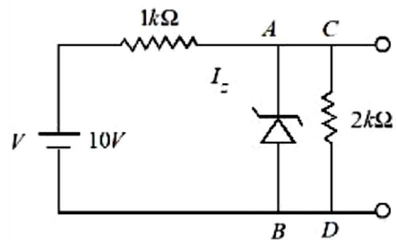
FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Monday 29th January, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

SECTION-A

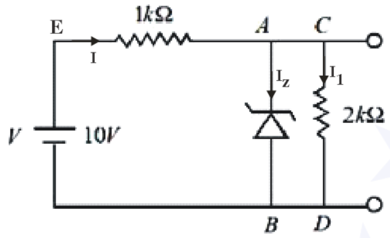
31. In the given circuit, the breakdown voltage of the Zener diode is 3.0 V. What is the value of I_z?



(1) 3.3 mA (2) 5.5 mA

(3) 10 mA (4) 7 mA

Ans. (2)



Sol.

$V_z = 3V$

Let potential at B = 0 V

Potential at $E(V_E) = 10$ V

$V_C = V_A = 3$ V

$I_z + I_1 = I$

$$I = \frac{10-3}{1000} = \frac{7}{1000} A$$

$$I_1 = \frac{3}{2000} A$$

Therefore $I_z = \frac{7-1.5}{1000} = 5.5mA$

32. The electric current through a wire varies with time as $I = I_0 + \beta t$. where $I_0 = 20$ A and $\beta = 3$ A/s. The amount of electric charge crossed through a section of the wire in 20 s is :

(1) 80 C (2) 1000 C

(3) 800 C (4) 1600 C

Ans. (2)

Sol. Given that

Current $I = I_0 + \beta t$

$I_0 = 20A$

$\beta = 3A/s$

$I = 20 + 3t$

$$\frac{dq}{dt} = 20 + 3t$$

$$\int_0^q dq = \int_0^{20} (20+3t)dt$$

$$q = \int_0^{20} 20dt + \int_0^{20} 3tdt$$

$$q = \left[20t + \frac{3t^2}{2}\right]_0^{20} = 1000 \text{ C}$$

33. Given below are two statements:

Statement I : If a capillary tube is immersed first in cold water and then in hot water, the height of capillary rise will be smaller in hot water.

Statement II : If a capillary tube is immersed first in cold water and then in hot water, the height of capillary rise will be smaller in cold water.

In the light of the above statements, choose the ***most appropriate*** from the options given below

(1) Both **Statement I** and **Statement II** are true

(2) Both **Statement I** and **Statement II** are false

(3) **Statement I** is true but **Statement II** is false

(4) **Statement I** is false but **Statement II** is true

Ans. (3)

Sol. Surface tension will be less as temperature increases

$$h = \frac{2T\cos\theta}{\rho gr}$$

Height of capillary rise will be smaller in hot water and larger in cold water.

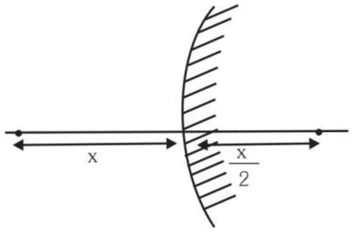
34. A convex mirror of radius of curvature 30 cm forms an image that is half the size of the object. The object distance is :

(1) –15 cm (2) 45 cm

(3) –45cm (4) 15 cm

Ans. **(1)**

Sol.



Given R = 30 cm

f = R/2 = +15 cm

Magnification (m) $= \pm\frac{1}{2}$

For convex mirror, virtual image is formed for real object.

Therefore, m is +ve

$$\frac{1}{2} = \frac{f}{f-u}$$

u = – 15 cm

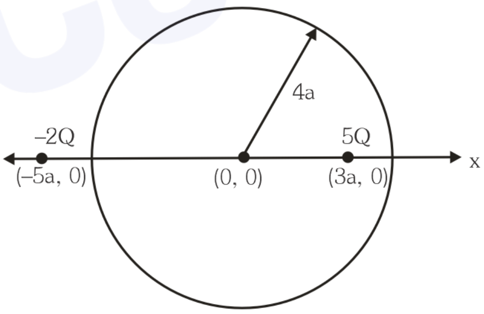
35. Two charges of 5Q and –2Q are situated at the points (3a, 0) and (–5a, 0) respectively. The electric flux through a sphere of radius '4a' having center at origin is :

(1) $\frac{2Q}{\varepsilon_0}$ (2) $\frac{5Q}{\varepsilon_0}$

(3) $\frac{7Q}{\varepsilon_0}$ (4) $\frac{3Q}{\varepsilon_0}$

Ans. **(2)**

Sol.



5Q charge is inside the spherical region

flux through sphere = $\frac{5Q}{\varepsilon_0}$

36. A body starts moving from rest with constant acceleration covers displacement S_1 in first (p – 1) seconds and S_2 in first p seconds. The displacement $S_1 + S_2$ will be made in time :

(1) $(2p+1)s$

(2) $\sqrt{(2p^2-2p+1)}s$

(3) $(2p-1)s$

(4) $(2p^2-2p+1)s$

Ans. **(2)**

Sol. S_1 in first (p – 1) sec

S_2 in first p sec

$$S_1 = \frac{1}{2}a(p-1)^2$$

$$S_2 = \frac{1}{2}a(p)^2$$

$$S_1 + S_2 = \frac{1}{2}at^2$$

$(p-1)^2 + p^2 = t^2$

$$t = \sqrt{2p^2+1-2p}$$

37. The potential energy function (in J) of a particle in a region of space is given as U = $(2x^2 + 3y^3 + 2z)$. Here x, y and z are in meter. The magnitude of x - component of force (in N) acting on the particle at point P (1, 2, 3) m is :

(1) 2 (2) 6

(3) 4 (4) 8

Ans. **(3)**

Sol. Given $U = 2x^2 + 3y^3 + 2z$

$$F_x = -\frac{\partial U}{\partial x} = -4x$$

At x = 1 magnitude of F_x is 4N

38. The resistance $R = \frac{V}{I}$ where $V = (200 \pm 5)V$ and $I = (20 \pm 0.2)A$, the percentage error in the measurement of R is :

(1) 3.5%

(2) 7%

(3) 3%

(4) 5.5%

Ans. **(1)**

Sol. $R = \frac{V}{I}$

According to error analysis

$$\frac{dR}{R} = \frac{dV}{V} + \frac{dI}{I}$$

$$\frac{dR}{R} = \frac{5}{200} + \frac{0.2}{20}$$

$$\frac{dR}{R} = \frac{7}{200}$$

% error $\frac{dR}{R} \times 100 = \frac{7}{200} \times 100 = 3.5\%$

39. A block of mass 100 kg slides over a distance of 10 m on a horizontal surface. If the co-efficient of friction between the surfaces is 0.4, then the work done against friction (in J) is :

(1) 4200

(2) 3900

(3) 4000

(4) 4500

Ans. **(3)**

Sol. Given m = 100 kg

s = 10 m

$\mu = 0.4$

As $f = \mu mg = 0.4 \times 100 \times 10 = 400$ N

Now $W = f.s = 400 \times 10 = 4000$ J

40. Match List I with List II

	List I		List II
A.	$\oint \vec{B}.\vec{dl} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$	I.	Gauss' law for electricity
B.	$\oint \vec{E}.\vec{dl} = \frac{d\phi_B}{dt}$	II.	Gauss' law for magnetism
C.	$\oint \vec{E}.\vec{dA} = \frac{Q}{\varepsilon_0}$	III.	Faraday law
D.	$\oint \vec{B}.\vec{dA} = 0$	IV.	Ampere – Maxwell law

Chose the correct answer from the options given below

(1) A-IV, B-I, C-III, D-II

(2) A-II, B-III, C-I, D-IV

(3) A-IV, B-III, C-I, D-II

(4) A-I, B-II, C-III, D-IV

Ans. **(3)**

Sol. Ampere – Maxwell law

$\rightarrow \int \vec{B}.\vec{dl} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{}$

Faraday law $\rightarrow \oint \vec{E}.\vec{dl} = \frac{d\phi_B}{dt}$

Gauss' law for electricity $\rightarrow \oint \vec{E}.\vec{dA} = \frac{Q}{\varepsilon_0}$

Gauss ' law for magnetism $\rightarrow \oint \vec{B}.\vec{dA} = 0$

41. If the radius of curvature of the path of two particles of same mass are in the ratio 3:4, then in order to have constant centripetal force, their velocities will be in the ratio of:

(1) $\sqrt{3} : 2$ (2) $1 : \sqrt{3}$

(3) $\sqrt{3} : 1$ (4) $2 : \sqrt{3}$

Ans. **(1)**

Sol. Given $m_1 = m_2$

and $\frac{r_1}{r_2} = \frac{3}{4}$

As centripetal force $F = \frac{mv^2}{r}$

In order to have constant (same in this question) centripetal force

$F_1 = F_2$

$$\frac{m_1v_1^2}{r_1} = \frac{m_2v_2^2}{r_2}$$

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} = \frac{\sqrt{3}}{2}$$

42. A galvanometer having coil resistance 10 Ω shows a full scale deflection for a current of 3mA. For it to measure a current of 8A, the value of the shunt should be:

(1) $3 \times 10^{-3}\Omega$ (2) $4.85 \times 10^{-3}\Omega$

(3) $3.75 \times 10^{-3}\Omega$ (4) $2.75 \times 10^{-3}\Omega$

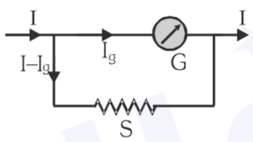
Ans. **(3)**

Sol. Given G = 10 Ω

$I_g = 3mA$

I = 8A

In case of conversion of galvanometer into ammeter.



We have $I_gG = (I - I_g)S$

$$S = \frac{I_gG}{I - I_g}$$

$$S = \frac{(3 \times 10^{-3})10}{8 - 0.003} = 3.75 \times 10^{-3}\Omega$$

43. The de-Broglie wavelength of an electron is the same as that of a photon. If velocity of electron is 25% of the velocity of light, then the ratio of K.E. of electron and K.E. of photon will be:

(1) $\frac{1}{1}$ (2) $\frac{1}{8}$

(3) $\frac{8}{1}$ (4) $\frac{1}{4}$

Ans. **(2)**

Sol. For photon

$$E_P = \frac{hc}{\lambda_p} \Rightarrow \lambda_p = \frac{hc}{E_P}$$

For electron

$$\lambda_e = \frac{h}{m_ev_e} = \frac{hv_e}{2K_e}$$

Given $v_e = 0.25$ c

$$\lambda_e = \frac{h \times 0.25c}{2K_e} = \frac{hc}{8K_e}$$

Also $\lambda_p = \lambda_e$

$$\frac{hc}{E_p} = \frac{hc}{8K_e}$$

$$\frac{K_e}{E_p} = \frac{1}{8}$$

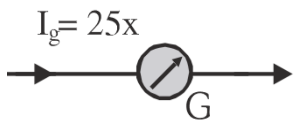
44. The deflection in moving coil galvanometer falls from 25 divisions to 5 division when a shunt of 24Ω is applied. The resistance of galvanometer coil will be :

(1) 12Ω (2) 96Ω

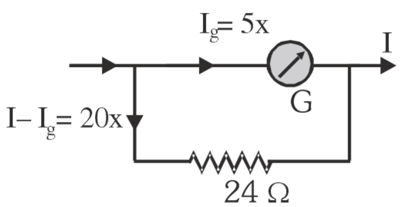
(3) 48Ω (4) 100Ω

Ans. **(2)**

Sol. Let x = current/division



After applying shunt



Now 5x × G = 20x × 24

G = 4 × 24

G = 96Ω

45. A biconvex lens of refractive index 1.5 has a focal length of 20 cm in air. Its focal length when immersed in a liquid of refractive index 1.6 will be:

(1) – 16 cm

(2) – 160 cm

(3) + 160 cm

(4) + 16 cm

Ans. **(2)**

Sol. $\mu_1 = 1.5$

$\mu_m = 1.6$

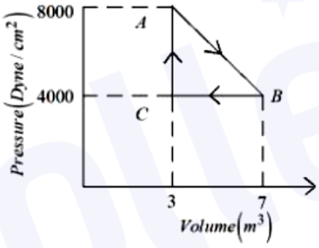
$f_a = 20$ cm

As $\frac{f_m}{f_a} = \frac{(\mu_1 - 1)\mu_m}{(\mu_1 - \mu_m)}$

$\frac{f_m}{20} = \frac{(1.5-1)1.6}{(1.5-1.6)}$

$f_m = -160$ cm

46. A thermodynamic system is taken from an original state A to an intermediate state B by a linear process as shown in the figure. It's volume is then reduced to the original value from B to C by an isobaric process. The total work done by the gas from A to B and B to C would be :



(1) 33800 J (2) 2200 J

(3) 600 J (4) 1200 J

Ans. **(BONUS)**

Sol.

Work done AB = $\frac{1}{2}$ (8000 + 6000) Dyne/cm^2 × 4m^3 = (6000Dyne/cm^2) × 4m^3

Work done BC = –(4000 Dyne/cm^2) × 4m^3

Total work done = 2000 Dyne/cm^2 × 4m^3

$$= 2\times10^3 \times \frac{1}{10^5}\frac{N}{cm^2}\times 4m^3$$

$$= 2\times10^{-2}\times\frac{N}{10^{-4}m^2}\times 4m^3$$

= 2 × 10^2 × 4 Nm = 800 J

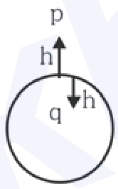
47. At what distance above and below the surface of the earth a body will have same weight, (take radius of earth as R.)

(1) $\sqrt{5}R - R$ (2) $\frac{\sqrt{3}R - R}{2}$

(3) $\frac{R}{2}$ (4) $\frac{\sqrt{5}R - R}{2}$

Ans. **(4)**

Sol. $\frac{gR^2}{\quad)^2}$



$g_q = g\left(1 - \frac{h}{R}\right)$

$g_p = g_q$

$\frac{g}{\left(1+\frac{h}{R}\right)^2} = g\left(1 - \frac{h}{R}\right)$

$\left(1 - \frac{h^2}{R^2}\right)\left(1 + \frac{h}{R}\right) = 1$

Take $\frac{h}{R} = x$

So

$x^3 - x + x^2 = 0$

$x = \frac{\sqrt{5}-1}{2}$

$h = \frac{R}{2}\left(\sqrt{5}-1\right)$

48. A capacitor of capacitance 100 μF is charged to a potential of 12 V and connected to a 6.4 mH inductor to produce oscillations. The maximum current in the circuit would be :

(1) 3.2 A (2) 1.5 A

(3) 2.0 A (4) 1.2 A

Ans. (2)

Sol. By energy conservation

$$\frac{1}{2}CV^2 = \frac{1}{2}LI_{max}^2$$

$$I_{max} = \sqrt{\frac{C}{L}}\,V$$

$$= \sqrt{\frac{100\times10^{-6}}{6.4\times10^{-3}}}\times12$$

$$= \frac{12}{8} = \frac{3}{2} = 1.5\text{ A}$$

49. The explosive in a Hydrogen bomb is a mixture of ${}_1H^2$, ${}_1H^3$ and ${}_3Li^6$ in some condensed form. The chain reaction is given by

${}_3Li^6 + {}_0n^1 \rightarrow {}_2He^4 + {}_1H^3$

${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + {}_0n^1$

During the explosion the energy released is approximately

[Given : M(Li) = 6.01690 amu. M (${}_1H^2$) = 2.01471 amu. M (${}_2He^4$) = 4.00388amu, and 1 amu = 931.5 MeV]

(1) 28.12 MeV (2) 12.64 MeV

(3) 16.48 MeV (4) 22.22 MeV

Ans. (4)

Sol. ${}_3Li^6 + {}_0n^1 \rightarrow {}_2He^4 + {}_1H^3$

${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + {}_0n^1$

$${}_3Li^6 +{}_1H^2 \rightarrow 2\left({}_2He^4\right)$$

Energy released in process

$Q = \Delta mc^2$

Q = [M(Li)+ M (${}_1H^2$) –2 × M(${}_2He^4$)] × 931.5 MeV

Q = [6.01690+2.01471–2 × 4.00388] × 931.5 MeV

Q = 22.216 MeV

Q = 22.22 MeV

50. Two vessels A and B are of the same size and are at same temperature. A contains 1g of hydrogen and B contains 1g of oxygen. P_A and P_B are the pressures of the gases in A and B respectively, then $\frac{P_A}{P_B}$ is :

(1) 16 (2) 8 (3) 4 (4) 32

Ans. (1)

Sol. $$\frac{P_AV_A}{P_BV_B} = \frac{n_ART_A}{n_BRT_B}$$

Given V_A =V_B

And $T_A = T_B$

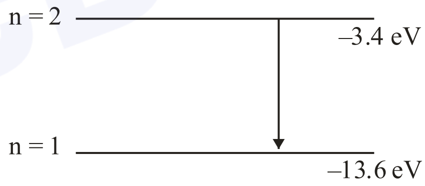
$$\frac{P_A}{P_B} = \frac{n_A}{n_B}$$

$$\frac{P_A}{P_B} = \frac{1/2}{1/32} = 16$$

SECTION-B

51. When a hydrogen atom going from n = 2 to n = 1 emits a photon, its recoil speed is $\frac{x}{5}$ *m/s*. Where x = ______ . (Use : mass of hydrogen atom = 1.6 × 10^{-27} kg)

Ans. (17)



Sol.

ΔE = 10.2 eV

Recoil speed(v) = $\frac{\Delta E}{mc}$

$$= \frac{10.2\text{eV}}{1.6\times10^{-27}\times3\times10^8}$$

$$= \frac{10.2\times1.6\times10^{-19}}{1.6\times10^{-27}\times3\times10^8}$$

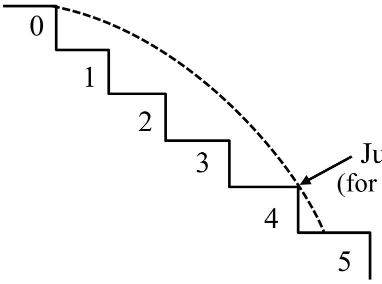
$v = 3.4\text{ m/s} = \frac{17}{5}\text{ m/s}$

Therefore, x = 17

52. A ball rolls off the top of a stairway with horizontal velocity u. The steps are 0.1 m high and 0.1 m wide. The minimum velocity u with which that ball just hits the step 5 of the stairway will be $\sqrt{x}\ ms^{-1}$ where x = ___________ [use g = 10 m/s^2].

Ans. **(2)**

Sol.



The ball needs to just cross 4 steps to just hit 5th step

Therefore, horizontal range (R) = 0.4 m

R = u.t

Similarly, in vertical direction

$$h = \frac{1}{2}gt^2$$

$$0.4 = \frac{1}{2}gt^2$$

$$0.4 = \frac{1}{2}g\left(\frac{0.4}{u}\right)^2$$

$u^2 = 2$

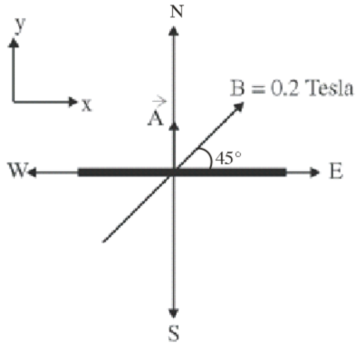
$u = \sqrt{2}$ m/s

Therefore, x = 2

53. A square loop of side 10 cm and resistance 0.7Ω is placed vertically in east-west plane. A uniform magnetic field of 0.20 T is set up across the plane in north east direction. The magnetic field is decreased to zero in 1 s at a steady rate. Then, magnitude of induced emf is $\sqrt{x} \times 10^{-3}$V. The value of x is ________.

Ans. **(2)**

Sol.



$$\vec{A} = (0.1)^2\,\hat{j}$$

$$\vec{B} = \frac{0.2}{\sqrt{2}}\hat{i} + \frac{0.2}{\sqrt{2}}\hat{j}$$

Magnitude of induced emf

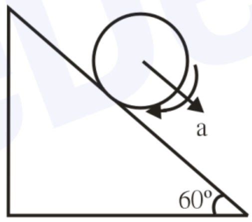
$$e = \frac{\Delta\phi}{\Delta t} = \frac{\vec{B}\cdot\vec{A} - 0}{1} = \sqrt{2} \times 10^{-3}\,V$$

54. A cylinder is rolling down on an inclined plane of inclination 60°. It's acceleration during rolling down will be $\frac{x}{\sqrt{3}}$ *m/s*^{2}, where x = ________.

(use g = 10 m/s^2).

Ans. **(10)**

Sol.



For rolling motion, $a = \frac{g\sin\theta}{1 + \frac{I_{cm}}{MR^2}}$

$$a = \frac{g\sin\theta}{1 + \frac{1}{2}}$$

$$= \frac{2 \times 10 \times \frac{\sqrt{3}}{2}}{3}$$

$$= \frac{10}{\sqrt{3}}$$

Therefore x = 10

55. The magnetic potential due to a magnetic dipole at a point on its axis situated at a distance of 20 cm from its center is 1.5×10^{-5}Tm. The magnetic moment of the dipole is__________Am^2.

(Given : $\frac{\mu_0}{4\pi} = 10^{-7}TmA^{-1}$)

Ans. **(6)**

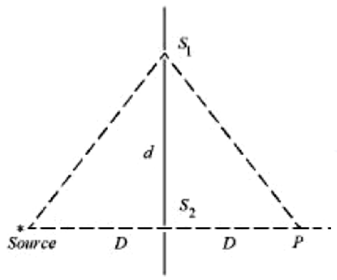
Sol. $V = \frac{\mu_0}{4\pi}\frac{M}{r^2}$

$\Rightarrow 1.5 \times 10^{-5} = 10^{-7} \times \frac{M}{\left(20 \times 10^{-2}\right)^2}$

$\Rightarrow M = \frac{1.5 \times 10^{-5} \times 20 \times 20 \times 10^{-4}}{10^{-7}}$

$M = 1.5 \times 4 = 6$

56. In a double slit experiment shown in figure, when light of wavelength 400 nm is used, dark fringe is observed at P. If D = 0.2 m. the minimum distance between the slits S_1 and S_2 is ______ mm.



Ans. **(0.20)**

Sol. Path difference for minima at P

$2\sqrt{D^2 + d^2} - 2D = \frac{\lambda}{2}$

$\therefore \sqrt{D^2 + d^2} - D = \frac{\lambda}{4}$

$\therefore \sqrt{D^2 + d^2} = \frac{\lambda}{4} + D$

$\Rightarrow D^2 + d^2 = D^2 + \frac{\lambda^2}{16} + \frac{D\lambda}{2}$

$\Rightarrow d^2 = \frac{D\lambda}{2} + \frac{\lambda^2}{16}$

$\Rightarrow d^2 = \frac{0.2 \times 400 \times 10^{-9}}{2} + \frac{4 \times 10^{-14}}{4}$

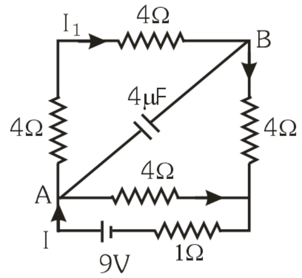
$\Rightarrow d^2 \approx 400 \times 10^{-10}$

$\therefore d = 20 \times 10^{-5}$

$\Rightarrow d = 0.20 mm$

57. A 16Ω wire is bend to form a square loop. A 9V battery with internal resistance 1Ω is connected across one of its sides. If a 4μF capacitor is connected across one of its diagonals, the energy stored by the capacitor will be $\frac{x}{2}$μJ. where x = _________.

Ans. **(81)**



Sol.

$I = \frac{V}{R_{eq}}$ $I = \frac{V}{R_{eq}} = \frac{9}{1 + \frac{12 \times 4}{12 + 4}} = \frac{9}{4}$

$I_1 = \frac{9}{4} \times \frac{4}{16} = \frac{9}{16}$

$V_A - V_B = I_1 \times 8 = \frac{9}{16} \times 8 = \frac{9}{2}$ **V**

$\therefore U = \frac{1}{2} \times 4 \times \frac{81}{4}$μJ

$\therefore U = \frac{81}{2}$μJ

$\therefore x = 81$

58. When the displacement of a simple harmonic oscillator is one third of its amplitude, the ratio of total energy to the kinetic energy is $\frac{x}{8}$, where x = _________.

Ans. **(9)**

Sol. Let total energy = E = $\frac{1}{2}KA^2$

$U = \frac{1}{2}K\left(\frac{A}{3}\right)^2 = \frac{KA^2}{2 \times 9} = \frac{E}{9}$

$KE = E - \frac{E}{9} = \frac{8E}{9}$

Ratio $\frac{Total}{KE} = \frac{E}{\frac{8E}{9}} = \frac{9}{8}$

x = 9

59. An electron is moving under the influence of the electric field of a uniformly charged infinite plane sheet S having surface charge density $+\sigma$. The electron at t = 0 is at a distance of 1 m from S and has a speed of 1 m/s. The maximum value of σ if the electron strikes S at t = 1 s is $\alpha\left[\frac{m \in_0}{e}\right]\frac{C}{m^2}$ the value of α is

Ans. (8)

Sol. $u = 1\ m/s;\ a = -\frac{\sigma e}{2\varepsilon_0 m}$

$t = 1\ s$

$S = -1\ m$

Using $S = ut + \frac{1}{2}at^2$

$-1 = 1\times1 - \frac{1}{2}\times\frac{\sigma e}{2\varepsilon_0 m}\times(1)^2$

$\therefore \sigma = 8\frac{\varepsilon_0 m}{e}$

$\therefore \alpha = 8$

60. In a test experiment on a model aeroplane in wind tunnel, the flow speeds on the upper and lower surfaces of the wings are 70 ms^{-1} and 65 ms^{-1} respectively. If the wing area is 2 m^2 the lift of the wing is _______ N.

(Given density of air = 1.2 kg m^{-3})

Ans. (810)

Sol. $F = \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)A$

$F = \frac{1}{2}\times1.2\times\left(70^2 - 65^2\right)\times2$

= 810 N