GS2025 Physics entrance question paper(IPhD)

Section A

A1 Consider the triangle subtended on the surface of a sphere of radius 1 by joining the points $(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0)$, $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$, and $(0, 0, 1)$ with arcs of great circles. The area subtended by this triangle on the surface of the sphere is given by:

(Hint: Drawing a figure might help.)

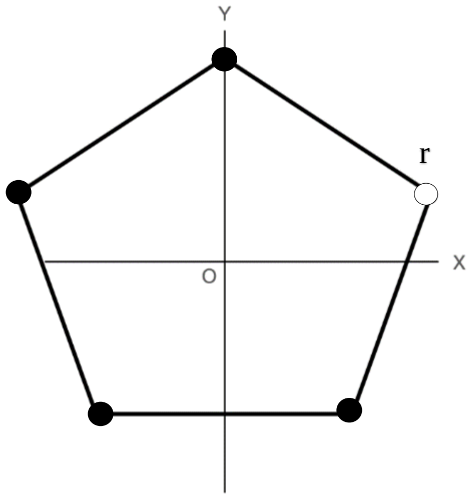
(a) $\pi/3$

(b) $\sqrt{3}\pi/2$

(c) $\sqrt{3}\pi$

(d) $2\pi/3$

A2 The Figure on the right shows a regular pentagon. The black solid circles on its vertices represent point charges with charge $-q$. There is no charge at the position of the white circle at **r** (measured from the origin, O, placed at the centre of the pentagon). The electric field at O is given by:



(a) $$\mathbf{E} = \frac{-q\,\mathbf{r}}{4\pi\epsilon_0 r^3}$$

(b) $$\mathbf{E} = \frac{-4q\,\mathbf{r}}{4\pi\epsilon_0 r^3}$$

(c) $$\mathbf{E} = \frac{q\,\mathbf{r}}{4\pi\epsilon_0 r^3}$$

(d) $$\mathbf{E} = \frac{-4q(\sin\frac{\pi}{10}\,\hat{\mathbf{x}} + \cos\frac{\pi}{10}\hat{\mathbf{y}})}{4\pi\epsilon_0 r^2}$$

A3 Consider a two dimensional insulating solid crystal. At low temperature, how does the specific heat at constant area $c_a = \frac{d\mathcal{E}}{dT}$, where $\mathcal{E}$ is the energy per unit area, depend on T?

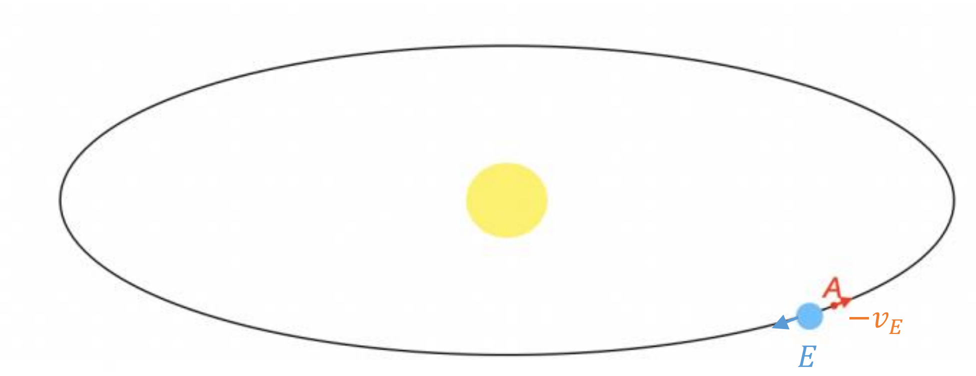
(a) $c_a \sim T^2$

(b) $c_a \sim T^3$

(c) $c_a \sim T$

(d) c_a is independent of T

A4 The Figure below shows a rocket (red arrow) launched from the earth which is now at a point A where the Earth's gravitational field is negligible. The rocket thrusters have stopped. In the rest frame of the Sun, the velocity of the rocket at A is same in magnitude but opposite in direction to that of the earth was, when it was at the same point. Which of the following statements is correct?



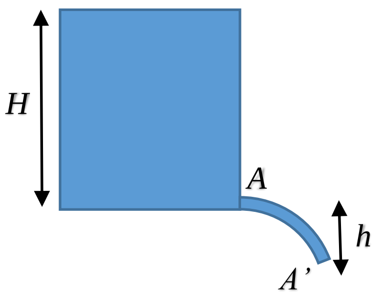
(a) The rocket will move exactly on the earth's elliptical orbit shown in the figure and eventually collide with the earth

(b) The rocket will eventually escape the Sun's gravitational field

(c) The rocket will eventually reverse its direction and follow the earth

(d) The rocket will turn towards the sun and eventually collide with it

A5 Water is flowing out of a small horizontal opening of area, A, at the bottom of a tank of height H. The flow is a laminar flow under the influence of gravity. What is the area, A', of the stream transverse to the fluid velocity, at a height h below the opening? (Neglect atmospheric pressure and dissipation effects. The thickness of the stream is negligible compared to H and h.)



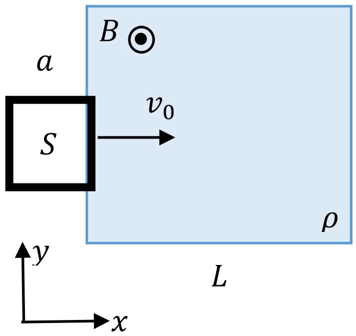
(a) $A\sqrt{\frac{H}{h+H}}$

(b) $A\frac{H}{h+H}$

(c) $A(1+\frac{h}{H})$

(d) $A\sqrt{1+\frac{h}{H}}$

A6 A small metallic wire with mass m and electric resistance R is bent into a closed square shape S with sides a. It passes through a region ρ of length $L > a$ with magnetic field $B\hat{z}$. The initial velocity $v_0\hat{x}$ of the square is large enough that it emerges out of ρ from the right. What is the final velocity of S after it completely emerges from ρ?



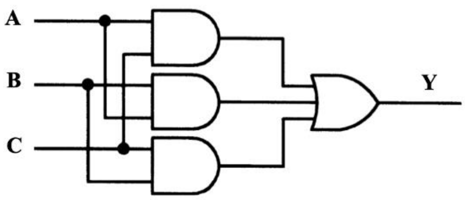
(a) $v_0\left(1-\frac{a^3B^2}{mRv_0}\right)^2\hat{x}$

(b) $v_0e^{-\frac{a^2B^2L}{mRv_0}}\hat{x}$

(c) $v_0(1-\frac{a^3B^2}{mRv_0})\hat{x}$

(d) $v_0\hat{x}$

A7 The output pulse train Y of the circuit shown on the right, with three synchronized input trains,



$$A = 00001111$$

$$B = 00110011$$

$$C = 01010101$$

will be:

(a) 00010111

(b) 00100111

(c) 01010101

(d) 00010001

A8 Consider a stationary electron in a uniform, time-independent magnetic field of strength $B_0/4$ oriented in the $\hat{z}$-direction. The Hamiltonian for this system is expressed as

$$H = -\frac{e}{m}\boldsymbol{S}.\boldsymbol{B}$$

where $\boldsymbol{S}$ is the spin-$1/2$ operator for electrons. The initial electron spin is oriented in the $\hat{x}$-direction. The spin precession frequency of the electrons is:

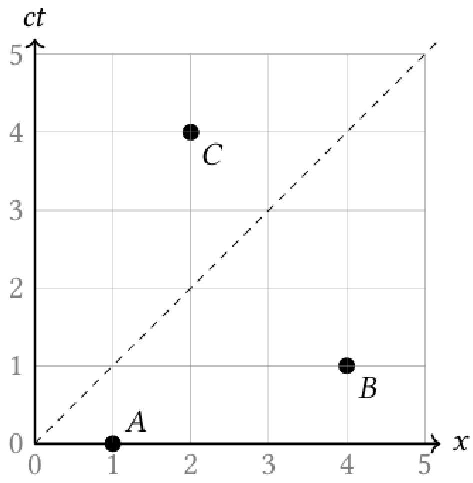
(a) $\frac{|e|\hbar B_0}{4m}$

(b) $\frac{|e|\hbar B_0}{8m}$

(c) $\frac{|e|\hbar B_0}{2m}$

(d) 0

A9 Consider the following space-time diagram which indicates three events A, B and C for an inertial observer. Which of the following statements is true?



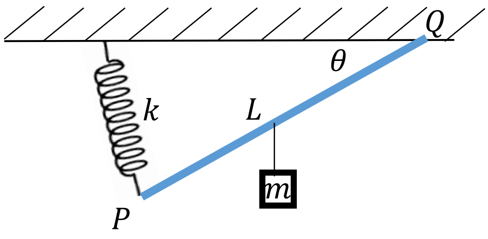
(a) It is always possible to find an inertial observer for whom events A and B are simultaneous. However, no inertial observer can be found for whom events A and C are simultaneous.

(b) It is always possible to find an inertial observer for whom events A and C are simultaneous. However, no inertial observer can be found for whom events A and B are simultaneous.

(c) It is always possible to find an inertial observer for whom events A and B are simultaneous. Similarly, an inertial observer can also be found for whom events A and C are simultaneous.

(d) It is impossible to find an inertial observer for whom events A and B are simultaneous. Similarly, no inertial observer can be found for whom events A and C are simultaneous.

A10 A massless rigid rod of length L is suspended with an ideal spring of spring constant k at one end P, and by a hinge on the other end, Q. The rest length of the spring is zero. A mass m is suspended from the mid-point of the rod. This results in tilting of the rod by angle θ. What is the angle θ?



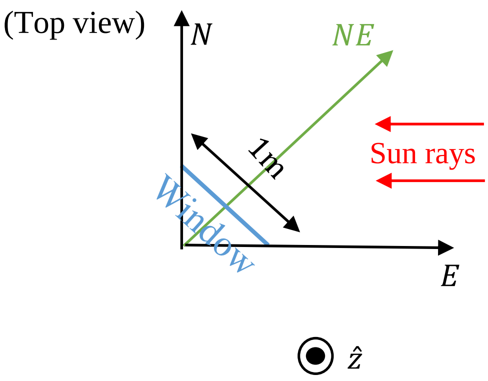
(a) $\tan^{-1}\left(\frac{mg}{2kL}\right)$

(b) $\sin^{-1}\left(\frac{mg}{2kL}\right)$

(c) $\cos^{-1}\left(\frac{mg}{kL}\right)$

(d) $\sec^{-1}\left(\frac{mg}{kL}\right)$

A11 There is an open window of dimension 1m × 1m on the north east (NE) facing wall of a house. At 9 AM, the sun shines through the window and illuminates a certain part of the floor of the house. What is the area A illuminated by the sun? (Assume that the sun rises in the east (E) at 6 AM and is directly overhead ($\hat{z}$) at 12 noon.)



(a) $\frac{1}{\sqrt{2}}$ m^2

(b) 1 m^2

(c) $\frac{1}{2}$ m^2

(d) $\sqrt{2}$ m^2

A12 For a one dimensional quantum harmonic oscillator, at time $t = 0$, the particle is in the ground state. What is the expectation value of the position and momentum operator at time t?

(a) $\langle x(t)\rangle = \langle p(t)\rangle = 0$

(b) $\langle x(t)\rangle = \sqrt{\frac{\hbar}{m\omega}} \sin \omega t \,, \langle p(t)\rangle = 0$

(c) $\langle x(t)\rangle = \sqrt{\frac{\hbar}{m\omega}} \sin \omega t \,, \langle p(t)\rangle = \sqrt{\hbar m\omega} \cos \omega t$

(d) $\langle x(t)\rangle = 0, \langle p(t)\rangle = \sqrt{\hbar m\omega} \cos \omega t$

A13 Consider two ideal gases A and B with atomic masses m_A and m_B respectively such that $m_A > m_B$. The two gases with same number of moles are kept at the same temperature and confined in containers with the same volume. Which of the gases will exert more pressure and molecules of which gas will have a higher RMS momentum?

(a) Both will exert the same pressure but molecules of Gas A will have more RMS momentum

(b) Gas A will exert more pressure and molecules of Gas B will have more RMS momentum

(c) Gas B will exert more pressure but molecules of Gas A will have more RMS momentum

(d) Both will exert the same pressure and molecules of both gases have the same RMS momentum

A14 For a given measurement of particles in a counter, a 10-minute data collection resulted in a statistical uncertainty of 2.5%. How much additional time must be allocated to reduce the statistical uncertainty to 0.5%?

(a) 240 minutes

(b) 40 minutes

(c) 250 minutes

(d) 50 minutes

A15 Laser light is incident normally on a thin film of material with a refractive index (n_s) larger than that of air ($n_a \approx 1$). As the wavelength of the laser light is varied, the intensity of the transmitted light through the film shows a peak at 633 nm. If the thickness of the film is 118 nm, the minimum n_s is closest to:

(a) 2.68

(b) 5.36

(c) 1.34

(d) 3.68

A16 The asymptotic expansion of the following function for $x \to \infty$

$$x \tanh^{-1} \frac{1}{x}$$

is given by:

(a) $1 + \frac{1}{3x^2} + \frac{1}{5x^4} + \frac{1}{7x^6} + \cdots$

(b) $1 - \frac{1}{3x^2} + \frac{1}{5x^4} - \frac{1}{7x^6} + \cdots$

(c) $x + \frac{1}{2x} + \frac{1}{4x^3} + \frac{1}{6x^5} + \cdots$

(d) $1 + \frac{1}{2x^2} + \frac{1}{4x^4} + \frac{1}{6x^6} + \cdots$

A17 The $n \times n$ $(n > 4)$ matrix M, with all entries equal to 1 has:

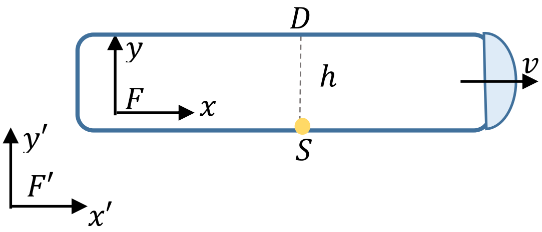
(a) Precisely $n - 1$ degenerate eigenvalues and one other non-degenerate eigenvalue

(b) Precisely $n - 2$ degenerate eigenvalues and two other non-degenerate eigenvalues

(c) Precisely 2 degenerate eigenvalues and $n - 2$ other non-degenerate eigenvalues

(d) No degenerate eigenvalues

A18 A spaceship is moving with a constant relativistic velocity $v\hat{x}'$ with respect to an inertial frame F'. In the frame F moving with spaceship, light is emitted from the source S and is detected at the detector D with displacement $h\hat{y}$ from S. In the frame F', what is the time t' taken for the light to reach from S to D?



(a) $$\frac{\left(\frac{h}{c}\right)}{\sqrt{1 - v^2/c^2}}$$

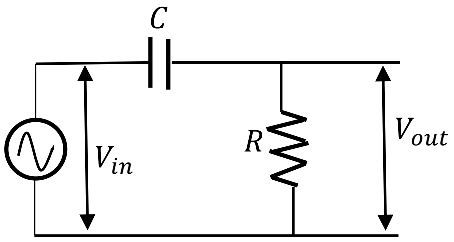
(b) $$\left(\frac{h}{c}\right)\sqrt{1 - v^2/c^2}$$

(c) $$\left(\frac{h}{c}\right)\sqrt{\frac{1 - v/c}{1 + v/c}}$$

(d) $$\left(\frac{h}{c}\right)$$

A19 The sinusoidal signal

$$V_{in} = V_i \sin(2\pi f t)$$



is given to a high-pass filter (see Figure). The output signal is given by

$$V_{out} = V_i\,|A| \sin(2\pi f t + \phi).$$

What is the value of $|A|$?

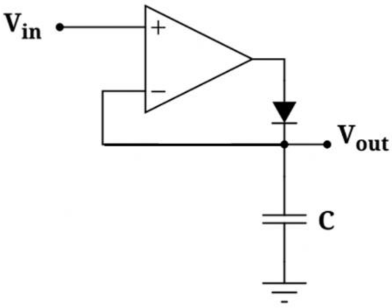
(a) $$\frac{1}{\left|1 + \left(\frac{1}{2\pi RCf}\right)^2\right|^{1/2}}$$

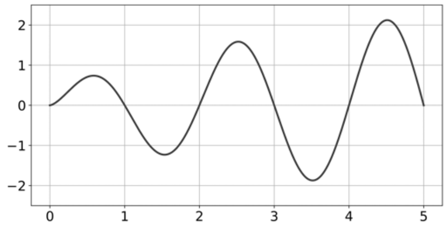
(b) $$\frac{1}{\left|1 + \left(\frac{1}{2\pi RCf}\right)\right|}$$

(c) $$\frac{1}{\left|1 + \left(\frac{1}{2\pi RCf}\right)^2\right|}$$

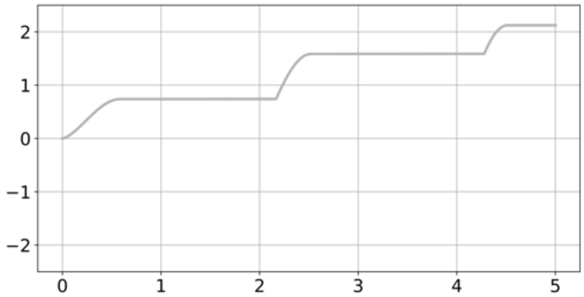
(d) $$\frac{1}{\left|1 + \left(\frac{1}{2\pi RCf}\right)\right|^{1/2}}$$

A20 For the circuit on the right, which graph represents V_{out} correctly for the V_{in} shown below?

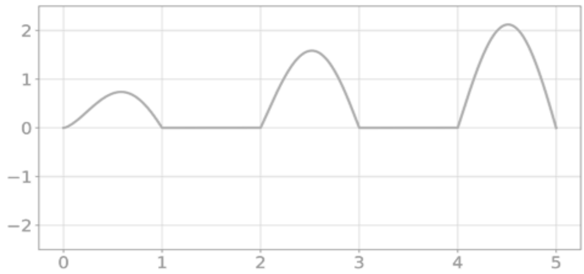




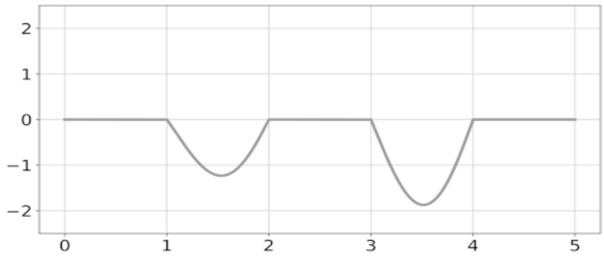
(a)



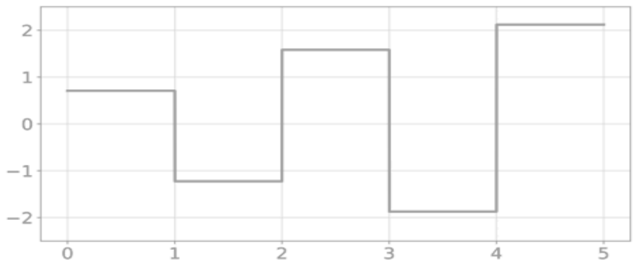
(b)



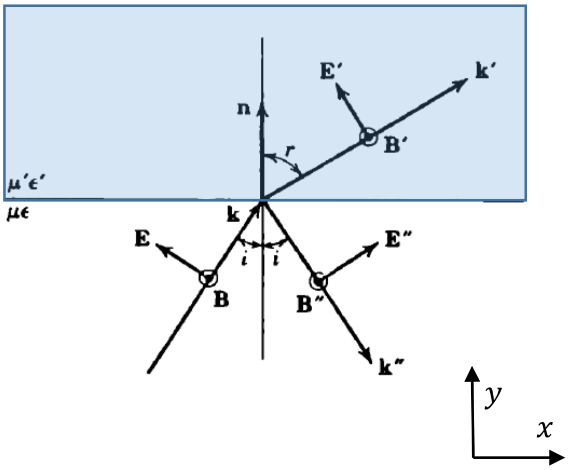
(c)



(d)



A21 Light in medium with electric permittivity ϵ and magnetic permeability μ is incident on a medium with electric permittivity ϵ' and magnetic permeability μ'. The angle of incidence is i. The E field is linearly polarized in the plane as shown, and the B field is in the $\hat{z}$ direction. Which of the following is a correct boundary condition on the fields?



(a) $\epsilon\,(E \sin i + E'' \sin i) = \epsilon' E' \sin r$

(b) $\epsilon\,(E \cos i - E'' \cos i) = \epsilon'\, E' \cos r$

(c) $E \sin i + E'' \sin i = E' \sin r$

(d) $\mu\,(B + B'') = \mu' B'$

A22 An experimental set-up needs to be kept in an environment with zero magnetic field by minimizing the Earth's magnetic field. This can be achieved by:

(a) Keeping the setup in a place completely covered with a sheet of metal of very high magnetic permeability

(b) Keeping the setup in a place completely covered with a sheet of metal of very high permittivity

(c) Keeping the setup at the centre of the interior of a long solenoid

(d) Keeping the setup in a place completely covered with a sheet of an insulating material

A23 Two types of particles A and B have the same mass, but are distinguished by an internal degree of freedom. A classical ideal gas in a volume V at temperature T contains (X) $2N$ particles of A-type and (Y) N particles of A-type and N particles of B-type. Which of the following is true?

(a) Pressure of (X) and (Y) are same; (Y) has more entropy than (X)

(b) Pressure of (X) and (Y) are same; (X) has more entropy than (Y)

(c) Pressure of (X) is greater than pressure of (Y); (X) has more entropy than (Y)

(d) Pressure of (X) is greater than pressure of (Y); (Y) has more entropy than (X)

A24 A negative electric charge $-q$ moves in a (classical) circular orbit at a non-relativistic speed v around a positive charge. The magnetic field at the centre of the circular orbit due to the negative charge is found to be B_1. Now, consider another situation where a negative charge $-2q$ moves in a circular orbit at the same speed v around the same positive charge. The magnetic field at the centre of the circular orbit in this case is B_2. What is the ratio B_2/B_1?

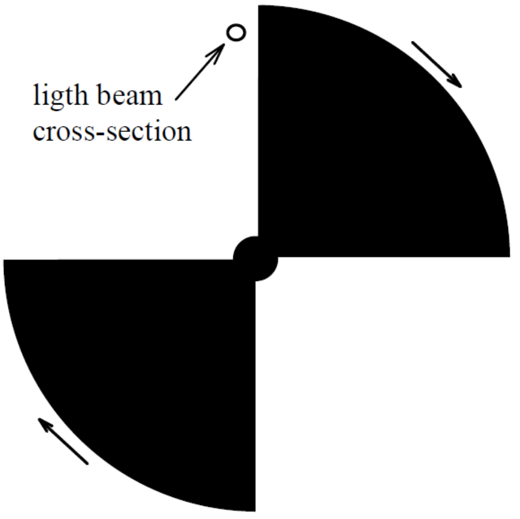
(a) $1/2$

(b) 1

(c) 2

(d) 4

A25 Consider a fan with blades rotating with frequency f, as shown in the Figure. It is used to periodically block a light beam of intensity I_0. The beam has a very small cross-sectional area and hits the blade near its outer edge, as shown. The transmitted beam is detected by a photo-detection unit which gives out a voltage signal V proportional to the transmitted intensity I. If this voltage signal pattern is displayed on an oscilloscope, what would best describe the signal pattern?



(a) $V_0\,[\frac{1}{2} + \sum_n \frac{4}{\pi n}\sin(2n\pi f t)], \qquad n = 2,6,10,14\ldots$

(b) $V_0 \sum_n [\cos^2(2n\pi f t) - \sin^2(2n\pi f t)], \qquad n = 2,6,10,14\ldots$

(c) $V_0\,[\frac{1}{2} + \frac{1}{2}\sin(4\pi f t)]$

(d) $V_0[\cos^2(4\pi f t)]$

Section B

B1 Let $|nlm\rangle$ denote the energy eigenstates of nonrelativistic hydrogen atoms without spin, and a_0 is the Bohr radius. The matrix element

$$\langle n = 2, l = 1, m_z = 0|\hat{x}|n = 2, l = 0, m_z = 0\rangle$$

is:

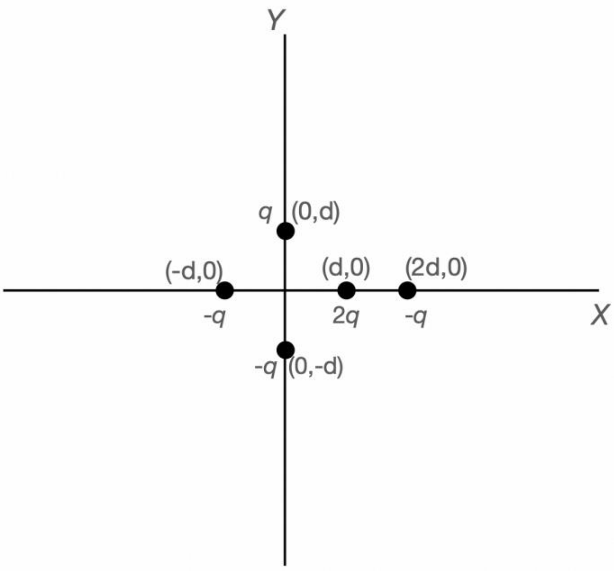
(a) 0

(b) $\sqrt{2}a_0$

(c) a_0

(d) $\sqrt{3}a_0$

B2 The given Figure shows some charges and their coordinates in the $x - y$ plane. The electric potential at a point, **r**, far from the origin, is given by:



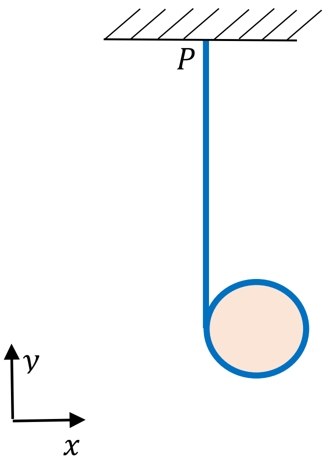
(a) $\phi = \dfrac{qd(\mathbf{r}.\hat{\mathbf{x}} + 2\,\mathbf{r}.\hat{\mathbf{y}})}{4\pi\epsilon_0 r^3}$

(b) $\phi = \dfrac{qd}{4\pi\epsilon_0 r^2}$

(c) $\phi = \dfrac{q\mathrm{d}\mathbf{r}.\hat{\mathbf{z}}}{4\pi\epsilon_0 r^3}$

(d) $\phi = \dfrac{qd\mathbf{r}.\hat{\mathbf{x}}}{4\pi\epsilon_0 r^3}$

B3 Consider a massless string with one end fixed on the ceiling (P). The total length of the string is L and it is initially completely wrapped around a solid uniform disc of radius R (assume $R \ll L$) and mass m. The disc is released from rest from the ceiling at time $t = 0$ and falls under gravity. The thread unwinds from the disc, always remaining tight. What is its velocity at a time t before the thread unwinds fully?



(a) $-\frac{2gt}{3}\,\hat{\boldsymbol{y}}$

(b) $-gt\,\hat{\boldsymbol{y}}$

(c) $-\frac{3gt}{4}\,\hat{\boldsymbol{y}}$

(d) $-\frac{gt}{2}\,\hat{\boldsymbol{y}}$

B4 A quantum particle is in the ground state of an infinite potential well of length L with

$$V(x) = \begin{cases} 0 & \text{for } x \in [0, L] \\ +\infty & \text{otherwise} \end{cases}$$

What is the expectation value of the operator,

$$\hat{O} = \hat{x}\hat{p} + \hat{p}\hat{x}$$

in this state?

(a) 0

(b) $i\hbar$

(c) $\hbar/2$

(d) $-i\hbar$

B5 Consider a random walker on a 2D plane which starts at the origin. At every step it either moves one unit along the positive x-axis with probability $1/2$ or along the positive y-axis with probablity $1/2$. The distance from the origin after n steps is denoted by r_n. What is the mean square displacement $\langle r_n^2 \rangle$?

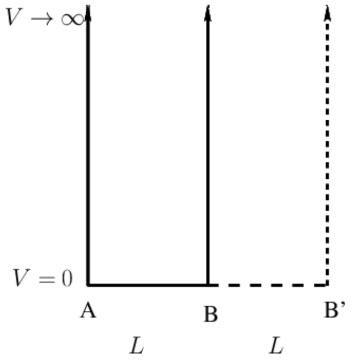
(a) $n(n+1)/2$

(b) $n(n-1)/2$

(c) n^2

(d) $n(n-1)$

B6 Consider a quantum particle of mass m in an infinite one-dimensional potential well of length L between points A and B. The particle is in the ground state with an energy E_g. The wall at B is suddenly shifted to B' where AB' has length $2L$. We measure the energy again, and obtain the value E_1. what is the probability that $E_1 \neq E_g$?



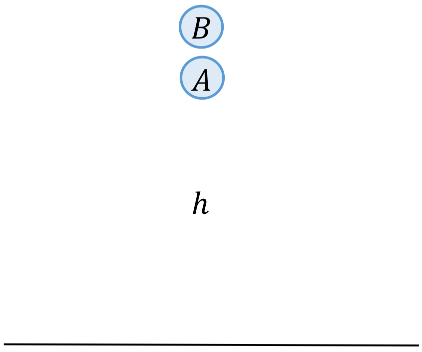
(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) 1

(d) 0

B7 A ball A is dropped on the floor from height h. It bounces up to height $h/4$ on the first bounce. Now identical balls A and B are dropped together from height h as shown. How high does the ball B bounce on the first bounce? Assume that the coefficient of restitution between balls A and B is 1. (Ignore the size and the small initial separation of the balls.)



(a) $h/4$

(b) h

(c) $h/2$

(d) $h/8$

B8 Consider a spherical planet with a radius $R = 6400$ km. The density $\rho(r)$ for r varies as $\rho(r) \propto r$, where r is the distance from the centre of the planet. A tunnel is dug through the centre, and the escape speed is measured at various distances r. At the planet's surface, the escape speed is found to be 11.2 km/sec, and at a distance of 3200 km from the centre, it is 12.7 km/sec. What is the escape speed at the centre of the planet?

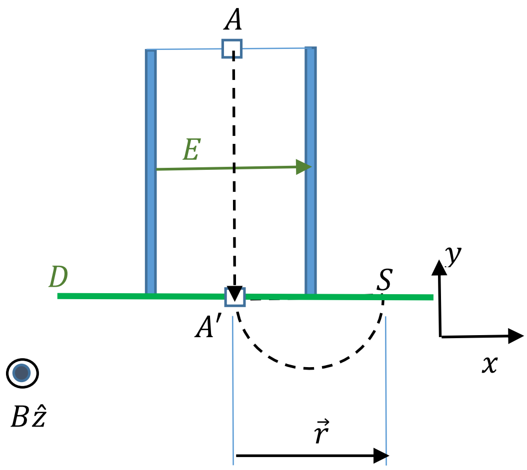
(a) 12.9 km/sec

(b) 14.2 km/sec

(c) 13.2 km/sec

(d) 0 km/sec

B9 There is a uniform electric field $E\hat{x}$ between two parallel plates of a capacitor (parallel to the xz plane). The plates are placed in a uniform magnetic field $B\hat{z}$ which fills the entire region (inside and outside the capacitor). Charged particles enter the capacitor through a small aperture A. They exit from a small aperture A' at the other end, if they do not deviate from a straight line path. There is a detector plate D in the xz plane passing through A'. D detects where the particles impinge. What is the displacement vector $\vec{r}$ between the impact point S and the aperture A' for a particle with mass m and charge q? (This device is a simple version of a mass spectrometer.)



(a) $\dfrac{-2mE}{qB^2}\hat{x}$

(b) $\dfrac{-mE}{qB^2}\hat{x}$

(c) $\dfrac{-mE}{2qB^2}\hat{x}$

(d) $\dfrac{mE}{qB^2}\hat{x}$

B10 A stream of electrons, each having an energy of 0.5 eV, impinges on a pair of extremely thin slits separated by 10 μm. The distance between adjacent minima on a screen 20 m behind the slits would be closest to:

(a) 3.48 mm

(b) 1.74 mm

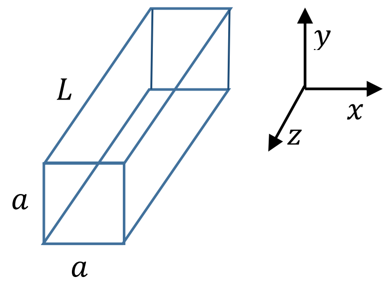
(c) 6.96 cm

(d) 5 m

B11 A very long square pipe with length L and cross-sectional area a^2 $(L \gg a)$ has ideal conducting walls. A travelling mode with

$$E_z(\vec{r}, t) = A \sin\frac{\pi x}{a} \sin\frac{\pi y}{a} e^{ikz-i\omega t}$$

is excited in the pipe. What is the relation between k and ω? (Assume $\epsilon \approx \epsilon_0, \mu \approx \mu_0$.)



(a) $k = \sqrt{\frac{\omega^2}{c^2} - \frac{2\pi^2}{a^2}}$

(b) $k = \sqrt{\frac{\omega^2}{c^2} + \frac{2\pi^2}{a^2}}$

(c) $k = \frac{\omega}{c}$

(d) $k = \sqrt{\frac{\omega^2}{c^2} + \frac{\pi^2}{a^2}}$

B12 The general solution of the equation

$$\frac{d^3y}{dx^3} + k^3y = 0 \ (k > 0)$$

is given by::

(a) $C_1 e^{-kx} + C_2 e^{\frac{kx}{2}} \cos(\sqrt{3}\, kx/2) + C_3 e^{\frac{kx}{2}} \sin(\sqrt{3}\, kx/2)$

(b) $C_1 e^{-kx} + C_2 e^{\frac{-kx}{2}} \cos(\sqrt{3}\, kx/2) + C_3 e^{\frac{-kx}{2}} \sin(\sqrt{3}\, kx/2)$

(c) $C_1 e^{-kx} + C_2 e^{\frac{kx}{2}} \cos(\sqrt{3}\, kx/2) + C_3 e^{\frac{kx}{2}} \sin(\, kx/2)$

(d) $C_1 e^{-kx} + C_2 e^{\frac{kx}{2}} \cos(\, kx/2) + C_3 e^{\frac{kx}{2}} \sin(\sqrt{3}\, kx/2)$

B13 Radiation from the Big-Bang observed today has a black-body spectrum with $T = 2.7$ K and its energy density as a function of the wavelength peaks at $\lambda = 1.1$ mm. This radiation is red-shifted to longer wave lengths compared to the black-body spectrum when the universe was hotter, say at 270 K. What was the photon energy corresponding to the wavelength at which the energy density peaked, when the universe was at 270 K?

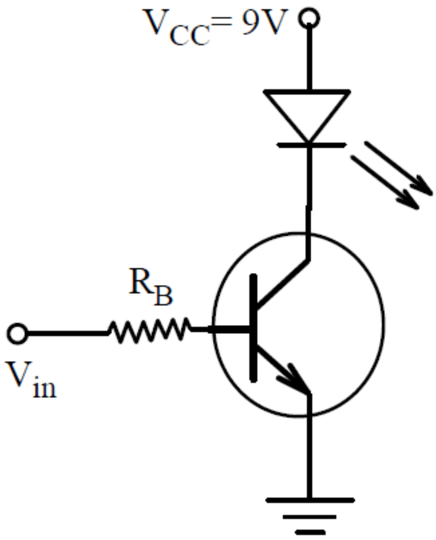
(a) 0.12 eV

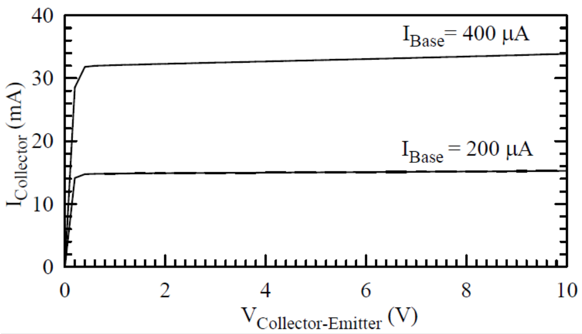
(b) 1.2 meV

(c) 23 meV

(d) 0.23 eV

B14 Consider the Collector-Emitter characteristics of a silicon NPN transistor in the Figure below. The circuit on the right is for lighting an LED with an input voltage $V_{in} = 1$V. The LED needs 20mA current, that will be provided by the transistor. A forward biased silicon PN junction has a 0.7V drop across it. What is the closest value of resistor R_B needed for this purpose?





(a) 1.3 $k\Omega$

(b) 560 Ω

(c) 4.2 $k\Omega$

(d) 15 $k\Omega$

B15 A classical ideal gas at temperature T is placed in a spherically symmetric potential

$$V(r) = c\,r^3$$

What is $\langle V(r) \rangle$ per particle?

(a) kT

(b) $3kT/2$

(c) $kT/2$

(d) $kT/3$