FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Tuesday 09th April, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

SECTION-A

31. A proton, an electron and an alpha particle have the same energies. Their de-Broglie wavelengths will be compared as:

(1) $\lambda_e > \lambda_\alpha > \lambda_p$ (2) $\lambda_\alpha < \lambda_p < \lambda_e$

(3) $\lambda_p < \lambda_e < \lambda_\alpha$ (4) $\lambda_p > \lambda_e > \lambda_\alpha$

Ans. (2)

Sol. $\lambda_{DB} = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mk}}$

$\Rightarrow \lambda_{DB} \;\alpha\; \dfrac{1}{\sqrt{m}}$

$\Rightarrow \lambda_a < \lambda_p < \lambda_e$

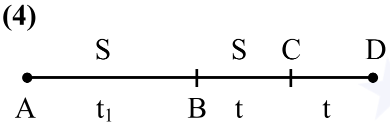
32. A particle moving in a straight line covers half the distance with speed 6 m/s. The other half is covered in two equal time intervals with speeds 9 m/s and 15 m/s respectively. The average speed of the particle during the motion is :

(1) 8.8 m/s (2) 10 m/s

(3) 9.2 m/s (4) 8 m/s

Ans. (4)

Sol.



BD $\Rightarrow$ S = 9t + 15t = 24t

AB $\Rightarrow$ S = $6t_1$ = 24t $\Rightarrow$ t_1 = 4t

$< \text{speed} > = \dfrac{\text{dist.}}{\text{time}} = \dfrac{48t}{2t + t_1}$

$= \dfrac{48t}{2t + 4t} \Rightarrow \dfrac{48t}{6t} \Rightarrow 8 \text{ m/s}$

33. A plane EM wave is propagating along x direction. It has a wavelength of 4 mm. If electric field is in y direction with the maximum magnitude of 60 Vm^{-1}, the equation for magnetic field is:

(1) $B_z = 60\sin\left[\dfrac{\pi}{2}\left(x - 3\times10^8 t\right)\right]\hat{k}T$

(2) $B_z = 2\times10^{-7}\sin\left[\dfrac{\pi}{2}\times10^3\left(x - 3\times10^8 t\right)\right]\hat{k}T$

(3) $B_x = 60\sin\left[\dfrac{\pi}{2}\left(x - 3\times10^8 t\right)\right]\hat{i}T$

(4) $B_z = 2\times10^{-7}\sin\left[\dfrac{\pi}{2}\left(x - 3\times10^8 t\right)\right]\hat{k}T$

Ans. (2)

Sol. E = BC $\Rightarrow$ 60 = B × 3 × 10^8

$\Rightarrow$ B = 2 × 10^{-7}

Also C = fλ

$\Rightarrow 3 \times 10^8 = f \times 4 \times 10^{-3}$

$\Rightarrow f = \dfrac{3}{4}\times10^{11}$

$\Rightarrow \omega = 2\pi f = \dfrac{3}{4}\times2\pi\times10^{11}$

$\Rightarrow \omega = \dfrac{\pi}{2}\times10^3\,C$

$\Rightarrow$ Electric field $\Rightarrow$ y direction

Propagation $\Rightarrow$ x direction

Magnetic field $\Rightarrow$ z-direction

34. Given below are two statements:

Statement (I): When an object is placed at the centre of curvature of a concave lens, image is formed at the centre of curvature of the lens on the other side.

Statement (II): Concave lens always forms a virtual and erect image.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) **Statement I** is false but **Statement II** is true.

(2) Both **Statement I** and **Statement II** are false.

(3) **Statement I** is true but **Statement II** is false.

(4) Both **Statement I** and **Statement II** are true.

NTA Ans. (1)

Allen Ans. (2)

Sol. $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

$\dfrac{1}{v} - \dfrac{1}{-2f} = \dfrac{1}{-f}$

$\Rightarrow \dfrac{1}{v} = \dfrac{-1}{2f} \Rightarrow v = -2f$

$\dfrac{1}{v} = \dfrac{1}{u} + \dfrac{1}{f} \Rightarrow$ Virtual image of Real object.

In statement II, it is not mentioned that object is real or virtual hence Statement II is false.

35. A light emitting diode (LED) is fabricated using GaAs semiconducting material whose band gap is 1.42 eV. The wavelength of light emitted from the LED is:

(1) 650 nm (2) 1243 nm

(3) 875 nm (4) 1400 nm

Ans. (3)

Sol. $\lambda = \frac{1240}{1.42} = 875$ nm (Approx)

36. A sphere of relative density σ and diameter D has concentric cavity of diameter d. The ratio of $\frac{D}{d}$, if it just floats on water in a tank is:

(1) $\left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{3}}$ (2) $\left(\frac{\sigma+1}{\sigma-1}\right)^{\frac{1}{3}}$

(3) $\left(\frac{\sigma-1}{\sigma}\right)^{\frac{1}{3}}$ (4) $\left(\frac{}{\sigma+2}\right)$

Ans. (1)

Sol. weight (w) = $\frac{4}{3}\pi\left(\frac{D^3-d^3}{8}\right)\sigma g$

Buoyant force (F_b) = $1\times\frac{4}{3}\pi\left(\frac{D^3}{8}\right)\cdot g$

For Just Float $\Rightarrow w = F_b$

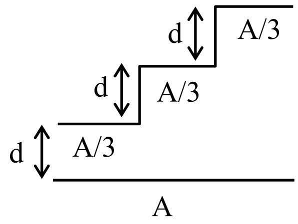
$\Rightarrow (D^3 - d^3)\sigma = D^3$

$\Rightarrow 1-\frac{d^3}{D^3}=\frac{1}{\sigma}$

$\Rightarrow 1-\frac{1}{\sigma}=\left(\frac{d}{D}\right)^3$

$\Rightarrow \left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{3}}=\left(\frac{D}{d}\right)$

37. A capacitor is made of a flat plate of area A and a second plate having a stair-like structure as shown in figure. If the area of each stair is $\frac{A}{3}$ and the height is d, the capacitance of the arrangement is:



(1) $\frac{11\varepsilon_0 A}{18d}$ (2) $\frac{13\varepsilon_0 A}{17d}$

(3) $\frac{11\varepsilon_0 A}{20d}$ (4) $\frac{18\varepsilon_0 A}{11d}$

Ans. (1)

Sol. All capacitor are in parallel combination.

Also effective area is common area only

$\Rightarrow C_{eq} = C_1 + C_2 + C_3$

$\Rightarrow C_{eq} = \frac{A\varepsilon_0}{3d}+\frac{A\varepsilon_0}{3(2d)}+\frac{A\varepsilon_0}{3(3d)}$

$\Rightarrow C_{eq} = \frac{A\varepsilon_0}{3}\left(\frac{11}{6d}\right)$

$\Rightarrow C_{eq} = \frac{11A\varepsilon_0}{18d}$

38. A light unstretchable string passing over a smooth light pulley connects two blocks of masses m_1 and m_2. If the acceleration of the system is $\frac{g}{8}$, then the ratio of the masses $\frac{m_2}{m_1}$ is:

(1) 9 : 7 (2) 4 : 3

(3) 5 : 3 (4) 8 : 1

Ans. (1)

Sol. $a_{sys} = \left(\frac{m_2-m_1}{m_1+m_2}\right)g = \frac{g}{8}$

$\Rightarrow \frac{m_2}{m_1}=\frac{9}{7}$

39. The dimensional formula of latent heat is:

(1) $[M^0LT^{-2}]$ (2) $[MLT^{-2}]$

(3) $[M^0L^2T^{-2}]$ (4) $[ML^2T^{-2}]$

Ans. (3)

Sol. Latent heat is specific heat

$$\Rightarrow \frac{ML^2T^{-2}}{M} = M^0L^2T^{-2}$$

40. The volume of an ideal gas (γ = 1.5) is changed adiabatically from 5 litres to 4 litres. The ratio of initial pressure to final pressure is:

(1) $\frac{4}{5}$ (2) $\frac{16}{25}$

(3) $\frac{8}{5\sqrt{5}}$ (4) $\frac{2}{\sqrt{5}}$

Ans. (3)

Sol. For Adiabatic process

$$P_iV_i = P_fV_f^{\gamma}$$

$$P_i(5)^{1.5} = P_f(4)^{1.5}$$

$$\frac{P_i}{P_f} = \left(\frac{4}{5}\right)^{\frac{3}{2}} = \frac{4}{5}\cdot\left(\frac{4}{5}\right)^{\frac{1}{2}} \Rightarrow \sqrt{}$$

41. The energy equivalent of 1g of substance is:

(1) 11.2×10^{24} MeV (2) 5.6×10^{12} MeV

(3) 5.6 eV (4) 5.6×10^{26} MeV

Ans. (4)

Sol. $E = mC^2$

$\Rightarrow E = (1 \times 10^{-3}) \times (3 \times 10^8)^2$ J

$\Rightarrow E = (10^{-3})(9 \times 10^{16})(6.241 \times 10^{18})$ eV

$E = 56.169 \times 10^{31}$ eV

$E \approx 5.6 \times 10^{26}$ MeV

42. An astronaut takes a ball of mass m from earth to space. He throws the ball into a circular orbit about earth at an altitude of 318.5 km. From earth's surface to the orbit, the change in total mechanical energy of the ball is $x\frac{GM_em}{21R_e}$. The value of x is (take R_e = 6370 km):

(1) 11 (2) 9

(3) 12 (4) 10

Ans. (1)

Sol. $h = 318.5 \approx \left(\frac{R_e}{20}\right)$

$$T{\cdot}E_i = \frac{-GM_em}{R_e}$$

$$T{\cdot}E_f = \frac{-GM_em}{2(R_e+h)} = \frac{-GM_em}{2\left(R_e+\frac{R_e}{20}\right)}$$

$$\Rightarrow T{\cdot}E_f = \frac{-10GM_em}{21R_e}$$

Change in total mechanical energy

$= TE_f - TE_i$

$$= \frac{GM_em}{Re}\left[1-\frac{10}{21}\right] = \frac{11GM_em}{21Re}$$

43. Given below are two statements:

Statement (I) : When currents vary with time, Newton's third law is valid only if momentum carried by the electromagnetic field is taken into account.

Statement (II) : Ampere's circuital law does not depend on Biot-Savart's law.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) Both **Statement I** and **Statement II** are false.

(2) **Statement I** is true but **Statement II** is false.

(3) **Statement I** is false but **Statement II** is true.

(4) Both **Statement I** and **Statement II** are true.

Ans. (2)

Sol. Conceptual.

44. A particle of mass m moves on a straight line with its velocity increasing with distance according to the equation $v = \alpha\sqrt{x}$, where α is a constant. The total work done by all the forces applied on the particle during its displacement from x = 0 to x = d, will be:

(1) $\frac{m}{2\alpha^2 d}$ (2) $\frac{md}{2\alpha^2}$

(3) $\frac{m\alpha^2 d}{2}$ (4) $2m\alpha^2 d$

Ans. (3)

Sol. $v = \alpha\sqrt{x}$

at $x = 0 : v = 0$

& at $x = d$; $v = \alpha\sqrt{d}$

$W.D = K_f - K_i$

$W.D = \frac{1}{2}m\left(\alpha\sqrt{d}\right)^2 - \frac{1}{2}m(0)^2$

$\Rightarrow W.D = \frac{m\alpha^2 d}{2}$

45. A galvanmeter has a coil of resistance 200 Ω with a full scale deflection at 20 μA. The value of resistance to be added to use it as an ammeter of range (0–20) mA is:

(1) 0.40 Ω (2) 0.20 Ω

(3) 0.50 Ω (4) 0.10 Ω

Ans. (2)

Sol. $G = 200\ \Omega$

$i_g = 20\ \mu A$

$i = i_g\left(\frac{G}{S}+1\right)$

$\Rightarrow 20 \times 10^{-3} = 20 \times 10^{-6}\left(\frac{200}{S}+1\right)$

$\Rightarrow \frac{200}{S} = 999$

$\Rightarrow S \approx 0.2\ \Omega$

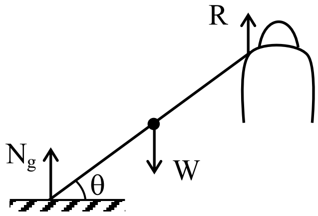
46. A heavy iron bar, of weight W is having its one end on the ground and the other on the shoulder of a person. The bar makes an angle θ with the horizontal. The weight experienced by the person is:

(1) $\frac{W}{2}$ (2) W

(3) W cos θ (4) W sin θ

Ans. (1)

Sol.



R = net reaction force by shoulder

Balancing torque about pt of contact on ground:

$W\left(\frac{L}{2}\cos\theta\right) = R(L\cos\theta)$

$\Rightarrow R = \frac{W}{2}$

47. One main scale division of a vernier caliper is equal to m units. If n^{th} division of main scale coincides with $(n + 1)^{th}$ division of vernier scale, the least count of the vernier caliper is:

(1) $\frac{n}{(n+1)}$ (2) $\frac{m}{(n+1)}$

(3) $\frac{1}{(n+1)}$ (4) $\frac{m}{n(n+1)}$

Ans. (2)

Sol. n MSD = (n + 1) VSD

$\Rightarrow 1\ VSD = \frac{n}{n+1}\ MSD$

L·C = 1 MSD – 1 VSD

$L{\cdot}C = m - m\left(\frac{n}{n+1}\right)$

$L{\cdot}C = m\left(\frac{n+1-n}{n+1}\right)$

$\Rightarrow L{\cdot}C = \left(\frac{m}{n+1}\right)$

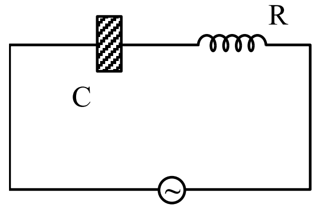
48. A bulb and a capacitor are connected in series across an ac supply. A dielectric is then placed between the plates of the capacitor. The glow of the bulb:

(1) increases (2) remains same

(3) becomes zero (4) decreases

Ans. (1)

Sol.



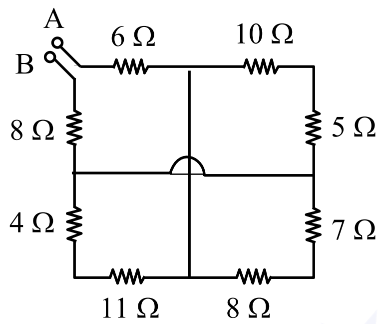
$Z=\sqrt{R^2+X_C^2}$ & $X_C=\frac{1}{WC}$

due to dielectric

$C\uparrow \Rightarrow X_c\downarrow \Rightarrow Z\downarrow$

So, current increases & thus bulb will glow more brighter.

49. The equivalent resistance between A and B is:

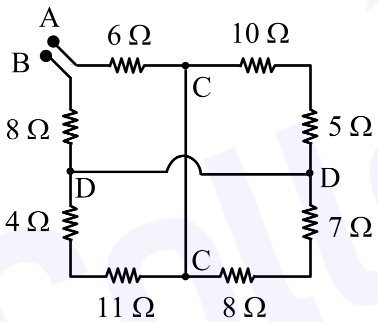


(1) 18 Ω (2) 25 Ω

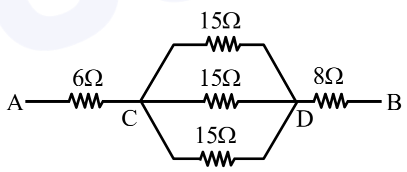
(3) 27 Ω (4) 19 Ω

Ans. (4)

Sol.



$\Downarrow$



$\Downarrow$

$\Rightarrow R_{eq}=6\Omega+5\Omega+8\Omega=19\Omega$

50. A sample of 1 mole gas at temperature T is adiabatically expanded to double its volume. If adiabatic constant for the gas is $\gamma=\frac{3}{2}$, then the work done by the gas in the process is:

(1) $RT\left[2-\sqrt{2}\right]$ (2) $\frac{R}{T}\left[2-\sqrt{2}\right]$

(3) $RT\left[2+\sqrt{2}\right]$ (4) $\frac{T}{R}\left[2+\sqrt{2}\right]$

Ans. (1)

Sol. $TV^{\gamma-1}=\text{constant}$

$\Rightarrow T(V)^{\frac{3}{2}-1}=T_f(2V)^{\frac{3}{2}-1}$

$\Rightarrow TV^{\frac{1}{2}}=T_f(2)^{\frac{1}{2}}(V)^{\frac{1}{2}}$

$\Rightarrow T_f=\left(\frac{T}{\sqrt{2}}\right)$

Now, W.D. $=\frac{nR\Delta T}{1-\gamma}=\frac{1\cdot R\left[\frac{T}{\sqrt{2}}-T\right]}{1-\frac{3}{2}}$

$\Rightarrow$ W.D. $=2RT\left[1-\frac{1}{\sqrt{2}}\right]$

$\Rightarrow$ W.D. $=RT\left[2-\sqrt{2}\right]$

SECTION-B

51. If $\vec{a}$ and $\vec{b}$ makes an angle $\cos^{-1}\left(\frac{5}{9}\right)$ with each other, then $|\vec{a}+\vec{b}|=\sqrt{2}|\vec{a}-\vec{b}|$ for $|\vec{a}|=n|\vec{b}|$

The integer value of n is ________.

Ans. (3)

Sol. $\cos\theta=\frac{5}{9}$

$\frac{\vec{a}\cdot\vec{b}}{ab}=\frac{5}{9}$ ....(1)

$|\vec{a}+\vec{b}|=\sqrt{2}|\vec{a}-\vec{b}|$

$a^2+b^2+2\vec{a}\cdot\vec{b}=2a^2+2b^2-4\vec{a}\cdot\vec{b}$

$6\vec{a}\cdot\vec{b}=a^2+b^2$

$$6\times\frac{5}{9}ab = a^2 + b^2$$

$$\frac{10}{3}ab = a^2 + b^2 \quad \& \quad a = nb$$

$$\frac{10}{3}nb^2 = n^2b^2 + b^2$$

$$3n^2 - 10n + 3 = 0$$

$$n = \frac{1}{3} \text{ and } n = 3$$

integer value n = 3

52. At the centre of a half ring of radius R = 10 cm and linear charge density 4n C m^{-1}, the potential is x π V. The value of x is ________.

Ans. (36)

Sol. Potential at centre of half ring

$$V = \frac{KQ}{R}$$

$$V = \frac{K\lambda\pi R}{R}$$

$$V = K\lambda\pi \Rightarrow V = 9 \times 10^{9} \times 4 \times 10^{-9}\pi$$

$$V = 36\pi$$

53. A star has 100% helium composition. It starts to convert three ^4He into one ^{12}C via triple alpha process as $^4He + {}^4He + {}^4He \rightarrow {}^{12}C + Q$. The mass of the star is 2.0×10^{32} kg and it generates energy at the rate of 5.808×10^{30} W. The rate of converting these ^4He to ^{12}C is $n \times 10^{42}\ s^{-1}$, where n is _______.
[Take, mass of ^4He = 4.0026 u, mass of ^{12}C = 12 u]

NTA Ans. (5)

Allen Ans. (15)

Sol. $^4He + {}^4He + {}^4He \rightarrow {}^{12}C + Q$

power generated = $\frac{N}{t}Q$

where, N → No. of reaction/sec.

$$Q = (3m_{He} - m_C)C^2$$

$$Q = (3 \times 4.0026 - 12)(3 \times 10^8)^2$$

Q = 7.266 MeV

$$\frac{N}{t} = \frac{\text{power}}{Q} = \frac{5.808\times10^{30}}{7.266\times10^{6}\times1.6\times10^{-19}}$$

$$\frac{N}{t} = 5\times10^{42}$$

rate of conversion of ^4He into $^{12}C = 15 \times 10^{42}$

Hence, n = 15

54. In a Young's double slit experiment, the intensity at a point is $\left(\frac{1}{4}\right)^{th}$ of the maximum intensity, the minimum distance of the point from the central maximum is ________ μm.
(Given : λ = 600 nm, d = 1.0 mm, D = 1.0 m)

Ans. (200)

Sol. $I = I_0\cos^2\left(\frac{\Delta\phi}{2}\right)$

$$\frac{I_0}{4} = \cos^2\left(\frac{\Delta\phi}{2}\right)$$

$$\Delta\phi = \frac{2\pi}{3}$$

$$\frac{2\pi}{\lambda}\left(\frac{yd}{D}\right) = \frac{2\pi}{3}$$

$$y = \frac{\lambda D}{3d} = \frac{600\times10^{-9}\times1}{3\times10^{-3}} = 2 \times 10^{-4}\text{ m}$$

55. A string is wrapped around the rim of a wheel of moment of inertia 0.40 kgm^2 and radius 10 cm. The wheel is free to rotate about its axis. Initially the wheel is at rest. The string is now pulled by a force of 40 N. The angular velocity of the wheel after 10 s is x rad/s, where x is _______.

Ans. (100)

Sol. $\tau = FR = I\alpha \Rightarrow 40 \times 0.1 = 0.4\alpha$

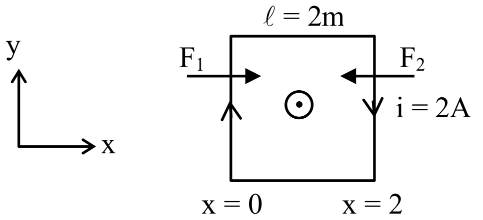
$\alpha = 10$ rad/s^2

$W_f = 10 \times 10 = 100$ rad/s

56. A square loop of edge length 2 m carrying current of 2 A is placed with its edges parallel to the x-y axis. A magnetic field is passing through the x-y plane and expressed as $\vec{B} = B_0(1+4x)\hat{k}$, where B_0 = 5 T. The net magnetic force experienced by the loop is _______ N.

Ans. (160)

Sol.



$B(x = 0) = B_0$, $B(x = 2) = 9B_0$

Also, $F = i\ell B$

$\Rightarrow F_1 = i\ell B_0$ & $F_2 = 9i\ell B_0$

$F = F_2 - F_1 = 8i\ell B_0 = 8 \times 2 \times 2 \times 5$

$F = 160$ N

57. Two persons pull a wire towards themselves. Each person exerts a force of 200 N on the wire. Young's modulus of the material of wire is 1×10^{11} N m^{-2}. Original length of the wire is 2 m and the area of cross section is 2 cm^2. The wire will extend in length by ______ μm.

Ans. (20)

Sol.

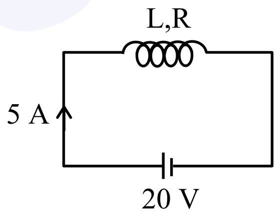
$$\frac{F}{A} = Y\frac{\Delta\ell}{\ell} \Rightarrow \Delta\ell = \frac{F\ell}{AY}$$

$$\Delta\ell = \frac{200 \times 2}{2 \times 10^{-4} \times 10^{11}} = 2 \times 10^{-5} = 20\mu m$$

58. When a coil is connected across a 20 V dc supply, it draws a current of 5 A. When it is connected across 20 V, 50 Hz ac supply, it draws a current of 4 A. The self inductance of the coil is ______ mH. (Take π = 3)

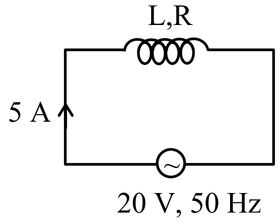
Ans. (10)

Sol. **<u>Case-I</u>:**



$$i = \frac{20}{R} \Rightarrow R = 4\Omega$$

<u>Case-II</u>:



$$i = \frac{20}{Z}$$

$$4 = \frac{20}{\sqrt{R^2 + X_L^2}} \Rightarrow \sqrt{R^2 + X_L^2} = 5$$

$$R^2 + X_L^2 = 25 \Rightarrow X_L = 3\ \Omega$$

$$L = \frac{3}{2\pi f} = \frac{1}{2 \times 50} = \frac{1000}{100} \text{mH}$$

$L = 10$ mH

59. The position, velocity and acceleration of a particle executing simple harmonic motion are found to have magnitudes of 4 m, 2 ms^{-1} and 16 ms^{-2} at a certain instant. The amplitude of the motion is $\sqrt{x}$ m where x is _______.

Ans. (17)

Sol. $x = 4$ m, $V = 2$ m/s, $a = 16$ m/s^2

$|a| = \omega^2 x$

$\Rightarrow 16 = \omega^2(4)$

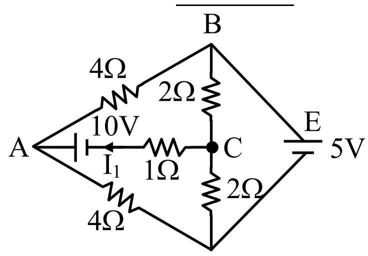
$\omega = 2$ rad/s

$v = \omega\sqrt{A^2 - x^2}$

$$A = \sqrt{\frac{v^2}{\omega^2} + x^2} \Rightarrow A = \sqrt{\frac{4}{4} + 16}$$

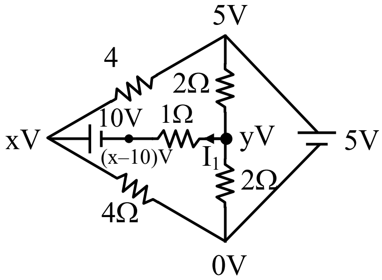
$A = \sqrt{17}$ m

60. The current flowing through the 1 Ω resistor is $\frac{n}{10}$ A. The value of n is _________.



Ans. (25)

Sol.



$$\frac{y-5}{2}+\frac{y-0}{2}+\frac{y-x+10}{1}=0$$

$y - 5 + y + 2y - 2x + 20 = 0$

$4y - 2x + 15 = 0$ ....(i)

$$\frac{x-5}{4}+\frac{x-0}{4}+\frac{x-10-y}{1}=0$$

$x - 5 + x + 4x - 40 - 4y = 0$

$6x - 4y - 45 = 0$ ..(i)

$-2x + 4y + 15 = 0$ ..(ii)

$4x - 30 = 0$

$x = \frac{15}{2}$ & $4y - 15 + 15 = 0$

$y = 0$

$$i = \frac{y-x+10}{1}$$

$$i = \frac{0-7.5+10}{1}$$

$i = 2.5A = \frac{n}{10}A$

$n = 25$