FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Monday 08th April, 2024) **TIME : 3 : 00 PM to 6 : 00 PM**

SECTION-A

1. If the image of the point (-4, 5) in the line $x + 2y = 2$ lies on the circle $(x + 4)^2 + (y-3)^2 = r^2$, then r is equal lo :

(1) 1 (2) 2

(3) 75 (4) 3

Ans. (2)

Sol. Image of point (–4, 5)

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = -2\left(\frac{ax_1 + by_1 + c}{a^2 + b^2}\right)$$

Line : $x + 2y - 2 = 0$

$$\frac{x + 4}{1} = \frac{y - 5}{2} = -2\left(\frac{-4 + 10 - 2}{1^2 + 2^2}\right)$$

$$= \frac{-8}{5}$$

$$x = -4 - \frac{8}{5} = -\frac{28}{5}$$

$$y = -\frac{16}{5} + 5 = \frac{9}{5}$$

Point lies on circle $(x + 4)^2 + (y - 3)^2 = r^2$

$$\frac{64}{25} + \left(\frac{9}{5} - 3\right)^2 = r^2$$

$$\frac{100}{25} = r^2, \boxed{r = 2}$$

2. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + \lambda\hat{k}$ be three vectors. Let $\vec{r}$ be a unit vector along $\vec{b} + \vec{c}$. If $\vec{r} \cdot \vec{a} = 3$, then 3λ is equal to :

(1) 27 (2) 25

(3) 25 (4) 21

Ans. (2)

Sol. $\vec{r} = k(\vec{b} + \vec{c})$

$\vec{r} \cdot \vec{a} = 3$

$\vec{r}.\vec{a} = k(\vec{b}.\vec{a} + \vec{c}.\vec{a})$

$3 = k(2 + 6 - 15 + 3 - 2 + 3\lambda)$

$3 = k(-6 + 3\lambda)$ …(1)

$\vec{r} = k(5\hat{i} + 2\hat{j} - (5 - \lambda)\hat{k})$

$|\vec{r}| = k\sqrt{25 + 4 + 25 + \lambda^2 - 10\lambda} = 1$ …(2)

$k = \frac{3}{-6 + 3\lambda} = \frac{1}{-2 + \lambda}$ put in (2)

$4 + \lambda^2 - 4\lambda = 54 + \lambda^2 - 10\lambda$

$6\lambda = 50$

$3\lambda = 25$

3. If $\alpha \neq a,\ \beta \neq b,\ \gamma \neq c$ and $\begin{vmatrix} \alpha & b & c \\ a & \beta & c \\ a & b & \gamma \end{vmatrix} = 0$, then $\frac{a}{\alpha - a} + \frac{b}{\beta - b} + \frac{\gamma}{\gamma - c}$ is equal to :

(1) 2 (2) 3

(3) 0 (4) 1

Ans. (3)

Sol. $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$

$$\begin{vmatrix} \alpha - a & b - \beta & 0 \\ 0 & \beta - b & c - \gamma \\ a & b & \gamma \end{vmatrix} = 0$$

$(\alpha - a)(\gamma(\beta - b) - b(c - \gamma)) - (b - \beta)(-a(c - \gamma)) = 0$

$\gamma(\alpha - a)(\beta - b) - b(\alpha - a)(c - \gamma) + a(b - \beta)(c - \gamma)$

$$\frac{\gamma}{\gamma - c} + \frac{b}{\beta - b} + \frac{a}{\alpha - a} = 0$$

4. In an increasing geometric progression ol positive terms, the sum of the second and sixth terms is $\frac{70}{3}$ and the product of the third and fifth terms is 49. Then the sum of the 4^{th}, 6^{th} and 8^{th} terms is :-

(1) 96 (2) 78

(3) 91 (4) 84

Ans. (3)

Sol. $T_2 + T_6 = \frac{70}{3}$

$ar + ar^5 = \frac{70}{3}$

$T_3 . T_5 = 49$

$ar^2 . ar^4 = 49$

$a^2r^6 = 49$

$ar^3 = +7,\ a = \frac{7}{r^3}$

$ar(1 + r^4) = \frac{70}{3}$

$\frac{7}{r^2}(1+r^4) = \frac{70}{3}, r^2 = t$

$\frac{1}{t}(1+t^2) = \frac{10}{3}$

$3t^2 - 10t + 3 = 0$

$t = 3, \frac{1}{3}$

Increasing G.P. $r^2 = 3,\ r = \sqrt{3}$

$T_4 + T_6 + T_8$

$= ar^3 + ar^5 + ar^7$

$= ar^3(1 + r^2 + r^4)$

$= 7(1 + 3 + 9) = 91$

5. The number of ways five alphabets can be chosen from the alphabets of the word MATHEMATICS, where the chosen alphabets are not necessarily distinct, is equal to :

(1) 175 (2) 181

(3) 177 (4) 179

Ans. (4)

Sol. AA, MM, TT, H, I, C, S, E

(1) All distinct

$^8C_5 \to 56$

(2) 2 same, 3 different

$^3C_1 \times {^7C_3} \to 105$

(3) 2 same I^{st} kind, 2 same 2^{nd} kind, 1 different

$^3C_2 \times {^6C_1} \to 18$

Total $\to 179$

6. The sum of all possible values of $\theta \in [-\pi, 2\pi]$, for which $\frac{1 + i\cos\theta}{1 - 2i\cos\theta}$ is purely imaginary, is equal to

(1) 2π (2) 3π

(3) 5π (4) 4π

Ans. (2)

Sol. $Z = \frac{1 + i\cos\theta}{1 - 2i\cos\theta}$

$Z = -\bar{Z} \Rightarrow \frac{1+i\cos\theta}{1-2i\cos\theta} = -\left(\overline{\frac{1+i\cos\theta}{1-2i\cos\theta}}\right)$

$(1+ i\cos\theta)(\overline{1-2i\cos\theta}) = -(1 - 2i\cos\theta)(\overline{1+i\cos\theta})$

$(1+i\cos\theta)(1 + 2i\cos\theta) = -(1 - 2i\cos\theta)(1 - i\cos\theta)$

$1 + 3i\cos\theta - 2\cos^2\theta = -(1 - 3i\cos\theta - 2\cos^2\theta)$

$2 - 4\cos^2\theta = 0$

$\Rightarrow \cos^2\theta = \frac{1}{2} \Rightarrow \theta = -\frac{\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

sum $= 3\pi$

7. If the system of equations $x + 4y - z = \lambda$, $7x + 9y + \mu z = -3$, $5x + y + 2z = -1$ has infinitely many solutions, then $(2\mu + 3\lambda)$ is equal to :

(1) 2 (2) –3

(3) 3 (4) –2

Ans. (2)

Sol. $\Delta = \begin{vmatrix} 1 & 4 & -1 \\ 7 & 9 & \mu \\ 5 & 1 & 2 \end{vmatrix} = 0$

$\Rightarrow (18-\mu) - 4(14-5\mu) - (7 - 45) = 0 \Rightarrow \mu = 0$

$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$ (For infinite solution)

$\Delta_x = \begin{vmatrix} \lambda & 4 & -1 \\ -3 & 9 & \mu \\ -1 & 1 & 2 \end{vmatrix} = 0$

$\lambda(18 - \mu) - 4(-6 + \mu) - 1(-3 + 9) = 0$

$18\lambda + 24 - 6 = 0 \Rightarrow \lambda = -1$

8. If the shortest distance between the lines $\frac{x-\lambda}{2}=\frac{y-4}{3}=\frac{z-3}{4}$ and $\frac{x-2}{4}=\frac{y-4}{6}=\frac{z-7}{8}$ is $\frac{13}{\sqrt{29}}$, then a value of λ is :

(1) $-\frac{13}{25}$ (2) $\frac{13}{25}$

(3) 1 (4) –1

Ans. (3)

Sol. $\left.\begin{array}{l}\bar{r}_1=(\lambda\hat{i}+4\hat{j}+3\hat{k})+\alpha(2\hat{i}+3\hat{j}+4\hat{k})\\ \bar{r}_2=(2\hat{i}+4\hat{j}+7\hat{k})+\beta(2\hat{i}+3\hat{j}+4\hat{k})\end{array}\right\}\begin{array}{l}\bar{b}=2\hat{i}+3\hat{j}+4\hat{k}\\ \bar{a}_1=+\lambda\hat{i}+4\hat{j}+3\hat{k}\\ \bar{a}_2=2\hat{i}+4\hat{j}+7\hat{k}\end{array}$

$$\text{Shortest dist.}=\frac{|\bar{b}\times(\bar{a}_2-\bar{a}_1)|}{|b|}=\frac{13}{\sqrt{29}}$$

$$\frac{|(2\hat{i}+3\hat{j}+4\hat{k})\times((2-\lambda)\hat{i}+4\hat{k})|}{\sqrt{29}}=\frac{13}{\sqrt{29}}$$

$$|-8\hat{j}-3(2-\lambda)\hat{k}+12\hat{i}+4(2-\lambda)\hat{j}|=13$$

$$|12\hat{i}-4\lambda\hat{j}+(3\lambda-6)\hat{k}|=13$$

$$144+16\lambda^2+(3\lambda-6)^2=169$$

$$16\lambda^2+(3\lambda-6)^2=25=\lambda\Rightarrow=1$$

9. If the value of $\frac{3\cos36^\circ+5\sin18^\circ}{5\cos36^\circ-3\sin18^\circ}$ is $\frac{a\sqrt{5}-b}{c}$, where a, b, c are natural numbers and gcd(a, c) = 1, then a + b + c is equal to :

(1) 50 (2) 40

(3) 52 (4) 54

Ans. (3)

Sol. $$\frac{3\left(\frac{\sqrt{5}+1}{4}\right)+5\left(\frac{\sqrt{5}-1}{4}\right)}{5\left(\frac{\sqrt{5}+1}{4}\right)-3\left(\frac{\sqrt{5}-1}{4}\right)}=\frac{8\sqrt{5}-2}{2\sqrt{5}+8}$$

$$=\frac{4\sqrt{5}-1}{\sqrt{5}+4}\times\frac{\sqrt{5}-4}{\sqrt{5}-4}$$

$$=\frac{20-16\sqrt{5}-\sqrt{5}+4}{-11}$$

$$=\frac{17\sqrt{5}-24}{11}\Rightarrow a=17,\ b=27,\ c=11$$

$a+b+c=52$

10. Let y = y(x) be the solution curve of the differential equation $\sec y\frac{dy}{dx}+2x\sin y=x^3\cos y$, $y(1)=0$. Then $y(\sqrt{3})$ is equal to :

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{12}$

Ans. (3)

Sol. $\sec^2 y\frac{dy}{dx}+2x\sin y\sec y=x^3\cos y\sec y$

$\sec^2 y\frac{dy}{dx}+2x\tan y=x^3$

$\tan y=t\Rightarrow\sec^2 y\frac{dy}{dx}=\frac{dt}{dx}$

$\frac{dt}{dx}+2xt=x^3$, If $=e^{\int 2x\,dx}=e^{x^2}$

$te^{x^2}=\int x^3.e^{x^2}dx+c$

$x^2=Z\Rightarrow t.e^Z=\frac{1}{2}\int e^Z.ZdZ=\frac{1}{2}[e^Z.Z-e^Z]+c$

$2\tan y=(x^2-1)+2ce^{-x^2}$

$y(1)=0\Rightarrow c=0\Rightarrow y(\sqrt{3})=\frac{\pi}{4}$

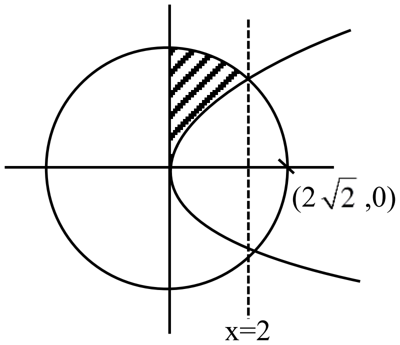
11. The area of the region in the first quadrant inside the circle $x^2+y^2=8$ and outside the pnrabola $y^2=2x$ is equal to :

(1) $\frac{\pi}{2}-\frac{1}{3}$ (2) $\pi-\frac{2}{3}$

(3) $\frac{\pi}{2}-\frac{2}{3}$ (4) $\pi-\frac{1}{3}$

Ans. (2)

Sol.



Required area = Ar(circle from 0 to 2) – ar(para from 0 to 2)

$$= \int_0^2 \sqrt{8 - x^2}\, dx - \int_0^2 \sqrt{2x}\, dx$$

$$= \left[\frac{x}{2}\sqrt{8 - x^2} + \frac{8}{2}\sin^{-1}\frac{x}{2\sqrt{2}}\right]_0^2 - \sqrt{2}\left[\frac{x\sqrt{x}}{3/2}\right]_0^2$$

$$= \frac{2}{2}\sqrt{8 - 4} + \frac{8}{2}\sin^{-1}\frac{2}{2\sqrt{2}} - \frac{2\sqrt{2}}{3}\left(2\sqrt{2} - 0\right)$$

$$\Rightarrow\ 2 + 4 \cdot \frac{\pi}{4} - \frac{8}{3} = \pi - \frac{2}{3}$$

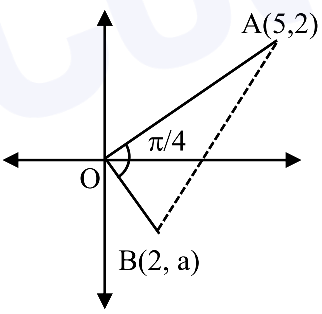
12. If the line segment joining the points (5, 2) and (2, a) subtends an angle $\frac{\pi}{4}$ at the origin, then the absolute value of the product of all possible values of a is :

(1) 6 (2) 8

(3) 2 (4) 4

Ans. (4)

Sol.



$$m_{OA} = \frac{2}{5}$$

$$m_{OB} = \frac{a}{2}$$

$$\tan\frac{\pi}{4} = \left|\frac{\frac{2}{5} - \frac{a}{2}}{1 + \frac{2}{5}\cdot\frac{a}{2}}\right|$$

$$1 = \left|\frac{4 - 5a}{10 + 2a}\right|$$

$$4 - 5a = \pm(10 + 2a)$$

$4 - 5a = 10 + 2a$	$4 - 5a = -10 - 2a$
$\Rightarrow 7a + 6 = 0$	$3a = 14$
$\Rightarrow a = -\frac{6}{7}$	$a = +\frac{14}{3}$

$$-\frac{6}{7} \times \frac{14}{3} = -4$$

13. Let $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 11\hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that

$\left(\vec{a} + \vec{b}\right) \times \vec{c} = \vec{c} \times \left(-2\vec{a} + 3\vec{b}\right)$.

If $\left(2\vec{a} + 3\vec{b}\right).\vec{c} = 1670$, then $|\vec{c}|^2$ is equal to :

(1) 1627 (2) 1618

(3) 1600 (4) 1609

Ans. (2)

Sol. $\left(\vec{a}+\vec{b}\right)\times\vec{c} - \vec{c}\times\left(-2\vec{a}+3\vec{b}\right) = 0$

$\left(\vec{a}+\vec{b}\right)\times\vec{c} + \left(-2\vec{a}+3\vec{b}\right)\times\vec{c} = 0$

$\Rightarrow \left(\vec{a}+\vec{b}\right) - 2\vec{a} + 3\vec{b}) \times \vec{c} = 0$

$\Rightarrow \vec{c} = \lambda(4\vec{b} - \vec{a})$

$\Rightarrow\ = \lambda\left(44\hat{i} - 4\hat{j} + 4\hat{k} - 4\hat{i} + \hat{j} - \hat{k}\right)$

$= \lambda\left(40\hat{i} - 3\hat{j} + 3\hat{k}\right)$

Now

$\left(8\hat{i} - 2\hat{j} + 2\hat{k} + 33\hat{i} - 3\hat{j} + 3\hat{k}\right) . \lambda(40\hat{i} - 3\hat{j} + 3\hat{k}) = 1670$

$\Rightarrow \left(41\hat{i} - 5\hat{j} + 5\hat{k}\right).(40\hat{i} - 3\hat{j} + 3\hat{k}) \times \lambda = 1670$)

$\Rightarrow (1640 + 15 + 15)\lambda = 1670 \Rightarrow \lambda = 1$

so $\vec{c} = 40\hat{i} - 3\hat{j} - 3\hat{k}$

$\Rightarrow |\vec{c}|^2 = 1600 + 9 + 9 = 1618$

14. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, $a > 0$ has a local maximum at $x = \alpha$ and a local minimum $x = \alpha^2$, then α and α^2 are the roots of the equation :

(1) $x^2 - 6x + 8 = 0$ (2) $8x^2 + 6x - 8 = 0$

(3) $8x^2 - 6x + 1 = 0$ (4) $x^2 + 6x + 8 = 0$

Ans. (1)

Sol. $f'(x) = 6x^2 - 18ax + 12a^2 = 0$ with roots α and α^2

$\alpha + \alpha^2 = 3a$ & $\alpha \times \alpha^2 = 2a^2$

$\downarrow$

$(\alpha + \alpha^2)^3 = 27a^3$

$\Rightarrow 2a^2 + 4a^4 + 3(3a)(2a^2) = 27a^3$

$\Rightarrow 2 + 4a^2 + 18a = 27a$

$\Rightarrow 4a^2 - 9a + 2 = 0$

$\Rightarrow 4a^2 - 8a - a + 2 = 0$

$\Rightarrow (4a - 1)(a - 2) = 0 \Rightarrow a = 2$

so $6x^2 - 36x + 48 = 0$

$\Rightarrow x^2 - 6x + 8 = 0$ (1)

If we take $a = \frac{1}{4}$ then $\alpha = \frac{1}{2}$ which is not possible

15. There are three bags X, Y and Z. Bag X contains 5 one-rupee coins and 4 five-rupee coins; Bag Y contains 4 one-rupee coins and 5 five-rupee coins and Bag Z contains 3 one-rupee coins and 6 five-rupee coins. A bag is selected at random and a coin drawn from it at random is found to be a one-rupee coin. Then the probability, that it came from bag Y, is :

(1) $\frac{1}{3}$ (2) $\frac{1}{2}$

(3) $\frac{1}{4}$ (4) $\frac{5}{12}$

Ans. (1)

Sol.

X	Y	Z
5 one & 4 five	4 one & 5 five	3 one & 6 five

$$P = \frac{4/9}{5/9 + 4/9 + 3/9} = \frac{4}{12} = \frac{1}{3}$$

16. Let $\int_{\alpha}^{\log_e 4} \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}$. Then e^{α} and $e^{-\alpha}$ are the roots of the equation :

(1) $2x^2 - 5x + 2 = 0$ (2) $x^2 - 2x - 8 = 0$

(3) $2x^2 - 5x - 2 = 0$ (4) $x^2 + 2x - 8 = 0$

Ans. (1)

Sol. $\int_{\alpha}^{\log_e 4} \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}$

Let $e^x - 1 = t^2$

$e^x\,dx = 2t\,dt$

$= \int \frac{2dt}{t^2 + 1}$

$= 2\tan^{-1}t$

$= 2\tan^{-1}\left(\sqrt{e^x - 1}\right)\Big|_{\alpha}^{\log_e 4}$

$= 2\left[\tan^{-1}\sqrt{3} - \tan^{-1}\sqrt{e^{\alpha} - 1}\right] = \frac{\pi}{6}$

$= \frac{\pi}{3} - \tan^{-1}\sqrt{e^{\alpha} - 1} = \frac{\pi}{12}$

$\Rightarrow \tan^{-1}\sqrt{e^{\alpha} - 1} = \frac{\pi}{4}$

$e^{\alpha} = 2$ $e^{-\alpha} = \frac{1}{2}$

$x^2 - \left(2 + \frac{1}{2}\right)x + 1 = 0$

$2x^2 - 5x + 2 = 0$

17. Let $f(x) = \begin{cases} -a & \text{if } -a \le x \le 0 \\ x + a & \text{if } 0 < x \le a \end{cases}$

where $a > 0$ and $g(x) = (f|x|) - |f(x)|)/2$.

Then the function $g : [-a, a] \to [-a, a]$ is

(1) neither one-one nor onto.

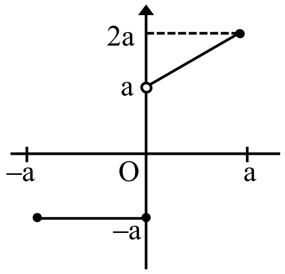
(2) both one-one and onto.

(3) one-one.

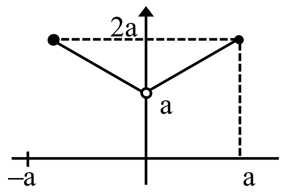
(4) onto

Ans. (1)

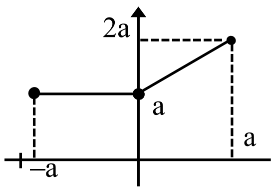
Sol. y = f(x)



y = f|x|



y = |f(x)|



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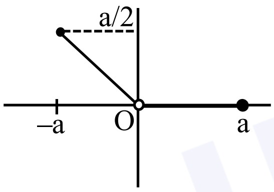
$g(x) = \frac{f(|x|) - |f(x)|}{2}$



18. Let A= {2, 3, 6, 8, 9, 11} and B = {1, 4, 5, 10, 15} Let R be a relation on A × B define by (a, b)R(c, d) if and only if 3ad – 7bc is an even integer. Then the relation R is

(1) reflexive but not symmetric.

(2) transitive but not symmetric.

(3) reflexive and symmetric but not transitive.

(4) an equivalence relation.

Ans. (3)

Sol. A = {2, 3, 6, 8, 9, 11} (a, b)R (c, d)

B = {1, 4, 5, 10, 15} 3ad – 7bc

Reflexive : (a, b) R(a, b)

⇒ 3ab – 7ba = – 4ab always even so it is reflexive.

Symmetric : If 3ad – 7bc = Even

Case-I : odd odd

Case-II : even even

(c, d) R(a, b) ⇒ 3bc – 3ab

Case-I : odd odd

Case-II : even even

so symmetric relation

Transitive :

Set (3, 4)R (6, 4) Satisfy relation

Set (6, 4)R(3, 1) Satisfy relation

but (3, 4) R(3, 1) does not satisfy relation

so not transitive.

19. For a, b > 0, let

$$f(x) = \begin{cases} \frac{\tan((a+1)x) + b\tan x}{x}, & x < 0 \\ 3, & x = 0 \\ \frac{\sqrt{ax + b^2x^2} - \sqrt{ax}}{b\sqrt{a}\,x\sqrt{x}}, & x > 0 \end{cases}$$

be a continous function at x = 0. Then $\frac{b}{a}$ is equal to

(1) 5 (2) 4

(3) 8 (4) 6

Ans. (4)

Sol. $\lim_{x\to 0} f(x) = f(0) = 3$

$$\lim_{x\to 0^+} \frac{\sqrt{ax+b^2x^2} - \sqrt{ax}}{b\sqrt{a}\,x\sqrt{x}} = 3$$

$$\lim_{x\to 0^+} \frac{ax + b^2x^2 - ax}{b\sqrt{a}\,x^{3/2}\left(\sqrt{ax+b^2x^2} + \sqrt{ax}\right)}$$

$$\lim_{x\to 0^+} \frac{b^2}{b\sqrt{a}\left(\sqrt{a+b^2x} + \sqrt{a}\right)}$$

$$\frac{b}{\sqrt{a}.2\sqrt{a}} \Rightarrow \frac{b}{2a} = 3 \Rightarrow \frac{b}{a} = 6$$

20. If the term independent of x in the expansion of $\left(\sqrt{a}x^2 + \frac{1}{2x^3}\right)^{10}$ is 105, then a^2 is equal to :

(1) 4 (2) 9

(3) 6 (4) 2

Ans. (1)

Sol. $\left(\sqrt{a}x^2 + \frac{1}{2x^3}\right)^{10}$

General term $= {}^{10}C_r \left(\sqrt{a}x^2\right)^{10-r}\left(\frac{1}{2x^3}\right)^r$

$20 - 2r - 3r = 0$

$r = 4$

${}^{10}C_4 a^3 . \frac{1}{16} = 105$

$a^3 = 8$

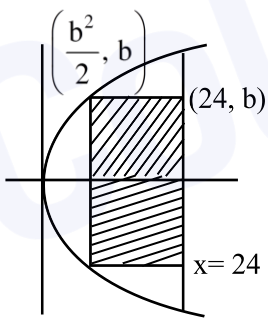
$a^2 = 4$

SECTION-B

21. Let A be the region enclosed by the parabola $y^2 = 2x$ and the line $x = 24$. Then the maximum area of the rectangle inscribed in the region A is ________ .

Ans. (128)

Sol.



$A = 2\left(24 - \frac{b^2}{2}\right).b$

$\frac{dA}{db} = 0 \Rightarrow b = 4$

$A = 2(24 - 8)4$

$= 128$

22. If $\alpha = \lim_{x \to 0^+}\left(\frac{e^{\sqrt{\tan x}} - e^{\sqrt{x}}}{\sqrt{\tan x} - \sqrt{x}}\right)$ and $\beta = \lim_{x \to 0}(1 + \sin x)^{\frac{1}{2}\cot x}$ are the roots of the quadratic equation $ax^2 + bx - \sqrt{e} = 0$, then 12 $\log_e(a + b)$ is equal to ________ .

Ans. (6)

Sol. $\alpha = \lim_{x \to 0^+} e^{\sqrt{x}} \frac{\left(e^{\sqrt{\tan x} - \sqrt{x}} - 1\right)}{\sqrt{\tan x} - \sqrt{x}}$

$= 1$

$\beta = \lim_{x \to 0}(1 + \sin x)^{\frac{1}{2}\cot x}$

$= e^{1/2}$

$x^2 - \left(1 + \sqrt{e}\right) + \sqrt{e} = 0$

$ax^2 + bx - \sqrt{e} = 0$

On comparing

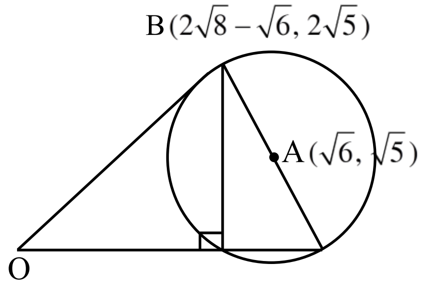
$a = -1, b = \sqrt{e} + 1$

$12\ \ell n(a + b) = 12 \times \frac{1}{2} = 6$

23. Let S be the focus of the hyperbola $\frac{x^2}{3} - \frac{y^2}{5} = 1$, on the positive x-axis. Let C be the circle with its centre at $A\left(\sqrt{6}, \sqrt{5}\right)$ and passing through the point S. if O is the origin and SAB is a diameter of C then the square of the area of the triangle OSB is equal to -

Ans. (40)

Sol.

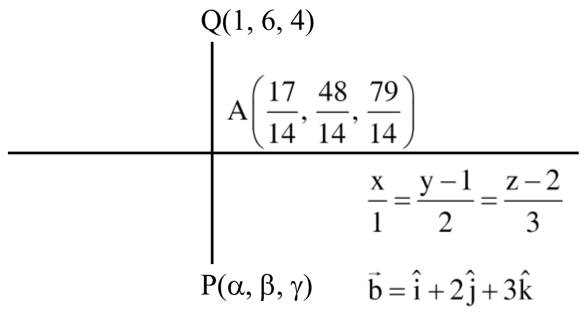


Area $= \frac{1}{2}(OS)\ h = \frac{1}{2}\sqrt{8}\ 2\sqrt{5} = \sqrt{40}$

24. Let P(α, β, γ) be the image of the point Q(l, 6, 4) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$. Then 2α + β + γ is equal to ________ .

Ans. (11)

Sol.



A(t, 2t + 1, 3t + 2)

$\overrightarrow{QA}=(t-1)\hat{i}+(2t-5)\hat{j}+(3t-2)\hat{k}$

$\overrightarrow{QA}.\vec{b}=0$

(t – 1) + 2(2t – 5) + 3(3t – 2) = 0

14t = 17

$\alpha=\frac{20}{14}$ $\quad\beta=\frac{12}{14}$ $\quad\gamma=\frac{102}{14}$

$2\alpha+\beta+\gamma=\frac{154}{14}=11$

25. An arithmetic progression is written in the following way

2
5 8
11 14 17
20 23 26 29
____
____

The sum of all the terms of the 10^{th} row is _______ .

Ans. (1505)

Sol. 2, 5, 11, 20, …..

General term $=\frac{3n^2-3n+4}{2}$

$T_{10}=\frac{3(100)-3(10)+4}{2}$

= 137

10 terms with c.d. = 3

$\text{sum}=\frac{10}{2}\left(2(137)+9(3)\right)$

= 1505

26. The number of distinct real roots of the equation |x + 1| |x + 3| – 4 |x + 2| + 5 = 0, is _________ .

Ans. (2)

Sol. |x + 1| |x + 3| - 4|x + 2| + 5 = 0

case–1

$x \le -3$

(x + 1) (x + 3) + 4(x + 2) + 5 = 0

$x^2 + 4x + 3 + 4x + 8 + 5 = 0$

$x^2 + 8x + 16 = 0$

$(x + 4)^2 = 0$

x = –4

case-2

$-3 \le x \le -2$

$-x^2 - 4x - 3 + 4x + 8 + 5 = 0$

$-x^2 + 10 = 0$

$x=\pm\sqrt{10}$

case-3

$-2 \le x \le -1$

$-x^2 - 4x - 3 - 4x - 8 + 5 = 0$

$-x^2 - 8x - 6 = 0$

$x^2 + 8x + 6 = 0$

$x=\frac{-8\pm2\sqrt{10}}{2}=-4\pm\sqrt{10}$

case-4

$x \ge -1$

$x^2 + 4x + 3 - 4x - 8 + 5 = 0$

$x^2 = 0$

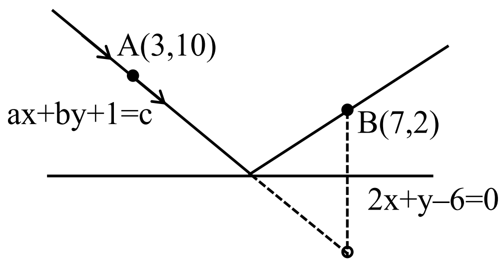
x = 0

No. of solution = 2

27. Let a ray of light passing through the point (3, 10) reflects on the line 2x + y = 6 and the reflected ray passes through the point (7, 2). If the equation of the incident ray is ax + by + 1 = 0, then $a^2 + b^2 + 3ab$ is equal to_.

Ans. (1)

Sol.



For B' $\frac{x-7}{2}=\frac{y-2}{1}=-2\left(\frac{14+2-6}{5}\right)$

$\frac{x-7}{2}=\frac{y-2}{1}=-4$

$x=-1 \quad y=-2 \quad B'(-1,-2)$

incident ray AB'

$M_{AB'}=3$

$y+2=3(x+1)$

$3x-y+1=0$

$a=3 \quad b=-1$

$a^2+b^2+3ab=9+1-9=1$

28. Let a, b, c ∈ N and a < b < c. Let the mean, the mean deviation about the mean and the variance of the 5 observations 9, 25, a, b, c be 18, 4 and $\frac{136}{5}$, respectively. Then 2a + b – c is equal to _______ .

Ans. (33)

Sol. a, b, c ∈ N a < b < c

$\bar{x}=\text{mean}=\frac{9+25+a+b+c}{5}=18$

$a+b+c=56$

$\text{Mean deviation}=\frac{\Sigma|x_i-\bar{x}|}{n}=4$

$=9+7+|18-a|+|18-b|+|18-c|=20$

$=|18-a|+|18-b|+|18-c|=4$

$\text{Variance}=\frac{\Sigma|x_i-\bar{x}|^2}{n}=\frac{136}{5}$

$=81+49+|18-a|^2+|18-b|^2+|18-c|^2=136$

$=(18-a)^2+(18-b)^2+(18-c)^2=6$

Possible values $(18-a)^2=1$, $(18-b)^2=1$ $(18-c)^2=4$

a < b < c

so 18–a=1 18–b=–1 18–c=–2

a=17 b=19 c=20

a + b + c = 56

2a + b – c 34 = 19 – 20 = 33

29. Lei $\alpha|x|=|y|e^{xy-\beta}$, α, β ∈ N be the solution of the differential equation xdy – ydx + xy(xdy+ydx) = 0, y(l) = 2. Then α + β is equal to _

Ans. (4)

Sol. $a|x|=|y|\,e^{yx-\beta}$, a, b ∈ N

$xdy-ydx+xy(xdy+ydx)=0$

$\frac{dy}{y}-\frac{dx}{x}+(xdy+ydx)=0$

$\ell n|y|-\ell n|x|+xy=c$

$y(1)=2$

$\ell n|2|-0+2=c$

$c=2+\ell n2$

$\ell n|y|-\ell n|x|+xy=2+\ell n2$

$\ell n|x|=\ell n\left|\frac{y}{2}\right|-2+xy$

$|x|=\left|\frac{y}{2}\right|e^{xy-2}$

$2|x|=|y|e^{xy-2}$

$\alpha=2 \quad \beta=2 \quad \alpha+\beta=4$

30. If $\int\frac{1}{\sqrt[5]{(x-1)^4\,(x+3)^6}}dx=A\left(\frac{\alpha x-1}{\beta x+3}\right)^B+C$, where C is the constant of integration, then the value of α + β + 20AB is _______ .

Ans. (7)

Sol. $\int\frac{1}{\sqrt[5]{(x-1)^4\,(x+3)^6}}dx=A\left(\frac{\alpha x-1}{\beta x+3}\right)^B+C$

$I=\int\frac{1}{(x-1)^{4/5}\,(x+3)^{6/5}}dx$

$I=\int\frac{1}{\left(\frac{x-1}{x+3}\right)^{4/5}(x+3)^2}dx$

$\left(\frac{x-1}{x+3}\right)=t \Rightarrow \frac{4}{(x+3)^2}dx=dt \qquad t^{-4/5+1}$

$I=\frac{1}{4}\int\frac{1}{t^{4/5}}dt=\frac{1}{4}\,\frac{t^{1/5}}{1/5}+c$

$I=\frac{5}{4}\left(\frac{x-1}{x+3}\right)^{1/5}+C$

$A=\frac{5}{4} \qquad \alpha=\beta=1 \qquad B=\frac{1}{5}$

$\alpha+\beta+20AB=2+20\times\frac{5}{4}\times\frac{1}{5}=7$