FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Thursday 01st February, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

SECTION-A

31. With rise in temperature, the Young's modulus of elasticity

(1) changes erratically

(2) decreases

(3) increases

(4) remains unchanged

Ans. (2)

Sol. Conceptual questions

32. If R is the radius of the earth and the acceleration due to gravity on the surface of earth is $g = \pi^2$ m/s^2, then the length of the second's pendulum at a height h = 2R from the surface of earth will be,:

(1) $\frac{2}{9}$m

(2) $\frac{1}{9}$m

(3) $\frac{4}{9}$m

(4) $\frac{8}{9}$m

Ans. (2)

Sol. $g' = \frac{GMe}{(3R)^2} = \frac{1}{9}g$

$T = 2\pi\sqrt{\frac{\ell}{g'}}$

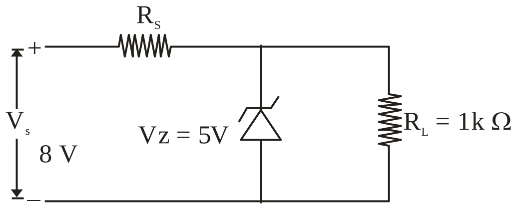
Since the time period of second pendulum is 2 sec.

T = 2 sec

$2 = 2\pi\sqrt{\frac{\ell}{g}9}$

$\boxed{\ell = \frac{1}{9}\text{m}}$

33. In the given circuit if the power rating of Zener diode is 10 mW, the value of series resistance R_s to regulate the input unregulated supply is :

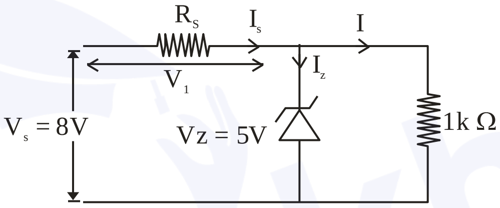


(1) 5kΩ (2) 10Ω

(3) 1kΩ (4) 10kΩ

Ans. (BONUS)

Sol.



Pd across R_s

$V_1 = 8 - 5 = 3V$

Current through the load resistor

$I = \frac{5}{1\times10^3} = 5mA$

Maximum current through Zener diode

$I_{z\ max.} = \frac{10}{5} = 2mA$

And minimum current through Zener diode

$I_{z\ min.} = 0$

$\therefore I_{s\ max.} = 5 + 2 = 7mA$

And $R_{s\ min} = \frac{V_1}{I_{s\ max.}} = \frac{3}{7}k\Omega$

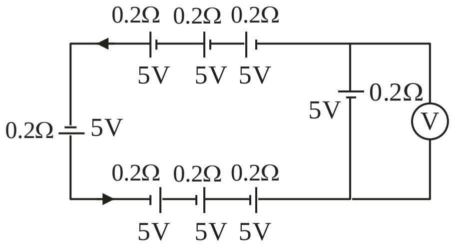
Similarly

$I_{s\ min.} = 5mA$

And $R_{s\ max.} = \frac{V_1}{I_{s\ min.}} = \frac{3}{5}k\Omega$

$\therefore \frac{3}{7}k\Omega < R_s < \frac{3}{5}k\Omega$

34. The reading in the ideal voltmeter (V) shown in the given circuit diagram is :



(1) 5V (2) 10V

(3) 0 V (4) 3V

Ans. (3)

Sol. $i = \frac{E_{eq}}{r_{eq}} = \frac{8 \times 5}{8 \times 0.2}$

$I = 25A$

$V = E - ir$

$= 5 - 0.2 \times 25$

$= 0$

35. Two identical capacitors have same capacitance C. One of them is charged to the potential V and other to the potential 2V. The negative ends of both are connected together. When the positive ends are also joined together, the decrease in energy of the combined system is :

(1) $\frac{1}{4}CV^2$

(2) $2\ CV^2$

(3) $\frac{1}{2}CV^2$

(4) $\frac{3}{4}CV^2$

Ans. (1)

Sol. $V_C = \frac{q_{net}}{C_{net}} = \frac{CV + 2CV}{2C}$

$V_C = \frac{3V}{2}$

Loss of energy

$= \frac{1}{2}CV^2 + \frac{1}{2}C(2V)^2 - \frac{1}{2}2C\left(\frac{3V}{2}\right)^2$

$= \left(\frac{CV^2}{4}\right)$

36. Two moles a monoatomic gas is mixed with six moles of a diatomic gas. The molar specific heat of the mixture at constant volume is :

(1) $\frac{9}{4}R$ (2) $\frac{7}{4}R$

(3) $\frac{3}{2}R$ (4) $\frac{5}{2}R$

Ans. (1)

Sol. $C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$

$= \frac{2 \times \frac{3}{2}R + 6 \times \frac{5}{2}R}{2 + 6}$

$= \frac{9}{4}R$

37. A ball of mass 0.5 kg is attached to a string of length 50 cm. The ball is rotated on a horizontal circular path about its vertical axis. The maximum tension that the string can bear is 400 N. The maximum possible value of angular velocity of the ball in rad/s is,:

(1) 1600 (2) 40

(3) 1000 (4) 20

Ans. (2)

Sol. $T = m\omega^2 \ell$

$400 = 0.5\omega^2 \times 0.5$

$\omega = 40$ rad/s.

38. A parallel plate capacitor has a capacitance C = 200 pF. It is connected to 230 V ac supply with an angular frequency 300 rad/s. The rms value of conduction current in the circuit and displacement current in the capacitor respectively are :

(1) 1.38 μA and 1.38 μA

(2) 14.3 μA and 143 μA

(3) 13.8 μA and 138 μA

(4) 13.8 μA and 13.8 μA

Ans. (4)

Sol. $I = \frac{V}{X_C} = 230 \times 300 \times 200 \times 10^{-12} = 13.8\ \mu A$

39. The pressure and volume of an ideal gas are related as $PV^{3/2} = K$ (Constant). The work done when the gas is taken from state A (P_1, V_1, T_1) to state B (P_2, V_2, T_2) is :

(1) $2(P_1V_1 - P_2V_2)$

(2) $2(P_2V_2 - P_1V_1)$

(3) $2(\sqrt{P_1}V_1 - \sqrt{P_2}V_2)$

(4) $2(P_2\sqrt{V_2} - P_1\sqrt{V_1})$

Ans. **(1 or 2)**

Sol. For PV^x = constant

If work done by gas is asked then

$$W = \frac{nR\Delta T}{1-x}$$

$$\text{Here } x = \frac{3}{2}$$

$\therefore$ W =

$$-\frac{}{2}$$

$= 2(P_1V_1 - P_2V_2)$ ….. Option (1) is correct

If work done by external is asked then

$W = -2(P_1V_1 - P_2V_2)$ ….. Option (2) is correct

40. A galvanometer has a resistance of 50 Ω and it allows maximum current of 5 mA. It can be converted into voltmeter to measure upto 100 V by connecting in series a resistor of resistance

(1) 5975 Ω

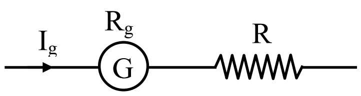
(2) 20050 Ω

(3) 19950 Ω

(4) 19500 Ω

Ans. **(3)**

Sol.



$$R = \frac{V}{I_g} - R_g = \frac{100}{5\times10^{-3}} - 50$$

$= 20000 - 50$

$= 19950\Omega$

41. The de Broglie wavelengths of a proton and an α particle are λ and 2 λ respectively. The ratio of the velocities of proton and α particle will be :

(1) 1 : 8

(2) 1 : 2

(3) 4 : 1

(4) 8 : 1

Ans. **(4)**

Sol. $\lambda = \frac{h}{p} = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda}$

$$\frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} \times \frac{\lambda_\alpha}{\lambda_p}$$

$= 4 \times 2 = 8$

42. 10 divisions on the main scale of a Vernier calliper coincide with 11 divisions on the Vernier scale. If each division on the main scale is of 5 units, the least count of the instrument is :

(1) $\frac{1}{2}$

(2) $\frac{10}{11}$

(3) $\frac{50}{11}$

(4) $\frac{5}{11}$

Ans. **(4)**

Sol. 10 MSD = 11 VSD

$$1\text{ VSD} = \frac{10}{11}\text{MSD}$$

$LC = 1MSD - 1VSD$

$$= 1\text{ MSD} - \frac{10}{11}\text{MSD}$$

$$= \frac{1MSD}{11}$$

$$= \frac{5}{11}\text{ units}$$

43. In series LCR circuit, the capacitance is changed from C to 4C. To keep the resonance frequency unchanged, the new inductance should be :

(1) reduced by $\frac{1}{4}L$

(2) increased by 2L

(3) reduced by $\frac{3}{4}L$

(4) increased to 4L

Ans. **(3)**

Sol. $\omega' = \omega$

$$\frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{LC}}$$

$\therefore$ L'C' = LC

L'(4C) = LC

$L' = \frac{L}{4}$

$\because$ Inductance must be decreased by $\frac{3L}{4}$

44. The radius (r), length (l) and resistance (R) of a metal wire was measured in the laboratory as

$r = (0.35 \pm 0.05)$ cm

$R = (100 \pm 10)$ ohm

$l = (15 \pm 0.2)$ cm

The percentage error in resistivity of the material of the wire is :

(1) 25.6% (2) 39.9%

(3) 37.3% (4) 35.6%

Ans. **(2)**

Sol. $\rho = R\frac{\rho}{\ell}$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta R}{R} + 2\frac{\Delta r}{r} + \frac{\Delta\ell}{\ell}$$

$$= \frac{10}{100} + 2\times\frac{0.05}{0.35} + \frac{0.2}{15}$$

$$= \frac{1}{10} + \frac{2}{7} + \frac{1}{75}$$

$$\frac{\Delta\rho}{\rho} = 39.9\%$$

45. The dimensional formula of angular impulse is :

(1) $[M L^{-2} T^{-1}]$ (2) $[M L^2 T^{-2}]$

(3) $[M L\ T^{-1}]$ (4) $[M L^2 T^{-1}]$

Ans. **(4)**

Sol. Angular impulse = change in angular momentum.

[Angular impulse] = [Angular momentum] = [mvr]

$= [M L^2 T^{-1}]$

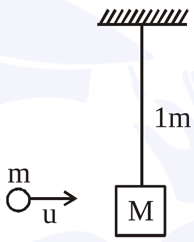
46. A simple pendulum of length 1 m has a wooden bob of mass 1 kg. It is struck by a bullet of mass 10^{-2} kg moving with a speed of 2×10^2 ms^{-1}. The bullet gets embedded into the bob. The height to which the bob rises before swinging back is. (use g = 10 m/s^2)

(1) 0.30 m (2) 0.20 m

(3) 0.35 m (4) 0.40 m

Ans. **(2)**

Sol.



$mu = (M + m)V$

$10^{-2} \times 2 \times 10^2 \cong 1 \times V$

$V \cong 2$m/s

$h = \frac{V^2}{2g} = 0.2$ m

47. A particle moving in a circle of radius R with uniform speed takes time T to complete one revolution. If this particle is projected with the same speed at an angle θ to the horizontal, the maximum height attained by it is equal to 4R. The angle of projection θ is then given by :

(1) $\sin^{-1}\left[\frac{2gT^2}{\pi^2 R}\right]^{\frac{1}{2}}$ (2) $\sin^{-1}\left[\frac{\pi^2 R}{2gT^2}\right]^{\frac{1}{2}}$

(3) $\cos^{-1}\left[\frac{2gT^2}{\pi^2 R}\right]^{\frac{1}{2}}$ (4) $\cos^{-1}\left[\frac{\pi R}{2gT^2}\right]^{\frac{1}{2}}$

Ans. **(1)**

Sol. $\frac{2\pi R}{T} = V$

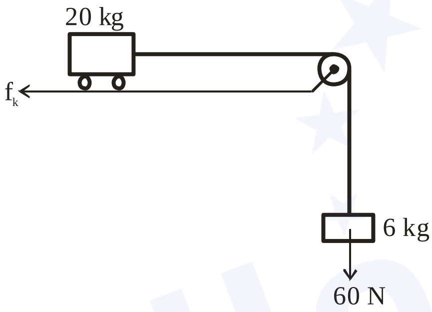
Maximum height $H = \frac{v^2 \sin^2\theta}{2g}$

$$4R = \frac{4\pi^2 R^2}{T^2 2g}\sin^2\theta$$

$$\sin\theta = \sqrt{\frac{2gT^2}{\pi^2 R}}$$

$$\theta = \sin^{-1}\left(\frac{2gT^2}{\pi^2 R}\right)^{\frac{1}{2}}$$

48. Consider a block and trolley system as shown in figure. If the coefficient of kinetic friction between the trolley and the surface is 0.04, the acceleration of the system in ms^{-2} is :
(Consider that the string is massless and unstretchable and the pulley is also massless and frictionless) :



(1) 3 (2) 4
(3) 2 (4) 1.2

Ans. **(3)**

Sol. $f_k = \mu N = 0.04 \times 20g = 8$ Newton

$$a = \frac{60-8}{26} = 2 m/s^2$$

49. The minimum energy required by a hydrogen atom in ground state to emit radiation in Balmer series is nearly :
(1) 1.5 eV (2) 13.6 eV
(3) 1.9 eV (4) 12.1 eV

Ans. **(4)**

Sol. Transition from n = 1 to n = 3
$\Delta E = 12.1eV$

50. A monochromatic light of wavelength 6000Å is incident on the single slit of width 0.01 mm. If the diffraction pattern is formed at the focus of the convex lens of focal length 20 cm, the linear width of the central maximum is :
(1) 60 mm
(2) 24 mm
(3) 120 mm
(4) 12 mm

Ans. **(2)**

Sol. Linear width

$$W = \frac{2\lambda d}{a} = \frac{2\times 6\times 10^{-7}\times 0.2}{1\times 10^{-5}}$$

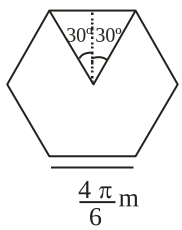
$= 2.4 \times 10^{-2} = 24$ mm

SECTION-B

51. A regular polygon of 6 sides is formed by bending a wire of length 4 π meter. If an electric current of $4\pi\sqrt{3}$ A is flowing through the sides of the polygon, the magnetic field at the centre of the polygon would be $x \times 10^{-7}$ T. The value of x is ____.

Ans. **(72)**

Sol.



$$B = 6\left(\frac{\mu_0 I}{4\pi r}\right)(\sin 30^\circ + \sin 30^\circ)$$

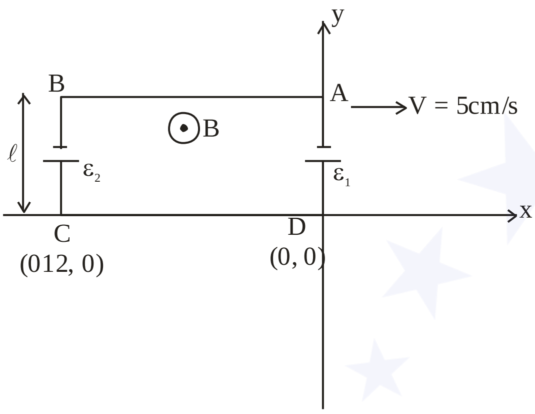
$$= 6\frac{10^{-7}\times 4\pi\sqrt{3}}{\left(\frac{\sqrt{3}\times 4\pi}{2\times 6}\right)}$$

$= 72 \times 10^{-7}T$

52. A rectangular loop of sides 12 cm and 5 cm, with its sides parallel to the x-axis and y-axis respectively moves with a velocity of 5 cm/s in the positive x axis direction, in a space containing a variable magnetic field in the positive z direction. The field has a gradient of 10^{-3}T/cm along the negative x direction and it is decreasing with time at the rate of 10^{-3} T/s. If the resistance of the loop is 6 mΩ, the power dissipated by the loop as heat is _______ $\times 10^{-9}$ W.

Ans. **(216)**

Sol.



B_0 is the magnetic field at origin

$$\frac{dB}{dx} = -\frac{10^{-3}}{10^{-2}}$$

$$\int_{B_0}^{B} dB = -\int_0^x 10^{-1}dx$$

$$B - B_0 = -10^{-1}x$$

$$B = \left(B_0 - \frac{x}{10}\right)$$

Motional emf in AB = 0

Motional emf in CD = 0

Motional emf in AD = $\varepsilon_1 = B_0 \ell v$

Magnetic field on rod BC B

$$= \left(B_0 - \frac{(-12 \times 10^{-2})}{10}\right)$$

Motional emf in BC = $\varepsilon_2 = \left(B_0 + \frac{12 \times 10^{-2}}{10}\right)\ell \times v$

$\varepsilon_{eq} = \varepsilon_2 - \varepsilon_1 = 300 \times 10^{-7}$ V

For time variation

$$(\varepsilon_{eq})' = A\frac{dB}{dt} = 60 \times 10^{-7}\text{ V}$$

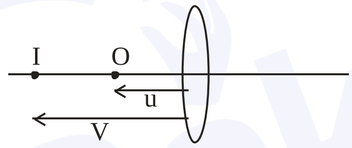
$$(\varepsilon_{eq})_{net} = \varepsilon_{eq} + (\varepsilon_{eq})' = 360 \times 10^{-7}\text{ V}$$

$$\text{Power} = \frac{\left(\varepsilon_{eq}\right)_{net}^2}{R} = 216 \times 10^{-9}\text{ W}$$

53. The distance between object and its 3 times magnified virtual image as produced by a convex lens is 20 cm. The focal length of the lens used is __________ cm.

Ans. **(15)**

Sol.



$v = 3u$

$v - u = 20$ cm

$2u = 20$ cm

$u = 10$ cm

$$\frac{1}{(-30)} - \frac{1}{(-10)} = \frac{1}{f}$$

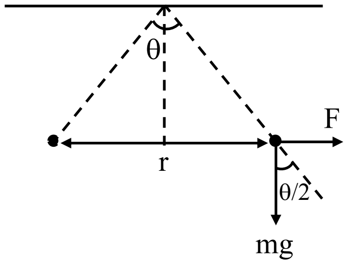
$f = 15$ cm

54. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle θ with each other. When suspended in water the angle remains the same. If density of the material of the sphere is 1.5 g/cc, the dielectric constant of water will be _________

(Take density of water = 1 g/cc)

Ans. **(3)**

Sol.



In air $\tan\frac{\theta}{2} = \frac{F}{mg} = \frac{q^2}{4\pi\varepsilon_0 r^2 mg}$

In water $\tan\frac{\theta}{2} = \frac{F'}{mg'} = \frac{q^2}{4\pi\varepsilon_0\varepsilon_r r^2 mg_{eff}}$

Equate both equations

$\varepsilon_0 g = \varepsilon_0\ \varepsilon_r\ g\left[1-\frac{1}{1.5}\right]$

$\varepsilon_r = 3$

55. The radius of a nucleus of mass number 64 is 4.8 fermi. Then the mass number of another nucleus having radius of 4 fermi is $\frac{1000}{x}$, where x is ______.

Ans. **(27)**

Sol. $R = R_0A^{1/3}$

$R^3 \propto A$

$\left(\frac{4.8}{4}\right)^3 = \frac{64}{A}$

$= \frac{64}{A} = (1.2)^3$

$\frac{64}{A} = 1.44\times1.2$

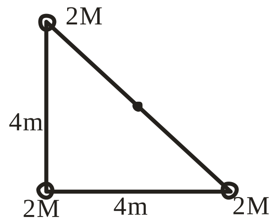
$A = \frac{64}{1.44\times1.2} = \frac{1000}{x}$

$x = \frac{144\times12}{64} = 27$

56. The identical spheres each of mass 2M are placed at the corners of a right angled triangle with mutually perpendicular sides equal to 4 m each. Taking point of intersection of these two sides as origin, the magnitude of position vector of the centre of mass of the system is $\frac{4\sqrt{2}}{x}$, where the value of x is ___________

Ans. **(3)**

Sol.



Position vector $\vec{r}_{COM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3}$

$\vec{r}_{COM} = \frac{2M\times0 + 2M\times4\hat{i} + 2M\times4\hat{j}}{6M}$

$\vec{r} = \frac{4}{3}\hat{i} + \frac{4}{3}\hat{j}$

$|\vec{r}| = \frac{4\sqrt{2}}{3}$

$x = 3$

57. A tuning fork resonates with a sonometer wire of length 1 m stretched with a tension of 6 N. When the tension in the wire is changed to 54 N, the same tuning fork produces 12 beats per second with it. The frequency of the tuning fork is _______ Hz.

Ans. **(6)**

Sol. $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

$f_1 = \frac{1}{2}\sqrt{\frac{6}{\mu}}$ $\qquad f_2 = \frac{1}{2}\sqrt{\frac{54}{\mu}}$

$\frac{f_1}{f_2} = \frac{1}{3}$ $\qquad f_2 - f_1 = 12$

$f_1 = 6HZ$

58. A plane is in level flight at constant speed and each of its two wings has an area of 40 m^2. If the speed of the air is 180 km/h over the lower wing surface and 252 km/h over the upper wing surface, the mass of the plane is ________kg. (Take air density to be 1 kg m^{-3} and g = 10 ms^{-2})

Ans. **(9600)**

Sol. $A = 80\ m^2$

Using Bernonlli equation

$$A(P_2 - P_1) = \frac{1}{2}\rho\left(V_1^2 - V_2^2\right)A$$

$$mg = \frac{1}{2} \times 1\ (70^2 - 50^2) \times 80$$

$$mg = 40 \times 2400$$

$$m = 9600\ kg$$

59. The current in a conductor is expressed as $I = 3t^2 + 4t^3$, where I is in Ampere and t is in second. The amount of electric charge that flows through a section of the conductor during t = 1s to t = 2s is ____________ C.

Ans. **(22)**

Sol. $$q = \int_1^2 i\ dt = \int_1^2 (3t^2 + 4t^3)dt$$

$$q = \left(t^3 + t^4\right)\Big|_1^2$$

$$q = 22C$$

60. A particle is moving in one dimension (along x axis) under the action of a variable force. It's initial position was 16 m right of origin. The variation of its position (x) with time (t) is given as $x = -3t^3 + 18t^2 + 16t$, where x is in m and t is in s. The velocity of the particle when its acceleration becomes zero is _________ m/s.

Ans. **(52)**

Sol. $x = 3t^3 + 18t^2 + 16t$

$\mathbf{v} = -9t^2 + 36 + 16$

$a = -18t + 36$

$a = 0$ at $t = 2s$

$v = -9(2)^2 + 36 \times 2 + 16$

$v = 52\ m/s$