

**SECTION-A**

1. Consider the system of linear equations  
 $x + y + z = 5$ ,  $x + 2y + \lambda^2 z = 9$ ,  
 $x + 3y + \lambda z = \mu$ , where  $\lambda, \mu \in \mathbb{R}$ . Then, which of the following statement is NOT correct?  
 (1) System has infinite number of solution if  $\lambda = 1$  and  $\mu = 13$   
 (2) System is inconsistent if  $\lambda = 1$  and  $\mu \neq 13$   
 (3) System is consistent if  $\lambda \neq 1$  and  $\mu = 13$   
 (4) System has unique solution if  $\lambda \neq 1$  and  $\mu \neq 13$

Ans. (4)

Sol. 
$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \lambda^2 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 2\lambda^2 - \lambda - 1 = 0$$

$$\lambda = 1, -\frac{1}{2}$$

$$\begin{vmatrix} 1 & 1 & 5 \\ 2 & \lambda^2 & 9 \\ 3 & \lambda & \mu \end{vmatrix} = 0 \Rightarrow \mu = 13$$

Infinite solution  $\lambda = 1$  &  $\mu = 13$

For unique sol<sup>n</sup>  $\lambda \neq 1$

For no sol<sup>n</sup>  $\lambda = 1$  &  $\mu \neq 13$

If  $\lambda \neq 1$  and  $\mu \neq 13$

Considering the case when  $\lambda = -\frac{1}{2}$  and  $\mu \neq 13$  this will generate no solution case

2. For  $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$ , let  $3\sin(\alpha + \beta) = 2\sin(\alpha - \beta)$  and a real number  $k$  be such that  $\tan\alpha = k\tan\beta$ . Then the value of  $k$  is equal to :

(1)  $-\frac{2}{3}$

(2)  $-5$

(3)  $\frac{2}{3}$

(4)  $5$

Ans. Bonus

Sol. 
$$\begin{aligned} &3\sin\alpha \cos\beta + 3\sin\beta \cos\alpha \\ &= 2\sin\alpha \cos\beta - 2\sin\beta \cos\alpha \\ &5\sin\beta \cos\alpha = -\sin\alpha \cos\beta \end{aligned}$$

$$\tan\beta = -\frac{1}{5}\tan\alpha$$

$$\tan\alpha = -5\tan\beta$$

Not possible as  $\tan\alpha, \tan\beta$  are positive

$\Rightarrow$  Data inconsistent

3. Let  $A(\alpha, 0)$  and  $B(0, \beta)$  be the points on the line  $5x + 7y = 50$ . Let the point  $P$  divide the line segment  $AB$  internally in the ratio  $7 : 3$ . Let  $3x - 25 = 0$  be a directrix of the ellipse  $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and the corresponding focus be  $S$ . If from  $S$ , the perpendicular on the  $x$ -axis passes through  $P$ , then the length of the latus rectum of  $E$  is equal to

(1)  $\frac{25}{3}$

(2)  $\frac{32}{9}$

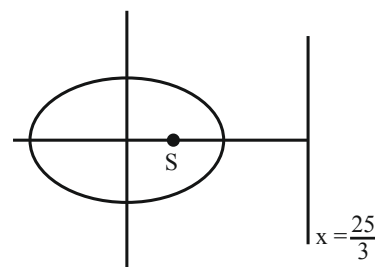
(3)  $\frac{25}{9}$

(4)  $\frac{32}{5}$

Ans. (4)

$A = (10, 0)$

Sol. 
$$B = \left(0, \frac{50}{7}\right) \left. \vphantom{B} \right\} P = (3, 5)$$



$ae = 3$

$$\frac{a}{e} = \frac{25}{3}$$

$a = 5$

$b = 4$

Length of LR  $= \frac{2b^2}{a} = \frac{32}{5}$

4. Let  $\vec{a} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ ,  $\alpha, \beta \in \mathbb{R}$ . Let a vector  $\vec{b}$  that the angle between  $\vec{a}$  and  $\vec{b}$  is  $\frac{\pi}{4}$  and  $|\vec{b}|^2 = 6$ , If  $\vec{a} \cdot \vec{b} = 3\sqrt{2}$ , then the value of  $(\alpha^2 + \beta^2)|\vec{a} \times \vec{b}|^2$  is equal to
- (1) 90 (2) 75  
(3) 95 (4) 85

Ans. (1)

Sol.  $|\vec{b}|^2 = 6$ ;  $|\vec{a}| |\vec{b}| \cos\theta = 3\sqrt{2}$

$$|\vec{a}|^2 |\vec{b}|^2 \cos^2 \theta = 18$$

$$|\vec{a}|^2 = 6$$

$$\text{Also } 1 + \alpha^2 + \beta^2 = 6$$

$$\alpha^2 + \beta^2 = 5$$

to find

$$(\alpha^2 + \beta^2) |\vec{a}|^2 |\vec{b}|^2 \sin^2 \theta$$

$$= (5)(6)(6) \left(\frac{1}{2}\right)$$

$$= 90$$

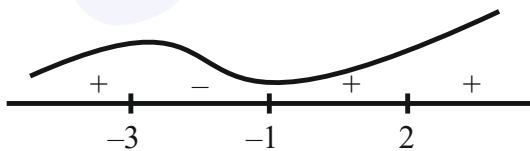
5. Let  $f(x) = (x+3)^2(x-2)^3$ ,  $x \in [-4, 4]$ . If  $M$  and  $m$  are the maximum and minimum values of  $f$ , respectively in  $[-4, 4]$ , then the value of  $M - m$  is :
- (1) 600 (2) 392  
(3) 608 (4) 108

Ans. (3)

Sol.  $f'(x) = (x+3)^2 \cdot 3(x-2)^2 + (x-2)^3 \cdot 2(x+3)$

$$= 5(x+3)(x-2)^2(x+1)$$

$$f'(x) = 0, x = -3, -1, 2$$



$$f(-4) = -216$$

$$f(-3) = 0, f(4) = 49 \times 8 = 392$$

$$M = 392, m = -216$$

$$M - m = 392 + 216 = 608$$

$$\text{Ans} = '3'$$

6. Let  $a$  and  $b$  be two distinct positive real numbers. Let  $11^{\text{th}}$  term of a GP, whose first term is  $a$  and third term is  $b$ , is equal to  $p^{\text{th}}$  term of another GP, whose first term is  $a$  and fifth term is  $b$ . Then  $p$  is equal to
- (1) 20 (2) 25  
(3) 21 (4) 24

Ans. (3)

Sol. 1<sup>st</sup> GP  $\Rightarrow t_1 = a, t_3 = b = ar^2 \Rightarrow r^2 = \frac{b}{a}$

$$t_{11} = ar^{10} = a(r^2)^5 = a \cdot \left(\frac{b}{a}\right)^5$$

$$2^{\text{nd}} \text{ G.P.} \Rightarrow T_1 = a, T_5 = ar^4 = b$$

$$\Rightarrow r^4 = \left(\frac{b}{a}\right) \Rightarrow r = \left(\frac{b}{a}\right)^{1/4}$$

$$T_p = ar^{p-1} = a \left(\frac{b}{a}\right)^{\frac{p-1}{4}}$$

$$t_{11} = T_p \Rightarrow a \left(\frac{b}{a}\right)^5 = a \left(\frac{b}{a}\right)^{\frac{p-1}{4}}$$

$$\Rightarrow 5 = \frac{p-1}{4} \Rightarrow p = 21$$

7. If  $x^2 - y^2 + 2hxy + 2gx + 2fy + c = 0$  is the locus of a point, which moves such that it is always equidistant from the lines  $x + 2y + 7 = 0$  and  $2x - y + 8 = 0$ , then the value of  $g + c + h - f$  equals
- (1) 14 (2) 6  
(3) 8 (4) 29

Ans. (1)

Sol. Cocus of point  $P(x, y)$  whose distance from

Gives

$$X + 2y + 7 = 0 \text{ \& } 2x - y + 8 = 0 \text{ are equal is}$$

$$\frac{x+2y+7}{\sqrt{5}} = \pm \frac{2x-y+8}{\sqrt{5}}$$

$$(x+2y+7)^2 - (2x-y+8)^2 = 0$$

Combined equation of lines

$$(x - 3y + 1)(3x + y + 15) = 0$$

$$3x^2 - 3y^2 - 8xy + 18x - 44y + 15 = 0$$

$$x^2 - y^2 - \frac{8}{3}xy + 6x - \frac{44}{3}y + 5 = 0$$

$$x^2 - y^2 + 2hxy + 2gx + 2fy + c = 0$$

$$h = \frac{4}{3}, g = 3, f = -\frac{22}{3}, c = 5$$

$$g + c + h - f = 3 + 5 - \frac{4}{3} + \frac{22}{3} = 8 + 6 = 14$$

8. Let  $\vec{a}$  and  $\vec{b}$  be two vectors such that  $|\vec{b}| = 1$  and  $|\vec{b} \times \vec{a}| = 2$ . Then  $|(\vec{b} \times \vec{a}) - \vec{b}|^2$  is equal to

(1) 3

(2) 5

(3) 1

(4) 4

Ans. (2)

Sol.  $|\vec{b}| = 1$  &  $|\vec{b} \times \vec{a}| = 2$

$$(\vec{b} \times \vec{a}) \cdot \vec{b} = \vec{b} \cdot (\vec{b} \times \vec{a}) = 0$$

$$\left( \begin{array}{c} | \\ | \\ | \\ | \end{array} \right)$$

$$= 4 + 1 = 5$$

9. Let  $y = f(x)$  be a thrice differentiable function in  $(-5, 5)$ . Let the tangents to the curve  $y = f(x)$  at  $(1, f(1))$  and  $(3, f(3))$  make angles  $\frac{\pi}{6}$  and  $\frac{\pi}{4}$ , respectively with positive  $x$ -axis. If

$$27 \int_1^3 ((f'(t))^2 + 1) f''(t) dt = \alpha + \beta \sqrt{3}$$

where  $\alpha, \beta$  are integers, then the value of  $\alpha + \beta$  equals

(1) -14

(2) 26

(3) -16

(4) 36

Ans. (2)

Sol.  $y = f(x) \Rightarrow \frac{dy}{dx} = f'(x)$

$$\left. \frac{dy}{dx} \right|_{(1, f(1))} = f'(1) = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} \Rightarrow f'(1) = \frac{1}{\sqrt{3}}$$

$$\left. \frac{dy}{dx} \right|_{(3, f(3))} = f'(3) = \tan \frac{\pi}{4} = 1 \Rightarrow f'(3) = 1$$

$$27 \int_1^3 ((f'(t))^2 + 1) f''(t) dt = \alpha + \beta \sqrt{3}$$

$$I = \int_1^3 ((f'(t))^2 + 1) f''(t) dt$$

$$f'(t) = z \Rightarrow f''(t) dt = dz$$

$$z = f'(3) = 1$$

$$z = f'(1) = \frac{1}{\sqrt{3}}$$

$$I = \int_{1/\sqrt{3}}^1 (z^2 + 1) dz = \left( \frac{z^3}{3} + z \right) \Big|_{1/\sqrt{3}}^1$$

$$= \left( \frac{1}{3} + 1 \right) - \left( \frac{1}{3} \cdot \frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{3}} \right)$$

$$= \frac{4}{3} - \frac{10}{9\sqrt{3}} = \frac{4}{3} - \frac{10}{27} \sqrt{3}$$

$$\sqrt{\left( \frac{4}{3} - \frac{10}{27} \sqrt{3} \right)} \quad \sqrt{\quad}$$

$$\alpha = 36, \beta = -10$$

$$\alpha + \beta = 36 - 10 = 26$$

10. Let P be a point on the hyperbola  $H: \frac{x^2}{9} - \frac{y^2}{4} = 1$ , in the first quadrant such that the area of triangle formed by P and the two foci of H is  $2\sqrt{13}$ . Then, the square of the distance of P from the origin is

(1) 18

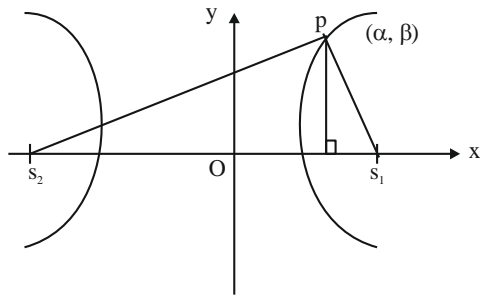
(2) 26

(3) 22

(4) 20

Ans. (3)

Sol.



$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$

$$a^2 = 9, b^2 = 4$$

$$b^2 = a^2(e^2 - 1) \Rightarrow e^2 = 1 + \frac{b^2}{a^2}$$

$$e^2 = 1 + \frac{4}{9} = \frac{13}{9}$$

$$e = \frac{\sqrt{13}}{3} \Rightarrow s_1 s_2 = 2ae = 2 \times 3 \times \frac{\sqrt{13}}{3} = 2\sqrt{13}$$

$$\text{Area of } \Delta P S_1 S_2 = \frac{1}{2} \times \beta \times s_1 s_2 = 2\sqrt{13}$$

$$\Rightarrow \frac{1}{2} \times \beta \times (2\sqrt{13}) = 2\sqrt{13} \Rightarrow \beta = 2$$

$$\frac{\alpha^2}{9} - \frac{\beta^2}{4} = 1 \Rightarrow \frac{\alpha^2}{9} - 1 = 1 \Rightarrow \alpha^2 = 18 \Rightarrow \alpha = 3\sqrt{2}$$

$$\begin{aligned} \text{Distance of P from origin} &= \sqrt{\alpha^2 + \beta^2} \\ &= \sqrt{18 + 4} = \sqrt{22} \end{aligned}$$

11. Bag A contains 3 white, 7 red balls and bag B contains 3 white, 2 red balls. One bag is selected at random and a ball is drawn from it. The probability of drawing the ball from the bag A, if the ball drawn in white, is :

- (1)  $\frac{1}{4}$                       (2)  $\frac{1}{9}$   
 (3)  $\frac{1}{3}$                       (4)  $\frac{3}{10}$

Ans. (3)

Sol.  $E_1$  : A is selected

A	B
3W 7R	3W 2R

$E_2$  : B is selected

E : white ball is drawn

$P(E_1/E) =$

$$\frac{P(E) \cdot P(E/E_1)}{P(E/E_1) + P(E_2) \cdot P(E)}$$

$$= \frac{3}{3+6} = \frac{1}{3}$$

12. Let  $f : R \rightarrow R$  be defined  $f(x) = ae^{2x} + be^x + cx$ . If  $f(0) = -1$ ,  $f'(\log_e 2) = 21$  and

$$\int_0^{\log_e 4} (f(x) - cx) dx = \frac{39}{2}$$

then the value of  $|a+b+c|$  equals :

- (1) 16                      (2) 10  
 (3) 12                      (4) 8

Ans. (4)

Sol.  $f(x) = ae^{2x} + be^x + cx$                        $f(0) = -1$

$$a + b = -1$$

$f'(x) = 2ae^{2x} + be^x + c$                        $f'(\ln 2) = 21$

$$8a + 2b + c = 21$$

$$\int_0^{\ln 4} (ae^{2x} + be^x) dx = \frac{39}{2}$$

$$\left[ \frac{ae^{2x}}{2} + be^x \right]_0^{\ln 4} = \frac{39}{2} \Rightarrow 8a + 4b - \frac{a}{2} - b = \frac{39}{2}$$

$$15a + 6b = 39$$

$$15a - 6a - 6 = 39$$

$$9a = 45 \Rightarrow a = 5$$

$$b = -6$$

$$c = 21 - 40 + 12 = -7$$

$$a + b + c = -8$$

$$|a + b + c| = 8$$

13. Let  $L_1 : \vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i} - \hat{j} + 2\hat{k}), \lambda \in \mathbb{R}$   
 $L_2 : \vec{r} = (\hat{j} - \hat{k}) + \mu(3\hat{i} + \hat{j} + p\hat{k}), \mu \in \mathbb{R}$  and  
 $L_3 : \vec{r} = \delta(\ell\hat{i} + m\hat{j} + n\hat{k}), \delta \in \mathbb{R}$

Be three lines such that  $L_1$  is perpendicular to  $L_2$  and  $L_3$  is perpendicular to both  $L_1$  and  $L_2$ . Then the point which lies on  $L_3$  is

- (1)  $(-1, 7, 4)$  (2)  $(-1, -7, 4)$   
 (3)  $(1, 7, -4)$  (4)  $(1, -7, 4)$

Ans. (1)

Sol.  $L_1 \perp L_2$   $L_3 \perp L_1, L_2$

$$3 - 1 + 2P = 0$$

$$P = -1$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & -1 \end{vmatrix} = -\hat{i} + 7\hat{j} + 4\hat{k}$$

$\therefore (-\delta, 7\delta, 4\delta)$  will lie on  $L_3$

For  $\delta = 1$  the point will be  $(-1, 7, 4)$

14. Let a and b be real constants such that the function

f defined by  $f(x) = \begin{cases} x^2 + 3x + a, & x \leq 1 \\ bx + 2, & x > 1 \end{cases}$  be

differentiable on  $\mathbb{R}$ . Then, the value of  $\int_{-2}^2 f(x) dx$

equals

- (1)  $\frac{15}{6}$  (2)  $\frac{19}{6}$   
 (3) 21 (4) 17

Ans. (4)

Sol. f is continuous  $f'(x) = 2x + 3, x < 1$

$$\therefore 4 + a = b + 2 \quad b, x > 1$$

$a = b - 2$  f is differentiable

$$\therefore b = 5$$

$$\therefore a = 3$$

$$\begin{aligned} & \int_{-2}^1 (x^2 + 3x + 3) dx + \int_1^2 (5x + 2) dx \\ &= \left[ \frac{x^3}{3} + \frac{3x^2}{2} + 3x \right]_{-2}^1 + \left[ \frac{5x^2}{2} + 2x \right]_1^2 \\ &= \left( \frac{1}{3} + \frac{3}{2} + 3 \right) - \left( \frac{-8}{3} + 6 - 6 \right) + \left( 10 + 4 - \frac{5}{2} - 2 \right) \\ &= 6 + \frac{3}{2} + 12 - \frac{5}{2} = 17 \end{aligned}$$

15. Let  $f: \mathbb{R} - \{0\} \rightarrow \mathbb{R}$  be a function satisfying

$$f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)} \text{ for all } x, y, f(y) \neq 0. \text{ If } f'(1) = 2024,$$

then

- (1)  $xf'(x) - 2024f(x) = 0$   
 (2)  $xf'(x) + 2024f(x) = 0$   
 (3)  $xf'(x) + f(x) = 2024$   
 (4)  $xf'(x) - 2023f(x) = 0$

Ans. (1)

Sol.  $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$   $f'(1) = 2024$   
 $f(1) = 1$

Partially differentiating w. r. t. x

$$f'\left(\frac{x}{y}\right) \cdot \frac{1}{y} = \frac{1}{f(y)} f'(x)$$

$y \rightarrow x$

$$f'(1) \cdot \frac{1}{x} = \frac{f'(x)}{f(x)}$$

$$2024f(x) = xf'(x) \Rightarrow xf'(x) - 2024f(x) = 0$$

16. If z is a complex number, then the number of common roots of the equation  $z^{1985} + z^{100} + 1 = 0$  and

$z^3 + 2z^2 + 2z + 1 = 0$ , is equal to :

- (1) 1 (2) 2  
 (3) 0 (4) 3

Ans. (2)

Sol.  $z^{1985} + z^{100} + 1 = 0$  &  $z^3 + 2z^2 + 2z + 1 = 0$

$$(z + 1)(z^2 - z + 1) + 2z(z + 1) = 0$$

$$(z + 1)(z^2 + z + 1) = 0$$

$$\Rightarrow z = -1, z = w, w^2$$

Now putting  $z = -1$  not satisfy

Now put  $z = w$

$$\Rightarrow w^{1985} + w^{100} + 1$$

$$\Rightarrow w^2 + w + 1 = 0$$

Also,  $z = w^2$

$$\Rightarrow w^{3970} + w^{200} + 1$$

$$\Rightarrow w + w^2 + 1 = 0$$

Two common root

17. Suppose  $2 - p$ ,  $p$ ,  $2 - \alpha$ ,  $\alpha$  are the coefficient of four consecutive terms in the expansion of  $(1+x)^n$ . Then the value of  $p^2 - \alpha^2 + 6\alpha + 2p$  equals

- (1) 4 (2) 10  
(3) 8 (4) 6

**Ans. (Bonus)**

**Sol.**  $2 - p$ ,  $p$ ,  $2 - \alpha$ ,  $\alpha$

Binomial coefficients are

${}^nC_r$ ,  ${}^nC_{r+1}$ ,  ${}^nC_{r+2}$ ,  ${}^nC_{r+3}$  respectively

$\Rightarrow {}^nC_r + {}^nC_{r+1} = 2$

$\Rightarrow {}^{n+1}C_{r+1} = 2 \dots\dots(1)$

Also,  ${}^nC_{r+2} + {}^nC_{r+3} = 2$

$\Rightarrow {}^{n+1}C_{r+3} = 2 \dots\dots(2)$

From (1) and (2)

${}^{n+1}C_{r+1} = {}^{n+1}C_{r+3}$

$\Rightarrow 2r + 4 = n + 1$

$n = 2r + 3$

${}^{2r+4}C_{r+1} = 2$

Data Inconsistent

18. If the domain of the function  $f(x) = \log_e$

$\left(\frac{2x+3}{4x^2+x-3}\right) + \cos^{-1}\left(\frac{2x-1}{x+2}\right)$  is  $(\alpha, \beta]$ , then the

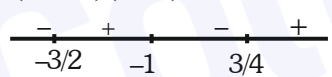
value of  $5\beta - 4\alpha$  is equal to

- (1) 10 (2) 12  
(3) 11 (4) 9

**Ans. (2)**

**Sol.**  $\frac{2x+3}{4x^2+x-3} > 0$  and  $-1 \leq \frac{2x-1}{x+2} \leq 1$

$\frac{2x+3}{(4x-3)(x+1)} > 0$   $\frac{+1}{x+2} \geq 0$  &  $\frac{x-3}{x+2} \leq 0$



$(-\infty, -2) \cup \left[\frac{-1}{3}, \infty\right) \dots\dots(1)$

$(-2, 3] \dots\dots(2)$

$\left[\frac{-1}{3}, 3\right] \dots\dots(3) \quad (1) \cap (2) \cap (3)$

$\left[\frac{3}{4}, 3\right]$

$\alpha = \frac{3}{4} \quad \beta = 3$

$5\beta - 4\alpha = 15 - 3 = 12$

19. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function defined

$f(x) = \frac{x}{(1+x^4)^{1/4}}$  and  $g(x) = f(f(f(f(x))))$  then

$18 \int_0^{\sqrt[2]{\sqrt{5}}} x^2 g(x) dx$

- (1) 33 (2) 36  
(3) 42 (4) 39

**Ans. (4)**

**Sol.**  $f(x) = \frac{x}{(1+x^4)^{1/4}}$

$f \circ f(x) = \frac{f(x)}{(1+f(x)^4)^{1/4}} = \frac{\frac{x}{(1+x^4)^{1/4}}}{\left(1 + \frac{x^4}{1+x^4}\right)^{1/4}} = \frac{x}{(1+2x^4)^{1/4}}$

$f(f(f(f(x)))) = \frac{x}{(1+4x^4)^{1/4}}$

$18 \int_0^{\sqrt[2]{\sqrt{5}}} \frac{x^3}{(1+4x^4)^{1/4}} dx$

Let  $1 + 4x^4 = t^4$

$16x^3 dx = 4t^3 dt$

$\frac{18}{4} \int_1^3 \frac{t^3 dt}{t}$

$= \frac{9}{2} \left(\frac{t^3}{3}\right)_1^3$

$= \frac{3}{2} [26] = 39$

20. Let  $R = \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$  be a non-zero  $3 \times 3$  matrix,

where  $x \sin \theta = y \sin \left(\theta + \frac{2\pi}{3}\right) = z \sin \left(\theta + \frac{4\pi}{3}\right)$

$\neq 0, \theta \in (0, 2\pi)$ . For a square matrix  $M$ , let trace  $(M)$  denote the sum of all the diagonal entries of  $M$ . Then, among the statements:

- (I) Trace  $(R) = 0$   
(II) If trace  $(\text{adj}(\text{adj}(R))) = 0$ , then  $R$  has exactly one non-zero entry.  
(1) Both (I) and (II) are true  
(2) Neither (I) nor (II) is true  
(3) Only (II) is true  
(4) Only (I) is true

**Ans. (3)**

**Sol.**  $x \sin \theta = y \sin \left( \theta + \frac{2\pi}{3} \right) = z \sin \left( \theta + \frac{4\pi}{3} \right) = \lambda$  (say),  $\lambda \neq 0$   
 $\Rightarrow x, y, z \neq 0$  and  $\sin \theta, \sin \left( \theta + \frac{2\pi}{3} \right), \sin \left( \theta + \frac{4\pi}{3} \right) \neq 0$

Also,

$$\sin \theta + \sin \left( \theta + \frac{2\pi}{3} \right) + \sin \left( \theta + \frac{4\pi}{3} \right) = 0 \quad \forall \theta \in \mathbb{R}$$

$$\Rightarrow x + y + z = \frac{-\lambda \left( \sin^2 \theta + \sin^2 \left( \theta + \frac{2\pi}{3} \right) + \sin^2 \left( \theta + \frac{4\pi}{3} \right) \right)}{2 \sin \theta \sin \left( \theta + \frac{2\pi}{3} \right) \sin \left( \theta + \frac{4\pi}{3} \right)} \neq 0$$

- (i) Trace (R) =  $x + y + z \neq 0$   
 $\Rightarrow$  Statement (i) is False
- (ii)  $\text{Adj}(\text{Adj}(R)) = |R| R$   
 Trace (Adj(Adj(R)))  
 =  $xyz(x + y + z) \neq 0$   
 $\Rightarrow$  Hypothesis of conditional statement (ii) is false  
 $\Rightarrow$  Conditional statement (ii) is vacuously true !!

### SECTION-B

21. Let  $Y = Y(X)$  be a curve lying in the first quadrant such that the area enclosed by the line  $Y - y = Y'(x)(X - x)$  and the co-ordinate axes, where  $(x, y)$  is any point on the curve, is always

$\frac{-y^2}{2Y'(x)} + 1$ ,  $Y'(x) \neq 0$ . If  $Y(1) = 1$ , then  $12Y(2)$  equals \_\_\_\_\_.

**Ans. (20)**

**Sol.**  $A = \frac{1}{2} \left( \frac{-y}{Y'(x)} + x \right) (y - xY'/x) = \frac{-y^2}{2Y'(x)} + 1$

$$\Rightarrow (-y + xY'(x))(y - xY'(x)) = -y^2 + 2Y'(x)$$

$$-y^2 + xyY'(x) + xyY'(x) - x^2 [Y'(x)]^2 = -y^2 + 2Y'(x)$$

$$2xy - x^2 Y'(x) = 2$$

$$\frac{dy}{dx} = \frac{2xy - 2}{x^2}$$

$$\frac{dy}{dx} - \frac{2}{x}y = \frac{-2}{x^2}$$

$$\text{I.F.} = e^{-2 \ln x} = \frac{1}{x^2}$$

$$y \cdot \frac{1}{x^2} = \frac{2}{3}x^{-3} + c$$

Put  $x = 1, y = 1$

$$1 = \frac{2}{3} + c \Rightarrow c = \frac{1}{3}$$

$$Y = \frac{2}{3} \cdot \frac{1}{X} + \frac{1}{3}X^2$$

$$\Rightarrow 12Y(2) = \frac{5}{3} \times 12 = 20$$

22. Let a line passing through the point  $(-1, 2, 3)$  intersect the lines  $L_1 : \frac{x-1}{3} = \frac{y-2}{2} = \frac{z+1}{-2}$  at

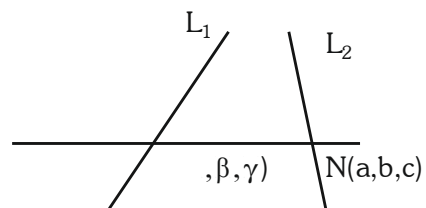
$M(\alpha, \beta, \gamma)$  and  $L_2 : \frac{x+2}{-3} = \frac{y-2}{-2} = \frac{z-1}{4}$  at  $N(a, b,$

$c)$ . Then the value of  $\frac{(\alpha + \beta + \gamma)^2}{(a + b + c)^2}$  equals \_\_\_\_\_.

**Ans. (196)**

**Sol.**  $M(3\lambda + 1, 2\lambda + 2, -2\lambda - 1) \therefore \alpha + \beta + \gamma = 3\lambda + 2$

$N(-3\mu - 2, -2\mu + 2, 4\mu + 1) \therefore a + b + c = -\mu + 1$



$$\frac{3\lambda + 2}{-3\mu - 1} = \frac{2\lambda}{-2\mu} = \frac{-2\lambda - 4}{4\mu - 2}$$

$$2\mu = \lambda$$

$$2\lambda\mu - \lambda = \lambda\mu + 2\mu$$

$$\lambda\mu = \lambda + 2\mu$$

$$\Rightarrow \lambda\mu = 2\lambda$$

$$\Rightarrow \mu = 2 \quad (\lambda \neq 0)$$

$$\therefore \lambda = 4$$

$$\alpha + \beta + \gamma = 14$$

$$a + b + c = -1$$

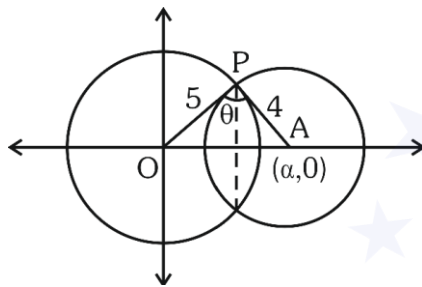
$$\frac{(\alpha + \beta + \gamma)^2}{(a + b + c)^2} = 196$$

23. Consider two circles  $C_1 : x^2 + y^2 = 25$  and  $C_2 : (x - \alpha)^2 + y^2 = 16$ , where  $\alpha \in (5, 9)$ . Let the angle between the two radii (one to each circle) drawn from one of the intersection points of  $C_1$  and  $C_2$  be  $\sin^{-1}\left(\frac{\sqrt{63}}{8}\right)$ . If the length of common chord of  $C_1$

and  $C_2$  is  $\beta$ , then the value of  $(\alpha\beta)^2$  equals \_\_\_\_\_.

**Ans. (1575)**

**Sol.**  $C_1 : x^2 + y^2 = 25$ ,  $C_2 : (x - \alpha)^2 + y^2 = 16$   
 $5 < \alpha < 9$



$$\theta = \sin^{-1}\left(\frac{\sqrt{63}}{8}\right)$$

$$\sin \theta = \frac{\sqrt{63}}{8}$$

$$\text{Area of } \triangle OAP = \frac{1}{2} \times \alpha \left(\frac{\beta}{2}\right) = \frac{1}{2} \times 5 \times 4 \sin \theta$$

$$\Rightarrow \alpha\beta = 40 \times \frac{\sqrt{63}}{8}$$

$$\alpha\beta = 5 \times \sqrt{63}$$

$$(\alpha\beta)^2 = 25 \times 63 = 1575$$

24. Let  $\alpha = \sum_{k=0}^n \left(\frac{{}^n C_k}{k+1}\right)^2$  and  $\beta = \sum_{k=0}^{n-1} \left(\frac{{}^n C_k \cdot {}^n C_{k+1}}{k+2}\right)$ .

If  $5\alpha = 6\beta$ , then  $n$  equals \_\_\_\_\_.

**Ans. (10)**

**Sol.** 
$$\alpha = \sum_{k=0}^n \frac{{}^n C_k \cdot {}^n C_k}{k+1} \cdot \frac{n+1}{n+1}$$

$$= \frac{1}{n+1} \sum_{k=0}^n {}^{n+1} C_{k+1} \cdot {}^n C_{n-k}$$

$$\alpha = \frac{1}{n+1} \cdot {}^{2n+1} C_{n+1}$$

$$\beta = \sum_{k=0}^{n-1} {}^n C_k \cdot \frac{{}^n C_{k+1}}{k+2} \cdot \frac{n+1}{n+1}$$

$$= \frac{1}{n+1} \sum_{k=0}^{n-1} {}^n C_{n-k} \cdot {}^{n+1} C_{k+2}$$

$$= \frac{1}{n+1} \cdot {}^{2n+1} C_{n+2}$$

$$\frac{\beta}{\alpha} = \frac{{}^{2n+1} C_{n+2}}{{}^{2n+1} C_{n+1}} = \frac{2n+1 - (n+2) + 1}{n+2}$$

$$\frac{\beta}{\alpha} = \frac{n}{n+2} = \frac{5}{6}$$

$$n = 10$$

25. Let  $S_n$  be the sum to  $n$ -terms of an arithmetic progression 3, 7, 11, ..... .

If  $40 < \left(\frac{6}{n(n+1)} \sum_{k=1}^n k\right) < 42$ , then  $n$  equals \_\_\_\_\_.

**Ans. (9)**

**Sol.**  $S_n = 3 + 7 + 11 + \dots + n$  terms

$$= \frac{n}{2} (6 + (n-1)4) = 3n + 2n^2 - 2n$$

$$= 2n^2 + n$$

$$\sum_{k=1}^n S_k = 2 \sum_{k=1}^n k^2 + \sum_{k=1}^n k$$

$$= 2 \cdot \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= n(n+1) \left[ \frac{2n+1}{3} + \frac{1}{2} \right]$$

$$= \frac{n(n+1)(4n+5)}{6}$$

$$\Rightarrow 40 < \frac{6}{n(n+1)} \sum_{k=1}^n S_k < 42$$

$$40 < 4n + 5 < 42$$

$$35 < 4n < 37$$

$$n = 9$$



26. In an examination of Mathematics paper, there are 20 questions of equal marks and the question paper is divided into three sections : A, B and C . A student is required to attempt total 15 questions taking at least 4 questions from each section. If section A has 8 questions, section B has 6 questions and section C has 6 questions, then the total number of ways a student can select 15 questions is \_\_\_\_\_ .

**Ans. (11376)**

**Sol.** If 4 questions from each section are selected Remaining 3 questions can be selected either in (1, 1, 1) or (3, 0, 0) or (2, 1, 0)

$$\begin{aligned} \therefore \text{Total ways} &= {}^8C_5 \cdot {}^6C_5 \cdot {}^6C_5 + {}^8C_6 \cdot {}^6C_5 \cdot {}^6C_4 \times 2 + \\ &{}^8C_5 \cdot {}^6C_6 \cdot {}^6C_4 \times 2 + {}^8C_4 \cdot {}^6C_6 \cdot {}^6C_5 \times 2 + {}^8C_7 \cdot {}^6C_4 \cdot {}^6C_4 \\ &= 56 \cdot 6 \cdot 6 + 28 \cdot 6 \cdot 15 \cdot 2 + 56 \cdot 15 \cdot 2 + 70 \cdot 6 \cdot 2 \\ &+ 8 \cdot 15 \cdot 15 \\ &= 2016 + 5040 + 1680 + 840 + 1800 = 11376 \end{aligned}$$

27. The number of symmetric relations defined on the set  $\{1, 2, 3, 4\}$  which are not reflexive is \_\_\_\_\_ .

**Ans. (960)**

**Sol.** Total number of relation both symmetric and

$$\text{reflexive} = 2^{\frac{n^2-n}{2}}$$

$$\text{Total number of symmetric relation} = 2^{\left(\frac{n^2+n}{2}\right)}$$

$\Rightarrow$  Then number of symmetric relation which are not reflexive

$$\Rightarrow 2^{\frac{n(n+1)}{2}} - 2^{\frac{n(n-1)}{2}}$$

$$\Rightarrow 2^{10} - 2^6$$

$$\Rightarrow 1024 - 64$$

$$= 960$$

28. The number of real solutions of the equation  $x(x^2 + 3|x| + 5|x-1| + 6|x-2|) = 0$  is \_\_\_\_\_ .

**Ans. (1)**

**Sol.**  $x = 0$  and  $x^2 + 3|x| + 5|x-1| + 6|x-2| = 0$

Here all terms are +ve except at  $x = 0$

So there is no value of  $x$

Satisfies this equation

Only solution  $x = 0$

No of solution 1.

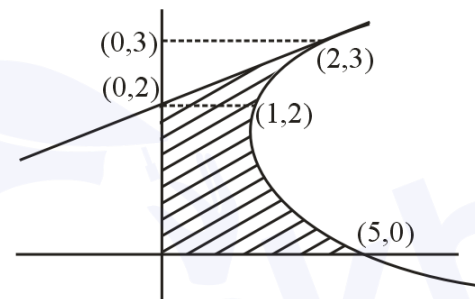
29. The area of the region enclosed by the parabola  $(y - 2)^2 = x - 1$ , the line  $x - 2y + 4 = 0$  and the positive coordinate axes is \_\_\_\_\_ .

**Ans. (5)**

**Sol.** Solving the equations

$$(y - 2)^2 = x - 1 \text{ and } x - 2y + 4 = 0$$

$$X = 2(y - 2)$$



$$\frac{x^2}{4} = x - 1$$

$$x^2 - 4x + 4 = 0$$

$$(x - 2)^2 = 0$$

$$x = 2$$

$$\text{Exclude area (w.r.t. y-axis)} = \int_0^3 x \, dy - \text{Area of } \Delta.$$

$$= \int_0^3 ((y-2)^2 + 1) \, dy - \frac{1}{2} \times 1 \times 2$$

$$= \int_0^3 (y^2 - 4y + 5) \, dy - 1$$

$$= \left[ \frac{y^3}{3} - 2y^2 + 5y \right]_0^3 - 1$$

$$= 9 - 18 + 15 - 1 = 5$$

30. The variance  $\sigma^2$  of the data

$x_i$	0	1	5	6	10	12	17
$f_i$	3	2	3	2	6	3	3

Is \_\_\_\_\_.

**Ans. (29)**

**Sol.**

$x_i$	$f_i$	$f_i x_i$	$f_i x_i^2$
0	3	0	0
1	2	2	2
5	3	15	75
6	2	12	72
10	6	60	600
12	3	36	432
17	3	51	867
	$\Sigma f_i = 22$		$\Sigma f_i x_i^2 = 2048$

$$\therefore \Sigma f_i x_i = 176$$

$$\text{So } \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{176}{22} = 8$$

$$\begin{aligned} \text{for } \sigma^2 &= \frac{1}{N} \Sigma f_i x_i^2 - (\bar{x})^2 \\ &= \frac{1}{22} \times 2048 - (8)^2 \\ &= 93.090964 \\ &= 29.0909 \end{aligned}$$