GOVERNMENT OF KARNATAKA
KARNATAKA SCHOOL EXAMINATION AND ASSESSMENT BOARD
WEIGHTAGE FRAMEWORK FOR MQP 3: II PU MATHEMATICS (35):2024-25

Chapter	CONTENT	Number of Teaching hours	PART A 1 mark		PART B 2 mark	PART C 3 mark	PART D 5 mark	PART E		Total
			MCQ	FB				6 mark	4 mark	
1	RELATIONS AND FUNCTIONS	9	1			1	1			9
2	INVERSE TRIGONOMETRIC FUNCTIONS	6	2		2					6
3	MATRICES	9	1	1	1		1			9
4	DETERMINANTS	12	1		1		1		1	12
5	CONTINUITY AND DIFFERENTIABILITY	20	2	1	1	1	1		1	17
6	APPLICATION OF DERIVATIVES	10	2	1	1	1				8
7	INTEGRALS	22	1	1	1	1	1	1		18
8	APPLICATION OF INTEGRALS	5					1			5
9	DIFFERENTIAL EQUATIONS	10	1		1		1			8
10	VECTOR ALGEBRA	11	2			2				8
11	THREE D GEOMETRY	8	1		1	1				6
12	LINEAR ROGRAMMING	7						1		6
13	PROBABILITY	11	1	1		2				8
	TOTAL	**140**	**15**	**5**	**9**	**9**	**7**	**2**	**2**	**120**

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Model Question Paper -3

II P.U.C MATHEMATICS (35):2024-25

Time : 3 hours **Max. Marks : 80**

Instructions :

1) *The question paper has five parts namely A, B, C, D and E. Answer all the parts.*
2) *PART A has 15 MCQ's ,5 Fill in the blanks of 1 mark each.*
3) *Use the graph sheet for question on linear programming in PART E.*

PART A

I. Answer ALL the Multiple Choice Questions **15×1 = 15**

1. The element needed to be added to the relation R={(1,1), (1,3), (2,2),(3,3) } on A = {1, 2, 3} so that the relation is neither symmetric nor transitive

A) (2, 3) B) (3, 1) C) (1, 2) D) (3, 2)

2. The graph of the function $y = \cos^{-1} x$ is the mirror image of the graph of the function y = cosx along the line

A) x = 0 B) y = x C) y = 1 D) y = 0

3. The value of $\tan^{-1}(\sqrt{3}) + \sec^{-1}(-2)$ is equal to

A) π B) $\frac{2\pi}{3}$ C) $-\frac{\pi}{3}$ D) $\frac{\pi}{3}$

4. If A and B are matrices of order 3 × 2 and 2 × 2 respectively, then which of the following are defined

A) AB B) BA C) A^2 D) A + B

5. A square matrix A is invertible if A is

A) Null matrix B) Singular matrix

C) skew symmetric matrix of order 3 D) Non-Singular matrix

6. If $y = \sin^{-1}(x\sqrt{x})$, then $\frac{dy}{dx} =$

A) $\frac{1}{\sqrt{1-x^3}}$ B) $\frac{2\sqrt{x}}{3\sqrt{1-x^3}}$ C) $\frac{3\sqrt{x}}{2\sqrt{1-x^3}}$ D) $\frac{-3\sqrt{x}}{2\sqrt{1-x^3}}$.

7. If $y = x^a + a^x + a^a$ for some fixed $a > 0$ and $x > 0$, then $\frac{dy}{dx} =$

(A) $ax^{a-1} + a^x \log a + aa^{a-1}$ B) $ax^{a-1} + a^x \log a$

C) $ax^{a-1} + xa^{x-1} + aa^{a-1}$ D) $ax^{a-1} + a^x \log a + a^a$.

8. Consider the following statements for the given function y=f(x) defined on an interval I and c$\in$ I, at x = c

I. $f'(c) = 0$ and $f''(c) < 0 \Rightarrow$ f attains local maxima
II. $f'(c) = 0$ and $f''(c) > 0 \Rightarrow$ f attains local minima
III. $f'(c) = 0$ and $f''(c) = 0 \Rightarrow$ f attains both maxima and minima

A) I and II are true B) I and III are true

C) II and III are true D) all are false

9. If each side of a cube is x units, then the rate of change of its surface area with respect to side is

A) 12x B) 6x C) $6x^2$ D) $3x^2$

10. Statement 1: The anti-derivative of $\left(\frac{1}{\sqrt{1+x^2}}\right)$ with respect to x is

$$\frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\log\left|x + \sqrt{1+x^2}\right| + C.$$

Statement 2**:** The derivative of $\frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\log\left|x + \sqrt{1+x^2}\right| + C$ with respect to x is $\frac{1}{\sqrt{1+x^2}}$.

A) Statement 1 is true, and Statement 2 is false.
B) Statement 1 is true, and Statement 2 is true, Statement 2 is correct explanation for Statement 1
C) Statement 1 is true, and Statement 2 is true, Statement 2 is not a correct explanation for Statement 1
D) Both statements are false.

11. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ is

A) 2 B) 3 C) 5 D) not defined

12. The position vector of a point which divides the join of points with position vectors $3\vec{a} - 2\vec{b}$ and $\vec{a} + \vec{b}$ externally in the ratio 2 : 1 is

A) $\frac{5\vec{a}}{3}$ B) $4\vec{a} - \vec{b}$ C) $4\vec{b} - \vec{a}$ D) $2\vec{a} + \vec{b}$

13. If a vector $\vec{a}$ makes angles with $\frac{\pi}{3}$ with $\hat{\imath}$ *and* $\frac{\pi}{4}$ with $\hat{\jmath}$ and an acute angle θ with $\hat{k}$, then θ is

A) $\frac{\pi}{6}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{2}$

14. Find the angle between the lines whose direction ratios are a, b, c and $b - c$, $c - a$, $a - b$ is

A) 45^0 B) 30^0 C) 60^0 D) 90^0

15. If A and B are two independent events such that $P(A) = \frac{1}{4}$ and $P(B) = \frac{1}{2}$ then P(neither A nor B)

A) $\frac{1}{3}$ B) $\frac{3}{8}$ C) $\frac{7}{8}$ D) $\frac{1}{2}$.

II. Fill in the blanks by choosing the appropriate answer from those given in the bracket (-2, $\frac{5}{2}$, 0, 1, 2, $\frac{3}{2}$) **5×1 = 5**

16. The number of all possible orders of matrices with 13 elements is ____

17. If $y = 5 \cos x - 3 \sin x$, then $\frac{d^2y}{dx^2} + y =$_____

18. If the function f given by $f(x) = x^2 + ax + 1$ is increasing on [1, 2], then the value of 'a' is greater than ___________

19. $\int_1^2 |x| dx =$________

20. If A and B are any two events such that P(A) + P(B) – P(A and B) =P(A), then P(A|B) is________

PART B

Answer any SIX questions **6 × 2=12**

21. Write the simplest form of $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$, $0 < x < \pi$.

22. Prove that $2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$.

23. If F(x)= $\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then show that $F(x)\,F(y) = F(x + y)$.

24. Find the equation of line joining (1, 2) and (3, 6) using determinants.

25. Differentiate $x^{\sin x}$, $x > 0$ with respect to x.

26. Find the intervals in which the function f given by $f(x) = x^2 e^{-x}$ is increasing.

27. Find $\int (x^2 + 1)\log x\, dx$.

28. Verify the function y =mx is the solution of $\frac{dy}{dx} - y = 0$, $x \neq 0$.

29. Find the distance between the lines $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$.

PART C

Answer any SIX questions **6 × 3= 18**

30. Let f : X →Y be a function. Define a relation R in X given by R = {(a, b): f(a) = f(b)}. Examine whether R is an equivalence relation or not.

31. If $x^3 + x^2y + xy^2 + y^3 = 81$, then find $\frac{dy}{dx}$.

32. The length x of a rectangle is decreasing at the rate of 3 cm/min and the width y is increasing at the rate of 2 cm/min. When $x = 10$ cm and $y = 6$ cm, find the rate of change of the perimeter of the rectangle.

33. Find the integral of $\frac{1}{a^2 + x^2}$ with respect to x .

34. If the vertices A, B and C of a triangle are $(1, 2, 3)$, $(-1, 0, 0)$ and $(0, 1, 2)$ respectively, then find the angle $\angle ABC$.

35. Find the area of the rectangle, whose vertices are $A\left(-\hat{i} + \frac{1}{2}\hat{j} + 4k\right)$, $B\left(\hat{i} + \frac{1}{2}j + 4k\right)$, $C\left(\hat{i} - \frac{1}{2}j + 4k\right)$ and $D\left(-\hat{i} - \frac{1}{2}j + 4k\right)$.

36. Find the vector equation of the line passing through the point (1, 2, – 4) and perpendicular to the two lines: $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

37. An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red.

38. Three coins are tossed simultaneously. Consider the Event E ‘three heads or three tails’, F ‘at least two heads and G ‘at most two heads’. Of the pairs (E,F), (E, G) and (F, G), which are independent? Which are dependent?

PART D

Answer any FOUR questions **4 × 5 = 20**

39. Let f : N →Y be a function defined as $f(x) = 4x + 3$, where, Y = {y ∈ N: $y = 4x + 3$ for some x ∈ N}. Show that f is invertible. Find the inverse of f.

40. If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$, calculate AB, AC and A(B + C). Verify that $A(B+C) = AB + AC$.

41. Solve the following system of linear equations by matrix method:

$2x + y + z = 1, \quad x - 2y - z = \frac{3}{2}$ and $\quad 3y - 5z = 9.$

42. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find $\frac{d^2y}{dx^2}$.

43. Find $\int \frac{x^4}{(x-1)(x^2+1)} dx$.

44. Find the area of the region bounded by the line $y = 3x + 2$, the x-axis and the ordinates $x = -1$ and $x = 1$ by integration method.

45. Find the equation of a curve passing through the origin given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the ordinates of the point.

PART E

Answer the following questions:

46. (a) Prove that $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_{0}^{a} f(x)dx, & \text{if } f(x) \text{ is even} \\ 0, & \text{if } f(x) \text{ is odd} \end{cases}$

and evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x \, dx$

OR

Solve the following linear programming problem graphically:

Minimize and maximize $Z = x + 2y$, subject to constraints

$x + 2y \geq 100, \; 2x - y \leq 0, \; 2x + y \leq 200 \; and \; x, y \geq 0.$

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47. If matrix A= $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ satisfying $A^3-6A^2+9A-4I=O$, then evaluate A^{-1}.

OR

If $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & If \; x \neq \frac{\pi}{2} \\ 3, & if \; x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, find k . **4**

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