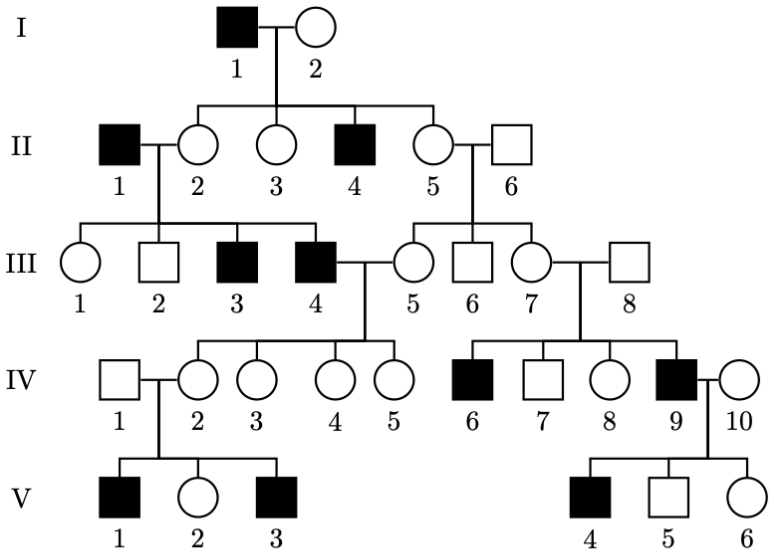
   A. Valves are absent in arteries, but present in veins and capillaries.

   B. Pulse can be felt in veins and capillaries but not in arteries.

   C. Valves are absent in arteries and capillaries but present in veins.

   D. Pulse can be felt in arteries and veins but not in capillaries.

3. In the given pedigree, circles represent females and squares represent males. Filled shapes indicate affected individuals, while unfilled shapes indicate unaffected individuals. Based on the pedigree analysis, consider the statements (i) to (iv).



(i) The inheritance pattern is autosomal recessive, and both I-2 and III-8 are carriers.

(ii) The inheritance pattern is autosomal recessive and the individual II-6 could be homozygous for the normal allele or heterozygous for the disease allele.

(iii) The inheritance pattern is autosomal recessive, and the individual II-2 is not a carrier.

(iv) The inheritance pattern is autosomal dominant and individual I-1 could be either homozygous or heterozygous for the disease allele.

The correct statement/s about the pedigree analysis is/are:

A. i and ii

B. only iii

C. ii and iii

D. only iv

4. In a plant, alleles ***P*** and ***p*** govern leaf colour (purple or green) whereas ***C*** and ***c*** govern shape of the leaf edge (sharp or rounded). A cross performed between two parent plants gave the following progeny.

Progeny phenotype	**Number of progeny**
Purple coloured; sharp edge	321
Purple coloured; rounded edge	101
Green coloured; sharp edge	310
Green coloured; rounded edge	107

Based on this information, the parental genotypes are:

A. *PpCc*, *ppCc*

B. ***PPCC***, ***ppcc***

C. ***ppCC***, ***ppcc***

D. ***ppCc***, ***PPCC***

5. The amino acid sequence of an unknown peptide can be determined using a series of proteolytic digestions. An eight-mer peptide contains 1 alanine (ALA), 2 arginines (ARG), 1 methionine (MET), 1 serine (SER), 1 tyrosine (TYR) and 2 valines (VAL) with ALA at the N-terminus and VAL at the C-terminus. This peptide was digested with trypsin (cleaves the peptide bond C-terminus to a LYS or ARG) and chymotrypsin (cleaves the peptide bond N-terminus to an aromatic amino acid) in two separate reactions. The amino acid composition of each of the fragments obtained is given below:

   Trypsin digestion: **fragment 1**: SER, ALA, ARG; **fragment 2**: VAL; **fragment 3**: TYR, ARG, MET, VAL

   Chymotrypsin digestion: **fragment 1**: VAL, ARG, ALA, SER; **fragment 2**: VAL, TYR, ARG, MET

   The correct amino acid sequence of the eight-mer peptide is:

   A. ALA-SER-ARG-VAL-TYR-MET-ARG-VAL

   B. ALA-ARG-SER-VAL-TYR-ARG-MET-VAL

   C. ALA-SER-ARG-TYR-MET-VAL-ARG-VAL

   D. ALA-SER-ARG-VAL-ARG-TYR-MET-VAL

6. The central dogma of molecular biology states that the information flow starts from genome. Consider the following statements:

   (i) Mutation in the DNA-dependent RNA polymerase reverses direction of the information flow.

   (ii) There is an amplification of the coded information from DNA to protein.

   (iii) The central dogma explains the flow of information in all known forms of life.

   (iv) The genome size determines the direction of the information flow.

   The statements that are correct about the central dogma are:

   A. i and ii

   B. ii and iii

   C. iii and iv

   D. i and iv

7. Following are the statements about sun leaves and shade leaves.

   (i) Chloroplasts are mostly restricted to palisade mesophyll cells in sun leaves but are evenly present in spongy and palisade mesophyll in shade leaves.

   (ii) The density of stomata is low in sun leaves and high in shade leaves.

   (iii) Starch accumulation in chloroplasts of sun leaves is higher compared to that in shade leaves.

   (iv) Sun leaves are larger and thinner, and shade leaves are smaller and thicker.

   The option with correct combination of statements is:

   A. ii and iv

   B. i and iii

   C. i and ii

   D. iii and iv

8. An experimenter attempting to measure the biodiversity of a region has made several measurements in different sampling areas of the region and plotted those values in a graph. The biodiversity of a region can be correctly represented by plotting _____.

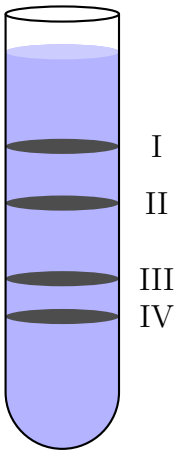
   A. number of different species *vs* different sampling areas

   B. number of individuals of a species *vs* different sampling areas

   C. number of primary producers in the region *vs* different sampling areas

   D. ratio of number of land species to aquatic species *vs* different sampling areas

9. Ribosomal RNA (rRNA) sequence analysis can be used for the identification of bacteria. An experimenter was analysing total cell lysate (homogenized mixture) of a plant tissue that is heavily infected with a bacterial pathogen. This lysate was used to prepare ribosome-enriched fraction (with large and small subunits dissociated), which was subjected to density gradient ultracentrifugation. The resulting band pattern is shown in the figure.



The correct set of bands that the researcher can use for the pathogen identification is:

A. Band I and III

B. Band II and III

C. Band I and II

D. Band III and IV

10. Accumulation of H_2O_2 is harmful for the survival of eukaryotic cells. The level of H_2O_2 in the cell is kept under control by _____.

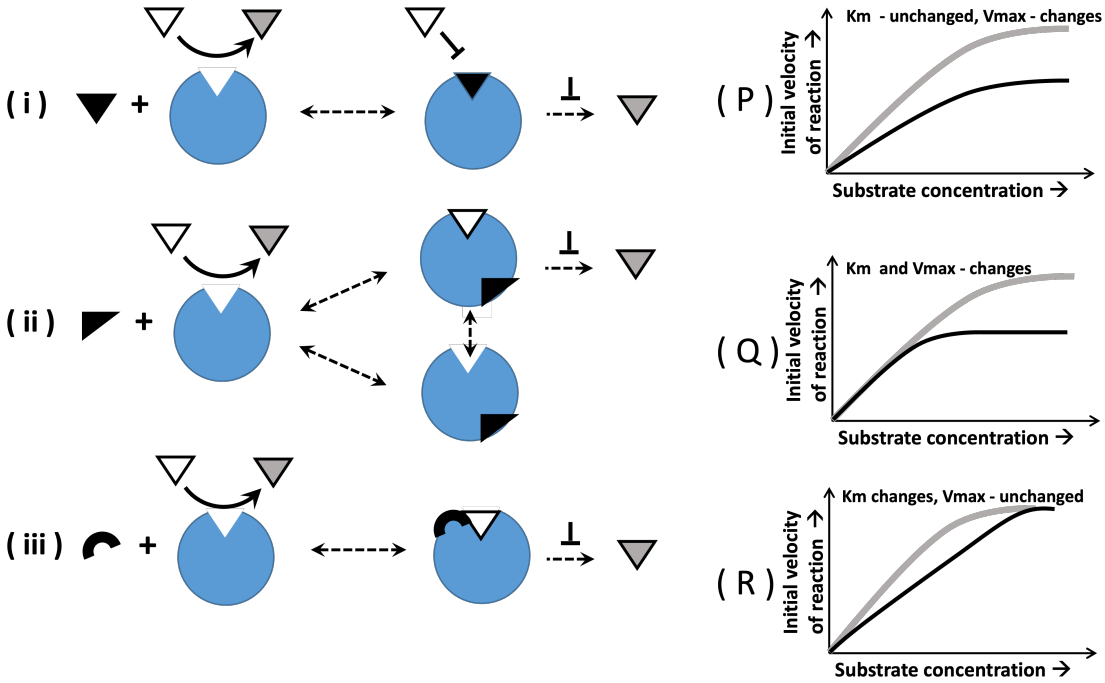
A. glyoxisomes with the enzyme glutathione reductase

B. peroxisomes with the enzyme citrate lyase

C. peroxisomes with the enzyme catalase

D. glyoxisomes with the enzyme glyceraldehyde dehydrogenase

11. Depicted in the figure are three possibilities of substrate (white triangle) conversion to the product (gray triangle) by an enzyme (blue) in the presence of different kinds of reversible inhibitors. The corresponding interactions with specific inhibitors (black shapes) are depicted schematically, where the dotted double-headed arrow indicates a reversible reaction and a blunt arrow ('—|') indicates inhibition. The Michaelis-Menton graphs (right side of the figure) show plots in absence (gray line) and presence of inhibitor (black line). Changes in K_m and V_{max} in each case are indicated. Based on your evaluation of the three modes of inhibition shown in the figure, the correctly matched combination of (i), (ii), (iii) and (P), (Q), (R) is:



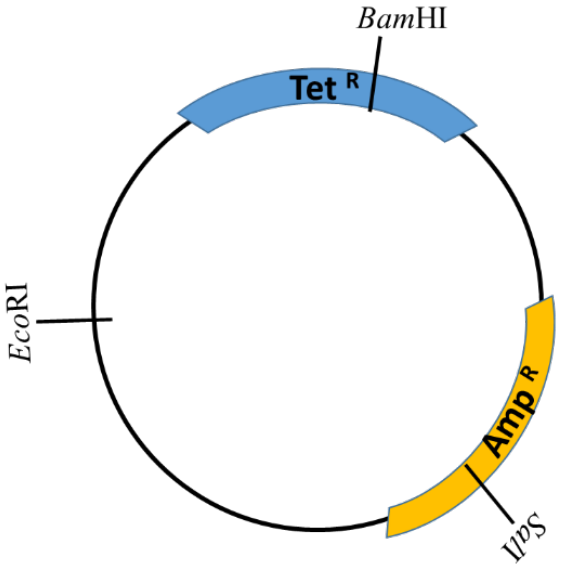
A. (i) R; (ii) Q; (iii) P.

B. (i) Q; (ii) P; (iii) R.

C. (i) P; (ii) R; (iii) Q.

D. (i) R; (ii) P; (iii) Q.

12. In the plasmid vector map Tet^R denotes tetracycline resistance gene and Amp^R denotes ampicillin resistance gene. A *Sal*I restriction enzyme-digested DNA fragment containing the gene of interest (insert) is ligated to the *Sal*I-digested plasmid vector and then transformed into strain of *E. coli* that is sensitive to both antibiotics (test reaction). The desired transformants (containing the recombinant plasmid) were obtained by plating this test reaction on tetracycline containing nutrient agar plates. An appropriate control for the above experiment that can demonstrate complete digestion of the plasmid vector by *Sal*I is:



A. Ligation of *BamH*I-digested plasmid vector with the *Sal*I-digested insert and transformed into *E. coli* followed by plating on ampicillin containing nutrient agar plate.

B. Ligation of *Sal*I-digested plasmid vector alone, and transformed into *E. coli* followed by plating on ampicillin containing nutrient agar plate.

C. Unligated *Sal*I-digested plasmid vector alone transformed into *E. coli* followed by plating on ampicillin containing nutrient agar plate.

D. Unligated *Sal*I-digested plasmid vector alone transformed into *E. coli* followed by plating on tetracycline containing nutrient agar plate.

13. A small group of large mammals that consisted of closely-related kins migrate into a new ecosystem. Despite inbreeding, the population is stable for many generations. However, due to a sudden appearance of a man-made geographical barrier, the group splits into two smaller groups, **P** and **Q**. The population soon begins to decline within the groups and conservationists decide to take action to increase population of these mammals within the newly divided ecosystems. The most appropriate intervention strategy, from the following options, would be _____.

    A. to have separate breeding programs within group **P** and group **Q**

    B. to create barriers to avoid predators

    C. to move all the individuals in one group to the location of the other group

    D. to create corridors to facilitate migration

14. A particular microbial pathogen that infects humans has a cell wall composed of peptidoglycan layer. An infection from this pathogen can easily be treated with an antibiotic that inhibits cell wall synthesis. A handwash containing this antibiotic was popularly used. In subsequent years, resistance to the antibiotic emerged in this pathogen. While one group evolved a modified cell wall, the other group evolved to survive without a cell wall. This type of selection is:

    A. Disruptive

    B. Stabilising

    C. Directional

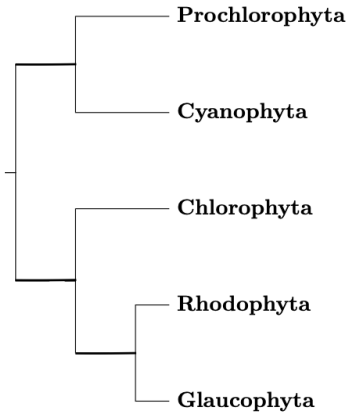
    D. Purifying

15. The table shows the presence of photosynthetic pigments in bacterial and algal groups.

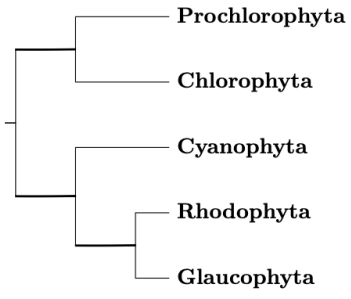
	Photosynthetic Bacteria		**Photosynthetic Algae**		
	Prochloro-phyta	Cyanophyta	Glauco-phyta	Rhodophyta	Chlorophyta
Pig-ments	Chloro-phyll **a** and Chloro-phyll **b**	Chlorophyll **a** and Phy-cobillins	Chlorophyll **a** and Phy-cobillins	Chlorophyll **a** and Phy-cobillins	Chloro-phyll **a** and Chlorophyll **b**

All eukaryotic chloroplasts are derived from a common ancestor. It is now well known that it was a result of a single endosymbiotic event. Assuming loss of gene/s during the course of evolution, the correct phylogenetic tree is:

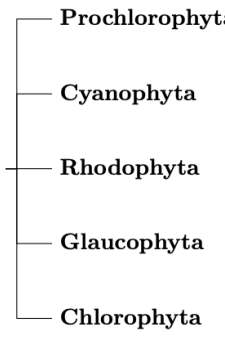
A.



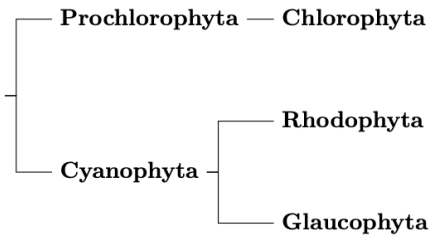
B.



C.



D.


Prochlorophyta
Chlorophyta
Rhodophyta
Cyanophyta
Glaucophyta

16. Chloroplasts of certain algal taxa such as Cryptophyta are surrounded by four-membrane envelope. The most likely reason for the origin of four-membrane enveloped chloroplast structure is _____.

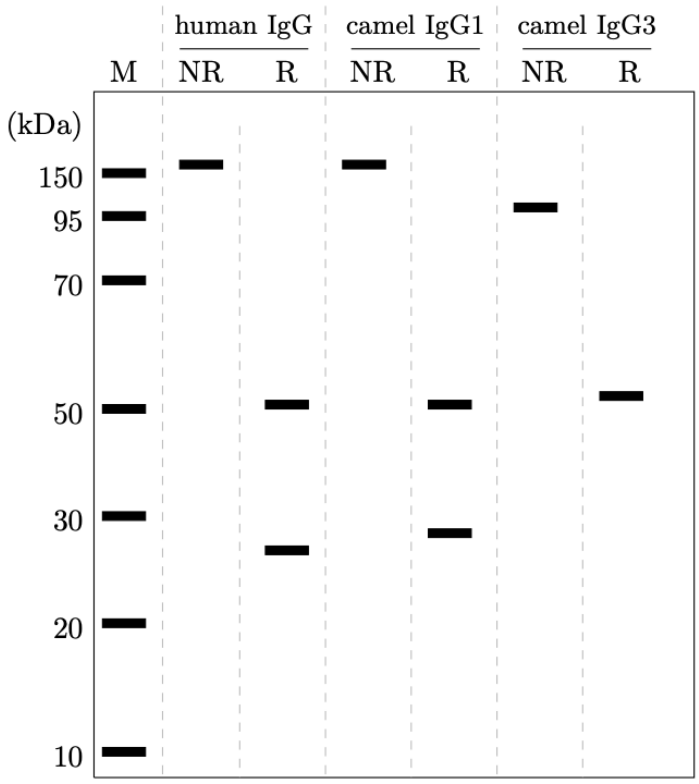
A. secondary endosymbiosis, where a photosynthetic eukaryote was engulfed by another eukaryote.

B. engulfment of the acquired chloroplast by host endoplasmic reticulum during the course of evolution.

C. fusion of the acquired chloroplast with host membrane vesicles during the course of evolution

D. acquisition of new genes to synthesize an additional membrane envelope around the acquired chloroplast

17. Camels have three different sub-classes of IgG (IgG1, IgG2 and IgG3), of which IgG2 and IgG3 are structurally different from the human IgG. A researcher purified camel IgG1 and IgG3, and carried out gel electrophoresis that separates polypeptides based on their molecular size. An IgG antibody purified from human serum is electrophoresed on the same gel. A schematic of the observed band pattern on the gel is shown in the figure, wherein lane M depicts molecular weight marker positions (kDa); R: samples were treated with a reducing agent (such as β-mercaptoethanol); NR: no reducing agent was used. Based on this information, the correct structure of camel IgG3 is _____



A. two heavy chains bonded by disulfide bonds.

B. one heavy and one light chain bonded by disulfide bonds.

C. two heavy and two light chains bonded by disulfide bonds.

D. two light chains bonded by disulfide bonds.

18. Interspecies interactions are well known among soil-dwelling bacteria belonging to the genera *Bacillus* and *Pseudomonas*. When co-cultured, *Pseudomonas chlororaphis* competes for space with *Bacillus subtilis*, infiltrates their colonies and kill them. Killing of vegetative cells of *B. subtilis* by *P. chlororaphis* requires physical contact and the type-VI secretion system (T6SS) to inject the toxins. *B. subtilis* undergo sporulation and use it as an escape/defense mechanism to avoid killing by *P. chlororaphis*. On the other hand, when *P. chlororaphis* are co-cultured with *B. amyloliquefaciens*, colonies have distinct zones with no physical contact. In this scenario, *P. chlororaphis* utilizes a different secretion system, T2SS, to secrete secondary metabolites that inhibit the growth of *B. amyloliquefaciens*. Bacillaene, a bacteriostatic compound, is produced by *B. amyloliquefaciens* that antagonizes *P. chlororaphis*. The interaction between *P. chlororaphis* with *B. subtilis* and *P. chlororaphis* with *B. amyloliquefaciens*, respectively, represents:

    A. predation and competition.

    B. commensalism and competition.

    C. competition and predation.

    D. commensalism and mutualism.

19. Mitochondria are present in species **P**, but absent in **Q**, **R** and **S**. The species **P**, **Q**, **R** and **S**, respectively, are:

A. Slime mould, Blue-green algae, Eubacteria and Mycoplasma.

B. Blue-green algae, Slime mould, Eubacteria and Mycoplasma.

C. Plant, Slime mould, Eubacteria and Blue-green algae.

D. Mycoplasma, Eubacteria, Blue-green algae and Slime mould.

20. A researcher has cloned a bacteriophage (phage) gene ***P*** into a bacterial vector under the control of a phage promoter. To express this gene in bacteria, a particular strain of bacteria that carries a phage RNA polymerase is used. The phage RNA polymerase expression in turn can be induced by addition of an inducer to the medium. Rifampicin specifically inhibits bacterial RNA polymerase and not phage RNA polymerase. Chloramphenicol binds bacterial ribosomes and blocks protein synthesis. The condition where the researcher will be able to express maximum amount of the protein **P** in bacteria is _____.

    A. addition of the inducer and rifampicin

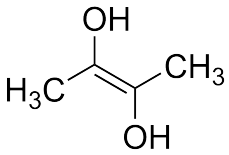
    B. addition of the inducer and chloramphenicol

    C. addition of rifampicin and chloramphenicol

    D. addition of only rifampicin

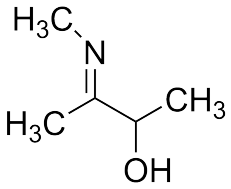
Chemistry - Set 2

1. **X** may exist in tautomeric form **Y**. **X** reacts with methylamine to form **Z**.



Y

The structure of **Z** is



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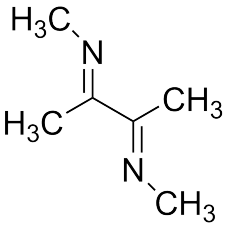

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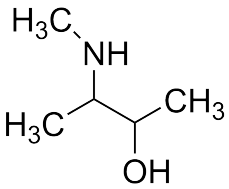

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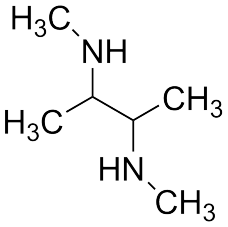
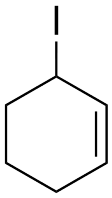


(d)

2. Cyclohexene on reaction with 1 equivalent of Cl_2 in the presence of light gives **P** (major product), which on reaction with 1 equivalent of NaI in dry acetone produces **Q**. The structure of **Q** is



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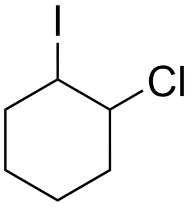

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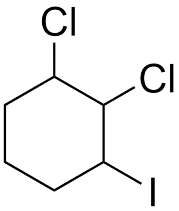

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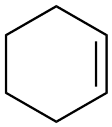
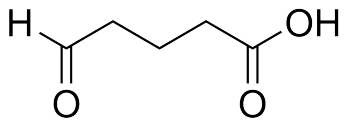


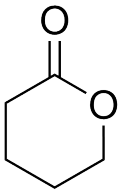
(d)

3. **X** on reaction with $NaBH_4$ gives **Y**, which upon treatment with catalytic amount of concentrated H_2SO_4 at 100 °C gives **Z** (major product).



X

The structure of **Z** is



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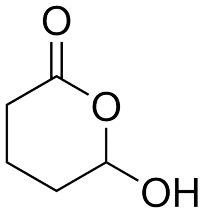

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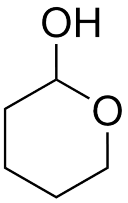

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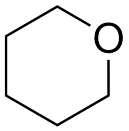
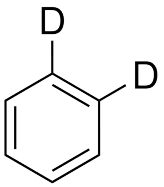
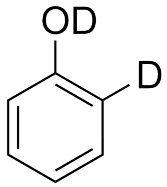


(d)

4. Phenol on treatment with dilute HNO_3 at room temperature gives a mixture of **P** and **Q**. The boiling point of **P** is lower than that of **Q**. **P** on reduction with Sn/HCl yields **R**. **R** on reaction with $NaNO_2$ and aqueous H_2SO_4 at 0−5 °C followed by reaction with an excess of C_2H_5OD gives **S**. The structure of **S** is



(a)



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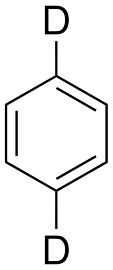

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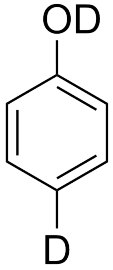
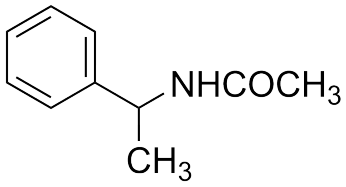


(d)

5. 2-Phenylpropanamide on treatment with Br_2/ aqueous NaOH gives **P**. **P** on reaction with ethanoic anhydride gives **Q**. The structure of **Q** is



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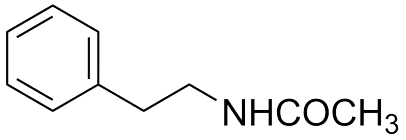

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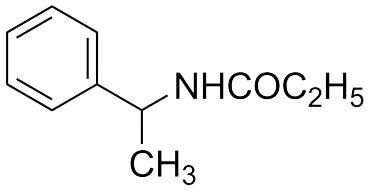

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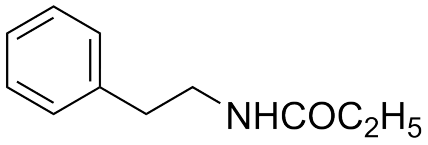
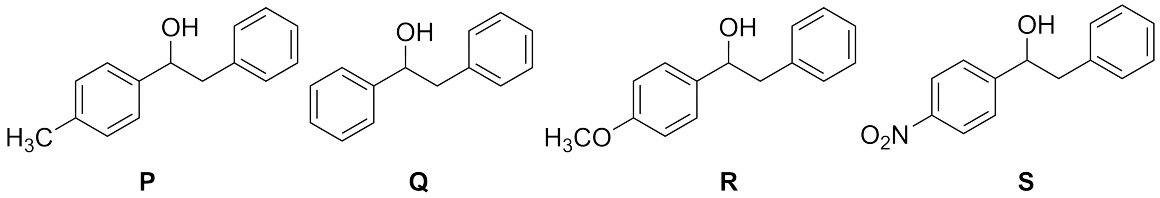


(d)

6. Consider the following compounds.



The correct order for the rate of acid catalyzed dehydration is

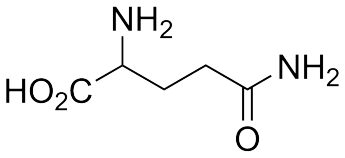
(a) P>Q>R>S

(b) R>P>Q>S

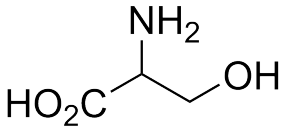
(c) S>R>P>Q

(d) S>P>Q>R

7. The amino acid whose aqueous solution turns red litmus blue is



(a)



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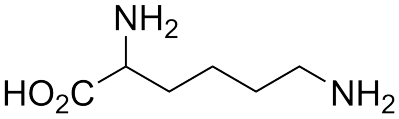

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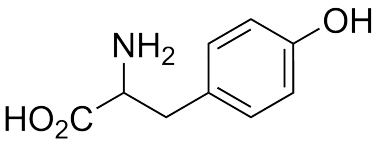


(d)

8. The correct order of reactivity of the metals Rb, Ba, Mo, and Ce with water is

   (a) Rb>Ba>Ce>Mo
   (b) Ce>Ba>Mo>Rb
   (c) Ba>Rb>Mo>Ce
   (d) Mo>Ce>Ba>Rb

9. Consider the coordination compounds $[Co(NH_3)_6]^{3+}$, $[CoCl(NH_3)_5]^{2+}$,$[Co(H_2O)(NH_3)_5]^{3+}$, and $[Co(CN)_6]^{3-}$, denoted respectively, by **P**, **Q**, **R**, and **S**. The correct order of wavelengths of light absorbed by the compounds is

   (a) **Q**>**P**>**R**>**S**

   (b) **Q**>**R**>**P**>**S**

   (c) **S**>**R**>**Q**>**P**

   (d) **S**>**R**>**P**>**Q**

10. The number of geometrical isomers possible for the complex $[Cr(NH_3)_2(H_2O)_2Cl_2]^+$ is

    (a) 3

    (b) 4

    (c) 5

    (d) 6

11. The element Seaborgium (Sg) with atomic number 106 has the electronic configuration [Ra] $5f^{14}$ $6d^4$ $7s^2$. The element with atomic number 114 is expected to be placed in

    (a) Group 13 (boron group)

    (b) Group 14 (carbon group)

    (c) Group 15 (nitrogen group)

    (d) Group 16 (oxygen group)

12. Cobalt(III) chloride on reaction with ammonia yields an octahedral compound **X**, which exists in two isomeric forms, one colored green and the other violet. Further, conductivity studies of **X** in solution reveal the presence of 1:1 electrolyte. The compound **X** is

    (a) $CoCl_3(NH_3)_6$

    (b) $CoCl_3(NH_3)_5$

    (c) $CoCl_3(NH_3)_4$

    (d) $CoCl_3(NH_3)_3$

13. Vanadium can exhibit different oxidation states V^{2+}, V^{3+}, V^{4+}, and V^{5+} in solution. The standard reduction potentials (E^0) for the vanadium couples V^{3+}/V^{2+}, V^{4+}/V^{3+}, and V^{5+}/V^{4+} are -0.25 volts, $+0.337$ volts and $+1.00$ volts, respectively. A solution of $(NH_4)_3VO_4$ in dilute HCl is treated with tin (E^0 for the couple Sn^{2+}/Sn is -0.14 volts). The oxidation state of vanadium in the final solution is

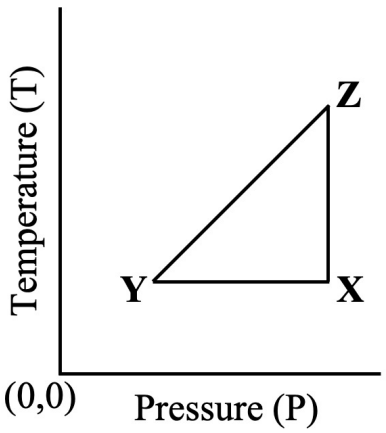
(a) $+5$

(b) $+4$

(c) $+3$

(d) $+2$

14. Consider n moles of an ideal gas undergoing a cyclic process $X(P_1,V_1,T_1) \rightarrow Y(P_2,V_2,T_2) \rightarrow Z(P_3,V_3,T_3) \rightarrow X(P_1,V_1,T_1)$ as shown below.



The correct option is

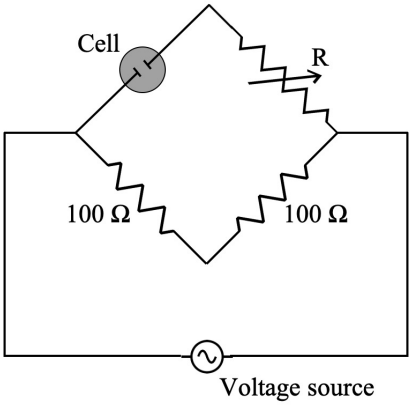
(a) The work done in the process Y→Z is zero

(b) The work done in the process Y→Z is $nRT_2 \ln \frac{P_2}{P_3}$

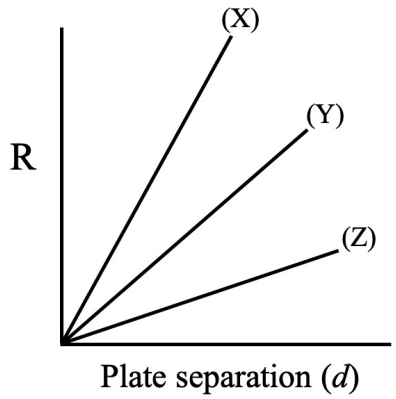
(c) In the process Z→X, ΔU is positive and ΔH is zero

(d) In the process X→Y, ΔU is zero and ΔH is positive

15. A Wheatstone bridge (Fig (i)) is setup with a conductivity measurement cell (shown in the figure as 'Cell') as one arm of the bridge. The cell has a provision of varying the separation (d) between the electrode plates keeping the area of the plates unchanged. R is resistance of the variable resistor required to achieve the balanced bridge condition. Fig (ii) (figure not drawn to scale) shows, for one molar aqueous solutions of different chloride electrolytes MCl (with different cations, M^+), the value of R as a function of d. In Fig (ii), the lines (X),(Y), and (Z) correspond, respectively, to MCl with M^+ as



(i)



(ii)

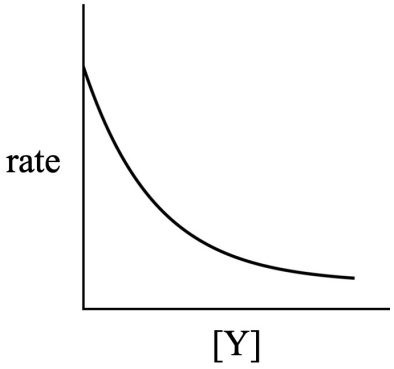
(a) H^+ , K^+, and Na^+

(b) Na^+, K^+ , and H^+

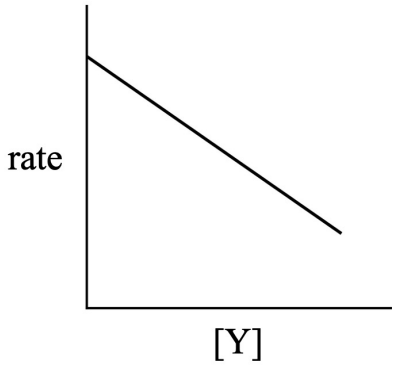
(c) H^+ , Na^+, and K^+

(d) K^+, Na^+ , and H^+

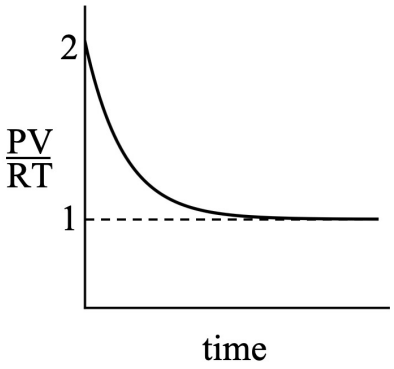
16. Consider the first order reaction $\mathbf{X}(g) \rightarrow 2\mathbf{Y}(g)$ studied as a function of time, at constant T and V. At any instant of time, the total pressure of the system is denoted by P while [**Y**] is the concentration of the species **Y**. Assuming ideal gas behaviour of the reaction mixture, and starting with 1 mole of pure $\mathbf{X}(g)$, the correct plot is



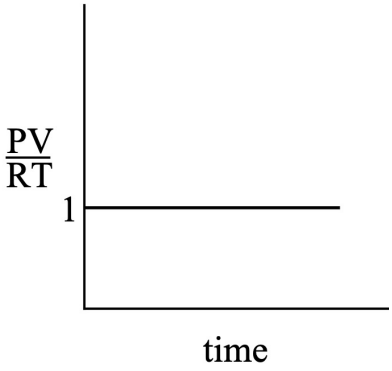
(a)



(b)



(c)



(d)

17. Solubility of a solute in some solvent is assumed to be expressed using an Arrhenius type equation where the activation energy is replaced by the enthalpy of solution and the Arrhenius constant is replaced by the solubility at infinite temperature.

    In a solubility study of two solutes **X** and **Y** in the same solvent it is found that when the temperature (keeping other conditions unchanged) is changed from T_1 to T_2, the solubility of **X** increases by a factor of 3 while that of **Y** increases by a factor of 2. The ratio of enthalpy of solution of **X** and that of **Y** is

    (a) 3/2

    (b) 9/4

    (c) ln(3/2)

    (d) ln3/ln2

18. It is known that 2s-2p mixing causes energy order swapping of certain molecular orbitals in some diatomic molecules. The molecular orbitals that swap their energies are

    (a) σ^*_{2p} and π^*_{2p}

    (b) σ^*_{2s} and σ_{2p}

    (c) σ_{2p} and π_{2p}

    (d) π^*_{2p} and π_{2p}

19. For the chemical reaction p$\mathbf{X}$+q$\mathbf{Y}$⇋r$\mathbf{Z}$, the standard free energy change is denoted by ΔG^0. For positive ΔG^0, the correct statement is

    (a) The reaction can never proceed in the forward direction, irrespective of initial conditions.

    (b) The reaction will always proceed in the backward direction, irrespective of initial conditions.

    (c) The reaction will proceed in the forward direction, if one has a mixture of X, Y and Z of unit concentration each.

    (d) The equilibrium constant is less than unity and the reaction can proceed in forward or backward direction, depending on the initial conditions.

20. Consider the reactions

Reaction 1: $\mathbf{X} \leftrightharpoons 3\mathbf{Y}$
Reaction 2: $\mathbf{P} \leftrightharpoons 2\mathbf{Q} + \mathbf{R}$

being performed in two different reaction vessels of identical fixed volume and at the same temperature. The initial concentrations of **X** and **P** are same. Also, the unreacted quantities of **X** and **P** at equilibrium are the same. The equilibrium constants (K_1 for reaction 1 and K_2 for reaction 2) are related as:

(a) $K_1 = K_2$

(b) $4K_1 = 27K_2$

(c) $2K_1 = 3K_2$

(d) $8K_1 = 9K_2$

Set-B

Notes and Notations

- Figures are for representational purposes only and are not drawn to scale.
- $\mathbb{N}$ denotes the set of all natural numbers.
- $\mathbb{Z}$ denotes the set of all integers.
- $\mathbb{R}$ denotes the set of all real numbers.
- $\mathbb{C}$ denotes the set of all complex numbers.
- For all $x \in \mathbb{R}$, $[x]$ denotes $\max\{m \in \mathbb{Z} : m \leq x\}$.
- The n^{th} order derivative of $f(x)$ is denoted by $f^{(n)}(x)$.

1. Let $f(X) = X^2 + 8X + 25$. Then the number of solutions in $\mathbb{R}$, of the equation

$$(f(X))^2 - 18f(X) + 80 = 0$$

is

A. 0

B. 1

C. 2

D. 4

2. Let $f : \mathbb{R} \to \mathbb{R}$ be a non-zero differentiable function such that $f(x^2) = xf(x)$. Then

   A. $f'(0) = f(0)$

   B. $f'(1) = f(1)$

   C. $f'(2) = f(2)$

   D. $f'(3) = f(3)$

3. Let N be the sum of all integers greater than 1000 and less than 2000. If $\frac{N}{x} \in \mathbb{N}$, then x is

   A. 5

   B. 7

   C. 11

   D. 13

4. The greatest common divisor of $n^2 + 4n + 3$ and $2n + 6$, where n is any natural number greater than 3, is equal to

   A. $n + 3$ for infinitely many values of n

   B. $n - 3$ for infinitely many values of n

   C. $n + 3$ for all n

   D. $n - 3$ for all n

5. Let a, b be the roots of the polynomial $3X^2 + 5X - 2$. Let A be any 3×3 matrix whose entries are from the set $S = \{a, b\}$. Then

   A. $\det(A) = 0$

   B. $0 \leq \det(A) \leq 48$

   C. $-48 \leq \det(A) \leq 0$

   D. $\det(A) \leq 30$

6. Let $f : \mathbb{Z} \to \mathbb{R}$ be a function such that $f(m+n)f(m)f(n) = 1$ for all m, n in $\mathbb{Z}$. Then

   A. $f(n+1) \geq f(n) \geq f(n-1)$ for all n

   B. $f(n+1) \leq f(n) \leq f(n-1)$ for all n

   C. $|f(n)| \leq 1$ for all n

   D. $f(n) > 0$ for all n

7. Let f be a polynomial such that $f(x) = x^3 + f'(0)x^2 + f''(1)x + 6$ for all $x \in \mathbb{R}$. Then

   A. $5f(1) + f(0) + f(2) = 4$

   B. $5f(0) + f(1) + f(2) = 3$

   C. $5f(2) + f(0) + f(1) = 5$

   D. $5f(0) + 5f(1) + f(2) = 4$

8. Let f and g be two distinct solutions of the ordinary differential equation

$$y'' + y = 0,$$

such that the function W defined by the determinant

$$W(x) = \begin{vmatrix} f(x) & g(x) \\ f'(x) & g'(x) \end{vmatrix}$$

is not identically zero. Then W satisfies

A. $W'' + W' + W = 0$

B. $W'' + W' - W = 0$

C. $W'' + W' = 0$

D. $W'' + W = 0$

9. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. For $0 < t < \pi$, let R_t be the region bounded by the curve $y = f(x)$, the line $x = 0$, the line $y = 0$ and the line $x = t$. Suppose the area of R_t is $\frac{t^2}{2}(1 - \cos t)$ for $0 < t < \pi$. Then

   A. $f(\frac{\pi}{2}) = \frac{\pi}{4}(1 + \frac{\pi}{4})$

   B. $f(\frac{\pi}{2}) = \frac{\pi}{2}(1 + \frac{\pi}{4})$

   C. $f(\frac{\pi}{2}) = \pi(1 + \frac{\pi}{8})$

   D. $f(\frac{\pi}{2}) = \frac{\pi}{4\sqrt{2}}(1 - \frac{\pi}{8})$

10. An example of a correct inequality is

A. $\int_0^1 x^{\frac{1}{3}}\ dx < \int_0^1 x^{\frac{1}{2}}\ dx$

B. $\int_0^1 x^3\ dx < \int_0^1 x^{\frac{1}{3}}\ dx$

C. $\int_1^2 x^3\ dx < \int_1^2 x^{\frac{1}{3}}\ dx$

D. $\int_1^2 x^{\frac{1}{2}}\ dx < \int_1^2 x^{\frac{1}{3}}\ dx$

11. Let $\alpha \geq 100$ and $N(\alpha)$ denote the total number of solutions of the equation $x^2 = \alpha|\sin x|$ in $(0, \infty)$. Then $N(\alpha)$ is

A. $\left[\frac{\sqrt{\alpha}}{\pi}\right]$

B. $\left[\frac{\sqrt{\alpha}}{\pi}\right] + 1$

C. $\left[\frac{2\sqrt{\alpha}}{\pi}\right] - 1$

D. $\left[\frac{2\sqrt{\alpha}}{\pi}\right]$

12. In a triangle ABC,

$$\sin A \sin B + \cos A \cos B \sin C = 1$$

and the length of one of its sides is 1. Let s be the sum of the other two sides of ABC. Then

A. s is always irrational and s^2 is always rational

B. s and s^2 are always irrational

C. either s^2 is rational or $(s-1)^2$ is rational

D. s is rational

13. Let $a, b \in \mathbb{R}$ and f be the function defined by

$$f(x) = \begin{cases} e^x, & \text{if } x < 5 \\ a + bx, & \text{if } x \geq 5. \end{cases}$$

If f is differentiable and f' is continuous on $\mathbb{R}$, then

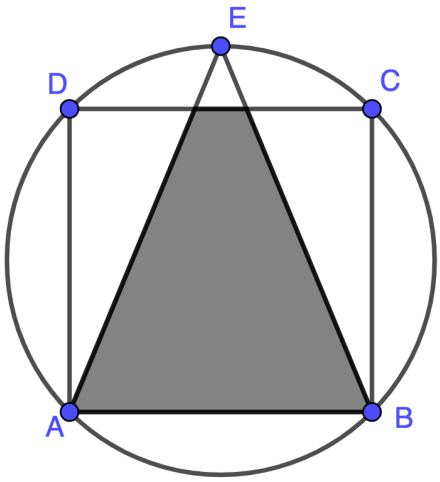
A. $a + 4b = 0$

B. $a - 4b = 0$

C. $a - 6b = 0$

D. $a + 6b = 0$

14. Let ABCD be a square inscribed in a circle of unit radius. Let E be the point on the circle such that ABE is an isosceles triangle, containing the centre of the circle (in its interior).



Then the area of the intersection of the square ABCD and the triangle ABE is

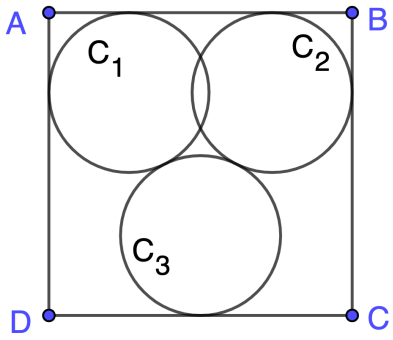
A. $\sqrt{5}/2$

B. $4 - 2\sqrt{2}$

C. less than 1

D. $\sqrt{3}$

15. Let $ABCD$ be a square with each side of length 1. Let C_1, C_2, C_3 be circles inside the square, such that C_1 touches side AB and AD, C_2 touches sides AB and BC, and C_3 touches the side CD and also the circles C_1 and C_2. Furthermore, suppose that C_1, C_2 and C_3 are of radius r.



Then r is equal to

A. $2 - \sqrt{3}$

B. $\frac{5}{2} - \sqrt{5}$

C. $\sqrt{3} - \sqrt{2}$

D. $2\sqrt{2} - \frac{5}{2}$

16. The sports club in a university changes its president every semester. Last year, the president for the first semester was chosen randomly from a collection of 80 students (boys and girls), out of whom 48 are boys. The probability that the president in the second semester will be of the same gender as the president in the first semester is 1/3. The records indicate that the president for the second semester was a girl. Given this information, the probability that the president in the first semester was a girl is

    A. 2/5

    B. 2/15

    C. 1/4

    D. 1/6

17. For $m, n \in \mathbb{Z}$, let $f_{m,n} : \mathbb{Z} \to \mathbb{Z}$ be the function defined as

$$f_{m,n}(x) = mx + n \quad \text{for all } \ x \in \mathbb{Z}.$$

Let $\mathcal{F}$ be the collection of all such functions, that is, $\mathcal{F} = \{f_{m,n} : m, n \in \mathbb{Z}\}$. Then

A. every element of $\mathcal{F}$ is bijective

B. $\mathcal{F}$ has infinitely many elements which are bijective and infinitely many elements which are not bijective

C. $\mathcal{F}$ has only finitely many elements which are bijective

D. no element of $\mathcal{F}$ is bijective

18. Let P and Q be the points of intersection of the hyperbola $x^2 - y^2 = 5$ and the line $2x + y = 5$. Let $(0, a)$ be the point where the perpendicular bisector of the segment PQ meets the y-axis. Then a is equal to

    A. $-10/3$

    B. $-21/6$

    C. $-11/3$

    D. $-23/6$

19. Let A be a nonzero 3×3 matrix with entries in $\mathbb{R}$. For any non-empty subset $\mathcal{S}$ of the set of all 3×3 matrices with entries in $\mathbb{R}$, we define a relation α_A on $\mathcal{S}$ by setting $X \alpha_A Y$ if $XA = AY$. Then α_A is an equivalence relation on $\mathcal{S}$

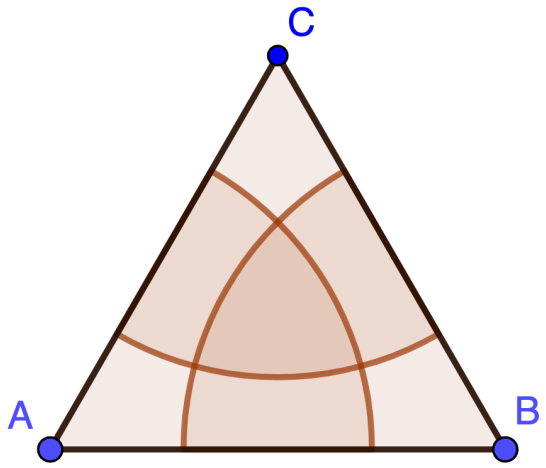
    A. for any subset $\mathcal{S}$

    B. if all matrices in $\mathcal{S}$ are symmetric

    C. if it is reflexive

    D. only if all elements of $\mathcal{S}$ are diagonal matrices

20. Let ΔABC be an equilateral triangle with each side being of length equal to 2. We construct three discs with centres at the points A, B and C respectively, each having radius equal to $\sqrt{2}$.



Then the area of the region common to all three discs is

A. $2\sqrt{3} - \pi$

B. $\sqrt{3} + \pi/2 - 3$

C. $\sqrt{3} + 16/9 - \pi$

D. $9/4 + \sqrt{3} - \pi$

NEST 2024 PHYSICS
Some Useful Constants

Acceleration due to gravity on Earth	g	≈ 10.0 m·s^{-2}
Boltzmann constant	k_B	1.38×10^{-23} J·K^{-1}
Current Mass of the Earth	M_e	6×10^{24} kg
Current Radius of the Earth	R_e	6.4×10^{6} m
Magnitude of the electron charge	e	1.60×10^{-19} C
Mass of the electron	m_e	9.11×10^{-31} kg
Mass of the proton	m_p	1.67×10^{-27} kg
Atomic Mass Unit	u	931.5 MeV/c^2
Permeability of free space	μ_0	1.26×10^{-6} H·m^{-1}
Permittivity of free space	ϵ_0	8.85×10^{-12} F·m^{-1}
Planck's constant	h	6.63×10^{-34} J·s
Avogadro Constant	N_A	6.022×10^{23} mol^{-1}
Speed of light in vacuum	c	3.00×10^{8} m·s^{-1}
Stefan-Boltzmann constant	σ	5.67×10^{-8} W·m$^{-2} \cdot$ K^{-4}
Universal Gas constant	R	8.31 J·K$^{-1} \cdot$ mol^{-1}
Universal Gravitational constant	G	6.67×10^{-11} kg^{-1}·m$^{3} \cdot$ s^{-2}
Wien's constant		2.90×10^{-3} m·K
$\ln 2$	$\approx$	0.69
$\ln 3$	$\approx$	1.10
$\ln 10$	$\approx$	2.30
$\ln 11$	$\approx$	2.40
π	$\approx$	3.14
Base of the Napierian logarithm e	$\approx$	2.72

PART-I (There are 20 questions, each with four choices. For each question select the one choice which best represents the answer.)

1. A particle of unit mass is moving along x-axis under the action of a force $F = (-1)^p kx^n$ where p and n are positive integers and k is a real positive constant. The position vs momentum plot of the particle is

   A. a hyperbola when $p = 1$ and $n = 2$.

   B. an ellipse when $p = 1$ and $n = 1$.

   C. a parabola when $p = 2$ and $n = 1$.

   D. a circle when $p = 0$ and $n = 2$.

2. A satellite in a circular orbit of radius 8 times the radius of the earth is completely stopped and then let go so that it falls radially towards the earth. A tunnel is built from the surface of the earth to its centre so that the satellite can pass through that tunnel without collision. Neglect the rotation of the earth and assume that the earth's gravity is the only relevant force. The speed of the satellite, in km.s^{-1}, on reaching the centre of the earth is closest to

A. 13

B. 0

C. 30

D. 20

3. Recent observations show that apart from the gravitational attraction, there is another force due to "dark energy" at large scales in the universe. This can be modelled in Newtonian physics by modifying the gravitational potential energy of a system of mass M and m separated by a distance r to include a universal repulsive force

$$V = -\frac{GMm}{r} - \frac{\Lambda m r^2}{6} \quad .$$

Here Λ is a positive constant called the cosmological constant. A particle is slightly displaced away from the equilibrium point. Select the correct option.

A. Acceleration is equal to Λ times the displacement from the equilibrium point and away from the equilibrium point.

B. Particle oscillates about the equilibrium point with angular frequency $\omega = \sqrt{\Lambda/3}$.

C. Equilibrium point is at $r = (6GM/\Lambda)^{1/3}$.

D. Acceleration is equal to $\Lambda/3$ times the displacement from the equilibrium point and towards the equilibrium point.

4. Consider a steel ball of mass 20 kg inside an ice cube which is floating on water in a container. The density of the steel is 8000 kg.m^{-3}. If the cross-sectional area of the container is 1 m^2 then the change in the water level when the ice melts is closest to

    A. 1.7 cm with water level rising

    B. 1.7 cm with water level falling

    C. 2.2 cm with water level rising

    D. 2.2 cm with water level falling

5. The energy of a one-dimensional system is given by

$$E = \int dx \left[\left(\frac{d\phi(x)}{dx} \right)^2 + ce^{a\phi(x)} + b\phi^4(x) \right]$$

where a, b and c are some non-zero constants. The dimension of the quantity b/a is given by

A. $[ML^{-1}T^{-2}]$

B. $[M^{-1}L^{-1}T^2]$

C. $[M^{-1/2}L^{-7/2}T]$

D. $[M^{-2}L^{-4}T^4]$

6. An electric charge distribution produces an electric field $\vec{E} = E_0 e^{-\alpha r}\vec{r}/r^3$. Here α is a non-zero constant. The charge Q contained in a spherical region of radius $1/\alpha$ is

A. $4\pi E_0 \epsilon_0 (e-1)/(e\alpha)$

B. $4\pi E_0 \epsilon_0 (e-1)/\alpha$

C. $4\pi E_0 \epsilon_0/\alpha$

D. $4\pi E_0 \epsilon_0/(e\alpha)$

7. A square loop of side length a is kept in the y-z plane with one of its arms lying along z-axis in a uniform magnetic field $\sqrt{2}\,(\hat{i}+\hat{j})$ T. If a current of 1.0 A passes through the loop in counter-clockwise direction, then instantaneous torque (in SI units) on the loop about the z-axis is

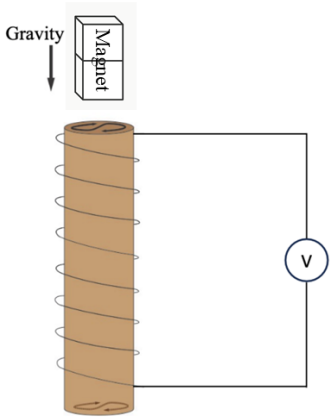
A. 0

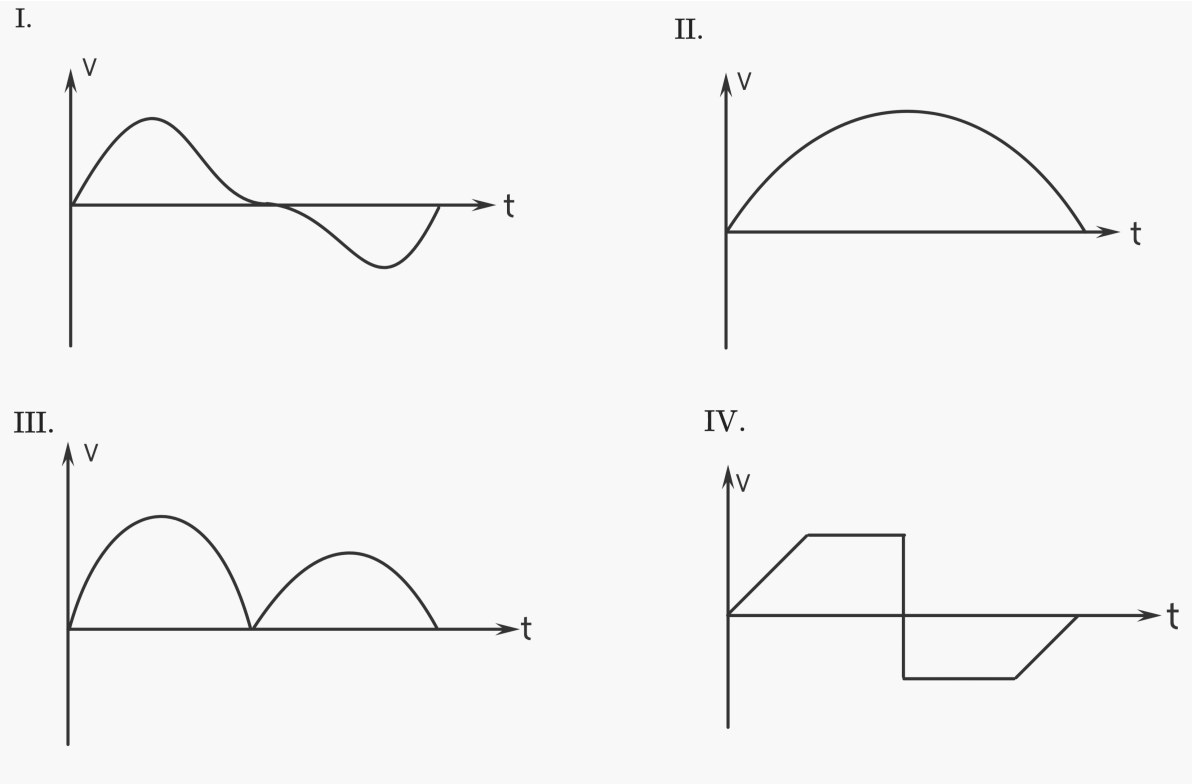
B. $\sqrt{2}a^2\hat{k}$

C. $a^2\hat{k}$

D. $a^2/\sqrt{2}\hat{k}$

8. A bar magnet is falling vertically under gravity through a solenoid. Select the figure that best describes the induced voltage V as a function of time t.





A. I

B. II

C. III

D. IV

9. Consider a region with uniform electric field and uniform magnetic field which are mutually perpendicular.Their magnitudes are 1.0 $V.m^{-1}$ and 0.1 T, respectively. A charged particle of mass 1.0 gram enters this region with a velocity perpendicular to the electric field. The particle moves with a constant velocity making an angle of 45° with the magnetic field. Neglecting gravity, the kinetic energy of the particle, in joules, on entering the region is

    A. 1.0

    B. 1.2

    C. 10.0

    D. 0.1

10. A resonating tube of length l, open at one end, is partially filled with water. When placed in gaseous *medium*-X, a water column of height $l/5$ generates the third harmonic using a tuning fork with frequency ν. When placed in gaseous *medium*-Y, a water column of height $l/3$ generates the fifth harmonic using the same tuning fork.The ratio $v_{\mathrm{X}} : v_{\mathrm{Y}}$ of the velocity of sound in *medium*-X to *medium*-Y is

    A. 2 : 1

    B. 4 : 1

    C. 1 : 4

    D. 1 : 2

11. The power of a system of two thin lenses which share a common axis and are separated by a distance d is given by $P = P_1 + P_2 - P_1 P_2 d$. Consider two thin lenses with focal lengths $f_1 = 30.0$ cm and $f_2 = 40.0$ cm. There is an object in front of the first lens at a distance $x > f_1$. A real image is created on the screen behind the second lens. If the image on the screen is of the same size regardless of the position of the object, the distance(in cm) between the lenses is

    A. 10.0

    B. 35.0

    C. 70.0

    D. 17.5

12. Consider a Young double slit experiment with pinholes at coordinates $(0, 0, a)$ and $(0, 0, -a)$ and a screen at $y = h$. When a light of wavelength λ is used, the shape of the second bright fringe is a hyperbola with eccentricity

A. a/λ

B. $\sqrt{2}$

C. ah/λ^2

D. $2a/\lambda$

13. Assume that the perfect gas law is applicable to an assembly of Hydrogen atoms. The temperature (in Kelvin) at which the average kinetic energy of the assembly is equal to the binding energy of a Hydrogen atom is closest to

A. 10^5

B. 10^2

C. 10^3

D. 10^4

14. In a room at temperature 20° C, a cup of coffee cools from 80° C to 75° C in 2 minutes.The time, in minutes, taken by the coffee to cool from 40° C to 35° C is closest to

    A. 2

    B. 6

    C. 4

    D. 8

15. An ideal gas undergoes expansion via various processes. Select the correct option.

    A. Temperature decreases in the isobaric process but increases in the adiabatic process.

    B. Temperature increases in both adiabatic and isobaric processes.

    C. Temperature decreases in adiabatic but increases in the isobaric process.

    D. Temperature decreases in both adiabatic and isobaric process.

16. Assume that when $^{235}_{92}$U undergoes fission, about 0.1 percent of the original mass is released as energy. When coal is burned, about 32.6×10^6 J.kg^{-1} of heat is liberated. The ratio of the amount of $^{235}_{92}$U to coal required to power a 100 MW electric power plant per day is closest to

A. 1kg of $^{235}_{92}$U: 3×10^3 kg of coal

B. 100 g of $^{235}_{92}$U : 3×10^5 kg of coal

C. 10 g of $^{235}_{92}$U: 3×10^6 kg of coal

D. 1 g of $^{235}_{92}$U : 3×10^4 kg of coal

17. Assuming that the Bohr's theory applies and the number of protons and neutrons in the nucleus is approximately same, the maximum atomic number for a hydrogenic atom is closest to

    A. 127

    B. 140

    C. 92

    D. 137

18. In a sequence of two consecutive steps, a hydrogen atom in its ground state absorbs two photons and finally reaches its $n = 7$ excited state. The energy of the first incident photon is found to be 12.47 eV, with an error margin of $\pm$ 0.32 eV. The energy, in eV, of second absorbed photon is closest to

A. 0.57

B. 1.45

C. 0.81

D. 3.40

19. Two identical samples of an ideal gas in identical conditions are separately heated by wires carrying equal currents for the same amount of time. In the first sample, the volume is kept constant while pressure becomes 8 times the initial pressure. In the second sample, the pressure is kept constant while the volume becomes 6 times the initial volume. The gas is composed of

    A. monoatomic molecules.

    B. rigid, diatomic molecules

    C. diatomic molecules with one vibrational mode.

    D. polyatomic molecules with one vibrational mode.

20. The radius of nucleus of an isotope ${}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}$ is $1.5\times10^{-15}\mathrm{A}^{1/3}$ m. Assume that the number of protons and neutrons in the nucleus is same and the mass $m_{proton} = m_{neutron}$. The approximate value of the nuclear density, in kg.m^{-3}, is closest to

A. 10^{17}

B. 10^{19}

C. 10^{13}

D. 10^{15}